

# Quantum Computing

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A Tutorial at the  
2005 Genetic and Evolutionary Computation Conference  
(GECCO-2005)

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## Overview

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- ◆ What is quantum computation?
- ◆ Why might it be important?
- ◆ How does/might it work?
- ◆ Simulating a quantum computer.
- ◆ Some quantum algorithms.
- ◆ Evolution of new quantum algorithms.
- ◆ Sources for more information.

## What is quantum computation?

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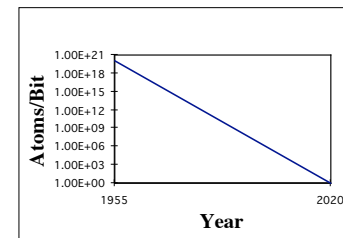
Computation with coherent atomic-scale dynamics.



The behavior of a quantum computer is governed  
by the laws of quantum mechanics.

## Why bother with quantum computation?

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- ◆ Moore's Law: the amount of information storable on a given amount of silicon has roughly doubled every 18 months. We hit the quantum level 2010 ~ 2020.
- ◆ Quantum computation is more powerful than classical computation. More can be computed in less time—the complexity classes are different!

## The power of quantum computation

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- ◆ In quantum systems *possibilities count*, even if they never happen!
- ◆ Each of exponentially many *possibilities* can be used to perform a part of a computation *at the same time*.

## Absurd but taken seriously

(not just quantum mechanics but also quantum computation)

- ◆ Under active investigation by many of the top physics labs around the world (including CalTech, MIT, AT&T, Stanford, Los Alamos, UCLA, Oxford, l'Université de Montréal, University of Innsbruck, IBM Research...)
- ◆ In the mass media (including *The New York Times*, *The Economist*, *American Scientist*, *Scientific American*, ...)
- ◆ Here.

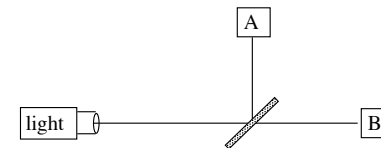
## Nobody understands quantum mechanics

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- ◆ “Anybody who is not shocked by quantum mechanics hasn’t understood it.” —Niels Bohr
- ◆ “No, you’re not going to be able to understand it. ... You see, my physics students don’t understand it either. That is because **I** don’t understand it. Nobody does. ... The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept Nature as She is—absurd.” —Richard Feynman

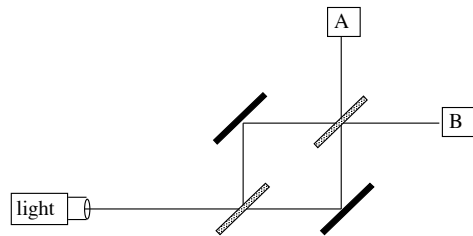
## A beam splitter

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Half of the photons leaving the light source arrive at detector A; the other half arrive at detector B.

## An interferometer



- ◆ Equal path lengths, rigid mirrors.
- ◆ Only one photon in the apparatus at a time.
- ◆ All of the photons leaving the light source arrive at detector B. WHY?

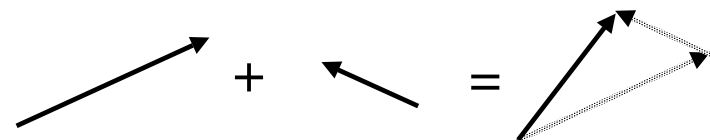
## Possibilities count

- ◆ There is an “amplitude” for each possible path that a photon can take.
- ◆ The amplitudes can interfere constructively and destructively, even though each photon takes only one path.
- ◆ The amplitudes at detector A interfere destructively; those at detector B interfere constructively.

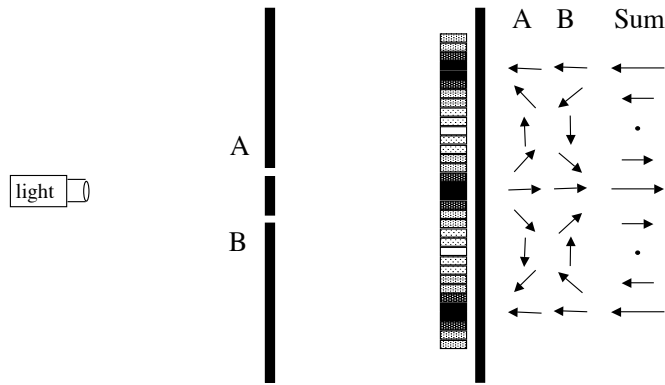
## Calculating interference

- ◆ “You will have to brace yourselves for this—not because it is difficult to understand, but because it is absolutely ridiculous: All we do is draw little arrows on a piece of paper—that’s all!” —Richard Feynman
- ◆ Arrows for each possibility.
- ◆ Arrows rotate; speed depends on frequency.
- ◆ Arrows flip 180° at mirrors, rotate 90° counter-clockwise when reflected from beam splitters.
- ◆ Add arrows and square the length of the result to determine the probability for any possibility.

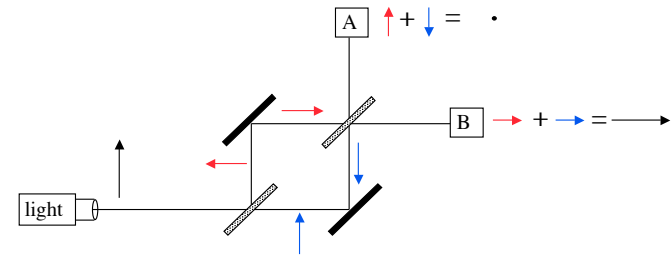
## Adding arrows



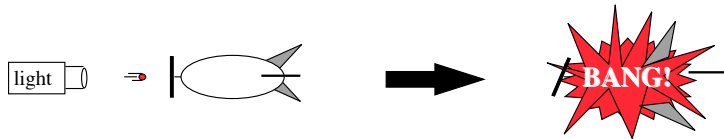
## Double slit interference



## Interference in the interferometer

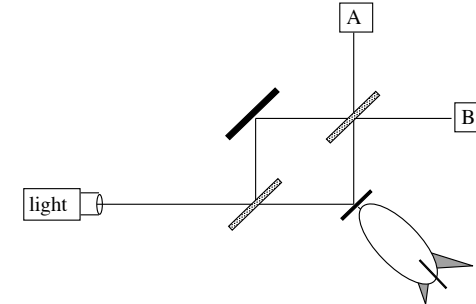


## A photon-triggered bomb



- ◆ A mirror is mounted on a plunger on the bomb's nose.
- ◆ A single photon hitting the mirror depresses the plunger and explodes the bomb.
- ◆ Some plungers are stuck, producing duds.
- ◆ How can you find a good, unexploded bomb?

## Elitzur-Vaidman bomb testing



- ◆ Possibilities count!
- ◆ Experimentally verified
- ◆ Can be enhanced to reduce or eliminate bomb loss  
[Kwiat, Weinfurter and Kasevich]

## Two interesting speedups

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- ◆ Grover's quantum database search algorithm finds an item in an unsorted list of  $n$  items in  $O(\sqrt{n})$  steps; classical algorithms require  $O(n)$ .
- ◆ Shor's quantum algorithm finds the prime factors of an  $n$ -digit number in time  $O(n^3)$ ; the best known classical factoring algorithms require at least time  $O(2^{n^{1/3} \log(n)^{2/3}})$ .

## Reminder: exponential savings is **very** good!

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Factor a 5,000 digit number:

- Classical computer (1ns/instr, ~today's best alg)
  - » **over 5 trillion years**  
(the universe is ~ 10–16 billion years old).
- Quantum computer (1ns/instr, ~Shor's alg)
  - » just over 2 minutes

## Quantum computing and the human brain

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- ◆ Penrose's argument
  - Brains do  $X$  (for  $X$  uncomputable)
  - Classical computers can't do  $X$
  - ∴ Brains aren't classical computers
  - First premise is false for all proposed  $X$ . For example, brains don't have knowably sound procedures for mathematical proof.
  - Would imply brains more powerful than quantum computers; new physics.

## Quantum consciousness?

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- ◆ Relation to consciousness etc. is much discussed, unclear at best. (Bohm, Penrose, Hameroff, others)
- ◆ “[Penrose's] argument seemed to be that consciousness is a mystery and quantum gravity is another mystery so they must be related.” (Hawking)

## Quantum information theory

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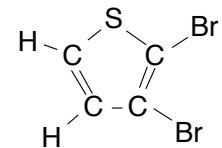
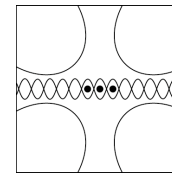
- ◆ Quantum cryptography: secure key distribution
- ◆ Quantum teleportation
- ◆ Quantum data compression
- ◆ Quantum error correction

Good introductions to these topics can be found in (Steane, 1998).

## Physical implementation

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- ◆ Ion traps
- ◆ Nuclear spins in NMR devices
- ◆ Optical systems
- ◆ So far: few qubits, impractical
- ◆ A lot of current research



## Languages and notations

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- ◆ Wave equations
- ◆ Wave diagrams
- ◆ Matrix mechanics
- ◆ Dirac's bra-ket notation ( $\langle\phi|\psi\rangle$ )
- ◆ Particle diagrams
- ◆ Amplitude diagrams
- ◆ Phasor diagrams
- ◆ QGAME programs

## Qubits

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- ◆ The smallest unit of information in a quantum computer is called a “qubit”.
- ◆ A qubit may be in the “on” (1) state or in the “off” (0) state or in any superposition of the two!

## State representation, 1 qubit

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- ◆ The state of a qubit can be represented as:

$$\alpha_0|0\rangle + \alpha_1|1\rangle$$

$\alpha_0$  and  $\alpha_1$  are complex numbers that specify the *probability amplitudes* of the corresponding states.

- ◆  $|\alpha_0|^2$  gives the probability that you will find the qubit in the “off” (0) state;  $|\alpha_1|^2$  gives the probability that you will find the qubit in the “on” (1) state.

## State representation, 2 qubits

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- ◆ The state of a two-qubit system can be represented as:

$$\alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle$$

$$\sum |\alpha|^2 = 1$$

- ◆ Measurement will always find the system in some (one) discrete state.

## Entanglement

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- ◆ Qubits in a multi-qubit system are not independent—they can become “entangled.” (We’ll see some examples.)
- ◆ To represent the state of  $n$  qubits one usually uses  $2^n$  complex number amplitudes.

## Measurement at the end of a computation

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- ◆  $\sum |\alpha|^2$ , for amplitudes of all states matching the output bit-pattern in question.
- ◆ This gives the probability that the particular output will be read upon measurement.
- ◆ Example:

$$0.316|00\rangle + 0.447|01\rangle + 0.548|10\rangle + 0.632|11\rangle$$

The probability to read the rightmost bit as 0 is  $|0.316|^2 + |0.548|^2 = 0.4$

## Partial measurement during a computation

- ◆ One-qubit measurement gates.
- ◆ Measurement changes the system.
- ◆ In simulation, branch computation for each possible measurement.

## Classical computation in matrix form

A state transition in a 4-bit system:

$$\begin{array}{c}
 \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 \alpha_0 \\
 \alpha_1 \\
 \alpha_2 \\
 \alpha_3 \\
 \alpha_4 \\
 \alpha_5 \\
 \alpha_6 \\
 \alpha_7 \\
 \alpha_8 \\
 \alpha_9 \\
 \alpha_{10} \\
 \alpha_{11} \\
 \alpha_{12} \\
 \alpha_{13} \\
 \alpha_{14} \\
 \alpha_{15}
 \end{bmatrix}
 \end{array}$$

## A quantum NOT gate

$$\begin{bmatrix}
 0 & 1 \\
 1 & 0
 \end{bmatrix}$$

Applied to a qubit:

$$\begin{bmatrix}
 0 & 1 \\
 1 & 0
 \end{bmatrix}
 \begin{bmatrix}
 \alpha_0 \\
 \alpha_1
 \end{bmatrix}
 =
 \begin{bmatrix}
 \alpha_1 \\
 \alpha_0
 \end{bmatrix}$$

$$\alpha_0|0\rangle + \alpha_1|1\rangle \rightarrow \alpha_1|0\rangle + \alpha_0|1\rangle$$

## Explicit matrix expansion

To expand gate matrix  $G$  for application to an  $n$ -qubit system:

- Create a  $2^n \times 2^n$  matrix  $M$ .
- Let  $Q$  be the set of qubits to which the operator is being applied, and  $Q'$  be the set of the remaining qubits.
- $M_{ij} = 0$  if  $i$  and  $j$  differ in positions in  $Q'$ .
- Otherwise concatenate bits from  $i$  in positions  $Q$  to produce  $i^*$ , and bits from  $j$  to produce  $j^*$ .  $M_{ij} = G_{i^*j^*}$ .

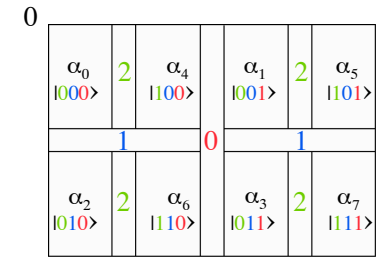


## Implicit matrix expansion

To apply gate matrix  $G$  to an  $n$ -qubit system:

- Let  $Q$  be the set of qubits to which the operator is being applied, and  $Q'$  be the set of the remaining qubits.
- For every combination  $C$  of 1 and 0 for qubits in  $Q'$ :
  - » Extract the column  $A$  of amplitudes that results from holding  $C$  constant and varying all qubits in  $Q$ .
  - »  $A' = G \times A$ .
  - » Install  $A'$  in place of  $A$  in the array of amplitudes.

## Amplitude diagrams



- ◆ Help to visualize amplitude distributions
- ◆ Scalable, hierarchical
- ◆ Can be shuffled to prioritize any qubits

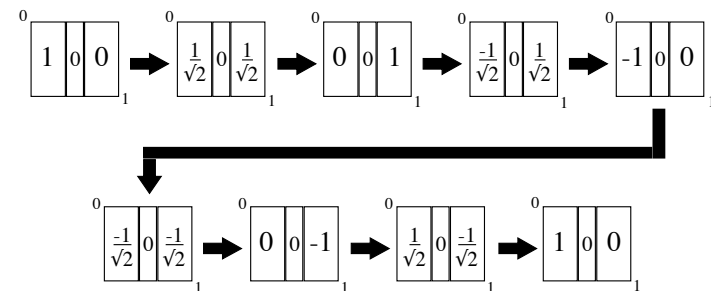
## A square-root-of-NOT (SRN) gate

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

- ◆ Applied once to a classical state, this ~randomizes the value of the qubit.
- ◆ Applied twice in a row, this is ~equivalent to NOT:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

## SRN amplitude diagrams



## Other quantum gates

◆ Rotation ( $U_\theta$ ): 
$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

◆ Hadamard ( $H$ ): 
$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

◆ Controlled NOT ( $CNOT$ ): 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

There are many small “complete” sets of gates  
[Barenco et al.].

## More quantum gates

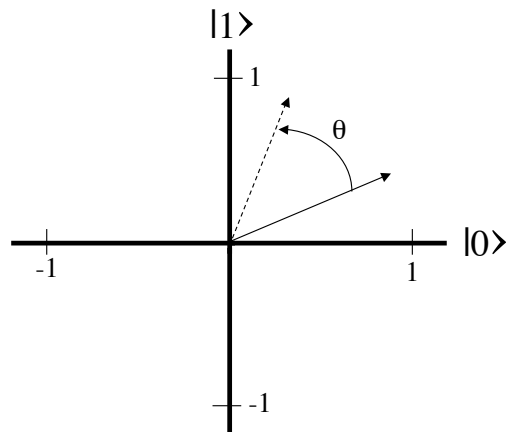
◆ Conditional phase: 
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\alpha} \end{bmatrix}$$

◆  $U2$ : 
$$\begin{bmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{bmatrix} \times \begin{bmatrix} \cos(\theta) & \sin(-\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \times \begin{bmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{bmatrix} \times \begin{bmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\alpha} \end{bmatrix}$$

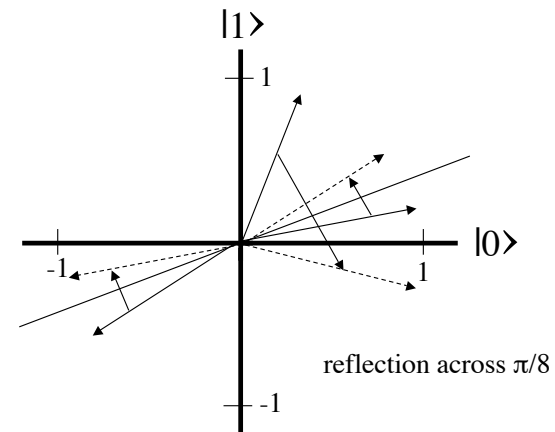
All gates must be unitary:  $U^\dagger U = U U^\dagger = I$ ,

where  $U^\dagger$  is the Hermitean adjoint of  $U$ , obtained by taking the complex conjugate of each element of  $U$  and then transposing the matrix.

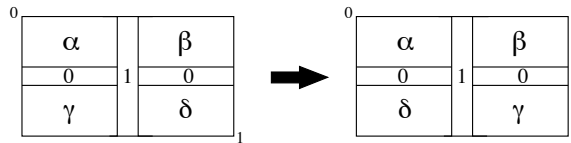
## Rotation polar plot for real vectors



## Hadamard polar plot for real vectors



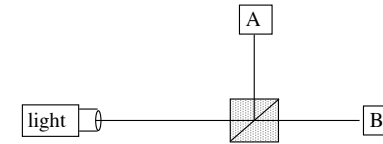
## CNOT amplitude diagrams



CNOT(0 [control], 1 [target])

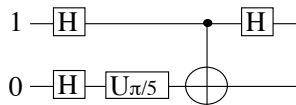
## Polarizing beam-splitter CNOT gate

[Cerf, Adami, and Kwiat]



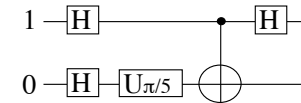
- ◆ Two qubits encoded in one photon, one in momentum (direction) and one in polarization.
- ◆ Polarization controls change in momentum.
- ◆ Cannot be scaled up directly, but demonstrates an implementation of a 2-qubit gate.

## Gate array diagrams

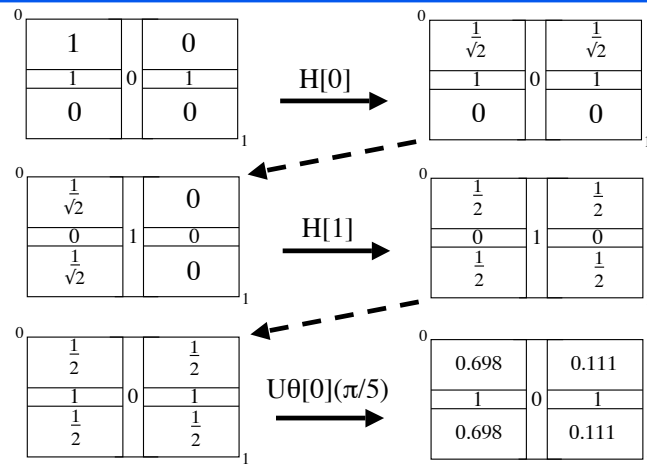


## Example execution trace

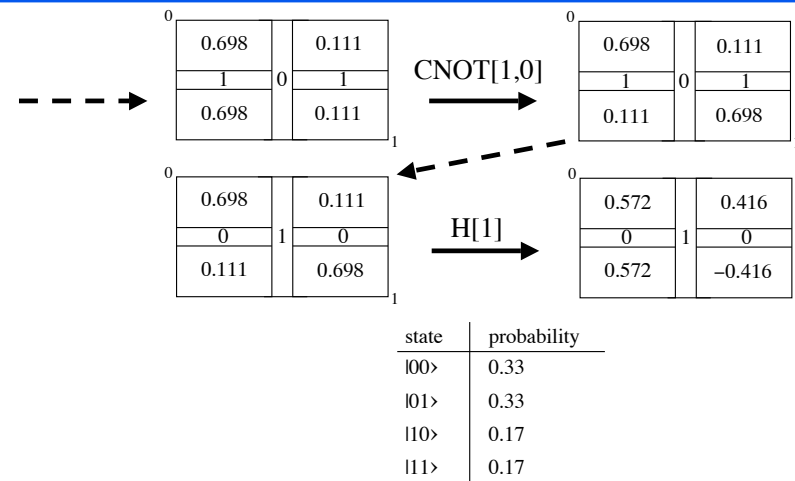
```
Hadamard qubit:0
Hadamard qubit:1
U-theta qubit:0 theta:pi/5
Controlled-not control:1 target:0
Hadamard qubit:1
```



## Trace, cont.



## Trace, cont.



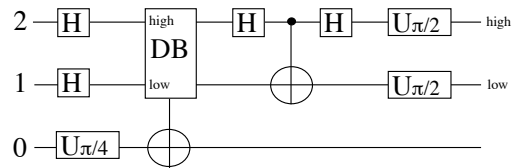
## The database search problem

- ◆ Given an unsorted database containing  $n$  items but only one “marked” item, find the address of the marked item with a minimal number of database calls.
- ◆ Lov Grover’s algorithm uses  $O(\sqrt{n})$  calls in general, and only one call for a 4-item database.

## Oracle problems

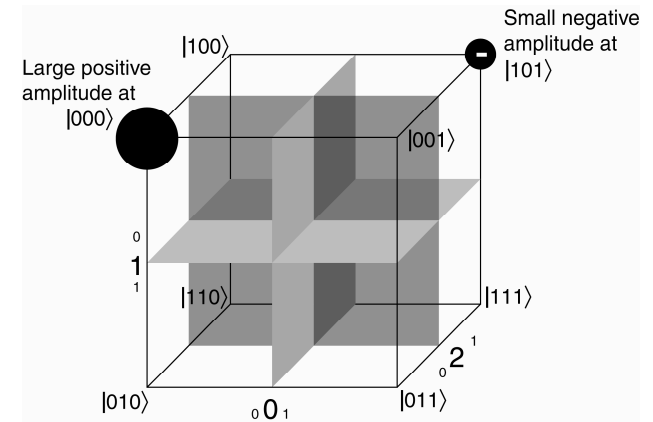
- ◆ The database search problem is an example of an “oracle problem.”
- ◆ We are given a “black box” or “oracle” function (in this case the database access function) and asked to find out if it has some particular property.
- ◆ Many other known quantum algorithms are for oracle problems.
- ◆ Often the oracle is “hard” to implement, so complexity is figured from the number of oracle calls.

## Grover's algorithm for a 4-item database

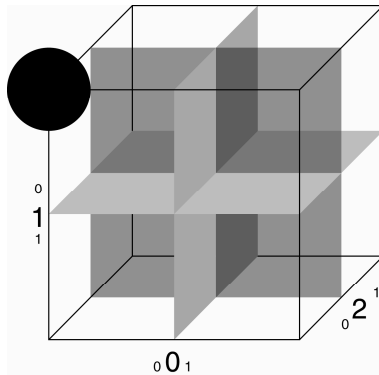


- ◆ Start in the state  $|000\rangle$ .
- ◆ Read answer from qubits 2 and 1.

## Cube diagram for a 3-qubit system

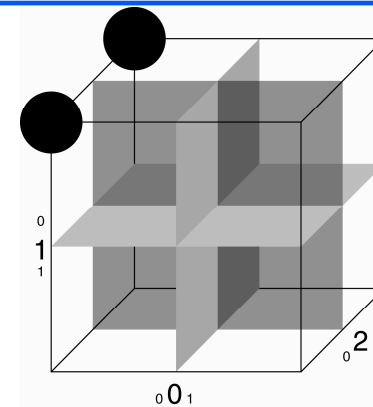


## (0) Grover's algorithm, item at 0,0



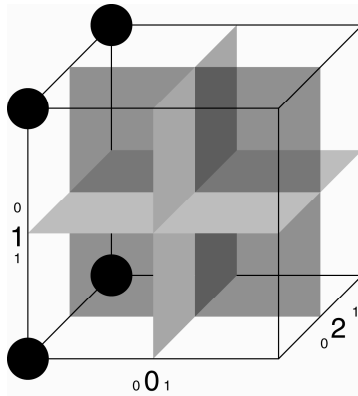
Initial State,  $|000\rangle$

## (1) Grover's algorithm, item at 0,0



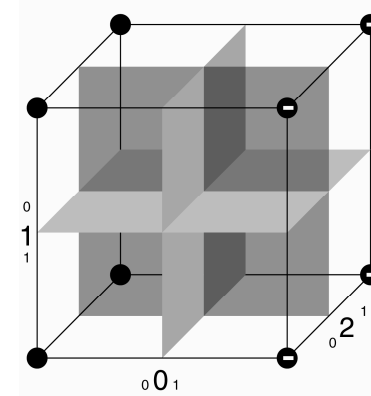
After Hadamard[2]

## (2) Grover's algorithm, item at 0,0



After Hadamard[1]

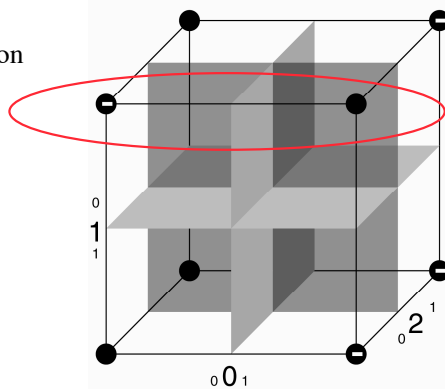
## (3) Grover's algorithm, item at 0,0



After  $U\theta[0](\pi/4)$

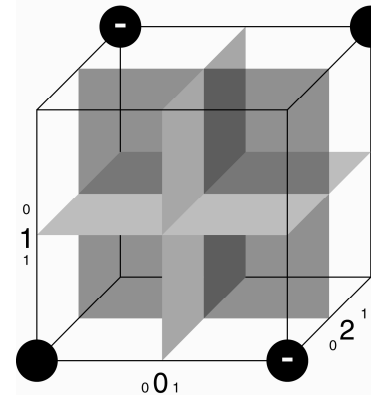
## (4) Grover's algorithm, item at 0,0

Note position  
of DB call  
effect.



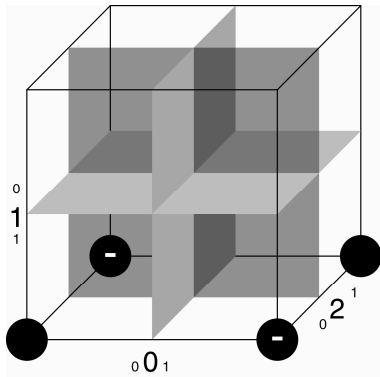
After Database Call [in: 2,1; out:0]

## (5) Grover's algorithm, item at 0,0



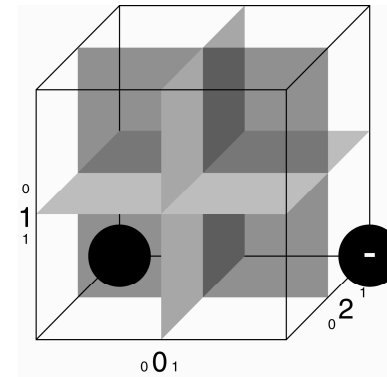
After Hadamard[2]

### (6) Grover's algorithm, item at 0,0



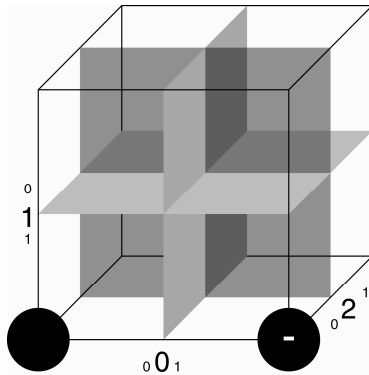
After CNOT [control: 2; target: 1]

### (7) Grover's algorithm, item at 0,0



After Hadamard[2]

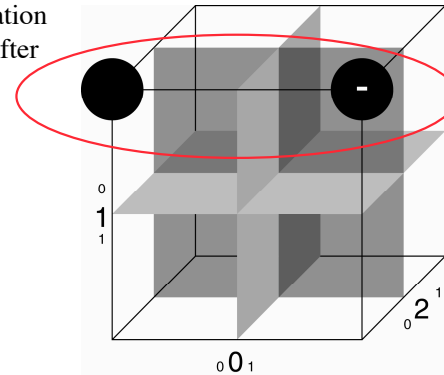
### (8) Grover's algorithm, item at 0,0



After  $U\theta[2](\pi/2)$

### (9) Grover's algorithm, item at 0,0

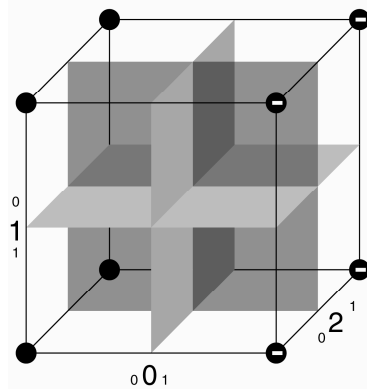
Note relation  
to state after  
DB call.



After  $U\theta[1](\pi/2)$ , Read output from qubits 2 (high) and 1(low)

### (3) Grover's algorithm, item at 0,1

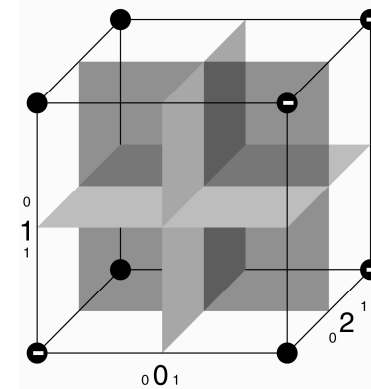
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After  $U\theta[0](\pi/4)$

### (4) Grover's algorithm, item at 0,1

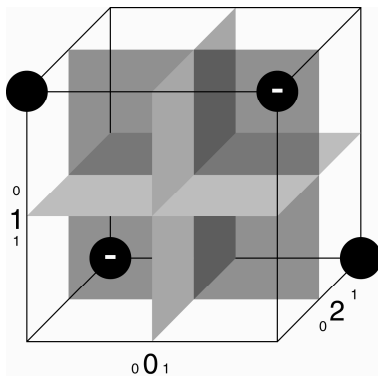
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After Database Call [in: 2,1; out:0]

### (5) Grover's algorithm, item at 0,1

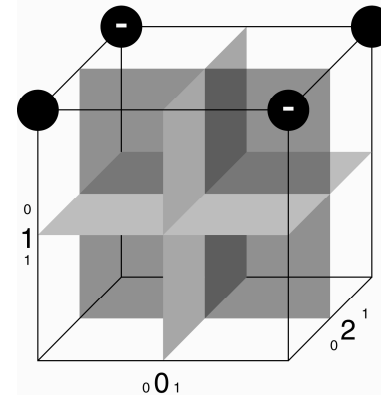
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After Hadamard[2]

### (6) Grover's algorithm, item at 0,1

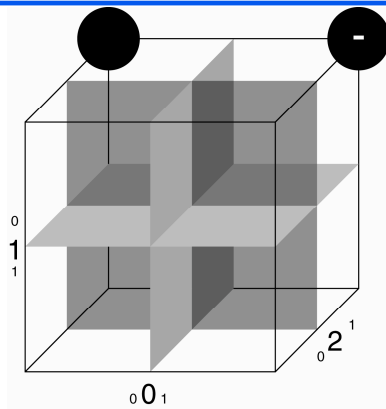
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After CNOT [control: 2; target: 1]

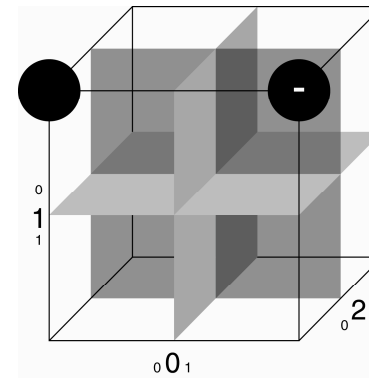


(7) Grover's algorithm, item at 0,1



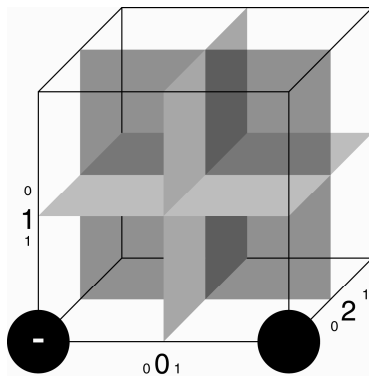
After Hadamard[2]

(8) Grover's algorithm, item at 0,1



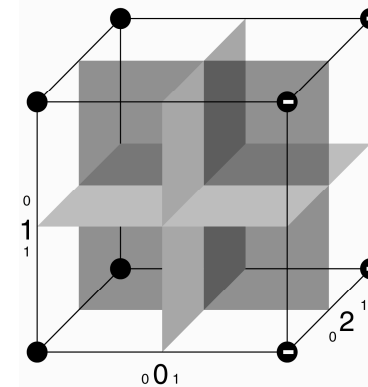
After  $U\theta[2](\pi/2)$

(9) Grover's algorithm, item at 0,1



After  $U\theta[1](\pi/2)$ , Read output from qubits 2 (high) and 1(low)

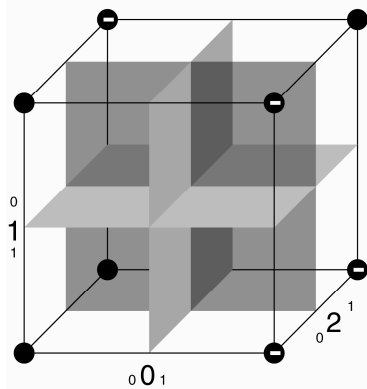
(3) Grover's algorithm, item at 1,0



After  $U\theta[0](\pi/4)$

#### (4) Grover's algorithm, item at 1,0

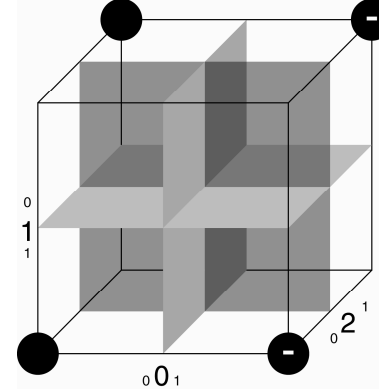
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After Database Call [in: 2,1; out:0]

#### (5) Grover's algorithm, item at 1,0

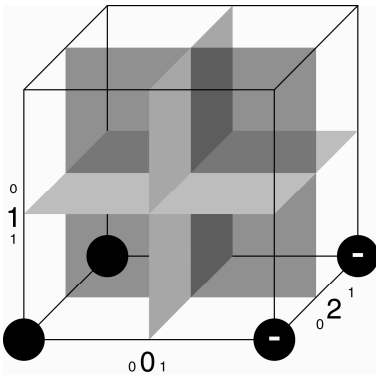
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After Hadamard[2]

#### (6) Grover's algorithm, item at 1,0

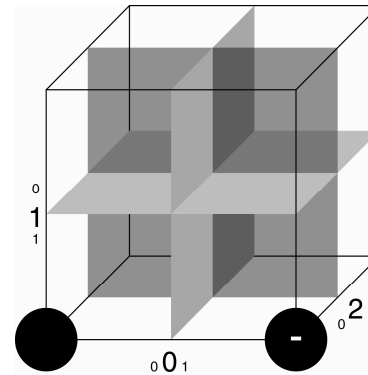
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After CNOT [control: 2; target: 1]

#### (7) Grover's algorithm, item at 1,0

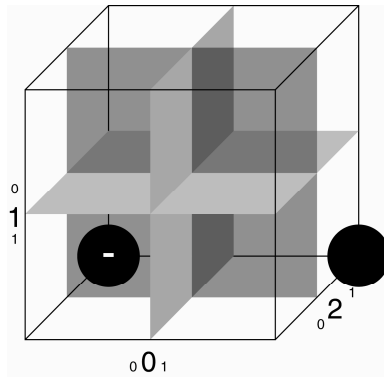
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After Hadamard[2]

### (8) Grover's algorithm, item at 1,0

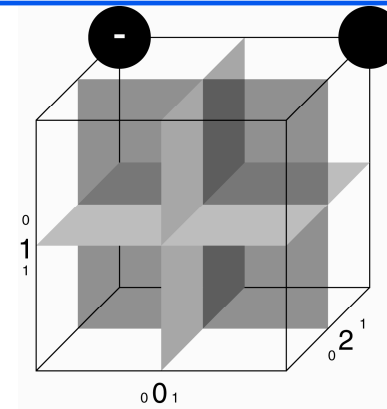
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After  $U\theta[2](\pi/2)$

### (9) Grover's algorithm, item at 1,0

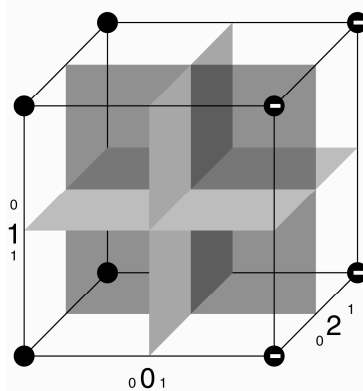
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After  $U\theta[1](\pi/2)$ , Read output from qubits 2 (high) and 1(low)

### (3) Grover's algorithm, item at 1,1

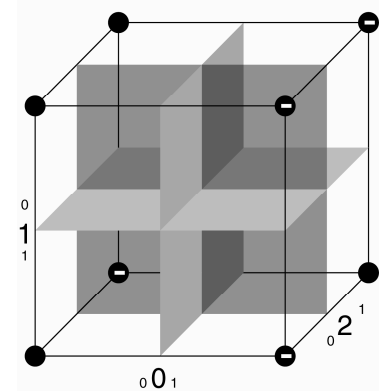
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After  $U\theta[0](\pi/4)$

### (4) Grover's algorithm, item at 1,1

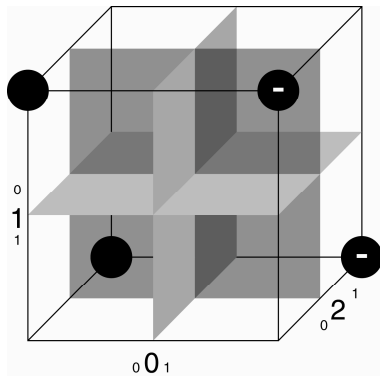
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After Database Call [in: 2,1; out:0]

(5) Grover's algorithm, item at 1,1

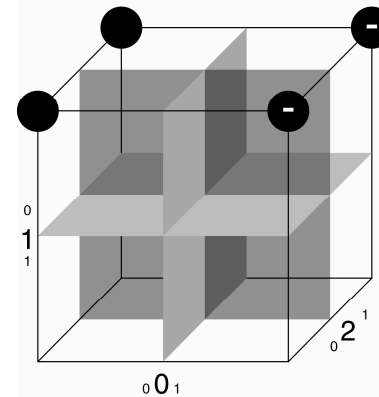
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After Hadamard[2]

(6) Grover's algorithm, item at 1,1

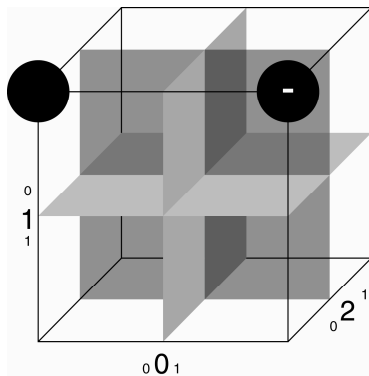
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After CNOT [control: 2; target: 1]

(7) Grover's algorithm, item at 1,1

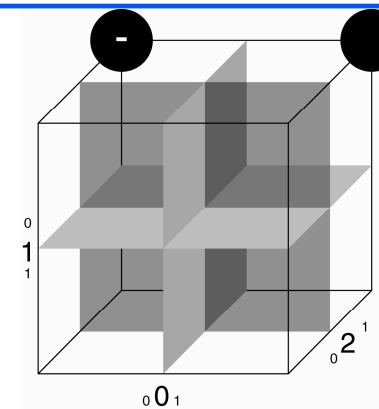
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After Hadamard[2]

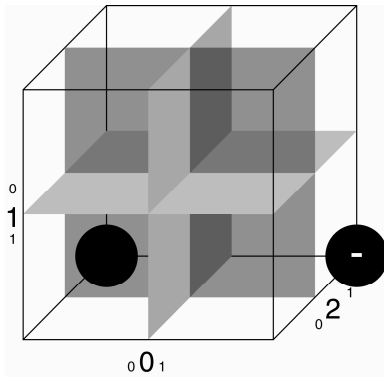
(8) Grover's algorithm, item at 1,1

---



After  $U_{\theta}[2](\pi/2)$

## (9) Grover's algorithm, item at 1,1

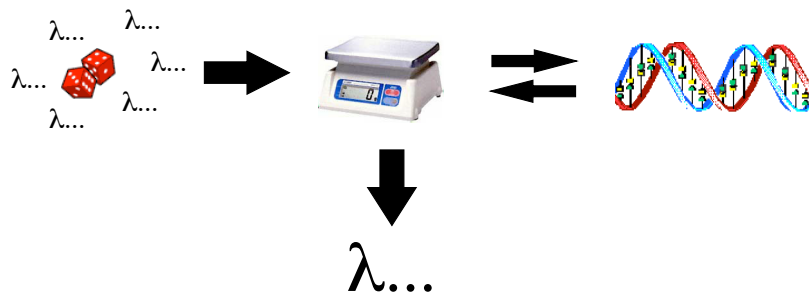


After  $U_{\theta}[1](\pi/2)$ , Read output from qubits 2 (high) and 1(low)

## Shor's algorithm

- ◆ hybrid algorithm to factor numbers
- ◆ quantum component helps to find the period  $r$  of a sequence  $a_1, a_2, \dots, a_i, \dots$ , given an oracle function that maps  $i$  to  $a_i$
- ◆ skeleton of the algorithm:
  - create a superposition of all oracle inputs
  - call the oracle function
  - apply a quantum Fourier transform to the input qubits
  - read the input qubits to obtain a random multiple of  $1/r$
  - repeat a small number of times to infer  $r$

## Genetic Programming (GP)



## GP for quantum computation

- ◆ Evolve:
  - gate arrays
  - programs that produce gate arrays
  - hybrid classical/quantum algorithms
  - input states or parameters
- ◆ Genome representation:
  - QGAME program
  - program (in any language) that generates a QGAME program
  - array of numbers

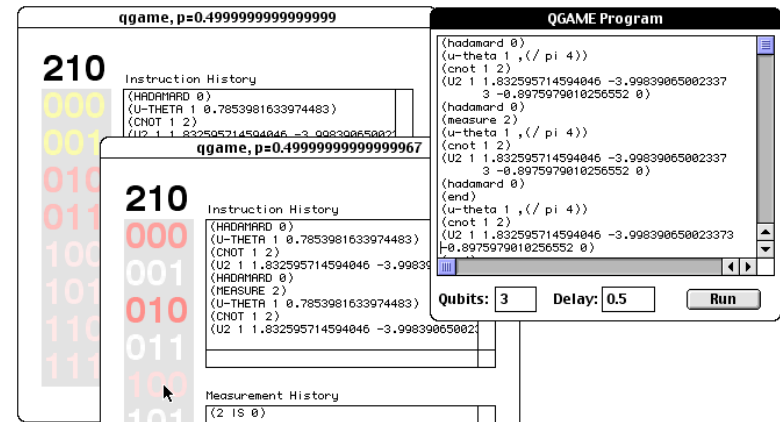
## Fitness

- ◆ Assessing the composite matrix
  - the trouble with oracles
- ◆ Assessing the results of simulation runs
- ◆ Criteria:
  - Error
  - Hits
  - Oracle calls
  - Number of gates

## Primitives; gate-array-producing programs

- ◆ Gates:  $H$ ,  $U_{\theta}$ ,  $CNOT$ ,  $ORACLE$ , ...
- ◆ Qubit indices
- ◆ Gate parameters (angles)
- ◆ Arithmetic operators
- ◆ Constants indicating problem size (num-qubits, num-input-qubits, num-output-qubits)
- ◆ Iteration structures, recursion, data structures, ...

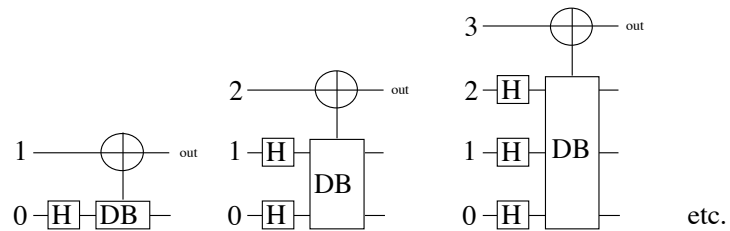
**QGAME** Quantum Gate and Measurement Emulator  
<http://hampshire.edu/l spectator/qgame.html>



## The scaling majority-on problem

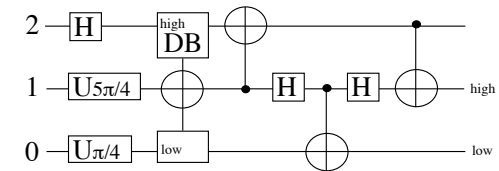
- ◆ Does the oracle answer “1” for a majority of inputs?
- ◆ Seek program that produces a gate array for any oracle size.

## Evolved scaling majority-on gate arrays

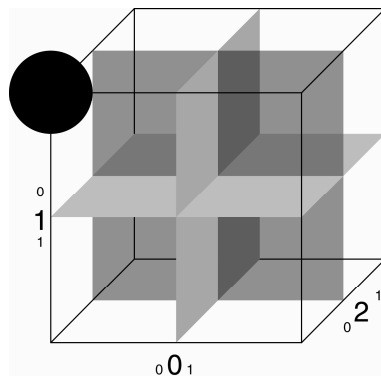


Not better than classical.

## Evolved database search gate array

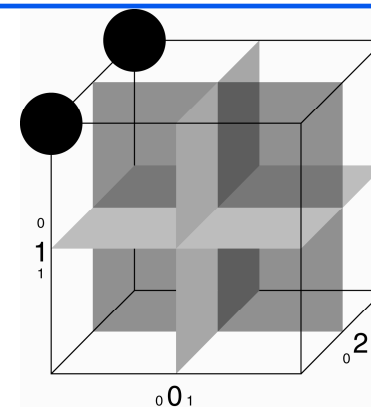


(0) Evolved quantum database algorithm,  
item at 0,0



Initial State,  $|000\rangle$

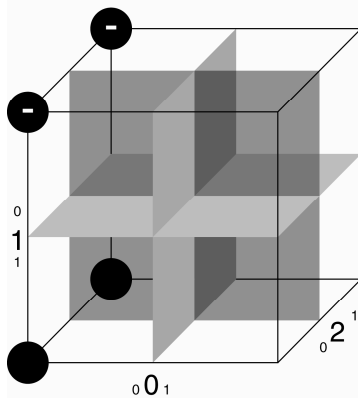
(1) Evolved quantum database algorithm,  
item at 0,0



After Hadamard [2]

(2) Evolved quantum database algorithm,  
item at 0,0

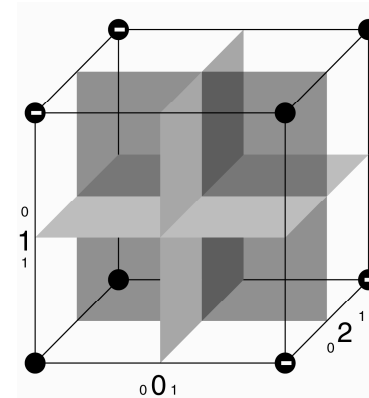
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After  $U\theta [1] (5\pi/4)$

(3) Evolved quantum database algorithm,  
item at 0,0

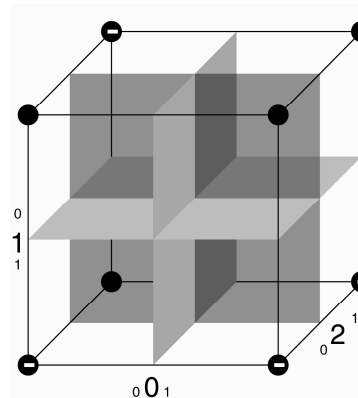
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After  $U\theta [0] (\pi/4)$

(4) Evolved quantum database algorithm,  
item at 0,0

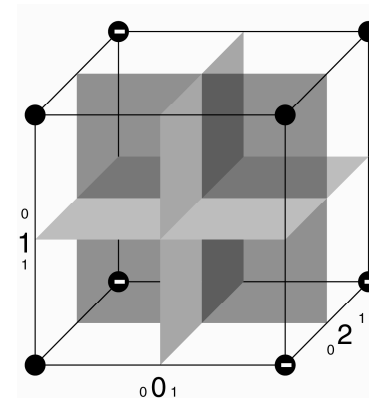
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After DB [in:2,0; out:1](item in 0,0)

(5) Evolved quantum database algorithm,  
item at 0,0

---

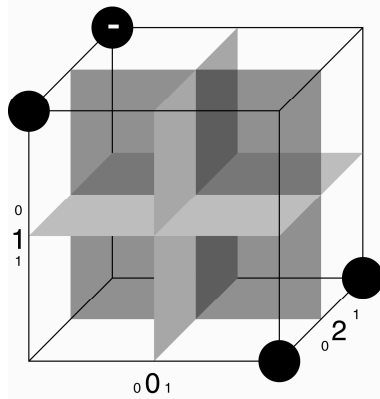


After CNOT [control: 1, target: 2]



(6) Evolved quantum database algorithm,  
item at 0,0

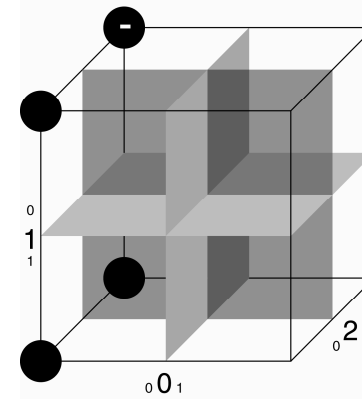
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After Hadamard [1]

(7) Evolved quantum database algorithm,  
item at 0,0

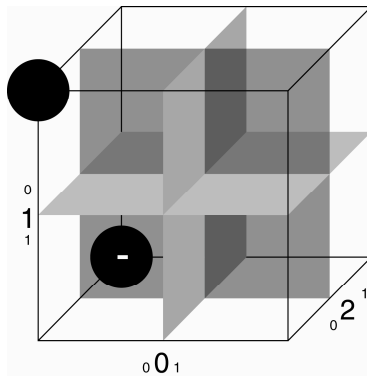
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After CNOT [control: 1, target: 0]

(8) Evolved quantum database algorithm,  
item at 0,0

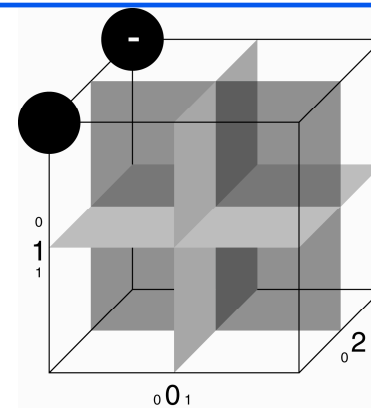
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After Hadamard [1]

(9) Evolved quantum database algorithm,  
item at 0,0

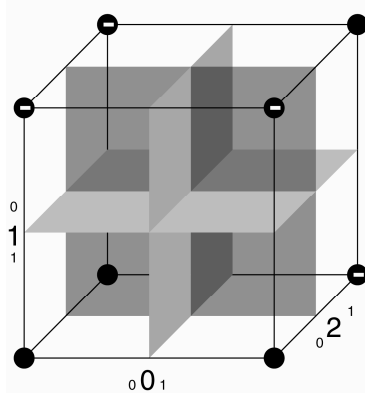
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After CNOT [control: 2, target: 1]  
Read output from qubits 1 (high) and 0(low)

(4) Evolved quantum database algorithm,  
item at 0,1

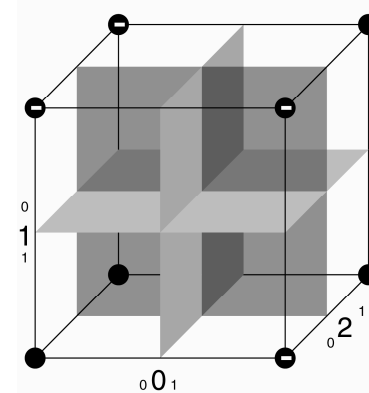
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After DB [in:2,0; out:1](item in 0,1)

(5) Evolved quantum database algorithm,  
item at 0,1

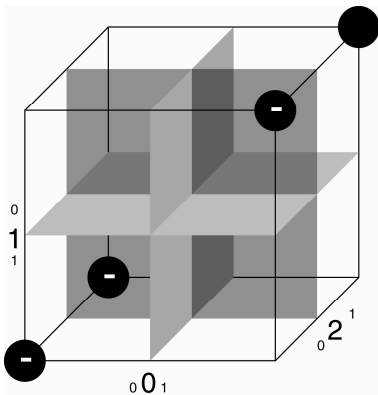
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After CNOT [control: 1, target: 2]

(6) Evolved quantum database algorithm,  
item at 0,1

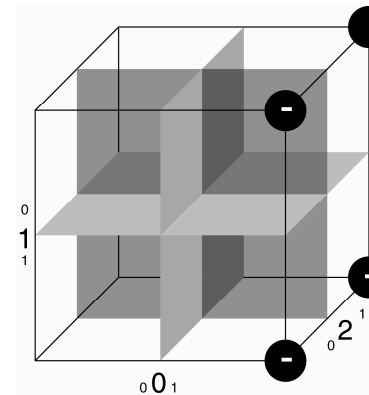
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After Hadamard [1]

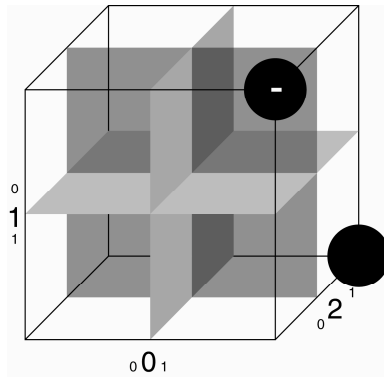
(7) Evolved quantum database algorithm,  
item at 0,1

---



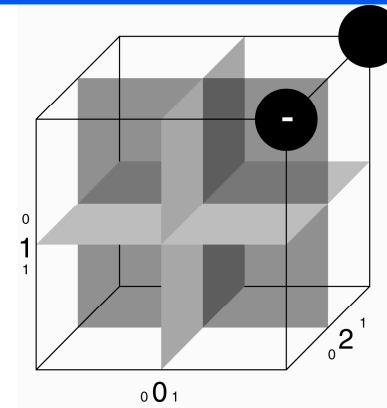
After CNOT [control: 1, target: 0]

### (8) Evolved quantum database algorithm, item at 0,1



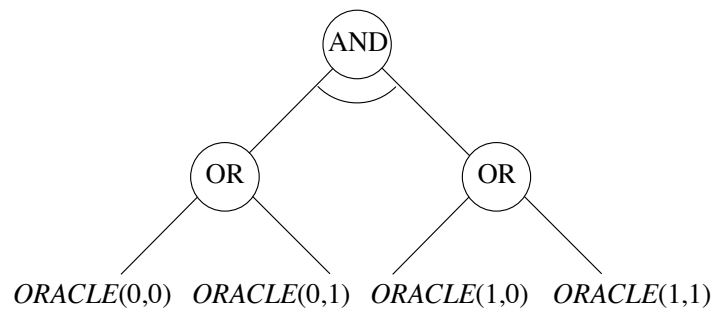
After Hadamard [1]

### (9) Evolved quantum database algorithm, item at 0,1

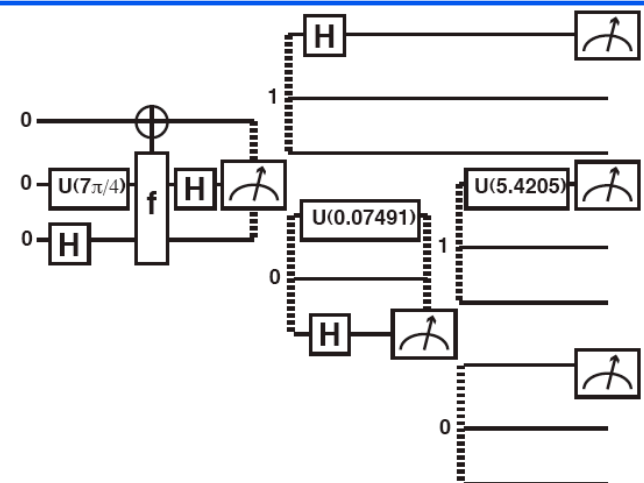


After CNOT [control: 2, target: 1]  
Read output from qubits 1 (high) and 0(low)

### The and-or tree problem



### Evolved and-or gate array



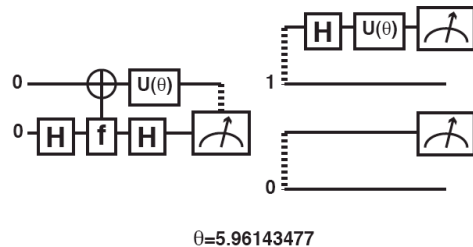
## Error/complexity measures

- ◆ *Las Vegas*  $\equiv$  always correct, but may answer “don’t know” with some probability
- ◆ *Monte Carlo*  $\equiv$  may err, with some probability
- ◆  $p_{max}^e \equiv$  worst case probability of error
- ◆  $q_{max}^e \equiv$  worst case expected queries
- ◆ *Exact*  $\equiv p_{max}^e = 0$

## Complexity of 2-bit AND/OR

- ◆ Classical Las Vegas:  $q_{max}^e = 3$ 
  - derived from [Saks and Wigderson 1986]
- ◆ Classical Monte Carlo: for  $q_{max}^e = 1, p_{max}^e \geq 1/3$ 
  - derived from [Santha 1991]
- ◆ Evolved Quantum Monte Carlo:  $p_{max}^e = 0.28732$

## Derived better-than-classical OR

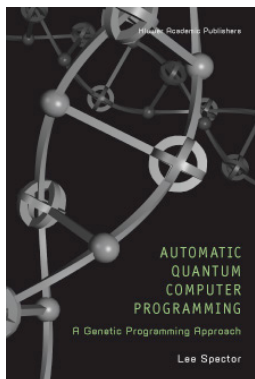


- ◆ Classical Monte Carlo: for  $q_{max}^e = 1, p_{max}^e \geq 1/6$  [Jozsa 1991, Beals 1998]
- ◆ Evolved algorithm  $q_{max}^e = 1, p_{max}^e = 1/10$

## GP/QC research directions

- ◆ Application to additional problems with incompletely understood quantum complexity
- ◆ Exploration of communication capacity of quantum gates
- ◆ Evolution of hybrid quantum/classical algorithms.
- ◆ Evolution guided by ease of physical implementation.
- ◆ QC applications in AI
  - general AI search?
  - and-or trees and Prolog: quantum logic machine?
  - Bayesian networks?
- ◆ Genetic programming *on* quantum computers.

## Book



### *Automatic Quantum Computer Programming: A Genetic Programming Approach*

Lee Spector. 2004.

Boston: Kluwer Academic Publishers.  
ISBN 1-4020-7894-3.

<http://hampshire.edu/lspector/aqcp/>

## Sources: selected books

- ♦ *Automatic Quantum Computer Programming: A Genetic Programming Approach*. By Lee Spector. Kluwer Academic Publishers. 2004.
- ♦ *Quantum Computation and Quantum Information*. By Michael A. Nielsen and Isaac L. Chuang. Cambridge University Press. 2000.
- ♦ *Schrödinger's Machines: The Quantum Technology Reshaping Everyday Life*. By Gerard J. Milburn. W.H. Freeman and Company. 1997.
- ♦ *Explorations in Quantum Computing*. By Colin P. Williams and Scott H. Clearwater. Springer-Verlag/Telos. 1997.
- ♦ *The Fabric of Reality*. By David Deutsch. Penguin Books. 1997.
- ♦ *The Large, the Small and the Human Mind*. By Roger Penrose, with Abner Shimony, Nancy Cartwright, and Stephen Hawking. Cambridge University Press. 1997.
- ♦ *QED: The Strange Theory of Light and Matter*. By Richard P. Feynman. Princeton University Press. 1985.

## Sources: selected articles

- ♦ A. Steane, 1998. "Quantum Computing," *Reports on Progress in Physics*, vol. 61, pp. 117-173. <http://xxx.lanl.gov/abs/quant-ph/9708022>
- ♦ P. Shor, 1998. "Quantum Computing," *Documenta Mathematica*, vol. Extra Volume ICM, pp. 467-486. <http://east.camel.math.ca/EMIS/journals/DMJDMV/xvol-icm/00/Shor.MAN.ps.gz>
- ♦ J. Preskill, 1997. "Quantum Computing: Pro and Con," Tech. Rep. CALT-68-2113, California Institute of Technology. <http://xxx.lanl.gov/abs/quant-ph/9705032>
- ♦ A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, N. Margolus, P. Shor, T. Sleator, J. Smolin, H. Weinfurter, 1995. "Elementary Gates for Quantum Computation," submitted to *Physical Review A*. <http://xxx.lanl.gov/abs/quant-ph/9503016>
- ♦ N.J. Cerf, C. Adami, P.G. Kwiat, 1998. "Optical Simulation of Quantum Logic," *Phys. Rev. A* 57, 1477. <http://xxx.lanl.gov/abs/quant-ph/9706022>
- ♦ L. Spector and H.J. Bernstein. 2003. "Communication Capacities of Some Quantum Gates, Discovered in Part through Genetic Programming," in *Proc. of the Sixth Intl. Conf. on Quantum Communication, Measurement, and Computing*, edited by J.H. Shapiro and O. Hirota. Princeton, NJ: Rinton Press, Inc. pp. 500-503. <http://hampshire.edu/lspector/pubs/spector-QCMC-prepress.pdf>
- ♦ H. Barnum, H.J. Bernstein, and L. Spector, 2000. Quantum circuits for OR and AND of ORs. *Journal of Physics A: Mathematical and General*, Vol. 33 No. 45 (17 November 2000), pp. 8047-8057. <http://hampshire.edu/lspector/pubs/jpa.pdf>
- ♦ L. Spector, H. Barnum, H.J. Bernstein, N. Swamy, 1999. "Quantum Computing Applications of Genetic Programming," in *Advances in Genetic Programming* 3, pp. 135-160, MIT Press.
- ♦ L. Spector, H. Barnum, H.J. Bernstein, N. Swamy, 1999. "Finding a Better-Than-Classical Quantum AND/OR Algorithm Using Genetic Programming," in *Proc. 1999 Congress on Evolutionary Computation*, IEEE Press.
- ♦ L. Spector, H. Barnum, H.J. Bernstein, 1998. "Genetic Programming for Quantum Computers," in *Genetic Programming 1998: Proceedings of the Third Annual Conference*, pp. 365-374, Morgan Kaufmann.

## Sources: selected WWW sites

- ♦ Oxford's Center for Quantum Computation: <http://www.qubit.org/>
- ♦ Stanford-Berkeley-MIT-IBM NMR Quantum Computation Project: <http://sqint.stanford.edu/>
- ♦ Quantum Information and Computation (Caltech - MIT - USC): <http://theory.caltech.edu/~quic/index.html>
- ♦ Quantum Computation at ISI/USC: [http://www.isi.edu/acal/quantum/quantum\\_intro.html](http://www.isi.edu/acal/quantum/quantum_intro.html)
- ♦ Los Alamos National Laboratory quantum physics e-print archive: <http://xxx.lanl.gov/form/quant-ph>
- ♦ John Preskill's Physics 229 course web page (many good links): <http://www.theory.caltech.edu/people/preskill/ph229/>
- ♦ Samuel L. Braunstein's on-line tutorial: <http://www.sees.bangor.ac.uk/~schmuel/comp/comp.html>
- ♦ NIST Ion Storage Group: <http://www.bldrdoc.gov/timefreq/ion/index.htm>
- ♦ QGAME, Quantum Gate And Measurement Emulator: <http://hampshire.edu/lspector/qgame.html>