Throughput and Delay Analysis of the IEEE 802.15.3 CSMA/CA Mechanism Considering the Suspending Events in Unsaturated Traffic Conditions

Xin Liu
Software School
Fudan University
Shanghai, China
liu_xin@fudan.edu.cn

Wenjun Zeng
Dept. of Computer Sciences
University of Missouri
Columbia, USA
zengw@missouri.edu

Abstract—Unlike in IEEE 802.11, the CSMA/CA traffic conditions in IEEE 802.15.3 are typically unsaturated. This paper presents an extended analytical model based on Bianchi’s model in IEEE 802.11, by taking into account the device suspending events, unsaturated traffic conditions, as well as the effects of error-prone channels. Based on this model we re-derive a close form expression of the average service time. The accuracy of the model is validated through extensive simulations. The analysis is also instructional for IEEE 802.11 networks under limited load.

Keywords- CSMA/CA; IEEE 802.15.3; MAC; markov chain; performance analysis

I. INTRODUCTION

The IEEE 802.15.3 [1] is designed for high-rate multimedia applications in wireless personal area networks (WPAN). It has defined a hybrid MAC protocol based on the TDMA and CSMA mechanisms [2]. The performance analysis of CSMA/CA is a key issue in IEEE 802.15.3. The corresponding analysis of IEEE 802.11 [3] can be used as a reference. However, in IEEE 802.11 networks, all of the traffic is transmitted by the CSMA/CA mechanism; thus the traffic condition is saturated in most cases. In contrast, in IEEE 802.15.3 networks most of the traffic consisting of synchronous data is sent by the TDMA mechanism, and only a small amount of traffic comprising commands and asynchronous data is transmitted by the CSMA/CA mechanism; hence the traffic condition is unsaturated frequently.

The modeling of CSMA/CA in IEEE 802.11 has been a research focus since the standards were proposed. The work in [4] gives the theoretical throughput limit of IEEE 802.11 based on a $p$-persistent variant. However, it does not consider the effect of contention window. In [5, 6] Bianchi analyzed the saturated throughput of IEEE 802.11 DCF based on markov chains and stochastic processes. Based on Bianchi’s model, [7] re-analyzes the throughput taking into account the frame retry limit. Some of the subsequent research is diverted on the analysis of IEEE 802.11e EDCF capturing the contention window differentiation [8-10]. A key common assumption in the above works is that each device always has a frame to transmit. This is unreasonable if the traffic load is limited, such as in IEEE 802.15.3 networks. To our best knowledge, little analysis is available for unsaturated conditions except in [11-14]. According to the specifications in [1, 3], whether or not the buffer has a frame to send after a successful transmission, the station will unconditionally back off. This was ignored in [12-14]. Furthermore, with the assumptions of traffic arriving in Poisson process and the buffering of multiple frames [12, 13], the transition probability from states having different retry times to the buffer empty state should not be identical (which unfortunately is assumed in [12, 13]) since the effective arriving time interval of the next frame is different. In [12, 13], the arriving time interval of the next frame is counted from the arriving moment of the current frame to the successful transmission moment of the current frame. It is different if the current frame ends with different retry times. Consequently we take the same assumption as in [11]: each device can buffer only one frame and data arrives with a Poisson process. Thus the effective traffic arriving interval of the next frame is counted from the successful transmission moment of the current frame and it is identical to the one-step state transition period. Moreover related research on average service time in IEEE 802.11 includes [9, 13-16]. All of them failed to take into account the suspending events during the back-off process. In this paper we extend the Bianchi’s model by considering the suspending events during the back-off process. Based on this model we re-derive the average service time in IEEE 802.15.3.

The paper is outlined as follows. Section II briefly reviews the IEEE 802.15.3 MAC mechanism. The extended analytical model is presented in section III. Section IV validates the model by comparing the analytical results with those obtained through simulations. Finally, concluding remarks are given in Section V.

II. IEEE 802.15.3 ACCESS MECHANISM

A. IEEE 802.15.3 MAC Protocol

In IEEE 802.15.3, the basic component is device and the basic network element is piconet. One device is required to assume the role of PNC (piconet coordinator). The time is divided into superframes. The superframe is divided into three parts as shown in Fig. 1: beacon frame, sent by PNC, used to
set time allocations and to communicate management information for the piconet; contention access period (CAP), accessing the channel in CSMA/CA manner, used to communicate commands and/or asynchronous data if it is present in the superframe; channel time allocation period (CTAP), adopting the TDMA protocol, composed of channel time allocations (CTAs), including management CTAs (MCTAs). CTAs are used for commands, isochronous streams and asynchronous data connections. MCTAs are a type of CTA that is used for communication between the DEVs and the PNC.

We eliminate the assumption that at least one frame is always available at each device. Furthermore we consider the effect of frame errors introduced by channel noise. We also do some simplifications in IEEE 802.15.3. PNC as a transmitter is not considered since PNC has the absolute access priority without the back-off process. The average frame delay of other devices caused by PNC insertion is limited to PNC transmission time. The device’s waiting time (SIFS duration) at the beginning of CAP is approximated with BIFS for simplicity.

A. The Modified Markov Chain Model

Consider a piconet with \( n \) devices and a PNC. Assume each device can buffer one frame and there is a constant probability \( q \) of at least one frame arriving per state, as discussed in section I. Let \( b(t) \) be the stochastic process representing the back-off time counter and \( r(t) \) be the stochastic process representing the back-off stage for a given device at slot time \( t \). We introduce the state \( \{(0, b(t)), e\} \) as the back-off process after a successful transmission if the buffer remains empty. The subsequent buffer empty state is denoted as \( \{(-1,0)\} \). \( u \) is the probability of channel sensed busy, and \( P_e \) is the frame-error probability. Different from previous models, the suspending probability \( s \) during back-off is supplemented. The modified markov chain model is depicted in Fig. 2.

Next we analyze the back-off and transmission process in detail. If the buffer has one frame to transmit, the device will enter the back-off process after BIFS idle duration and the 0-th back-off timer is \( C W_0 \). Note that only when the channel is idle for \( \sigma \) can the back-off timer decrease; if the channel is occupied, the back-off timer will suspend and stay in the current state. The probability is \( 1-s \) and \( s \), respectively, where \( s \) is the suspending probability, namely the channel sensed busy probability \( u \). The state transition period differs: if it is an idle slot, it lasts \( \sigma \); if the slot is occupied, it can be a successful transmission slot, a failed transmission slot due to frame error or collision. Transmission is attempted when the back-off timer reaches zero. The transmission is unsuccessful with probability \( p \) due to collision or frame error; or it is successful with probability \( 1-p \). In the model \( p \) is the transition probability from one row to the next row. In the former case the next back-off and retransmission cycle begins again until the transmission reaches the retry limit. In the latter case the device either has a new frame to transmit with probability \( q \), or enters the empty back-off process \( \{(0,k), e\} \) with probability \( 1-q \) if the buffer remains empty after a successful transmission. As to the state \( \{(0,k), e\} \), if no frame arrives, it switches to state \( \{(0,k-1), e\} \) or remains in the current state because of suspending; if a frame arrives, it

III. AN ANALYTICAL MODEL OF IEEE 802.15.3 CSMA/CA

In this section we present an analytical model of IEEE 802.15.3 based on Bianchi’s model [7]. We impose some assumptions similar to [7]: no hidden devices are considered; the collision probability of a frame is constant and independent of the number of retransmission.

B. IEEE 802.15.3 CSMA/CA Mechanism

The device first waits for back-off interframe space (BIFS) duration, from when the medium is determined to be idle before beginning the back-off algorithm. At the beginning of the CAP, the device may begin the back-off algorithm a short interframe space (SIFS) after the beacon transmission. Suppose the device selects \( BO_i \) as the back-off timer in the \( i \)-th back-off process, then \( BO_i = \sigma \times CW_i \), where \( CW_i \) is the \( i \)-th back-off window, \( CW_i = \text{rand}[0, W_i - 1] \), \( CW_{min} \leq W_i \leq CW_{max} \) and \( \sigma \) is the slot time. For the first transmission attempt of a frame, \( i \) is set to zero and \( W_i \) is set to \( CW_{min} \). With the retry count \( i \) increments, \( W_i \) is doubled up to \( CW_{max} \) after the \( m \)-th collisions, \( CW_{max} = CW_{min} \times 2^m \). Once \( W_i \) reaches \( CW_{max} \), it will remain at \( CW_{max} \) until it is reset. \( W_i \) is reset to \( CW_{min} \) either after every successful transmission or when retry count reaches the retry limit \( m \). The back-off timer \( BO_i \) is decremented only when the medium is idle for the entire duration of slot time; it is suspended when the channel is busy. The device resumes to continuously decrement the back-off timer until the channel is measured idle for a BIFS. The device may transmit the frame at the moment that the back-off timer expires. After correctly receiving a frame in the destination device, a positive ACK is sent to notify the source device after SIFS. If the ACK is not received, the source device assumes the transmitted frame is collided, and then it schedules a retransmission and re-enters the back-off process.

Figure 1. IEEE 802.15.3 Piconet superframe

<table>
<thead>
<tr>
<th>Beacon</th>
<th>Contention access period</th>
<th>Channel time allocation period</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>MCTA1</td>
</tr>
</tbody>
</table>

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switches to state \( \{(0, k)\} \) or \( \{(0, k - 1)\} \) similarly. The transition probability is \((1 - q)s\), \((1 - q)(1 - s)\), \(qs\) and \(q(1 - s)\) respectively. The transition probabilities for the states \( \{(0, k)\}\) and \(\{-1, 0\}\) are identical: whether there is a frame to come with the probability \(q\), whether the channel is sensed idle with the probability \(1 - u\), whether the transmission is successful with the probability \(1 - p\). From the above analysis, we can obtain the one-step transition probabilities.

\[
\begin{align*}
P(i, k) | (i, k) &= s \quad k \in [1, W_i - 1] \quad i \in [0, m] \\
P(i, k - 1) | (i, k) &= 1 - s \quad k \in [1, W_i - 1] \quad i \in [0, m] \\
P(0, 0) | (i, 0) &= \frac{(1 - p)q}{W_0} \quad k \in [0, W_0 - 1] \quad i \in [0, m - 1] \\
P(0, k) | (i, 0) &= \frac{(1 - p)(1 - q)}{W_0} \quad k \in [0, W_0 - 1] \quad i \in [0, m - 1] \\
P(0, 0) | (m, 0) &= \frac{q}{W_0} \quad k \in [0, W_0 - 1] \\
P(0, k) | (m, 0) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
\end{align*}
\]

\[
\begin{align*}
P(0, k) | (0, 0) &= s(1 - q) \quad k \in [1, W_0 - 1] \\
P(0, k - 1) | (0, 0) &= (1 - s)(1 - q) \quad k \in [1, W_0 - 1] \\
P(0, k) | (0, k) &= sq \quad k \in [1, W_0 - 1] \\
P(0, k - 1) | (0, k) &= q(1 - s) \quad k \in [1, W_0 - 1] \\
P((0, k) | (0, 0)) &= \frac{qu}{W_0} \quad k \in [0, W_0 - 1] \\
P((1, k) | (0, 0)) &= \frac{q(1 - u)p}{W_0} \quad k \in [0, W_0 - 1] \\
P((0, k) | (m, 0)) &= \frac{q(1 - u)(1 - p)}{W_0} \quad k \in [0, W_0 - 1] \\
P((0, k) | (0, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
P((0, k) | (m, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
P((1, k) | (0, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
P((1, k) | (m, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
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P((0, k) | (m, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
P((1, k) | (m, 0)) &= \frac{1 - q}{W_0} \quad k \in [0, W_0 - 1] \\
\end{align*}
\]
where $P\{ (i_1, k_1) \mid (i_0, k_0) \}$ is the short notation of $P\{ r(t + 1) = i_1, b(t + 1) = k_1 \mid r(t) = i_0, b(t) = k_0 \}$.

Let $b_{j,k}$, $b_{(k,j)}$ and $b_{-1,0}$ represent the stationary distribution and normalize them to 1, we can solve $b_{0,0}$.

$$b_{1,0} + b_{(0,0)_r} + \sum_{k=1}^{m-1} b_{(0,k)_r} + b_{0,0} + \sum_{k=1}^{m-1} b_{k,0} + \sum_{i=1}^{m} b_{i,0} + \sum_{i=1}^{m} \sum_{k=1}^{m-1} b_{i,k} = 1$$

The probability that a device transmits a frame in a slot time can be evaluated as:

$$\tau = \sum_{i=0}^{m} b_{i,0} + q(1-u)[b_{(0,0)_r} + b_{-1,0}]$$

$p_c$ is the probability that a frame encounters a collision.

$$p_c = u = s = 1 - (1-\tau)^{n-1}$$

$$p = p_c - (1 - p_c) \cdot p_c = 1 - (1-\tau)^{n-1} + p_c \cdot (1-\tau)^{n-1}$$

Combining equation (1)-(10), a nonlinear system with one unknown $\tau$ is formed. Due to the space limitation, we refer the readers to [7] for the detailed derivation process. We consider not only the unsaturated traffic condition but also the device suspending event, so the process to solve the equations is much more complicated than previous models. We can find the solution by using the symbolic math toolbox in MATLAB.

### B. Throughput

$P_{tr}$ is the probability that at least one transmission occurs.

$$P_{tr} = 1 - (1-\tau)^n$$

$P_s$ is the probability that exactly one device transmits, conditioned on the fact that at least one device transmits

$$P_s = \frac{n \tau (1-\tau)^{n-1}}{P_{tr}} = \frac{n \tau (1-\tau)^{n-1}}{1 - (1-\tau)^n}$$

The normalized throughput in an error-prone channel is

$$S_{norm} = \frac{P_s P_{tr} (1 - P_c) \cdot E[T_c]}{(1 - P_{tr}) P_s + P_{tr} (1 - P_c) T_s + P_{tr} (1 - P_c) T_c + P_{tr} P_c P_{tr} T_c}$$

If $P_c$ is zero, equation (13) agrees with [7] and [11]. $E[T_c]$ is the average frame payload transmission time; $T_s$, $T_c$, and $T_e$ are the average times that the medium is sensed busy due to a successful transmission, due to a collision and due to transmission errors respectively. Let $\delta$ be the propagation delay, $E[L_c]$ be the average frame length, and $E[L]$ be the average length of the longest frame payload involved in a collision.

$$T_s = BI\!S + PHY_{\text{prehdr}} + MAC_{\text{hdr}} + E[L_c] + \delta + SI\!S + ACK + \delta$$

$$T_c = BI\!S + PHY_{\text{prehdr}} + MAC_{\text{hdr}} + E[L_c]$$

$$T_e = BI\!S + PHY_{\text{prehdr}} + MAC_{\text{hdr}} + E[L_c] + \delta$$

### C. Frame Discard Probability

The frame discard probability $P_d$ is the probability of consecutive $m+1$ unsuccessful transmission attempts.

$$P_d = p^{m+1}$$

### D. Average Frame Delay

We denote as $T_d$ the average frame delay from the moment the back-off process is initiated until the frame is successfully transmitted. Let $N_c$ be the number of collisions experienced before a successful transmission, $T_{di}$ be the average frame delay at the condition that $N_c$ is $i$. Then $T_{di}$ can be divided into four parts: $i$ times back-off process, $i$ times failed transmission, $i+1$-th back-off process and the successful transmission, then the average frame delay is:

$$T_d = \sum_{i=0}^{m} P(N_c = i) \cdot T_{di} = \sum_{i=0}^{m} \frac{p^i (1-p)}{1-p^{m+1}} \cdot \left[ \sum_{k=0}^{i} T_{bk} + i \cdot T_f + T_e \right]$$

Note that the above average frame delay is derived provided that this frame is not discarded, so the weighted coefficient before $T_{di}$ is $\frac{P(N_c = i)}{1-p^{m+1}}$ and the sum of the coefficients is $\sum_{i=0}^{m} P(N_c = i)$, equaling to 1.

$$T_f$$ is the average failed transmission time.

$$T_f = \frac{p_c T_c + (1 - p_c) P_c T_c}{p}$$

$T_{bk}$ is the back-off time during the k+1-th back-off process. Let $T_{state}$ be the one-step state transition period in Fig. 2, $T_{decr}$ be the average time for the back-off timer to decrease by 1, namely the arriving time from state $(k, j)$ to $(k, j-1)$, $N_{decr}$ be the number of steps staying in state $(k, j)$ before jumping to $(k, j-1)$.

$$T_{bk} = \frac{W_t - 1}{2} \cdot T_{decr}$$
During the one-step state transition, the channel may be idle or occupied because of successful transmission, frame error or collision by the remaining \( n - 1 \) devices

\[
T_{state} = (1 - u) \sigma + u p_{tr} (1 - P_e) T_s + u p_{tr} P_e + u (1 - p_{tr}) T_c \tag{20}
\]

\( p_{tr} \) is the probability of exactly one transmission from the \( n - 1 \) remaining devices:

\[
p_{tr} = \frac{(n - 1) r (1 - r)^{n-2}}{p_c} \tag{21}
\]

After some algebraic operations, (16) becomes

\[
T_d = \frac{T_{state}}{2 (1 - s) (1 - p_{tr})} \left( 2^{m'} CW_{\text{min}} p^2 (p^m - p^{m'}) - p^{m+1} + p \right) (1 - p) + CW_{\text{min}} (m - m' + 2)
\]

\[
- CW_{\text{min}} - m - 1) + T_f \cdot p \left[ p^2 \left( \frac{1}{1 - p} - \frac{(m+1)p^m}{1 - p^{m+1}} \right) \right] + T_s.
\tag{22}
\]

### IV. Numerical Results

According to [1], the specific network configuration used in our simulations is listed in Table I. The base rate is 22Mbps and the data rate is 55Mbps. In order to verify the accuracy of our analytical model, the results obtained using the IEEE 802.15.3 network in NS-2 are also plotted in the following figures. Related IEEE 802.15.3 simulation environment in NS-2 is set up by Mustafa [17]. The superframe size is set to 15 microseonds and it consists of only the beacon frame and CAP to avoid the implication that CTAP may pose on the delay measurement.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma ) (( \mu s ))</td>
<td>17.273</td>
<td>( CW_{\text{min}} )</td>
<td>8</td>
</tr>
<tr>
<td>SIFS (sec)</td>
<td>10</td>
<td>( CW_{\text{max}} )</td>
<td>64</td>
</tr>
<tr>
<td>BIFS (( \mu s ))</td>
<td>17.273</td>
<td>( ACK ) (bytes)</td>
<td>10</td>
</tr>
<tr>
<td>PHY _pre (( \mu s ))</td>
<td>17.455</td>
<td>( d ) (( \mu s ))</td>
<td>1</td>
</tr>
<tr>
<td>PHY _hide (( \mu s ))</td>
<td>0.727</td>
<td>( L ) (bytes)</td>
<td>256</td>
</tr>
<tr>
<td>MAC _hide (( \mu s ))</td>
<td>3.636</td>
<td>( m )</td>
<td>3</td>
</tr>
</tbody>
</table>

The models in [7] and [15] are both based on Bianchi’s model but considering the retry limit in saturated traffic conditions. [7] derived the expression for the throughput and [15] derived the expression for the average frame delay. Our unsaturated traffic model is based on the model in [11], whereas [11] did not take into account the effect of device suspending and did not address the average frame delay. [16] gave the mean and standard deviation of the frame delay in saturated traffic conditions using one-dimensional markov chain without considering the suspending events. We compare the throughput analysis with [7] and [11] in Fig. 3 and compare the average frame delay with [15] and [16] in Fig. 4 to demonstrate the effect of considering unsaturated traffic conditions and suspending events in performance modeling.

All of the above works assume the channel is ideal, so the frame error rate \( P_e \) in our model is set to zero for fair comparison purpose.

In Fig. 3, the model in [7] assumes a saturated traffic condition and \( q \) is set to 0.1 in [11] and our model. All curves demonstrate the same trend: with the increased number of devices the throughput increases and achieves a maximum and then decreases. Once all of the bandwidth is occupied, the throughput can not increase with the number of devices. On the contrary the throughput will decrease with the increase of the collision probability. Comparing the models in [7] and [11], we can observe that the maximum throughput is reached with a smaller number of devices in the saturated traffic case than in the unsaturated traffic case. The larger the \( q \) is, the earlier the throughput becomes saturated.

As shown in Fig. 3, our analytical model matches the NS-2 simulation results much better than the others. When suspending event is not considered, the device at the back off state will jump faster to the state to transmit a frame than otherwise. This means the probability to transmit a frame in a given slot is larger when not considering the suspending event. If the network bandwidth is not occupied fully, larger probability to transmit a frame results in larger throughput. As the network becomes saturated with the increased number of devices, a larger probability to transmit a frame will lower the throughput. This suggests the performance analysis in [7] and [11] which ignores the suspending event during the back-off process is underestimated for large number of devices.

![Figure 3. Throughput for different number of devices](image-url)
unsaturated cases. In particular, let us compare the models in [15], [16] and ours in saturated conditions. As there exist suspending events, the back-off timer has more chance to stay in the current state. The larger the number of devices, the higher probability to stay in the current state. The average frame delay does not become saturated as suggested by the other two models with the increased number of devices even for a finite retry limit. This illuminates that there can not be too many devices in a piconet in order to maintain a low frame delay.

![Average frame delay for different number of devices](image)

**Figure 4.** Average frame delay for different number of devices

V. CONCLUSIONS

In this paper we propose an extended analytical model to study the performance of the IEEE 802.15.3 CSMA/CA mechanism. Both the back-off process after a successful transmission in unsaturated conditions and the suspending event during the back-off process are considered. The expression for the average frame delay is re-derived based on the modified model. The explicit analytical model has been validated to be in agreement with the computer simulations.

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[1] Part 15.3: Wireless medium access control (MAC) and physical layer (PHY) specifications for high rate wireless personal area networks (WPAN), IEEE Std 802.15.3, Sep. 2003


