Frequency-Dependent Reluctance Extraction

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ABSTRACT
A new methodology is presented to capture high frequency effects of interconnect, namely skin and proximity by using the reluctance (inverse inductance) method. As demonstrated in numerous publications that the reluctance method exhibits excellent locality and suitability of sparsification. The reluctance method results in great benefit in terms of efficiency of extraction and simulation. Most of the previous studies described the reluctance extraction method without taking frequency dependent effects into consideration. In this paper, we first show the differences in frequency response between formula-based inductance extraction and frequency dependent inductance extraction to demonstrate the need to capture high frequency effect. Then a novel frequency dependent reluctance extraction method is proposed by using a robust windowing policy, which is able to handle irregular geometries in VLSI applications. Experimental results demonstrate the superior runtime and accuracy over traditional partial inductance extraction.

1. INTRODUCTION
becomes an indispensable faster by more metal layers, inductance [2]. Signal integrity analysis has become an essential stage for today’s high-speed digital design. Various design issues arise from signal integrity engineering such as cross-talk, ground bounce, ringing from inductance coupling, noise margins, impedance matching, and decoupling are now critical to a successful design. While operating frequency approaches multi giga-hertz, correct and efficient modeling of on-chip inductance for VLSI circuits becomes an indispensable issue. The increasing visibility of inductance effect is not only as a result of higher switching frequency but also of many other factors: reduction of resistance by copper and capacitance by low-κ dielectric, denser geometries, and growing complexity of interconnect design.[1][2]

Inductance modeling has been posed as a challenging issue in VLSI circuit due to its difficulties in predetermined return paths. With the introduction of partial inductance under the partial element equivalent circuit (PEEC) model[3], it solved the return path problem with the assumption of return path at infinity. However, this assumption leads to a dense L matrix because of its long-range effect. Although far-away term maybe small but simple truncation of them may lead to an unstable system[4]. Various techniques have been introduced to alleviate this problem. FastHenry[5] speeds up the extraction process by multi-pole expansion. A. Pacelli [6] proposed to model circuit as a vector-potential equivalent circuit (VPEC) by magnetic resistance and vector-potential controlled voltage source. A interconnect modeling technique called wire duplication[7] has also been proposed.

Since the partial inductance model results in a dense inductance matrix, several studies proposed techniques to sparsify the partial inductance matrix. In the past five years, a new idea that sparsifies the inverse inductance matrix was proposed. This method benefits from the great locality and shielding effect and leads to superior sparsity compared to traditional inductance sparsification methods. Several works using different terminologies to name the inverse inductance matrix, such as K elements[8][9], susceptance[10], and reluctance[11]. We adopt the use of reluctance in the rest of the discussion in this paper.

Like norton equivalent method [9], approach [11]. And handle equal-length parallel conductors to handle irregular geometries.

Most of the existing inductance/reluctance matrix sparsification techniques did not consider any high frequency effect, such as skin and proximity effects. They assume the current density along a conductor is evenly distributed, and utilized formula-based inductance extraction[12][13] to calculate the inductance values. In this paper, we present a new methodology to extract frequency-dependent reluctance elements. Our algorithm not only benefits from the efficiency of sparse reluctance extraction, but also further captures high frequency effects of conductors.

The rest of this paper proceeds as follows. Section 2 discusses the background of inductance, reluctance, skin effect, and proximity effect. There are three parts in Section 3. First we compare the differences between the formula-based reluctance extraction[11] and frequency-dependent reluctance extraction, and justify the need via frequency response analysis. Then the proposed frequency-dependent reluctance extraction algorithm is described, and in the third part we show how to extract reluctance elements with the ground plane and how it affects the extraction value with an example. Sections 4 and 5 present numerical results and conclusions respectively.

2. BACKGROUND
In this section, we briefly introduce the background of inductance, reluctance, skin effect, and proximity effect.
2.1 Definitions of Inductance and Reluctance

Given an unit current along the aggressor conductor, the partial inductance is defined by the total flux flowing through the virtual loop formed by the victim conductor to infinity.[3]

\[ L_{ij} = \frac{\Phi_{ij}}{I_i}, \]

where the subscript \( i \) means the aggressor conductor, and \( j \) means the victim. The integral form of partial inductance formulation can be obtained from the Maxwell’s equations as follows.

\[ L_{ij} = \frac{\mu_0}{4\pi} \int_{a_i} \int_{a_j} \frac{dl_i \cdot dl_j}{r_{ij}} da_i da_j \]  

Equation (2) can be calculated using exact solution of the integration[12] or in a reduced form using Taylor series expansion[13].

Therefore, given \( n \) conductors, the corresponding linear system equations can be represented in matrix form:

\[ L \mathbf{I} = \Phi, \]

where \( L \in \mathbb{R}^{n \times n} \) is the partial inductance matrix, and \( \mathbf{I} \) and \( \Phi \) are \( n \times 1 \) current and flux vectors respectively. By inverting matrix \( L \), Equation (3) becomes

\[ K\Phi = \mathbf{I}, \]

where \( K = L^{-1} \) is the reluctance matrix.

The reluctance matrix has better locality than the inductance matrix, and demonstrates great shielding effect that does not exist in inductance matrix. Placing a conductor (shield) between two conductors, the mutual reluctance between these two conductor reduces significantly while their mutual inductance remains the same. Due to this property, the reluctance matrix \( K \) shows great potential to be sparsiﬁed. With some windowing policy, we can avoid the full inversion of the inductance matrix and directly extract the reluctance values within the window. The window-based reluctance extraction algorithm has been shown to be stable, accurate and efficient.[8][9][11][14]

2.2 Skin and Proximity Effects

As the frequency goes high, the current on a conductor is no longer evenly distributed, which leads to changes in resistance and inductance value. This high-frequency phenomenon can be characterized as skin effect and proximity effect. We briefly explain each effect in the following paragraphs.

At high frequencies, electro-magnetic wave is attenuated rapidly as it propagates in a good conductor. This skin effect is a tendency for high-frequency current to flow near the surface of a solid conductor. Intuitively speaking, since resistance is inversely proportion to the cross-section area, and skin effect causes currents crowded to the surface, this in fact reduces the cross-section area and thus increases the equivalent resistance. Skin effect can be reduced by using stranded rather than solid wire. This increases the effective surface area of the wire for a given wire gauge.

Similar to skin effect, when two conductors are close to each other, the high-frequency current will crowd to the surface that are close to each other. Current tends to follow through the path of least impedance. At low frequencies, the impedance is dominated by resistance of the interconnect. For a finite cross-section area of a interconnect, the current will spread out evenly over the cross-section to minimize the overall resistance. As switching frequency increases, the inductive part of the impedance, \( R + j\omega L \), starts to dominate the overall term. In order to minimize the impedance, the loop size must be reduced to lower the loop inductance \( L \). This results in current crowding on the return path and between signal lines. Figure 1 demonstrate the current distributions for (a) skin effect and (b) proximity effect.

The right panel shows the distribution of current within conductors. For a conductor system with \( k \) conductors, the impedance matrix at frequency \( \omega \) can be denoted as follows:

\[ Z_k(\omega) I_k(\omega) = V_k(\omega), \]

where \( I_k \) and \( V_k \) are branch current and voltage drop respectively, and \( Z_k(\omega) \) is:

\[ Z_k(\omega) = \begin{bmatrix} R_{11} + j\omega L_{11} & j\omega L_{12} & \cdots & j\omega L_{1k} \\ j\omega L_{21} & R_{22} + j\omega L_{22} & \cdots & j\omega L_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega L_{k1} & j\omega L_{k2} & \cdots & R_{kk} + j\omega L_{kk} \end{bmatrix}, \]

where \( R_{ii} \) and \( L_{ii} \) are the \( i \)th filament’s resistance and self inductance, \( L_{ij}(i \neq j) \) is the mutual inductance of conductors \( i \) and \( j \), and \( \omega = 2\pi f \) is the angular frequency.

Given a system, we activate the voltage drop \( V_i \) cross each conductor to 1 each time. By solving Equation (5) and assembling all current \( I_i \) running through each conductor, we essentially construct the effective admittance matrix, \( Y \in \mathbb{R}^{n \times n} \) (where \( n \) is the number of conductors), from those current vectors calculated above. To get the lumped circuit element of the effective resistance and inductance for each conductor, we invert admittance matrix \( Y \) and find the equivalent impedance matrix.

Figure 1: Current distribution on the cross-section of conductors (a) Skin effect (b) Proximity effect

In order to capture both skin and proximity effects, conductors have to be discretized into filaments (as shown in Figure 2) so as to capture the non-uniform distribution of current within conductors. For a conductor system with \( k \) filaments, the impedance matrix at frequency \( \omega \) can be denoted as follows:

\[ Z_k(\omega) I_k(\omega) = V_k(\omega), \]

where \( I_k \) and \( V_k \) are branch current and voltage drop respectively, and \( Z_k(\omega) \) is:

\[ Z_k(\omega) = \begin{bmatrix} R_{11} + j\omega L_{11} & j\omega L_{12} & \cdots & j\omega L_{1k} \\ j\omega L_{21} & R_{22} + j\omega L_{22} & \cdots & j\omega L_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega L_{k1} & j\omega L_{k2} & \cdots & R_{kk} + j\omega L_{kk} \end{bmatrix}, \]

where \( R_{ii} \) and \( L_{ii} \) are the \( i \)th filament’s resistance and self inductance, \( L_{ij}(i \neq j) \) is the mutual inductance of conductors \( i \) and \( j \), and \( \omega = 2\pi f \) is the angular frequency.

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Figure 2: An example of discretizing a conductor into filaments
3. FREQUENCY-DEPENDENT RELUCTANCE EXTRACTION

In this session, we will first justify the need for capturing high-frequency effects. The our proposed frequency-dependent reluctance extraction method is presented. In the third part, we will take the ground plane into consideration.

3.1 A Case Study: Need for Capturing High-Frequency Effects

New method: As published in [11], we need to employ a window selection algorithm so as to capture the significant effect. However, if we only use the formula-based inductance value to form the small window, we cannot capture the skin and proximity effect. Based on this notion, we propose to use the original window selection algorithm. But when forming the small window we expand that to include filaments of each conductor. Hence, skin and proximity effect are captured, as they are short-range effect. Therefore, an \( n \times n \) small window becomes a \( k \times k \) window where \( n \) is the number of conductors and \( k \) is the number of filaments. After that, we solve the effective \( R \) and \( L \) as mention in previous section, and then invert the effective \( L \) to obtain the reluctance matrix \( K \).

Differences: The method described in Section 2 captures the skin and proximity effects by cutting all the conductors in the system into filaments. This leads to a very large dense matrix and the factorization will consume a lot of computation time. Instead, we capture the short-range effect of skin and proximity effect in the small window using window selection algorithm (WSA)[11]. The main advantage is that inversion of \( K \) is much more computationally efficient than inversion of the entire system discretized into filaments. Because it only involves the inversion of an \( n \times n \) matrix resulted from the search window. In a nutshell, the main idea is to capture the high-frequency effect in a small window but expresses in the reluctance matrix domain, which exhibits locality. A summarized comparison between the original and frequency dependent extraction is shown in Table 1.

Using the following example, we show the difference between formula-based method and frequency-dependent method using a 5 x 5 bus structure. The parameters for this example are as follows: the width of each conductor is 5 \( \mu \)m, the space between two neighbor conductors is 1 \( \mu \)m, their length and thickness are 1000 \( \mu \)m and 0.36 \( \mu \)m respectively, conductivity(\( \sigma \)) is 4.996 x 10^6, and evaluated by formula based method.

\[
K = \begin{bmatrix}
 3.91 & -2.35 & -0.28 & -0.25 & -0.29 \\
-2.35 & 5.31 & -2.20 & -0.16 & -0.25 \\
-0.28 & -2.20 & 5.31 & -2.20 & -0.28 \\
-0.25 & -0.16 & -2.20 & 5.31 & -2.35 \\
-0.29 & -0.25 & -0.28 & -2.35 & 3.91 \\
\end{bmatrix} \times 10^9 H^{-1}
\]  

For the same conductor system, but evaluated at 30GHz, the reluctance and resistance values are as follows:

\[
K(\omega) = \begin{bmatrix}
 2.13 & -1.32 & -0.22 & -0.13 & -0.16 \\
-1.32 & 2.96 & -1.20 & -0.16 & -0.13 \\
-0.22 & -1.20 & 2.98 & -2.16 & -0.22 \\
-0.13 & -0.16 & -2.16 & 2.96 & -1.32 \\
-0.16 & -0.13 & -0.22 & -1.32 & 2.13 \\
\end{bmatrix} \times 10^9 H^{-1}
\]  

As stated above, we could notice the increase of resistance because of skin effect and the differences in reluctance resulting from the proximity and skin effects.

![Figure 3: Frequency response (10-100GHz) of the aggressor](image1.png)

![Figure 4: Frequency response (10-100GHz) of a far away victim](image2.png)

Figures 3 and 4 show the differences in frequency response for a system with 100 conductors. The formula-based L extraction data are produced by extracting the system using the formula-based L method and then perform AC analysis from 10GHz to 100GHz. For the frequency-dependent extraction data, we perform extraction for each frequency point, and then simulate the frequency response at that frequency.

Obviously there are discrepancies between the formula based L extraction and frequency dependent L extraction. Before
Frequency-dependent reluctance extraction algorithm is summarized as follows.

1. Divide all conductors into two sets, vertical and horizontal.
2. For the vertical set, sort all the conductors with their x coordinates.
3. For every conductor from left to right (or bottom to top for horizontal net),
   (a) Use window selection algorithm to pick nearby conductors that posed as strong attacker
   (b) Formulate $Z_k(\omega) = R_k(\omega) + j\omega L_k(\omega)$ for $i \rightarrow 1 \text{ to } n$
       do $V_{xi} \leftarrow -1$
       for $j \rightarrow 1 \text{ to } n$
       do $Y_{ji} \leftarrow \sum(I_m)$, filament $m \in$ conductor $j$
       where $k$ is the number of filaments and $n$ is the number of conductors in the small window
   (c) $Z_n(\omega) = Y_n^{-1}(\omega)$, where $Y \in \mathbb{R}^{n \times n}$, and $n$ is number of conductors in the small window
   (d) $R_n \leftarrow \text{Re}(Z_n(\omega))$. Form L by $\text{Im}(Z_n(\omega))/\omega$, then we set flux along the aggressor to one and others to zero. And solve the reluctance elements by inverting the small matrix corresponding to the window. The resulting elements form the column of $K_{asym}$ that corresponds to the aggressor.
4. Analogically repeat step 2 and 3 for the horizontal conductors.
5. Symmetrize by $K = \frac{1}{2}(K_{asym} + K_{asym}^T)$

Table 1: Comparison between formula-based and frequency-dependent reluctance extraction

<table>
<thead>
<tr>
<th></th>
<th>Formula-based extraction method</th>
<th>Frequency-dependent extraction method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>formula based</td>
<td>Use a small window to form the impedance matrix by filaments. Take the real part of effective impedance from inverting the $n \times n$ effective admittance matrix.</td>
</tr>
<tr>
<td>Reluctance</td>
<td>Use formula based $L$ and invert it</td>
<td>Use a small window to form the impedance matrix by filaments. Take imaginary part of effective impedance from inverting effective admittance matrix, and non-symmetrized $K$.</td>
</tr>
<tr>
<td>Advantage</td>
<td>Use window selection algorithm for sparsification</td>
<td>Use window selection algorithm for sparsification. Also capture skin and proximity effect.</td>
</tr>
<tr>
<td>Disadvantage</td>
<td>Cannot capture skin and proximity effect, which becomes a significant issue in high frequency.</td>
<td>Extra inversion involved: 1) Invert a $k \times k$ impedance matrix, where $k$ is the number of filaments. Sum up current for each conductor to form the admittance matrix. 2) Invert an $n \times n$ admittance matrix to get effective R and L values. Use the effective L to get $K$ matrix.</td>
</tr>
</tbody>
</table>

3.2 Modified reluctance extraction algorithm

The proposed frequency-dependent reluctance extraction algorithm is summarized as follows.

Frequency-dependent reluctance extraction

1. Divide all conductors into two sets, vertical and horizontal.

2. For the vertical set, sort all the conductors with their x coordinates.

3. For every conductor from left to right (or bottom to top for horizontal net),

   (a) Use window selection algorithm to pick nearby conductors that posed as strong attacker

   (b) Formulate $Z_k(\omega) = R_k(\omega) + j\omega L_k(\omega)$ for $i \rightarrow 1 \text{ to } n$
       do $V_{si} \leftarrow -1$
       for $j \rightarrow 1 \text{ to } n$
       do $Y_{ji} \leftarrow \sum(I_m)$, filament $m \in$ conductor $j$

   where $k$ is the number of filaments and $n$ is the number of conductors in the small window

4. Analogically repeat step 2 and 3 for the horizontal conductors.

5. Symmetrize by $K = \frac{1}{2}(K_{asym} + K_{asym}^T)$

3.3 Deal with Ground Plane

For certain digital design, an explicit ground plane is added to provide return paths for circuits as well as a shielding element to mitigate inductance-coupling effect. Our frequency reluctance extractor can also consider a ground plane as input. In the following example, we show a 5-conductor system that inductance as well as resistance value differs extracted with and without the ground plane. For the sake of simplicity, we show the data using the inductance elements extracted with and without the ground plane. For a setup that inductance as well as resistance value differences extracted with and without the ground plane. For the sake of simplicity, we show the data using the inductance elements instead of reluctance. The setup is as follows: the width for each conductor is $2 \mu m$, the thickness and length of a conductor are $0.36 \mu m$ and $1000 \mu m$ respectively, the conductivity($\sigma$) is $4.996 \times 10^7$, and evaluated at 30GHz.

$$L(\omega) = \begin{bmatrix}
16.2 + 12.5j & 0 + 9.68j & 0 + 8.39j & 0 + 7.60j & 0 + 7.10j \\
0 + 9.68j & 17.8 + 12.4j & 0 + 9.61j & 0 + 8.35j & 0 + 7.60j \\
0 + 8.39j & 0 + 9.61j & 18.1 + 12.3j & 0 + 9.61j & 0 + 8.40j \\
0 + 7.60j & 0 + 8.35j & 0 + 9.61j & 17.8 + 12.4j & 0 + 9.75j \\
0 + 7.10j & 0 + 9.61j & 0 + 9.68j & 16.2 + 12.5j & 0 + 9.75j \\
\end{bmatrix} \times 10^{-10} H$$

For the same setup, we evaluated again but with a ground plane in the center below the two conductors. The group plane setup is as follows: $200 \mu m$ wide, $600 \mu m$ long, the thickness is $0.28 \mu m$, and separate vertically by $0.38 \mu m$. The L matrix becomes:

$$L(\omega) = \begin{bmatrix}
16.4 + 12.1j & 0 + 9.30j & 0 + 8.0j & 0 + 7.21j & 0 + 6.66j \\
0 + 9.30j & 17.9 + 12.0j & 0 + 9.21j & 0 + 7.95j & 0 + 7.19j \\
0 + 8.00j & 0 + 9.21j & 18.2 + 11.9j & 0 + 9.20j & 0 + 7.97j \\
0 + 7.21j & 0 + 7.95j & 0 + 9.20j & 17.9 + 11.9j & 0 + 9.26j \\
0 + 6.66j & 0 + 7.19j & 0 + 7.97j & 0 + 9.26j & 16.3 + 12.0j \\
\end{bmatrix} \times 10^{-10} H$$

Figure 5: An example of ground plane discretization
As we could see, with the insertion of ground plane it serves as a shielding element and alters both the inductance and resistance value. In our implementation, we discretize the ground plane with both horizontal and vertical discretization, and attach two ports in two ends (Figure 5). Then we activate each conductor at a time similar to the algorithm mentioned above but skipped the ground plane activation. Instead, the ground plane ports are assigned to be zero. Then we collect the current to form the effective admittance matrix and proceed as before. Another way to obtain the inductance matrix with ground is to activate every conductor including ground and form the lump circuit including the ground. Then, we could do some post-processing steps to get the reduced size matrix. This technique is similar to floating a conductor in capacitance matrix (Q=CV), where those floated conductor’s charge is zero and we substitute those floated (unknown) charge in every row. Therefore, formulating it using admittance matrix and assigning those ground ports as zero actually could avoid steps in post-processing. In a sense, the reduced size L matrix is obtained on-the-fly.

metal filler. fabrication. be modeled

### 4. EXPERIMENTAL RESULT

We develop the frequency dependent reluctance extractor in C/C++ programming language. The extractor implements the Window-Selection-Algorithm and discretized the conductor within the search window into filaments. We run the simulations on an Intel Pentium IV 1.4 GHz system.

Table 2 shows the run time comparison between the exact solution using frequency dependent L extraction technique compared with frequency dependent reluctance method with different number of neighbors. In figure 6 and 7 shows the frequency response of the aggressor net and far-away net comparison between frequency dependent reluctance method and frequency dependent inductance method. As we could see, frequency dependent reluctance method has relatively good match compared with the exact solution (frequency dependent L method).

For each conductor the driving end has a voltage source connected to ground, and we also connect the driving end with a 1-Ohm resistor as the driving resistor. The end is connected with a 25f loading capacitor. We activate a driving source in the middle, called aggressor, with a 1-volt step function. Then we observe the voltage of both the aggressor and a far-away net (7 conductors away) voltage via frequency response.

Table 2: Run time of frequency-dependent reluctance extraction

<table>
<thead>
<tr>
<th># of cond.</th>
<th>FastHenry</th>
<th>Frequency Depen. Reluctance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>extract time</td>
<td>4 net ext. time</td>
</tr>
<tr>
<td>100</td>
<td>15 mins</td>
<td>2.91 s</td>
</tr>
<tr>
<td>300</td>
<td>2 hrs 17 mins</td>
<td>8.99 s</td>
</tr>
<tr>
<td>500</td>
<td>6 hrs 3 mins</td>
<td>15.94 s</td>
</tr>
<tr>
<td>1,000</td>
<td>112 mins</td>
<td>40.17 s</td>
</tr>
<tr>
<td>10,000</td>
<td>-</td>
<td>302.24 s</td>
</tr>
<tr>
<td>100,000</td>
<td>-</td>
<td>5 hrs 50 mins</td>
</tr>
</tbody>
</table>

Figure 6: Frequency response (10G-100GHz) comparison of the aggressor between frequency-dependent reluctance and frequency-dependent inductance

### 6. ACKNOWLEDGEMENT

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### 7. REFERENCES


Figure 7: Frequency response (10G-100GHz) comparison of a far away victim between frequency-dependent reluctance and frequency-dependent inductance


