A Time-domain RF Steady-State Method for Closely Spaced Tones

Jaijeet Roychowdhury
University of Minnesota
jr@ece.umn.edu

Abstract

Verifying circuits with two or more closely-spaced driving frequencies is important in RF and wireless communications, *e.g.*, in the design of down-conversion mixers. Existing steady-state calculation methods, like harmonic balance, rely on Fourier series expansions to find the difference-frequency components typically of interest. Time-domain methods are, however, better suited for circuits with strong nonlinearities such as switching. Towards this end, we present a purely time-domain method for direct computation of difference tones in closely-spaced multi-tone problems. Our approach is based on multiple artificial time scales for decoupling the tones driving the circuit. Our method relies on a novel multi-time reformulation that expresses circuit equations directly in terms of time-scales corresponding to difference tones. We apply the new technique to an RF-CMOS mixer to predict baseband bit-streams and down-conversion gain and distortion, in two orders of magnitude less CPU time than traditional time-stepping simulation.

Categories and Subject Descriptors

B.7.2 [**Integrated Circuits**]: Design Aids—*Analog Verification*

General Terms

Algorithms, Design, Verification

Keywords

MPDE, difference-frequency time scales, multi-time PDEs, harmonic balance, shooting, envelope, analog, mixed-signal, analog/RF simulation, RF switching mixers, artificial time scales, homotopy, continuation methods

1. INTRODUCTION

RF sections of wireless communication systems typically involve one or more stages of frequency conversion, with frequency tones that are either widely separated, or very closely spaced. While frequency up-conversion in transmitters involves widely separated frequencies, down-conversion circuits in receivers are typically driven by two or more very closely spaced frequencies. For example, a direct-conversion cellphone could feature tones at 1.8GHz, spaced only a few Mhz or 100s of kHz apart. It is the difference frequencies that are of primary interest in receiver design and verification, since they carry the information being received; hence design and verification tools must necessarily deal with the small frequencies together with the large ones.

As is well known, simulating circuits with disparate frequencies using time-stepping methods, such as those used for transient simulation in SPICE [5, 8] and similar tools, can be very inefficient, particularly when the time-varying steady-state solution is desired. Better methods exist for both widely-separated and closely-spaced driving tones. An important technique that is useful for both situations is harmonic balance (HB) (*e.g.*, [2–4]). HB expands all time-varying waveforms in the circuit in Fourier series components featuring the driving tones, their harmonics and mixes . Since sum and difference frequencies appear explicitly in the mix components, HB is able to accommodate both widely-separated and closely-spaced driving tones naturally. Unfortunately, Fourier series expansions are also the Achilles' heel of HB, for they are ill-suited to waveforms with sharp corners or peaks. Such waveforms arise in modern integrated-RF designs, such as switching RF circuits, desirable for their low power and noise characteristics. In these situations, *i.e.*, when strong nonlinearities are encountered, time-domain methods are usually preferred over frequency-domain ones like HB.

Purely time-domain methods for steady-state and RF calculations have, until recently, been limited to single-tone problems, with shooting and its variants commonly used (*e.g.*, [1, 6, 10]). Recently, artificial time scale approaches [2, 9] have addressed the multi-tone case, resulting in purely time-domain methods for quasi-periodic steady state and envelope simulation. So far, however, these techniques have concentrated only on the case of widely-separated driving tones.

In this work, we extend multi-time approaches to the case of closely-spaced driving tones. The key enabler towards this end is the use of *difference-frequency time scales* (or simply *difference time scales*). In contrast to the widely-used concept of difference frequencies, which is linked to sinusoidal basis functions and quasi-periodic waveforms, difference time scales capture the essential intuition of slow variations at difference frequencies without being limited to sinusoidal bases or quasi-periodicity. This makes it possible to obtain, for example, the shape of baseband or down-converted information signals (such as bit streams) directly in the time domain, without at any point involving frequency-domain representations or calculations.

We present applications of the new technique to balanced and unbalanced switching mixer circuits for direct-conversion receivers, similar to those reported recently in [7, 11]. We are able to obtain, directly, time-domain waveforms for sections of bit streams downconverted to baseband. Using pure-tone driving excitations, we are also able to obtain down-conversion gain and distortion figures. The new method is faster by about two orders of magnitude than periodic steady-state computation using single-time shooting.

Although we demonstrate the new technique on RF communication circuits in this paper, we note that the proposed method can be applied generally to other systems featuring closely-spaced tones, such as power conversion circuits and electro-optical communication systems.

In the remainder of the paper, we describe schemes for reformulating and solving the circuit equations in terms of difference-frequency time scales in Section 2. In Section 3, we describe the application of the new technique to a balanced switching mixer, and provide interpretations of the results obtained.

2. DIFFERENCE TIME SCALES

Multi-time circuit equations

We first recall the basic concepts of multi-time circuit formulations, following the approach of [9]. The equations of the circuit are represented as

$$\dot{q}(x(t)) + f(x(t)) + b(t) = 0, \tag{1}$$

where $q(\cdot)$ represents the linear and nonlinear charge/flux terms, $f(\cdot)$ the conductive terms, $b(t)$ the excitation, and $x(t)$ the unknowns (voltages and currents) in the circuit. When the excitation is multi-tone (whether closely or widely spaced), $b(t)$ can be expressed in terms of multi-variate functions involving several artificial time scales. For example, a two-tone signal may be expressed as

$$b(t) = \hat{b}(t,t), \tag{2}$$

where

$$\hat{b}(t_1,t_2) = \hat{b}(t_1+T_1, t_2+T_2), \tag{3}$$

is periodic in each of its two arguments, for some periods T_1 and T_2. The two tones of excitation are $f_1 = \frac{1}{T_1}$ and $f_2 = \frac{1}{T_2}$; they are widely separated if $f_1 \gg f_2$ (or vice-versa). If the tones are closely spaced, then typically, $f_1 \sim f_2 \gg |f_1 - f_2|$.

In [9], it has been shown that when the tones are widely separated, the multi-time representation (3) can be represented far more compactly, in terms of numerical samples, than the normal single-time form (2). This fact is applied towards solving the circuit equations (1), by assuming that the circuit unknowns $x(t)$ can also be expressed in a numerically compact multi-time form similar to (3). Finding this multi-time solution directly involves solution of the following *multi-time partial differential equation* (MPDE) corresponding to (1):

$$\frac{\partial q(\hat{x}(t_1,t_2))}{\partial t_1} + \frac{\partial q(\hat{x}(t_1,t_2))}{\partial t_2} + f(\hat{x}(t_1,t_2)) + \hat{b}(t_1,t_2) = 0. \tag{4}$$

In (4), $\hat{x}(t_1,t_2)$ is the multi-time representation of $x(t)$ that is sought. It has been shown that if $\hat{x}(t_1,t_2)$ satisfies (4), then $x(t) = \hat{x}(t,t)$ solves the original circuit equations (1). Regardless of whether the tones in the circuit are separated or not, this basic fact always holds.

When the driving tones are in fact widely separated, *e.g.*, $T_1 \ll T_2$, solving the MPDE (4) is preferred over solving (1) for two reasons. First, $\hat{b}$ and $\hat{x}$ can be represented much more efficiently than their single-time counterparts; and second, the t_1 and t_2 variations of the solution $\hat{x}(t_1,t_2)$ directly provide information of interest, namely the slowly-varying envelope, and the detailed shape of the fast variations at any point under the envelope.

Closely-spaced tones

When the tones are closely spaced, *i.e.*, $T_1 \sim T_2$, the MPDE (4) as it stands is still perfectly valid, however it is not immediately useful as in the widely separated case, even though $\hat{b}$ and $\hat{x}$ are still numerically far more compact than their single-time counterparts. The reason for this is that the frequencies of interest are typically *difference frequencies*, such as $f_1 - f_2$, $2f_1 - f_2$, *etc.*. The difference frequencies are much smaller than f_1 or f_2, and manifest themselves as baseband signals or as slow envelopes riding on much faster waveforms. The multi-time solution $\hat{x}$ does not directly provide the slow waveforms at these frequencies. As a concrete example, consider the ideal mixing operation

$$z(t) = x(t)y(t), \tag{5}$$

where $x(t) = \cos(2\pi f_1 t)$ and $y(t) = \cos(2\pi f_2 t)$, with $f_1 = 1$GHz and $f_2 = f_1 - 10$kHz. In other words, the two tones f_1 and f_2 are very closely spaced, with the difference tone $f_1 - f_2 = 10$kHz being much smaller. The mixing operation (5) generates this difference tone explicitly, since

$$z(t) = \frac{1}{2}\left[\cos(2\pi \times 10\text{kHz} \times t) + \cos(2\pi(f_1+f_2)t)\right], \tag{6}$$

and the high frequency component at $f_1 + f_2$ is usually eliminated by separate filtering. The multi-time representation of $z(t)$, given by

$$\hat{z}_1(t_1,t_2) = \cos(2\pi f_1 t_1)\cos(2\pi f_2 t_2), \tag{7}$$

does not directly provide this difference-frequency component.

For a multi-time method to be practically useful for the case of closely-spaced driving tones, it must be capable of directly solving for these slow waveforms in the time domain. We now outline how this can be achieved. The key insight is to realize that a given $\hat{b}$ satisfying (2) is not unique, *i.e.*, there are many functions $\hat{b}$, with *different* T_1 and T_2 values in (3), providing the same $b(t)$ by (2). We illustrate this with our ideal mixing example. For convenience, we first define

$$\hat{z}_s(t_{1s},t_{2s}) = \cos(2\pi t_{1s})\cos(2\pi t_{2s}) \tag{8}$$

to be a scaled multi-time representation of $z(t)$, periodic with period 1 in both its arguments. $\hat{z}_1$ in (7) can be expressed in terms of this as

$$\hat{z}_1(t_1,t_2) = \hat{z}_s(f_1 t_1, f_2 t_2). \tag{9}$$

Note that $\hat{z}_1$ above, with periods $T_1 = \frac{1}{f_1}$ and $T_2 = \frac{1}{f_2}$ very similar to each other, does not provide difference-frequency information directly; but as always required, it satisfies $z(t) = \hat{z}(t,t)$.

If we now define a difference frequency of interest to be

$$f_d = f_1 - f_2 = 10\text{kHz} \tag{10}$$

and the corresponding period $T_d = \frac{1}{f_d} = 0.1$ms, we can devise a new multi-time representation

$$\hat{z}_2(t_1,t_2) = \hat{z}_s(f_1 t_1, f_1 t_1 - f_d t_2). \tag{11}$$

Note that $\hat{z}_2$ is periodic with respect to t_1 and t_2, with periods T_1 and T_d, respectively, hence its changes along t_2 capture variations corresponding to difference-frequency time scales. Furthermore, it continues to satisfy the requirement $z(t) = \hat{z}_2(t,t)$, hence is as relevant to the underlying one-time problem as $\hat{z}_1(t_1,t_2)$. $\hat{z}_1$ and $\hat{z}_2$ are plotted in Figure 1 and Figure 2, respectively; note the explicit variation in the difference time scale, spanning 0.1ms, in the latter.

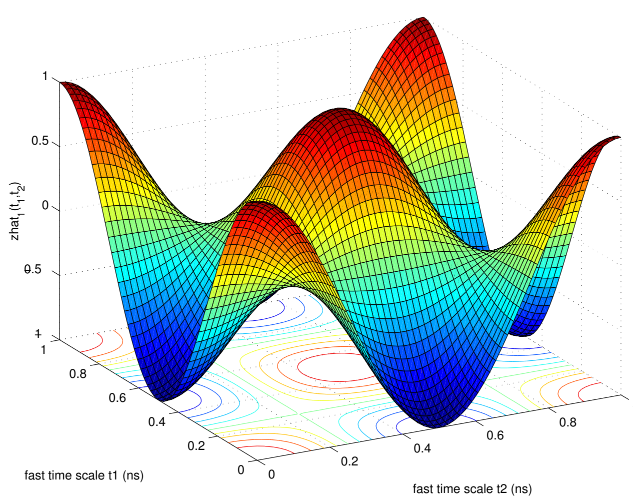

Figure 1: $\hat{z}_1(t_1,t_2)$

Note that the arguments $(f_1 t_1, f_1 t_1 - f_d t_2)$ in (11) can be interpreted geometrically as scaling and shearing of the original time scales (t_{1s}, t_{2s}) in (8). Furthermore, numerical compactness of representation is not affected by the shearing, as can be seen in Figure 2. Hence time-sheared multi-time representations for $\hat{b}$ and $\hat{x}$ can be applied to the MPDE (4), and a solution obtained directly in terms of the slow difference-frequency time scale of interest.

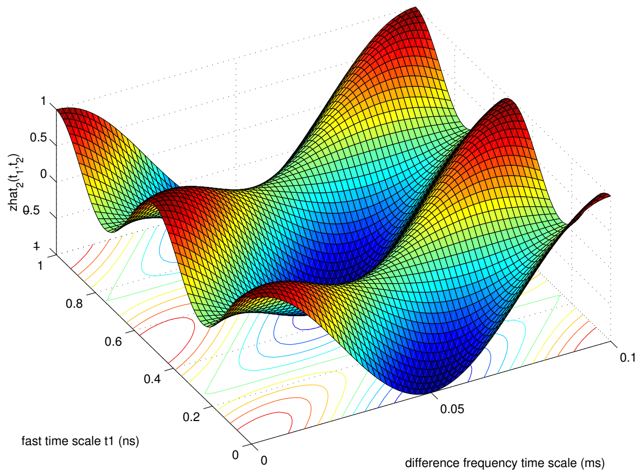

Figure 2: $\hat{z}_2(t_1,t_2)$

We also note that (10) is only one way of defining a difference-frequency time scale; depending on the application, other difference frequencies should be used. For example, in the balanced mixer circuit in Section 3, the local oscillator frequency $f_1 = 450$MHz is doubled internally within the circuit and then mixed with the information-carrying tone close to 900MHz. Hence the difference frequency of interest (at baseband) is

$$f_d = 2f_1 - f_2, \tag{12}$$

with a corresponding change to (11),

$$\hat{z}_2(t_1,t_2) = \hat{z}_s\left(f_1 t_1, 2f_1 t_1 - f_d t_2\right), \tag{13}$$

restoring the property $z(t) = \hat{z}_2(t,t)$.

An important point to note is that the signals need not be sinusoidal or near-sinusoidal. Indeed, high-frequency 'tones' involving modulation by bit streams can be used to drive a circuit. For example, the normalized-to-period-1 information-carrying tone driving the balanced mixer in Section 3 is given in multi-time form by

$$\hat{b}_i(t_{1s},t_{2s}) = 2\cos(2\pi \times 2t_{1s})\,\text{pulse}\left(2(2t_{1s} - t_{2s})\right). \tag{14}$$

Once reformulated in terms of difference time scales, the MPDE (4) may be solved by any of the time-domain numerical methods in [9].

3. APPLICATION AND EXAMPLES

Balanced LO-doubling down-conversion mixer

A CMOS balanced down-conversion mixer, adapted from a circuit proposed in [11], was analyzed using the above ideas. An important feature of this circuit is that the lower pair of MOSFETs constitutes a frequency doubler, generating a current at twice the LO frequency. This current feeds the differential pair formed by the upper two MOSFETs, resulting in mixing and down-conversion. The supplied LO signal in this case is a sinusoid at 450MHz; the RF signal 'tone' is as for the previous circuit, a modulated bit stream close to 900MHz. For this circuit, the natural choice for the difference frequency is given by (12), again resulting in a baseband frequency of 15kHz. The time-shearing function is given by (13). The multi-time form of the differential output voltage between the drains of the upper MOSFETs is shown in Figure 3. Once again, the time-domain shape of the bit-stream is evident from the variation along the difference-frequency time scale. Figure 4 shows the envelope along the difference-frequency time scale, corresponding to the actual baseband voltage of the output.

The sharp waveforms created by the action of the frequency doubler in the circuit are clearly visible in Figure 5, which depicts the

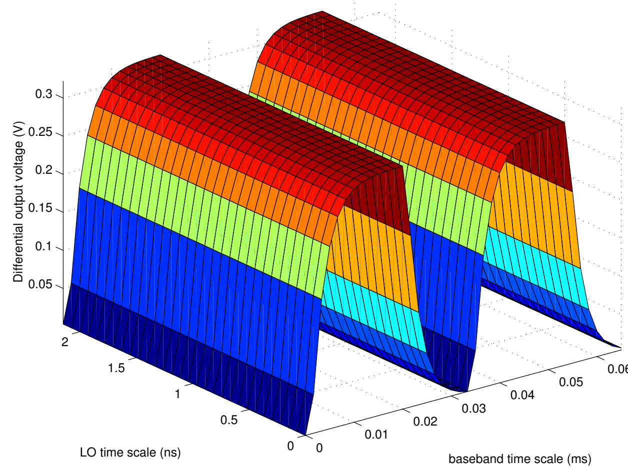

Figure 3: Balanced mixer: differential output voltage

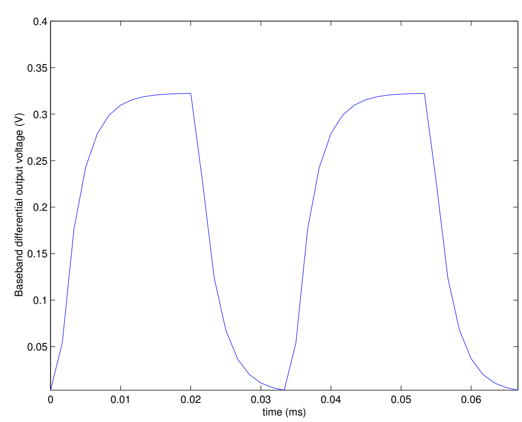

Figure 4: Balanced mixer: baseband differential output

multi-time voltage at the sources of the differential pair. The advantages of fully time-domain solution methods are best utilized in such situations. A small section of the actual voltage waveform, over a period of 5 LO cycles, is depicted in Figure 6.

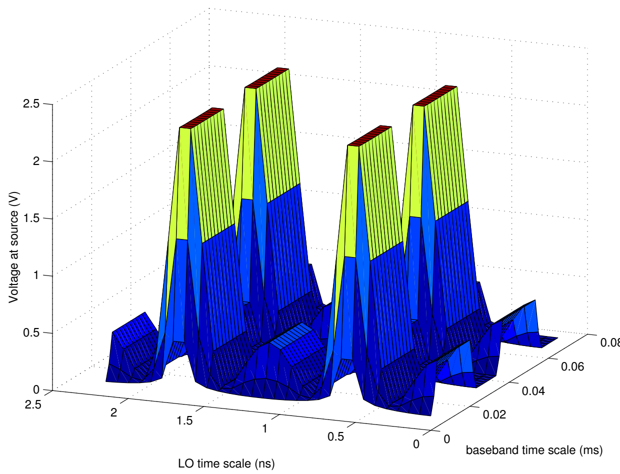

Figure 5: Balanced mixer: voltage at MOSFET sources

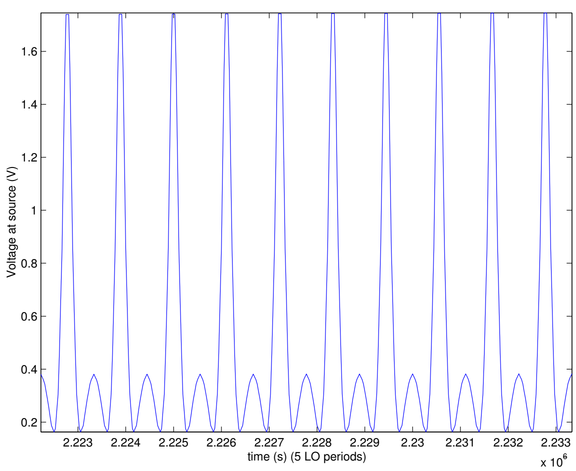

Figure 6: Balanced mixer: voltage at MOSFET sources

Computational speedup

The computational speedup of the new method stems from that relatively few grid points in the multi-time plane are sufficient to capture solution waveforms, compared to the number of time-points needed for the normal time representation. For the balanced mixer circuit above, we employed a 40×30 grid, resulting in 1200 grid points. Wall-clock time for the longest run (26 iterations) of Newton-Raphson on the balanced mixer circuit (given a good starting guess) was 1m3s[1]. In cases where Newton-Raphson did not converge, using continuation reliably obtained solutions in 10-20m.

The closest comparable traditional time-domain approach is shooting (or time-discretization) across one period of the difference frequency, but with time-steps small enough to capture the LO variations with sufficient accuracy, *i.e.*, 10 or more time-steps per LO period. This amounts to some 300000 or more time-steps over the difference-frequency period, resulting in an equation system more than $250\times$ larger. Using iterative linear solution methods and in the presence of similar nonlinear convergence behaviour, this results in a computational advantage of more than two orders of magnitude in favour of the new method described here. However, because it can be significantly more difficult to obtain nonlinear solution convergence for traditional shooting than for the multi-time approach in the presence of widely separated time scales, the real user-experienced speedup can be considerably larger.

We note that the speedup depends roughly linearly on extent of disparity between the LO time scale and the difference-frequency time scale. The break-even point (in terms of frequency separation) for computational speedup over single-time shooting is strongly dependent on implementation. We have noted that frequency disparities of 200 and above confer a speed advantage to multi-time methods.

Conclusion

Through the use of sheared, difference-frequency time scales, we have extended multi-time circuit solution formulations, previously useful only for widely-separated driving tones, to situations where driving signals are close in frequency. A purely time-domain method for solving the resulting equations has been presented and applied to compute the time-domain shapes of baseband bit-streams in down-conversion mixers. The new method is well-suited for estimating effects such as ISI and ACI in communication symbol streams. When baseband variations are much slower than those of the driving signals, it is also more than two orders of magnitude faster than comparable single-time methods.

[1] All times are on a single-CPU, 1.4GHz AMD Athlon system with 512MB of 133MHz DDR memory, running Linux 2.4.15.

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