Time-domain steady-state simulation of frequency-dependent components using multi-interval Chebyshev method

Baolin Yang
Cadence Design Systems*
byang@celestry.com

Joel Phillips
Cadence Berkeley Laboratories
jrp@cadence.com

ABSTRACT

Simulation of RF circuits often demands analysis of distributed component models that are described via frequency-dependent multi-port Y, Z, or S parameters. Frequency-domain methods such as harmonic balance are able to handle these components without difficulty, while they are more difficult for time-domain simulation methods to treat. In this paper, we propose a hybrid frequency-time approach to treat these components in steady-state time-domain simulations. Efficiency is achieved through the use of the multi-interval Chebyshev (MIC) simulation method and a low-order rational-fitting model for preconditioning matrix-implicit Krylov-subspace solvers.

Categories and Subject Descriptors

I.6 [**Computing Methodologies**]: Simulation and Modeling

General Terms

Algorithms

Keywords

RF circuit simulation, frequency dependent, S parameter

1. INTRODUCTION

Special-purpose RF circuit simulators exploit the sparsity of the spectrum in order to make the computations tractable[1]. RF communication circuits often contain components (particularly passive components) such as transmission lines, integrated inductors, and SAW filters, where distributed effects are important. Distributed components are those that are not conveniently described by a finite-dimensional (i.e. lumped) state-space model, for example because they have an infinite-dimensional state space. We will use the term to mean any component more conveniently described by a frequency-domain or convolutional representation.

*The author is now with Celestry Design Technologies, Inc.

DAC 2002, June 10-14, 2002, New Orleans, Louisiana, USA.

Circuit simulation methods solve circuit equations in either the time domain or frequency domain. Time-domain simulation methods discretize circuit equations using finite difference methods such as the second-order Gear method. The advantage of the time-domain methods is that they can select time-points based on localized error estimates and as a result can easily handle strongly nonlinear phenomena and sharp transitions in circuit waveforms. Frequency-domain simulation methods, such as the harmonic balance method, are popular for RF simulation and have the advantage of attaining spectral accuracy for smooth waveforms. Development of matrix-free Krylov-subspace algorithms [2, 3, 4] has made dedicated RF simulation tools even more popular as they can now be used to analyze circuits with thousands of devices.

In contrast to frequency-domain techniques, which have no more difficulty simulating distributed devices than lumped devices, a drawback of time-domain methods is that the simulation of distributed or passive components is more difficult. There are generally two approaches. The first approach is to apply a model generation tool[5, 6] to generate finite-dimensional state-space models for those components. These state-space models are easily simulated in the time domain. The second approach is to compute the impulse response of the distributed element, and apply a convolution approach at each timestep to obtain the time-domain response of the element. In transient analysis, one may use direct convolution approaches to simulate these components[7] without significant difficulty.

In RF simulation, however, such as periodic steady-state analysis via shooting-Newton methods[3], problems arise due to the fact that distributed components have infinite-dimensional state. First, it is not sufficient to calculate the state of the circuit at a single point of the periodic time interval in order to describe the periodic steady-state, the distributed state of the devices must be computed[8]. This in turn implies that the sensitivity calculation in shooting methods involves more than the two ending points of the periodic interval. Second, the distributed components destroy the block-banded structure of the Jacobian that is exploited in preconditioner computation[3].

In this paper, we propose a novel hybrid frequency-time method to include these distributed passive components in an RF simulator. Because we will take a matrix-implicit approach to equation solution, the distributed devices can be treated as black-boxes with a port-to-port operator given most naturally by frequency-dependent S/Y/Z-parameters. Since it is only necessary to evaluate this operator, not to write an explicit matrix, we propose an approach that is a dual to traditional harmonic balance. We formulate the equations in the time-domain, then use Fourier analysis to switch between candidate time-domain waveform solutions and the frequency-domain spectrum where we evaluate the frequency-domain devices. With this combined frequency/time-domain approach, we can inherit the advantages of both time-domain methods in describing hard nonlinearities, and harmonic balance methods in conveniently treating frequency-domain devices.

In principle, any time domain method can be used, but for traditional low-order polynomial methods such as second-order Gear, the Fourier transformation step is expensive. Therefore we advocate the multi-interval Chebyshev (MIC) method [9] as our time-domain simulation method, as we can preserve the accuracy of the frequency-domain model description, but with less computational time. Within the multi-interval Chebyshev context, the remainder of the paper gives details of the formulation, describes optimal partitioning of distributed device evaluation between time and frequency domains, and considers practical aspects such as effective preconditioner construction.

2. CIRCUIT EQUATIONS

2.1 Formulating Distributed Component Equations

Lumped circuit behavior is usually described by a set of N nonlinear differential algebraic equations (DAEs) that can be written, without loss of generality, as

$$\frac{dQ(v(t))}{dt} + I(v(t)) + u(t) = 0 \ , \tag{1}$$

where $Q(v(t)) \in \mathbb{R}^n$ represents contribution of the dynamic elements such as inductors and capacitors, $I(v(t) \in \mathbb{R}^n$ is the vector representing static elements such as resistors, $u(t) \in \mathbb{R}^n$ is the vector of inputs, and the vector $v(t) \in \mathbb{R}^n$ contains state variables such as node voltages. Distributed components must be described using a more general form. Suppose that there are K linear, time-invariant, but distributed devices, each of which possess a convolutional (impulse response) representation. Then we may formulate the time-domain circuit equations as

$$\frac{dQ(v(t))}{dt} + I(v(t)) + \left[\sum_{k=1}^{K} P_k \int_{-\infty}^{t} H_k(t-\tau) P_k^T v(\tau)\, d\tau\right] + u(t) = 0 \ , \tag{2}$$

Here $H : (-\infty, t] \to \mathbb{R}^{p_k \times p_k}$ is the $p_k \times p_k$ matrix of impulse responses for the kth distributed component. It is related to the rest of the circuit equations via a matrix $P_k \in \mathbb{R}^{n \times p}$. P_k^T is an operator that extracts the relevant portion of the state vector. That is, P_k is one in the (i, j) entry if the jth connection to the kth device is through the state variable with index i.

In RF simulation, we are usually interested in solving steady-state problems. The periodic steady-state problem is the simplest and serves as a model for all the others. It is also important to study because it forms the basis of more advanced analyses, such as cyclostationary noise analysis, multi-frequency distortion analysis, or envelope simulation. The periodic steady-state solution is the solution of Eq. (2) that also satisfies the two-point boundary condition $v(t+T) - v(t) = 0$ for some period T and all t. It is therefore sufficient to calculate the periodic steady-state solution in the interval $(0, T]$, i.e. we seek $v \in (0, T] \times \mathbb{R}^n$. This is useful because we may utilize the (discrete) Fourier transform $\mathcal{F} v : \mathbb{Z} \to \mathbb{C}$, and its inverse $\mathcal{F}^{-1} = \mathcal{F}^T$, in representing and manipulating signals. $\mathcal{F}$ maps v on $[0, T)$ to its frequency-domain representation, the Fourier series coefficients.

Without loss of generality, we can describe any physically-realizable distributed linear element as an N-port component described by an $N \times N$ frequency-dependent scattering (S) parameter matrix. As the convolution operation is diagonalized by the Fourier transform, we may write the circuit equations in terms of the frequency-dependent matrix $H(\omega)$ as

$$\frac{dQ(v(t))}{dt} + I(v(t)) + \left[\sum_{k=1}^{N} P_k \mathcal{F}^T H_k(\omega) \mathcal{F} P_k^T v\right] + u(t) = 0 \ . \tag{3}$$

Here $H_k(\omega)$ is the (continuous) Fourier transform of the kth impulse response matrix. The relation between the scattering parameter matrix $S(\omega)$ of the underlying element and the matrix $H(\omega)$ depends on the way the circuit equations are stamped. For simplicity of presentation, we consider a 2-port device with S-parameter description to be stamped in the MNA formulation. We denote the potential at the first port as e_1 and e_1' and the second port as e_2 and e_2'. The currents at the two ports are denoted as I_1 and I_2. The port resistance is R_1 and R_2. If we ignore contributions from other circuit elements (*), formally the circuit equation that describes the behavior of the 2-port and defines $H(\omega)$ is:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 1-S_{11} & S_{11}-1 & -S_{12}\sqrt{\frac{R_1}{R_2}} & S_{12}\sqrt{\frac{R_1}{R_2}} & -(S_{11}+1)R_1 & -S_{12}\sqrt{R_1R_2} \\ -S_{21}\sqrt{\frac{R_2}{R_1}} & S_{21}\sqrt{\frac{R_2}{R_1}} & 1-S_{22} & S_{22}-1 & -S_{21}\sqrt{R_1R_2} & -(S_{22}+1)R_2 \end{bmatrix} \cdot \begin{bmatrix} e_1 & e_1' & e_2 & e_2' & i_1 & i_2 \end{bmatrix}' = \begin{bmatrix} * & * & * & * & 0 & 0 \end{bmatrix}' \tag{4}$$

2.2 Discretization

In frequency-domain simulation methods one solves for the Fourier spectrum of the waveform instead of function values at a set of time points. It is straightforward to include distributed or passive components that may be described using frequency-dependent tabulated S-parameter. With the S-parameter information, one knows how to load the Jacobian of the circuit equation in frequency domain and set up the linear system. A linear solver, such as a Krylov-subspace iterative solver, can be used to solve the linear system.

In time-domain methods, on the other hand we solve for the waveform values, $u(t_j)$, at a set of time steps: $t_j,\ j = 0, 1, \cdots, M$. To perform the discretization at $t = t_k$, we seek an interpolating polynomial of degree p based on $p+1$ time points. The time derivative is approximated by evaluating the derivative of the interpolating polynomial at $t = t_k$. Thus the time-derivative of the approximate solution is required to match the differential equation at the point t_k. The time steps are not uniform and their variation can be big. It is difficult to simulate these components on this set of time points without loss of efficiency. We propose an approach that makes this happen. In our novel approach, advanced Krylov-subspace iterative solver and multi-domain Chebyshev method are used to achieve this purpose.

Here we give the basics of MIC method. Given the values, $u(t_j)$, at the Chebyshev Gauss-Lobatto collocation points, $t_j = \cos\frac{\pi j}{M},\ j = 0, \ldots, M$ in one subinterval, we seek an interpolating polynomial of degree M:

$$I_M u(t) = \sum_{k=0}^{M} \tilde{u}_k T_k(t) \ , \tag{5}$$

and the derivative of the polynomial can be computed by using backward recurrence relation for Chebyshev polynomials $T_k(t)$. We can also obtain a differentiation matrix D by noting that the interpolating polynomial can be expressed in terms of a Lagrange interpolation polynomial $g_j(t)$. The entries of the matrix D are computed by taking the analytical derivative of $g_j(t)$, $D_{kj} = g_j'(t_k)$. Refer to

[9] for the exact formulation of the matrix. As in harmonic balance, high accuracy is achievable because in each approximation interval all the desirable approximation properties of Chebyshev polynomials can be exploited.

3. HYBRID FREQUENCY-TIME DOMAIN SIMULATION

3.1 Matrix-Implicit Krylov Subspace Solution Technique

The major computational task for large-scale RF simulation is to solve the large linear systems generated from the circuit Jacobian matrix. In the nonlinear steady-state computation, a system must be solved at each step of Newton's method. The Jacobian J is composed of three pieces, the dynamic piece J_C, the static piece J_G, and the distributed piece J_D. With the Chebyshev discretization and

$$G(t_k) = \left.\frac{\partial I}{\partial v}\right|_{v(t_k)}, \quad C(t_k) = \left.\frac{\partial Q}{\partial v}\right|_{v(t_k)}, \tag{6}$$

we have

$$J_C = \begin{bmatrix} D^1 \otimes I^1 & & & D_0^1 \otimes I^1 \\ & & \ddots & \\ & D_0^2 \otimes I^2 & D^2 \otimes I^2 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} C(t_1) & & & \\ & C(t_2) & & \\ & & C(t_3) & \\ & & & \ddots \end{bmatrix} \tag{7}$$

where D^k is part of the Chebyshev differentiation matrix D in the kth interval ($[D_{ij}]|_{i,j=1,\cdots,M}$), D_0^k is $[D_{ij}]|_{i=1,\cdots,M;j=0}$, and I^k the identity matrix of dimension equal to the number of timepoints in the kth interval,

$$J_G = \begin{bmatrix} G(t_1) & & & \\ & G(t_2) & & \\ & & G(t_3) & \\ & & & \ddots \end{bmatrix} \textit{ and } J_D = \left[\sum_{k=1}^{N} P_k F^T H_k(\omega) F P_k^T\right]. \tag{8}$$

In small-signal and noise analysis, a related system must be solved at each frequency point of analysis. One notes that because of the frequency-dependent property of the distributed components, the transformation F is dependent on all time points in the simulation time interval. From Equation 3, it is clear that every p-port distributed element introduces p rows and columns into the Jacobian that are full. This can be seen by noting that the matrices PF^T and FP^T are dense. It is not possible to form and solve large systems of such a form efficiently using direct linear algebra. In our approach, as is now standard, a Krylov-subspace iterative solver such as GMRES is used. The use of these methods in circuit simulation are treated in detail elsewhere [2, 3, 4, 10].

We focus on the matrix-implicit treatment of the distributed components. To evaluate the term J_D, we need a procedure that takes time-domain waveforms as input, applies the Fourier transformation to transform them into the frequency domain, applies the transfer function of the distributed components, in terms of frequency-dependent S-parameters, to the frequency-domain result, and inverse-transforms to the time-domain. Such a procedure has as much flexibility in dealing with distributed-components as harmonic balance methods. In fact, there is more flexibility. We can extract any part of the transfer function that is best suited to evaluation in the time-domain and perform that part of the port-to-port transformation in time domain. For example, we can perform the delay transformation of transmission lines in the time domain and obtain much more accurate answers than possible by representing the delay using a sum of Fourier harmonics. All these operations fit in the matrix-vector product calculation of iterative solvers.

3.2 Time-Frequency Decomposition

Taking a two-port system as an example, we now explain the partitioning of the distributed component properties between the time- and frequency- domain representations. For each port in the distributed component, there is a port equation in the circuit equations. For a two-port system, we have

$$(1-S_{11})e_1 + (S_{11}-1)e_1' - S_{12}\sqrt{\frac{R_1}{R_2}}e_2 + S_{12}\sqrt{\frac{R_1}{R_2}}e_2' - (S_{11}+1)R_1 i_1 - S_{12}\sqrt{R_1 R_2}\, i_2 = 0\,, \tag{9}$$

for one port and a similar equation for the other port. Note that the S-parameter in this formula is frequency dependent. Hence the corresponding equations in time domain are more complicated and can be obtained using frequency-time-domain transformations. For periodic steady state analysis, the spectrum of solution, which we show how to compute in the next section, is at integer multiples of the fundamental frequency. We need the S-parameters at those frequencies and then we can obtain the equations in time domain.

Now we discuss decomposing the port equations into a frequency-independent part and a frequency-dependent S-parameter part. We decompose the first port equation into

$$A_1 = (1-\bar{S}_{11})e_1 - (1-\bar{S}_{11})e_1' - \bar{S}_{12}\sqrt{\frac{R_1}{R_2}}e_2 + \bar{S}_{12}\sqrt{\frac{R_1}{R_2}}e_2' - (1+\bar{S}_{11})R_1 i_1 - \bar{S}_{12}\sqrt{R_1 R_2}\, i_2 \tag{10}$$

and

$$B_1 = (\bar{S}_{11} - S_{11})e_1 + (S_{11} - \bar{S}_{11})e_1' - (S_{12} - \bar{S}_{12})\sqrt{\frac{R_1}{R_2}}e_2 + (S_{12} - \bar{S}_{12})\sqrt{\frac{R_1}{R_2}}e_2' - (S_{11} - \bar{S}_{11})R_1 i_1 - (S_{12} - \bar{S}_{12})\sqrt{R_1 R_2}\, i_2 \tag{11}$$

and enforce $A_1 + B_1 = 0$. Likewise we can decompose the second port equation.

In general, s-parameters with bar's in the split equations are the S-parameters of some time-domain effect, such as delay, DC offset, or linear scale. These time-domain effect can be calculated in time domain separately. One simple example is to choose constants for the approximation when S-parameters at high frequencies approach a constant. Thus, the frequency-dependent parts, B_1 and B_2, do not have any high-frequency effects, which can be calculated in frequency domain through Fourier transforms without exhibiting oscillations in sharp-transition regions of time-domain waveform, known as Gibbs phenomena.

3.3 Chebyshev Fourier Quadratures

It should be clear that, while the above procedure can be performed with time-domain periodic-steady-state solution method enforce periodicity explicitly, one key to efficient implementation is efficient evaluation of the Fourier transforms. The Chebyshev discretization achieves high accuracy with relatively few timepoints. With this property, one can afford to apply Fourier transformation and inverse Fourier transformation to transform the waveforms between frequency domain and time domain. The computational cost is $O(M^2)$, where M is the number of time steps. Because of the non-uniform timepoint distribution, we cannot base an implementation on the standard FFT, though we could use more sophisticated $O(M \log M)$ non-uniform Fourier analysis techniques[11].

In order to use the frequency-dependent S-parameters directly, the time-domain waveform is transformed into frequency domain. We use Chebyshev quadrature formulas to obtain the Fourier spectrum with high-order accuracy. The order of accuracy is basically equal to the number of Chebyshev collocation points, which in turn given by the number time steps in one Chebyshev time interval. Unlike low-order polynomial approximations such as Gear methods, the Chebyshev quadratures maintain spectral accuracy in this process. Assume the time steps we have are $t_i, i = 0, \cdots, i = M$. The integration formula is

$$\int_{t_0}^{t_M} v(t)dt = \sum_{i=0}^{M} w_i v(t_i) \tag{12}$$

where

$$w_i = \begin{cases} \frac{1}{M^2-1} & i = 0 \\ \frac{2}{M}\left(1 + \frac{(-1)^(i-1)}{M^2-1} + \sum_{k=1}^{\frac{M}{2}-1} \frac{2}{1-4k^2}\cos(2ki\frac{\pi}{M})\right) & i \leq \frac{M}{2} \\ w_{M-i} & \frac{M}{2} < i \leq M \end{cases} \tag{13}$$

Note that we are considering a periodic function. The Fourier spectrum can be calculated using the following formula:

$$\int_{t_0}^{t_M} v(t)e^{jk\frac{2\pi}{T}t}dt = \sum_{i=0}^{M} w_i v(t_i) e^{jk\frac{2\pi}{T}t_i} . \tag{14}$$

With the Fourier spectrum known, it is possible to perform the necessary part of the Jacobian and equation evaluation in the frequency domain, just as in harmonic balance. After we obtain the result of the operation in frequency domain, we then transform it back to the time domain using inverse Fourier transformation. Our time-domain simulation needs to have the result at M time points $t_i, i = 0, \cdots, M$. For all these transformations, the computational complexity of the multi-interval Chebyshev method and traditional low-order methods are the same, generally proportional to M or M^2. Because M for the Chebyshev methods is much less than that of traditional time-domain methods, the Chebyshev methods have a large advantage over traditional low-order methods.

This Fourier weight calculation method is subject to aliasing error. When a large k is required, the integration formula (14) may not have enough resolution, because M needs to be at least $2k$ for the formula to have any accuracy. Spectral accuracy is achieved when $\frac{M}{k}$ is large. However, one can interpolate the solution at $t_i, i = 0, \cdots, M$ to a finer Chebyshev grid. This interpolation keeps the high-order accuracy of the time-domain solution. Then one can perform integration of (14) on the fine grid to guarantee the accuracy. In practice, it is not necessary to really interpolate the solution. One only needs to pre-correct the integration weights, $w_i, i = 0, \cdots, M$, to realize this refinement in integration. This situation can occur when the time-domain waveform in an interval is much smoother than the Fourier harmonic to be evaluated.

4. PRECONDITIONING

4.1 Block-Lower Triangular Preconditioners

Rapid convergence of Krylov-subspace iterative methods requires construction of effective preconditioners. Shooting-based time-domain RF simulation methods[2, 3, 10] typically construct preconditioners by observing that that block-lower-triangular portion of the Jacobian is easily inverted, given that the diagonal blocks can be efficiently factored via sparse direct techniques. That is, to solve the preconditioned system $P^{-1}Jx = P^{-1}b$, the Jacobian is first decomposed into upper- and lower- block triangular pieces, $J = L + U$, with

$$U = \begin{bmatrix} 0 & & D_0^1 \otimes I^1 \\ & \ddots & \\ & & 0 \end{bmatrix} \begin{bmatrix} C(t_1) & & & \\ & C(t_2) & & \\ & & C(t_3) & \\ & & & \ddots \end{bmatrix}, \tag{15}$$

$L = \tilde{J}_C + J_G$, where

$$\tilde{J}_C = \begin{bmatrix} D^1 \otimes I^1 & & & \\ & & \ddots & \\ & D_0^2 \otimes I^2 & D^2 \otimes I^2 & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} C(t_1) & & & \\ & C(t_2) & & \\ & & C(t_3) & \\ & & & \ddots \end{bmatrix}. \tag{16}$$

The preconditioner is then $P = \tilde{J}_C + J_G = L$. As P is block-lower-triangular, inversion is easy as long as the diagonal blocks can be factored or inverted efficiently, which is the case here. The problem with applying a similar technique, when distributed elements are present, is the term J_D. In the matrix-implicit context, it is not even clear how to construct the lower-diagonal piece, nor how to invert it, much less that this would be an effective strategy.

In general, solutions at nodes of the distributed components are coupled in time because of the Fourier transformation done in the periodic time interval. Existing preconditioning techniques for periodic steady-state simulation can not solve this difficulty. In this paper, we propose to use low-order lumped models to construct the distributed part of the preconditioner.

4.2 Rational Fitting

The first idea in our preconditioning strategy is to first construct an approximate $\tilde{H}(\omega)$ that is close to $H(\omega)$. One possibility to construct an approximate $\tilde{H}(\omega)$ is to use rational approximation[5] of the underlying device parameters $S(\omega)$. We propose constructing a rational approximation $\tilde{S}(\omega)$ of $S(\omega)$. Since, because the rational approximation will be used for a preconditioner, it does not have to be extremely accurate, we may approximate $S(\omega)$ with a low-order model, which is cheap to construct. This approach has the advantage that we may use the block-lower-triangular preconditioners discussed in the previous section. Constructing a rational function model is equivalent to finding a state-space model of the form

$$\frac{dx}{dt} = Ax + Bu(t), \quad y = Cx + Du \tag{17}$$

such that the transfer function $\tilde{S}(\omega) = D + C(i\omega I - A)^{-1}B$ approximates $S(\omega)$. We do this by requiring that the mean-square error

$$E = \sum_k W(\omega_k)^2 ||\tilde{S}(\omega_k) - S(\omega_k)||_2 \tag{18}$$

be minimized[5]. In the above, $W(\omega_k)$ represent the weighting, at frequency point k, for the least-squares minimization.

4.3 Lumped Model Preconditioner

Ideally, the preconditioner would then be $P = \tilde{J}_C + J_G + \tilde{J}_D$, where

$$\tilde{J}_D = \left[\sum_{k=1}^{N} P_k F^T \tilde{H}_k(\omega) F P_k^T v.\right] \tag{19}$$

Unfortunately, even with the rational model technique, we cannot invert matrices of this form efficiently. However, as state-space models have finite-dimensional state space, the rational function models may be included in an extended version of the circuit equations. The extended circuit equations will include the states of the rational fitting model, and so the Jacobian of the extended system will be the Jacobian of the original system treating the distributed component as the rational fitting model. Adopting the J, L, U notation, let

$$J_R = L_R + U_R \tag{20}$$

be the Jacobian for the circuit equations including the rational fitting model, where R denotes rational fitting, and L_R and U_R denote the upper- and lower-, respectively, block-triangular pieces as defined above. If there are M timesteps in the Chebyshev discretization of the periodic interval, and n circuit equations, we have $J \in \mathbb{R}^{Mn \times Mn}$ but $J_R \in \mathbb{R}^{M(n+q) \times M(n+q)}$ where q is the total number of additional state space variables introduced by the rational approximation of the frequency response of all the distributed components. Thus we cannot simply take $P = L_R$, as L_R and J have different dimensions.

Define an operator $Q : \mathbb{R}^{Mn} \to \mathbb{R}^{M(n+q)}$ that embeds the dimension $M(n+q)$ space into the Mn one – it is a section of the identity matrix. Q^T likewise extracts the original state space from the extended one. The relation that the Jacobian of the extended system will be the Jacobian of the original system treating the distributed component as the rational fitting model can be expressed as

$$J = Q^T J_R Q. \tag{21}$$

To use L_R as the preconditioner for Jacobian J, we may use Q^T to extract the relevant components, so that

$$P^{-1} = Q^T L_R^{-1} Q. \tag{22}$$

Note that the inverse of L_R can be easily computed because it is a block lower-triangular system. Note that the preconditioner will also include an approximation to $\tilde{H}$. If the rational fitting model is a fairly good approximation of the original system, the GMRES solver should be efficient whenever the block-lower-triangular strategy is also effective.

Note that, because we use the rational model as a preconditioner, its accuracy does not affect the accuracy of the final solution. If we were to use rational approximation to construct an equivalent lumped model, instead of evaluating the distributed components, the rational model would need to very closely track the frequency response, meaning it would often be of very high order. We would also have to introduce physical constraints on the rational model – such as passivity – that are computationally expensive to enforce for high-order multi-port models. Both the model generation process, and the later time-domain simulation, would be more computationally expensive than the strategy proposed here, where rational approximations of orders two or three are often enough to give effective preconditioners.

5. NUMERICAL RESULTS

In this section we show the effectiveness of the hybrid frequency-time multi-interval Chebyshev (MIC) method in periodic steady state simulation of distributed elements including a spiral inductor, a transmission line, and a SAW filter. We compare the new approach with the pure rational-fitting model generation approach. Efficiency of the low-order rational fitting model in building a preconditioner for the hybrid frequency-time approach is shown through comparing the convergence history of GMRES solvers with and without the preconditioner.

In the first example, we simulate the periodic steady state of a spiral inductor driven by a pulse source. The spiral inductor is described with an S-parameter table. The frequency range of the S-parameters is from $50MHz$ to $10GHz$. The fundamental frequency of the periodic steady state is $1GHz$. We show the numerical result, waveform at the output, of three simulations. First, we use

Figure 1: Comparison of low-order rational fitting, high-order rational fitting, and hybrid frequency-time MIC with low-order rational fitting preconditioner.

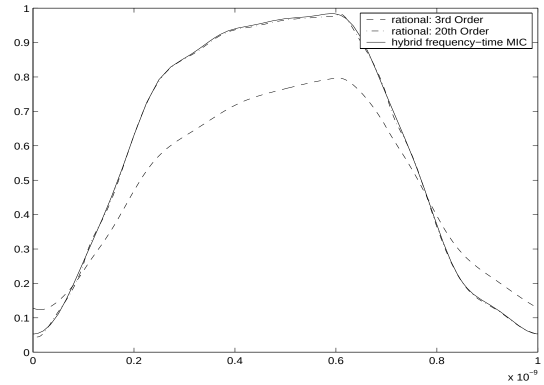
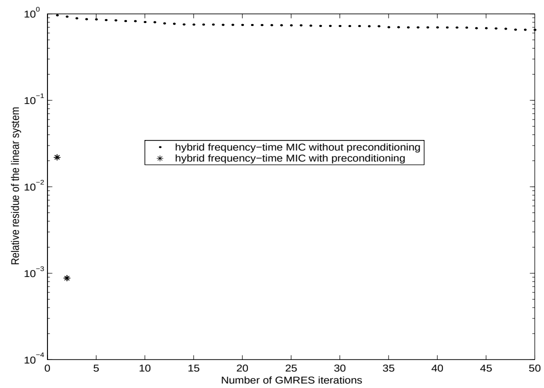


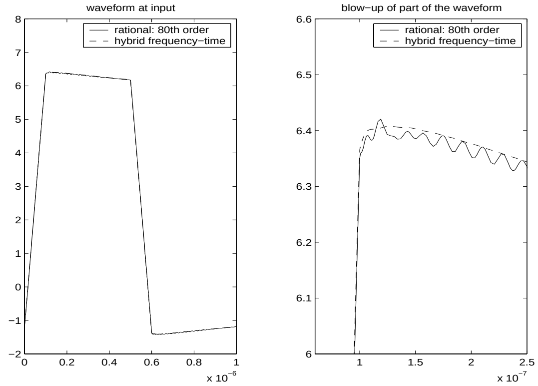
Figure 2: Comparison of GMRES convergence with and without preconditioner.



a third-order rational fitting model and shooting-Newton method to obtain the periodic steady state. The solution is very inaccurate compared to other results. Clearly, the third-order rational fitting model is inaccurate by itself. Second, we use a $20th$ order rational fitting model instead. The result is much more accurate than that of the first simulation. Last, we use the hybrid frequency-time MIC method with the third-order rational fitting model as the preconditioner. With no regard to the inaccuracy of the $3rd$-order model, the result of the new method is satisfactory. There is only a small discrepancy between the second and third simulation result, which may be due to the fact that the treatment of the S-parameters is done in fundamentally different ways and there is no guarantee that the two methods will agree in the frequency range outside of $[50MHz\ ,10GHz]$.

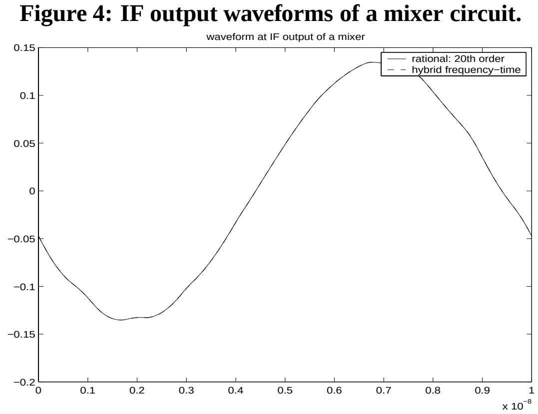
Although the accuracy of rational fitting model is satisfactory in this experiment, it is inefficient because of the high order used in doing rational fitting. The additional state-space equations added when the high-order rational fit model is used means that the circuit equations we need to solve are of much larger size. Indeed, the overall simulation time of the hybrid frequency-time domain method is 5 seconds while that of the 20th-order rational fitting method is 30 seconds. In Fig. 2 we plot the convergence history of the GMRES solver. We compare GMRES convergence with the $3rd$-order fitting model preconditioner and without the preconditioner. One can clearly see that GMRES convergence without the preconditioner is very slow while GMRES converges in a few itera-

Figure 3: Comparison of rational fitting and hybrid frequency-time in transmission line simulation.



tions with the preconditioner. The convergence of our linear solver is satisfactory in this and other experiments.

Figure 4: IF output waveforms of a mixer circuit.



In the second example, we apply the new method to simulate a transmission line with loss and dispersion due to skin effect and finite conductor thickness. We show the waveform of both the rational fitting based simulation and the hybrid frequency-time simulation. In Fig. 3, one can clearly see from the expanded plot on the right that the rational fitting model exhibits high-order oscillation in the waveform, while the waveform of hybrid frequency-time method does not have this problem. The spurious oscillation of rational fitting models is often present in this and other experiments as well as those given in the literature. The hybrid frequency-time approach is promising in that it gives more accurate results.

In the next example, we calculate the periodic steady-state of a mixer circuit. This is a nonlinear circuit that contains 4 bipolar transistors and a SAW filter described by measured S-parameters. We again compare the high-order rational fitting simulation and the hybrid frequency-time simulations. From Fig. 4 we observe that the waveforms of the two simulations are basically on top of each other, verifying the correctness of the result.

Finally, we have performed simulations of a mixer designed with dozens of microstrip transmission lines. In our simulations, we observe that the hybrid frequency-time domain method, using low-order rational fitting model only as preconditioner, has better convergence properties than the pure rational fitting method, which is what we would expect. We verified the simulation result with that of a long transient simulation.

In summary, the new method is promising in terms of accuracy, convergence, and efficiency in all of our numerical experiments.

6. CONCLUSIONS

The hybrid frequency-time domain approach proposed in this paper for the treatment of distributed components, such as described by frequency-dependent tabulated S-parameters, is an effective technique for steady-state time-domain simulation with such components. Because of the spectral accuracy of the Chebyshev discretization in time domain, efficiency and accuracy is achieved in the transformation between frequency and time domains for device evaluation involving frequency-dependent S-parameters. A matrix-implicit Krylov-subspace method is naturally and conveniently used as the linear solver. Numerical experiments show that the new method gives more accurate results and has better convergence properties, compared to the pure rational-fitting lumped model approach. Efficiency of the low-order rational fitting model as a preconditioner for the hybrid frequency-time method is also shown. The method proposed in this paper can be conveniently extended to other time-domain RF analyses, such as quasi-periodic steady-state analysis[10] and small-signal or noise analysis[2].

7. REFERENCES

[1] Ken Kundert, "Introduction to RF simulation and its applications," *IEEE J. Sol. State Circuits, to appear September 1999*, September 1999.

[2] R. Telichevesky, J. White, and K. Kundert, "Efficient AC and noise analysis of two-tone RF circuits," in *Proc. 33rd Design Automation Conference*, June 1996.

[3] R. Telichevesky, J. White, and K. Kundert, "Efficient steady-state analysis based on matrix-free Krylov-subspace methods," in *Proc. 32rd Design Automation Conference*, June 1995.

[4] D. Long, R. Melville, K. Ashby, and B. Horton, "Full chip harmonic balance," in *Proc. Custom Integrated Circuits Conference*, May 1997.

[5] Carlos P. Coelho, Joel R. Phillips, and L. Miguel Silveira, "Robust rational function approximation algorithm for model generation," in *Proceedings of the 36th Design Automation Conference*, New Orleans, June 1999, pp. 207–212.

[6] K. Nabors, Tze-Ting Fang, Hung-Wen Chang, K. S. Kundert, and J. K. White, "Lumped interconnect models via gaussian quadrature," in 34^{th} *ACM/IEEE Design Automation Conference*, 1997, pp. 40–45.

[7] J. E. Schutt-Aine and R. Mittra, "Scattering Parameter Transient Analysis of Transmissions Lines loaded with Nonlinear Terminations," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-36, pp. 529–536, 1988.

[8] K. Engelborghs, T. Luzyanina, K. J. in 't Hout, , and D. Roose, "Collocation methods for the computation of periodic solutions of delay differential equations," *SIAM J. Sci. Comput.*, vol. 22, pp. 1593–1609, 2000.

[9] B. Yang and J. Phillips, "A multi-interval chebyshev collocation method for efficient high-accuracy rf circuit simulation," in *Proc. 37th Design Automation Conference*, June 2000, pp. 178–183.

[10] D. Feng, J. Phillips, K. Nabors, K. Kundert, and J. White, "Efficient computation of quasi-periodic circuit operating conditions using a mixed frequency/time approach," in *Proc. 36th Design Automation Conference*, June 1999.

[11] A. Dutt and V. Rokhlin, "Fast fourier transforms for nonequispaced data II," *Appl. Comput. Harmon. Anal.*, vol. 2, pp. 85–100, 1995.