Analytical Dynamic Time Delay Model of Strongly Coupled RLC Interconnect Lines Dependent on Switching

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Abstract

In today’s UDSM(ultra-deep-sub-micron)-process-technology-based ICs, dynamic delay variations of strongly coupled lines (due to neighboring net switching activity) make static timing analysis problematic. In this paper, new analytical timing models for RLC coupled lines are presented and their accuracy is verified. Coupled interconnect lines are decoupled into an effective single line model by using effective capacitances and effective inductances corresponding to switching activity. Then signal transient waveforms of the effective single line models are determined by exploiting the TWA (Traveling-wave-based Waveform Approximation) technique. This is followed by single line analytical timing model development. It is shown that the models have excellent agreement with SPICE simulations for various circuit performance parameters such as line pitch, line length, driver/receiver size, IMD-thickness, and aspect ratio.

1 Introduction

In today’s UDSM process-based IC interconnect lines, capacitive as well as inductive coupling significantly modifies circuit performance due to the switching activity of the neighboring nets[1]-[6]. Since the interconnect lines play a dominant role in determining circuit performance, simple static timing analysis becomes a significant problem. As the physical line spacing becomes much tighter, the dynamic delay variation due to the switching activity of neighboring nets has to be accurately characterized. In order to verify the timing of the strongly coupled RC interconnect lines, in previous work, the coupled lines have been decoupled into an effective single line by using Miller-factor referred to as a switching factor (SF) [7]-[10], modeled by an iterative method [11], or modeled via an empirical fitting technique [12]. Thereby, a static timing analysis algorithm for coupled lines could be directly exploited by using an effective single line model. It is well known that lower and upper bounds of the switching factor are -1 and 3, respectively. However, these reported techniques may not be accurate enough for today’s strongly coupled RLC interconnect nets since new literature shows that the upper bound of the switching factor which occurs in the cross-coupled switching case is not 3 but larger than 3 [7]. In addition, the authors show that in cross-coupled switching interconnect lines, not only the coupling capacitance but also the self-capacitance changes effectively. Moreover, with faster edge rates and longer wire lengths, inductances have a substantial effect on the circuit timing [2][13][14]. To produce, in this work, more accurate timing analysis of strongly coupled RLC interconnect lines, the switching-based coupling capacitance variations, switching-based self-capacitance variations, and inductive effects have to be taken into account.

The authors present a new “closed form dynamic timing models” of coupled RLC interconnect lines which are highly desirable for efficient as well as accurate timing verification of extremely complicated UDSM-based VLSI circuit designs. First, the coupled RLC interconnect lines are decoupled into an effective isolated single line by using an effective capacitance and an effective inductance which are
functions of the switching activity of the neighboring nets. Next, the response waveforms for the effective single line model are determined by using the TWA-technique [15] which provides analytical waveform expressions. Then, an analytical timing model using the approximated waveform is derived. Finally, it is shown that the models have excellent agreement with SPICE simulations for various circuit performance parameters such as line pitch, line length, driver/receiver size, IMD-thickness, and aspect ratio. However, note, in order for the model to be closed form, TWA technique assumed a linear device model and a step input.

2 Decoupling of the Coupled RLC Interconnect Lines

In the frequency-domain, interconnect lines can be mathematically represented by using the Telegrapher’s equations. Then, the i-th line voltage can be written as

\[
\frac{d^2 V_i}{dx^2} = \sum_{j=1}^{n} Z_{ij} \left( \sum_{k=1}^{n} Y_{jk} V_k \right) = \sum_{j=1}^{n} Z_{ij} Y_{jk} V_k,
\]

(1)

where \( Z_{ij} \) and \( Y_{jk} \) are represented with per-unit-length (PUL) resistance, inductance, conductance, and capacitance. Assuming RLC transmission lines and negligible (PUL) resistance, inductance, conductance, and capacitance, the physical capacitances as follows

\[
[Z] \approx [R]_{diag} + j \omega [L], \quad [Y] \approx j \omega [D].
\]

(2)

Note, the capacitive coupling parameter matrix \([D]\) is not a physical capacitance but an admittance-based short-circuit capacitance. The components (i.e., \( D_{ij} \)) are related with the physical capacitances as follows

\[
D_{ii} = \sum_{j=1}^{n} C_{ij}, \quad D_{ij} = -C_{ij} \quad \text{for} \quad i \neq j.
\]

(3)

Considering 3-coupled lines (i.e., \( n = 3 \)), the center line (i.e., \( i = 2 \)) voltage which is the most sensitive to the crosstalk noise is given by

\[
\frac{d^2 V_2}{dx^2} = \sum_{j=1}^{3} \sum_{k=1}^{3} \left( j \omega R_{2j} D_{2k} + (j \omega)^2 L_{2j} D_{jk} \right) V_k.
\]

(4)

Note, since a single line voltage is given by

\[
\frac{d^2 V}{dx^2} = j \omega R C \cdot V + (j \omega)^2 L C \cdot V,
\]

(5)

(4) can be rewritten in a form similar to that of the single line model. That is, assuming \( R_{ij} = 0 \) for \( i \neq j \),

\[
\frac{d^2 V_2}{dx^2} = j \omega R_{22} \sum_{k=1}^{3} D_{2k} V_k + (j \omega)^2 \sum_{j=1}^{3} L_{2j} \sum_{k=1}^{3} D_{jk} V_k.
\]

(6)

Thus, the center line can be decoupled by using the effective transmission parameters which correspond to the single line parameters. Each switching characteristic is represented by the three switching symbols as “↑↑” (switching from logic 0 to logic 1), “↓↓” (switching from logic 1 to logic 0), and “0” (the quiet state). Considering 5-coupled lines, the switching pattern of “↑↑↑↑↑” generates lower bound (the fastest signal transient) delay. In contrast, the upper bound of the delay (the slowest signal transient) is induced by the switching pattern of “↓↓↓↓↓”. These are clearly understood from Fig. 1. Taking the crosstalk noise between the lines into account, the effective transmission line parameters of the three coupled lines can be estimated as follows

\[
C_{eff}^{↑↑↑} \approx C_{22},
\]

(7)

\[
L_{eff}^{↑↑↑} \approx L_{22} + \frac{2 C_{11}}{C_{22}},
\]

(8)

\[
C_{eff}^{010} \approx C_{22} + C_{11} \left( 1 - \frac{C_{21}/2}{C_{11} + C_{21}} \right) + C_{21} \left( 1 - \frac{C_{23}/2}{C_{33} + C_{23}} \right),
\]

(9)

\[
L_{eff}^{010} \approx L_{22} - L_{21} \frac{2 C_{21}}{C_{22} + 2 C_{21}} + L_{21} \left( \frac{C_{21}}{C_{22} + C_{21}} \right),
\]

(10)

\[
C_{eff}^{↑↑↓} \approx C_{22} \left( 1 - \frac{C_{21}}{C_{22} + C_{21}} \right) + 4 C_{21} \left( 1 - \frac{1}{C_{22} + C_{21}} \right),
\]

(11)

\[
L_{eff}^{↑↑↓} \approx L_{22} - L_{21} \frac{2 (C_{11} + 2 C_{21})}{C_{22} + 4 C_{21}} + L_{21} \left( \frac{C_{22} + 2 C_{21}}{C_{11} + 2 C_{21}} \right).
\]

(12)

The effective-single-line-based signal transients for the 3-coupled lines shown in Fig. 2 are compared with the generic 3-coupled line circuit models. The cross-sectional dimensions of the interconnect line (as in top-layer metals) are \( w = 1.84 \mu m \), \( s = 2.62 \mu m \), \( t = 1.23 \mu m \), and \( h = 1.23 \mu m \). As shown in Fig. 3, the signal transients based
on the effective single line model have excellent agreement with those of the generic coupled line model. Unlike the capacitive-coupled RC lines, inductive coupling effects may not be completely shielded from neighboring lines. Thus, more than 3-coupled lines have to be considered for the RLC-coupled line model. The effective inductance of the $n$-coupled lines corresponding to the switching activity of the neighboring nets is determined by

$$L_{\text{eff}}^{\text{3-coupled line}}(n) \approx L_{jj} + 2 \sum_{k=1}^{j} \left( L_{jk} C_{kk} C_{jj} + 2 C_{jk} C_{jj} \right),$$

(13)

in which the line number, $n = 2m + 1$ and $m$ is a positive integer. The subscript $j$ is defined as $j \equiv (n + 1) / 2$.

As shown in Fig. 4, the signal transients calculated from the equivalent single line inductance model have excellent agreement with the generic coupled line models, even for multi-coupled RLC lines.

### 3 TWA-based Waveform Approximation

Once the coupled lines are decoupled into an effective single line, the response waveform of the line needs to be determined in an analytical manner. In order to determine the analytical waveform model, the single line transfer function is represented approximately with three dominant poles as

$$H(s) \approx \frac{1}{b(s - s_1)(s - s_2)(s - s_3)},$$

(16)

where $b$ is a transfer function coefficient. Then, the 3-pole-based time-domain step response, $v_{03}(t)$, can be determined readily without any time-consuming numerical integration. For example, if the $s_i$'s are simple poles, $v_{03}(t)$ becomes

$$v_{03}(t) = 1 - \sum_{i=1}^{3} K_i e^{s_i t},$$

(17)

where $K_i$ is a residue corresponding to the pole $s_i$. However, the 3-pole-based time-domain response is not accurate enough for interconnect lines with prominent inductive effects. The physical description of the time-domain signal transient of transmission lines with prominent inductive effects can be modeled by using the TWA-technique which combines a frequency-domain-based approximation for low-frequency characteristics with a time-domain-based approximation for high-frequency characteristics of the system response [15]. In the TWA-technique, by assuming a step input, the low-frequency transient signal is represented with only 3-dominant poles. Then, the high-frequency characteristics of the transient signal are incorporated into an approximation function by exploiting the traveling wave characteristics and an RC-response estimation in the time-domain. A pulsed signal includes many frequency components from DC to very high frequency. Since the capaci-
Analytical Dynamic Delay Modeling

Since the TWA-based time-domain transient response provides an analytical expression, the interconnect line dynamic delay can be readily determined in a closed form. In the linear region, the time-domain response is given by [15]

\[
v_0(t) = \sum_{n=1}^{\infty} \frac{v_{o3}(2(n-1)t_f) - v_{o3}(2(n-1)t_f + t_f^*)}{t_f - t_f^*} \cdot [u(t-(n-1)t_f) - u(t-(n-1)t_f + t_f^*)]
\]

for \((2n-1)t_f - \delta \leq t \leq (2n-1)t_f + \delta\).\hspace*{1cm} (18)

In contrast, in the RC-response like region, the time-domain response is given by

\[
v_0(t) = \sum_{n=1}^{\infty} \left[ v_{o3}(2(n-1)t_f) - v_{o3}(2(n-1)t_f + t_f^*) + w_n \left( 1 - \exp \left( \frac{-2nt_f t_f}{\tau} \right) \right) \right]
\]

where \(v_{o3}(t < t_f^*) = 0\), \(\tau \approx R(C + C_L)\),

\[
w_n = \frac{v_{o3}(2(n-1)t_f + t_f^*) - v_{o3}(2(n-1)t_f) + v_{o3}(2(n-1)t_f + t_f^*)}{1 - \exp \left( \frac{-2nt_f}{\tau} \right)}.
\]

A delay time is found by solving the voltage equation in terms of a time that satisfies \(v_{o3}(t) = v_d\), where \(v_d\) is a specific output voltage level for a delay determination (i.e., \(v_d = 0.5V_{DD}\) for the 50\% delay and \(v_d = 0.9V_{DD}\) for 90\% delay, respectively). Since the above expressions are suitable for the switching cases of “↑↑↑” and “↑↑↓” which have no negative crosstalk noise signal, they can not be directly applied for the cross-coupled switching case. That is, for the switching pattern of “↑↑↑”, there is a negative notch in the response signal. In practice, the self-capacitance modulation effect due to the negative crosstalk noises can be fairly well modeled by introducing a shifting factor that shifts the input signal. The input signal shift due to the self-capacitance modulation effect can be simply modeled as a shifting factor that shifts the input signal. The input signal shift due to the self-capacitance modulation effect can be simply modeled as a shifting factor that shifts the input signal. The input signal shift due to the self-capacitance modulation effect can be simply modeled as a shifting factor that shifts the input signal. The input signal shift due to the self-capacitance modulation effect can be simply modeled as a shifting factor that shifts the input signal.

The analytical modeling of the single-line interconnect can be derived by using the TWA-based waveform expressions.

4 Analytical Dynamic Delay Modeling

The linear component is frequency-dependent, the reflection coefficient is inherently frequency-dependent. That is, with a capacitive load, the reflection coefficient is in the range of 1 (for DC) to -1 (for very high-frequency components). Thus, the sharp edge part (i.e., the part that is concerned with the high-frequency components) of the time-domain voltage response is blunted a bit. The linear region response is modeled with an RC-response-like exponential function, while the sharp transient part of the response wave-shape is modeled with a linear function. Defining the following parameters

\[
\delta \equiv \sqrt{L_{\text{line}}(C_{\text{line}} + C_L)} - \sqrt{L_{\text{line}}C_{\text{line}}},
\]

\[
t_f = \sqrt{L_{\text{line}}(C_{\text{line}} + C_L)}, \quad t_f^* \equiv \sqrt{L_{\text{line}}C_{\text{line}}},
\]

a linear region response and an RC-response-like region response can be determined. The linear region response is determined in the time interval between \((2n-1)t_f - \delta\) and \((2n-1)t_f + \delta\), where \(n = 1, 2, 3, \cdots\) (note, \(n\) is the reflection count). Then the waveform can be approximated with a linear function. In contrast, in the time interval between \((2n-1)t_f - \delta\) and \((2n+1)t_f - \delta\), the waveform can be modeled with an RC-response-like function. The TWA-based signal transient responses are compared with the SPICE simulations for the previous interconnect structures in Fig. 5. They show excellent agreement. Thus, the analytical models of the single-line interconnect can be derived by using the TWA-based waveform expressions.
Thus, the modified specific voltage level for determining
time delay, $v_{d-md}$, can be defined as

$$v_{d-md} = \frac{v_d + V_m}{1 + (1/3)m}.$$  \hfill (23)

Furthermore, shifting the input signal by “$-V_m$”, the
effective capacitance for the switching pattern of “$\uparrow \downarrow$” has to be
modified as

$$C_{ef-md} \approx \left[ C_{22} + 4C_{21} \left( 1 - \frac{1}{4} \frac{C_{21}}{C_{22} + C_{21}} \right) \right].$$  \hfill (24)

Thus, the time delay for a specific level of the output volt-
age can be determined readily by using a modified effective
capacitance and a modified output signal waveform expres-
sion. That is, the general form of the delay expression can be
calculated as follows. In the linear region, the time delay can be determined as

$$t_{d-linear} = (2n-1)t_f + \frac{\left( t_f - t_0 \right)}{v_m \left( (2n-1)t_f + \left( 2n-1 \right) t_f + t_0 \right)}$$

for $t_{d-linear} < (t_f + \delta).$  \hfill (25)

Similarly, in the RC-response like region, the time delay can be determined as

$$t_{d-nonlinear} = 2nt_f - t_f$$

$$\tau \ln \left[ 1 - \frac{v_{d-md} - 2v_m (2n-1)t_f + t_f + t_0}{v_m} \right]$$

for $t_{d-nonlinear} \geq (t_f + \delta),$  \hfill (26)

where $v_{d-md}$ is a function of $V_m$, $v_{d-md} (V_m)$. Note, $V_m$ is not zero for the cross-coupled switching case, “$\uparrow \downarrow$”. Without any cross-coupled switching between the lines, $V_m = 0$. The time delay for more than 3-coupled lines can be determined by using the same models as above. During the verification of the accuracy of the model, the microstrip line structure and the strip line structure of the multiple interconnect lines are considered. The models are tested by varying the circuit performance parameters such as line pitch, line length, driver/receiver size, IMD-thickness, and aspect ratio. The test conditions are summarized in Table 1. As shown in Fig. 6 to Fig. 9, the models have excellent agreement with SPICE simulations. In most cases, for all the parametric variations of the global lines (1mm ~ 1cm),
The error of the model is less than 5%. The deviation of the modeling of the delay of short local lines (less than 1mm) as compared to SPICE simulation is less than 5ps. Further, in order to show the accuracy of the model, experimental data for various analytic timing models are compared in Table 2 and Table 3. Table 3 shows that the models are more accurate than other models.

5 Conclusion

In this work, new dynamic time delay models corresponding to the switching activity of the neighboring nets were developed. Strongly coupled RLC interconnect lines were decoupled into an effective single line model with effective single transmission line model parameters, followed by analytical time delay models using the TWA-technique. It was shown that the models have excellent agreement with SPICE simulations. Since the models are very simple closed-form, the dynamic timing delay depending on the switching activity of the neighboring nets can be efficiently as well as accurately calculated without any significant modification of the existing CAD frameworks.

References