Tutorial on
Evolutionary Multiobjective Optimization
GECCO 2005

Eckart Zitzler

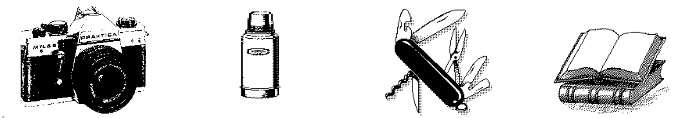
Computer Engineering (TIK), ETH Zurich, Switzerland

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Computer Engineering
and Networks Laboratory

Introductory Example: The Knapsack Problem

weight = 750g profit = 5	weight = 1500g profit = 8	weight = 300g profit = 7	weight = 1000g profit = 3



?

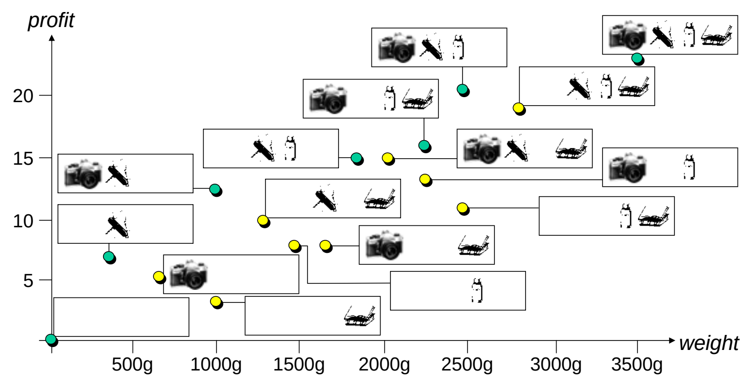
Single objective:

choose subset that
- maximizes overall profit
- w.r.t. a weight limit

Multiobjective:

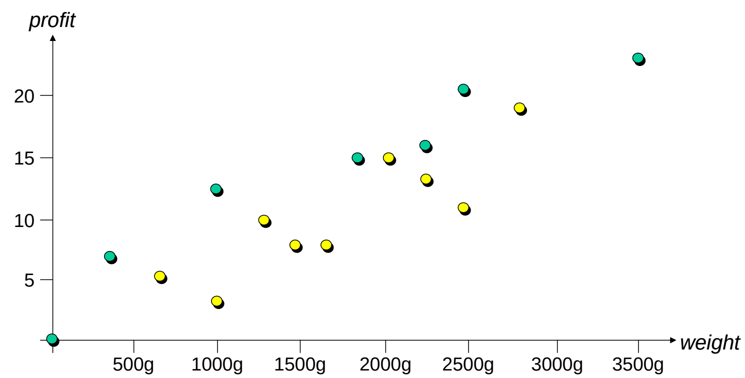
choose subset that
- maximizes overall profit
- minimizes overall weight

The Search Space



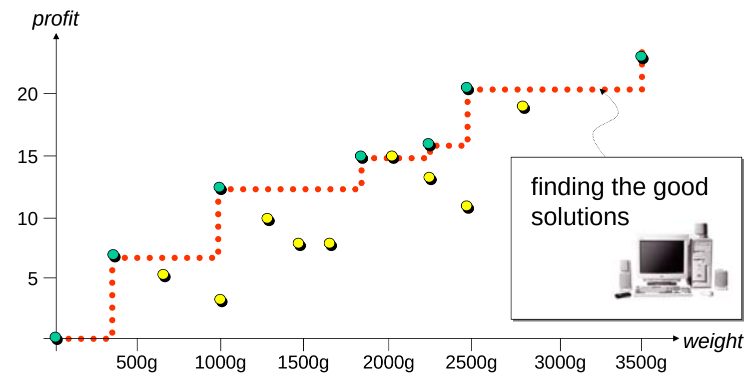
The Trade-off Front

Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)



The Trade-off Front

Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)

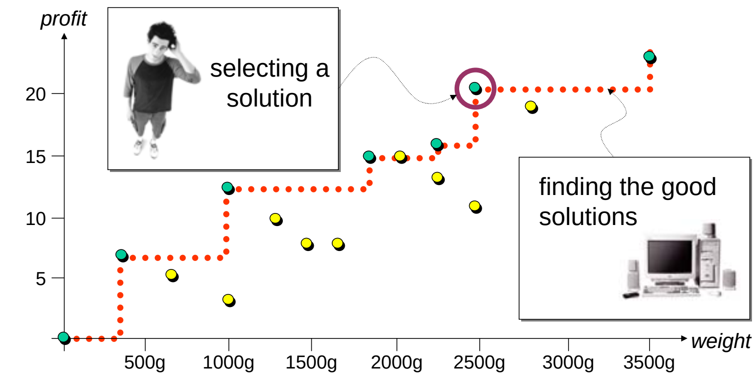


© Eckart Zitzler
ETH Zürich

The Trade-off Front

GECCO 2005
Tutorial on EMO

Observations: ❶ there is no single optimal solution, but
❷ some solutions (●) are better than others (●)

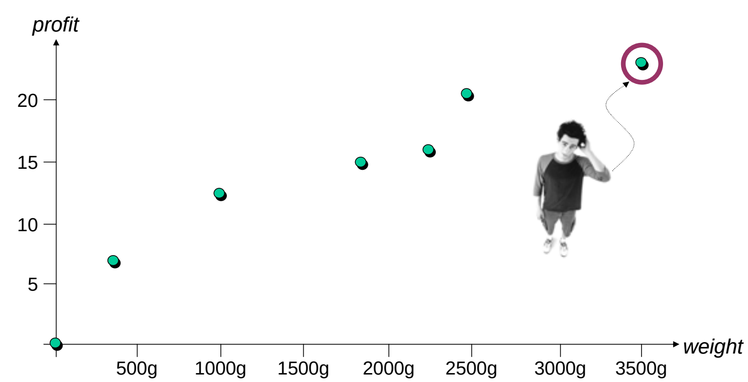


© Eckart Zitzler
ETH Zürich

Decision Making: Selecting a Solution

GECCO 2005
Tutorial on EMO

Approaches: • profit more important than cost (ranking)

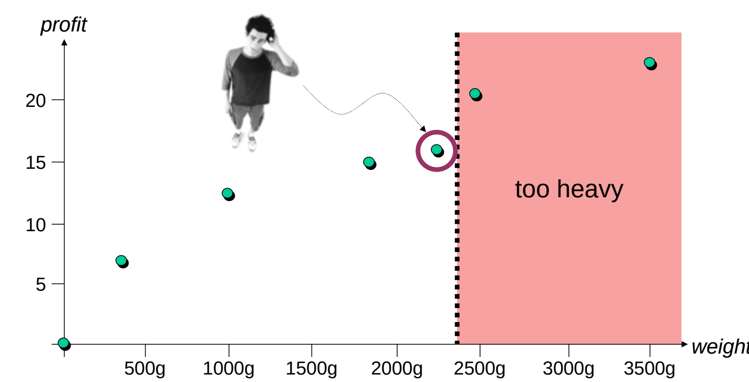


© Eckart Zitzler
ETH Zürich

Decision Making: Selecting a Solution

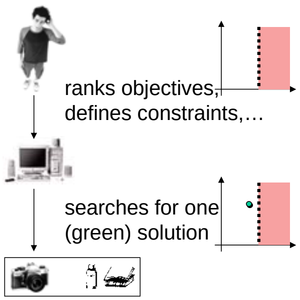
GECCO 2005
Tutorial on EMO

Approaches: • profit more important than cost (ranking)
• weight must not exceed 2400g (constraint)



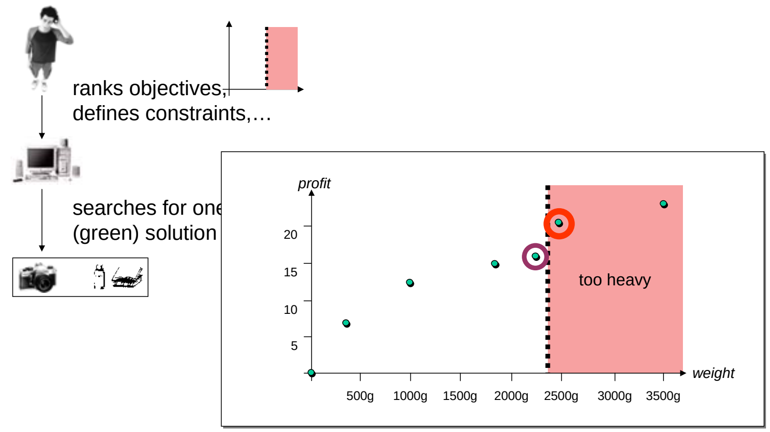
When to Make the Decision

Before Optimization:



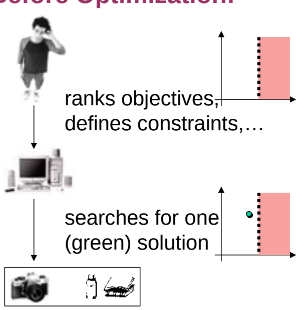
When to Make the Decision

Before Optimization:

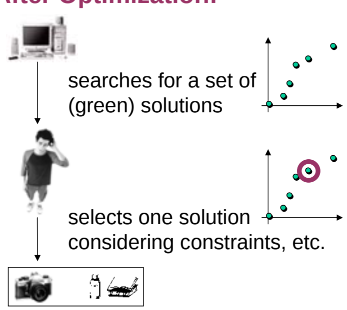


When to Make the Decision

Before Optimization:



After Optimization:



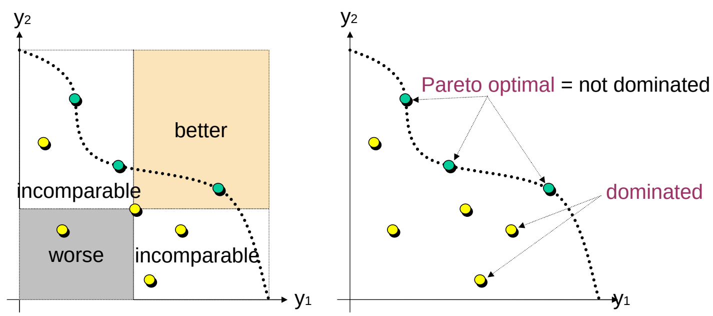
- decision making often easier
- EAs well suited

Outline

1. **Introduction:** Why multiple objectives make a difference
2. **Basic Principles:** Terms you need to know
3. **Algorithm Design:** Do it yourself
4. **Performance Assessment:** Once upon a time
5. **Applications Domains:** Where EMO is useful
6. **Further Information:** What else

The Concept of Pareto Optimality

Maximize $(y_1, y_2, \ldots, y_k) = f(x_1, x_2, \ldots, x_n)$



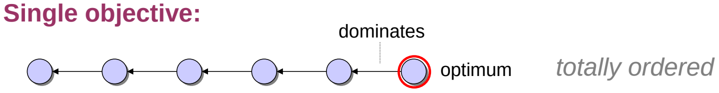
Pareto(-optimal) set = set of all Pareto-optimal solutions

© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

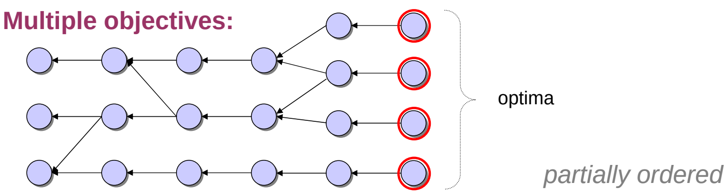
Total and Partial Orders

Pareto dominance = ❶ not worse in all objectives
❷ better in at least one objective

Single objective:



Multiple objectives:

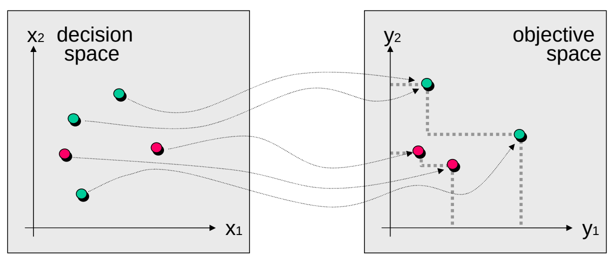


Note: other definitions of dominance possible...

© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

Decision and Objective Space

Pareto set — Pareto front
non-optimal decision vector — non-optimal objective vector



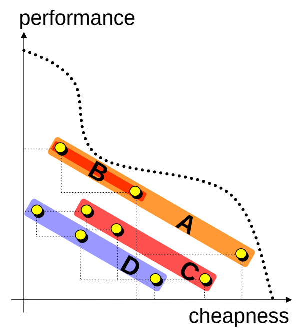
$(x_1, x_2, \ldots, x_n) \longrightarrow$ **f** $\longrightarrow (y_1, y_2, \ldots, y_k)$

search ← **evaluation**

© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

Pareto Set Approximations

Pareto set approximation (algorithm outcome) =
set of incomparable solutions



A is **weakly better** than **B**
= not worse in all objectives
and sets not equal

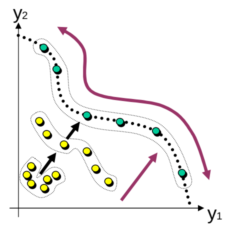
C is **better** than **D**
= better in at least one objective

A is **strictly better** than **C**
= better in all objectives

B is **incomparable** to **C**
= neither set weakly better

What Is the Optimization Goal?

- Find all Pareto-optimal solutions?
  - Impossible in continuous search spaces
  - How should the decision maker handle 10000 solutions?
- Find a representative subset of the Pareto set?
  - Many problems are NP-hard
  - What does representative actually mean?
- Find a good approximation of the Pareto set?
  - What is a good approximation?
  - How to formalize intuitive understanding:
    - ❶ close to the Pareto front
    - ❷ well distributed

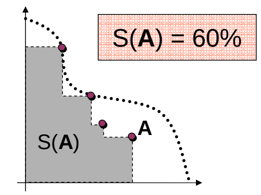


Preference Information

Preference information (here) = any additional information that refines the dominance relation on approximation sets (partial order → total order)

Example:

optimization goal
=
maximize size S of dominated objective space

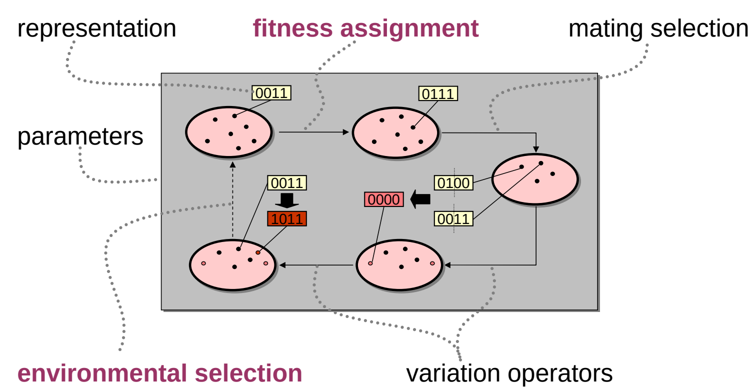


Note: every algorithm implicitly or explicitly makes assumptions about the decision maker's preferences (limited memory, selection)

Outline

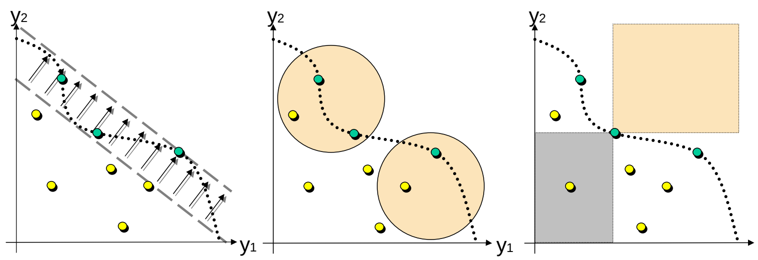
1. **Introduction:** Why multiple objectives make a difference
2. **Basic Principles:** Terms you need to know
3. **Algorithm Design:** Do it yourself
4. **Performance Assessment:** Once upon a time
5. **Applications Domains:** Where EMO is useful
6. **Further Information:** What else

Design Choices



Ranking Solutions

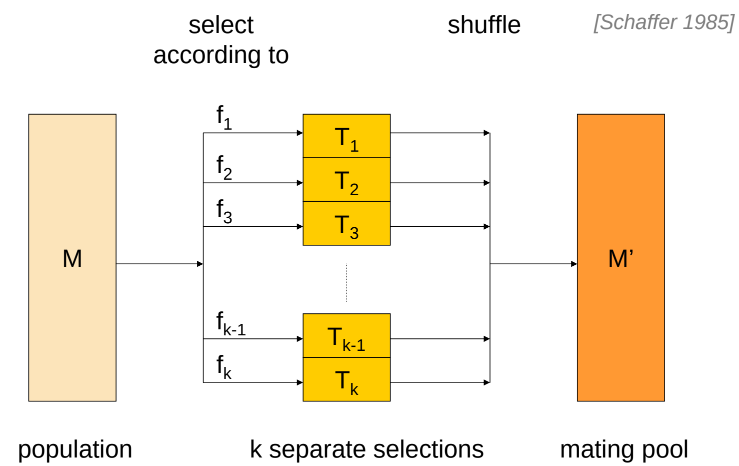
aggregation-based	**criterion-based**	**dominance-based**
weighted sum	*VEGA*	*SPEA2*



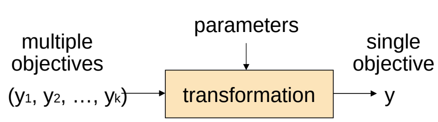
parameter-oriented
scaling-dependent

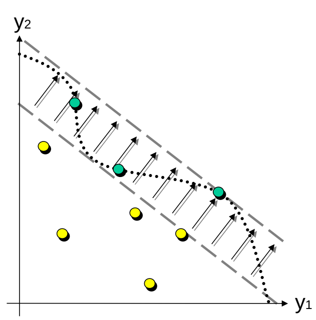
set-oriented
scaling-independent

Example: VEGA

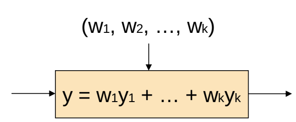


Aggregation-Based Ranking



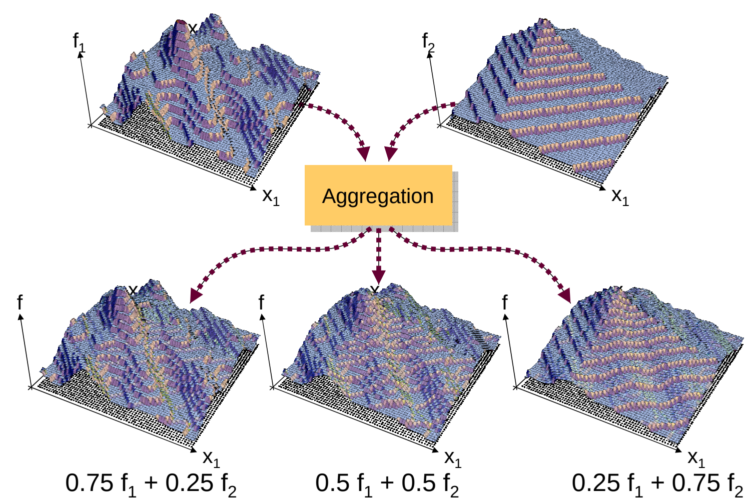


Example: weighting approach



Note: weights need to be varied during the run...

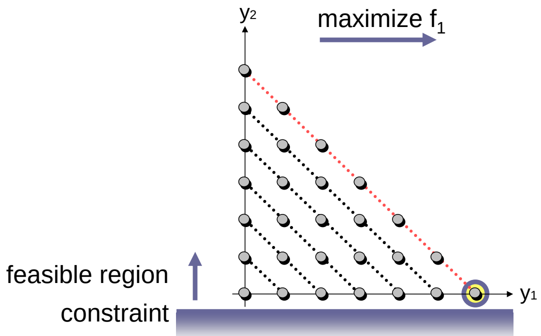
Example: Weighted Sum



Example: Multistart Constraint Method

Underlying concept:

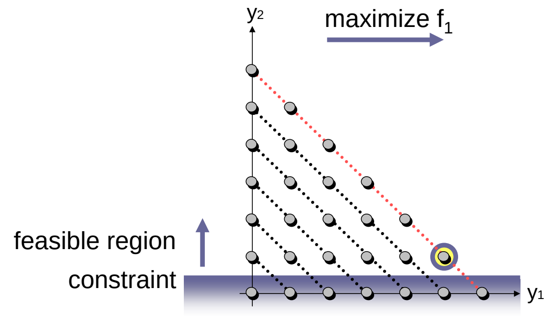
- Convert all objectives except of one into constraints
- Adaptively vary constraints



Example: Multistart Constraint Method

Underlying concept:

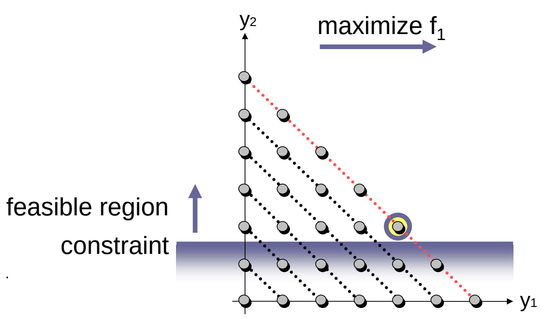
- Convert all objectives except of one into constraints
- Adaptively vary constraints



Example: Multistart Constraint Method

Underlying concept:

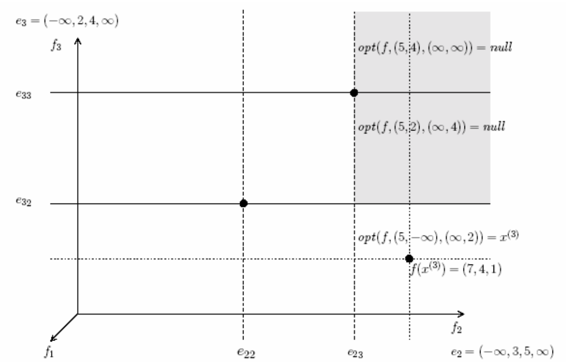
- Convert all objectives except of one into constraints
- Adaptively vary constraints



Example: Multistart Constraint Method (Cont'd)

Extension to n objectives: ECEA *[Laumanns et al. 2004]*

- f_1 is the objective to optimize
- The boxes are defined by constraints on f_2 and f_3



Dominance-Based Ranking

Types of information:

- **dominance rank** — by how many individuals is an individual dominated?
- **dominance count** — how many individuals does an individual dominate?
- **dominance depth** — at which front is an individual located?

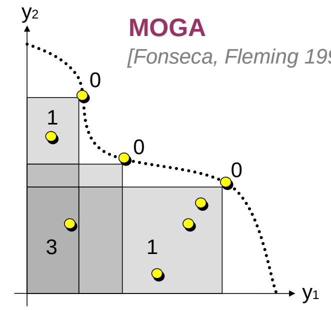
Examples:

- *MOGA, NPGA* — dominance rank
- *NSGA/NSGA-II* — dominance depth
- *SPEA/SPEA2* — dominance count + rank

Example: MOGA and SPEA2

MOGA

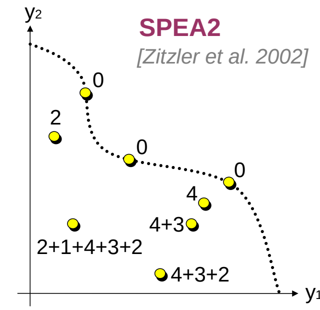
[Fonseca, Fleming 1993]



R (raw fitness) =
#dominating solutions

SPEA2

[Zitzler et al. 2002]



S (strength) =
#dominated solutions

R (raw fitness) =
$\sum$ strengths of dominators

Refining Rankings

ranks	**pure dominance rank**	**refined ranking**
1	0	0.245
2	0	0.311
3	0	0.329
4	1	
5	1	
6	2	
7	2	0
8	2	1
9	3	2

❶ density information based on Euclidean distance

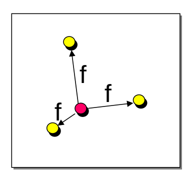
no selection pressure within equivalence classes

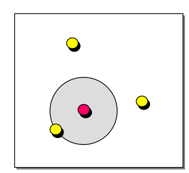
❷ modified dominance relation

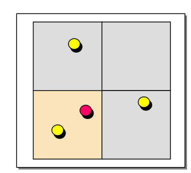
Methods Based On Euclidean Distance

Density estimation techniques: *[Silverman 1986]*

Kernel *MOGA*	**Nearest neighbor** *SPEA2*	**Histogram** *PAES*
density estimate = sum of f values where f is a function of the distance	density estimate = volume of the sphere defined by the nearest neighbor	density estimate = number of solutions in the same box



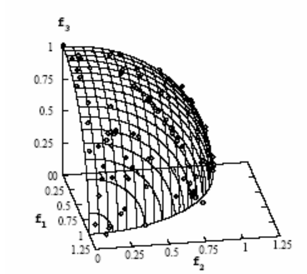


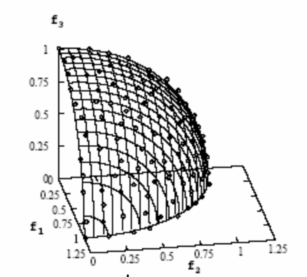


Computation Effort Versus Accuracy

Two Nearest Neighbor Variants

Objective-Wise	Euclidean Distance
NSGA-II	*SPEA2*

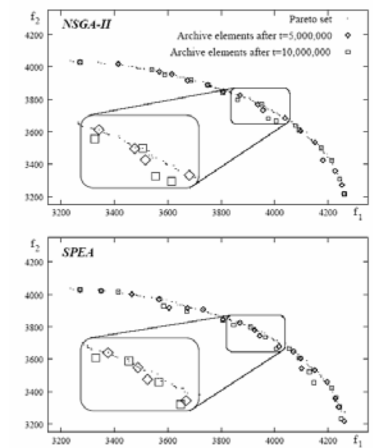




Objective-Wise	Euclidean Distance
faster	slower
good for 2 objectives	good for 3 objectives and more
[Deb et al. 2002]	*[Zitzler et al. 2002]*

The Problem of Deterioration

Observation:
The use of Euclidean distance can lead to deterioration



Knapsack problem
[Laumanns et al. 2002]

Refinement of Dominance Relations

Integration of Goals, Priorities, Constraints:
[Fonseca, Fleming 1998]

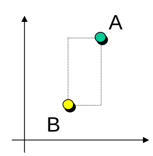
A is preferable over B $\Leftrightarrow$ $(\overset{u}{\boldsymbol{u}_p}\ {}_p\!< \overset{u}{\boldsymbol{v}_p}) \vee \{(\overset{u}{\boldsymbol{u}_p} = \overset{u}{\boldsymbol{v}_p})$
$\wedge [(\overset{u}{\boldsymbol{v}_p} \not\leq \overset{u}{\boldsymbol{g}_p}) \vee (\boldsymbol{u}_{1,\cdots,p-1} \underset{\boldsymbol{g}_{1,\cdots,p-1}}{\prec} \boldsymbol{v}_{1,\cdots,p-1})]\}$

Continuous dominance "relations": *[Zitzler et al. 2003]*

$I_{\varepsilon+}(A,B) = \min_i f_i(A) - f_i(B)$

$I_{\varepsilon+}(A,B) \geq 0$ and $I_{\varepsilon+}(B,A) < 0 \Leftrightarrow$ A dominates B

(binary additive epsilon quality indicator)



Example: IBEA

Question: How to continuous dominance "relations" for fitness assignment? *[Zitzler, Künzli 2004]*

Given: function I (binary quality indicator) with

A dominates B $\Leftrightarrow$ I(A, B) < I(B, A)

Idea: measure for "loss in quality" if A is removed

Fitness: $F'(\boldsymbol{x}^1) = \sum_{\boldsymbol{x}^2 \in P \setminus \{\boldsymbol{x}^1\}} I(\{\boldsymbol{x}^2\}, \{\boldsymbol{x}^1\})$

...corresponds to continuous extension of dominance rank
...blurrs influence of dominating and dominated individuals

Example: IBEA (Cont'd)

Fitness assignment: O(n^2)

Fitness: $$F(\boldsymbol{x}^1) = \sum_{\boldsymbol{x}^2 \in P \setminus \{\boldsymbol{x}^1\}} -e^{-I(\{\boldsymbol{x}^2\},\{\boldsymbol{x}^1\})/\kappa}$$

- parameter κ is problem- and indicator-dependent
- no additional diversity preservation mechanism

Mating selection: O(n)

- binary tournament selection, fitness values constant

Environmental selection: O(n^2)

- iteratively remove individual with lowest fitness
- update fitness values of remaining individuals after each deletion

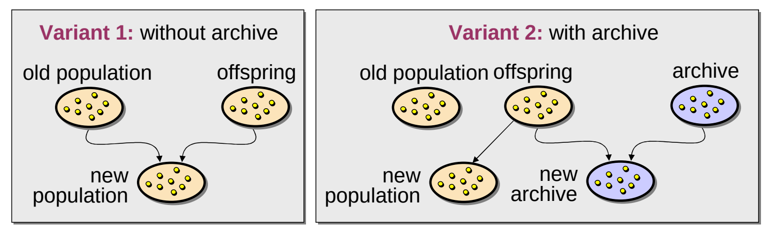
Further Design Aspects

- **Constraint handling:**
  How to integrate constraints into fitness assignment?
- **Archiving / environmental selection:**
  How to keep a good approximation?
- **Hybridization:**
  How to integrate, e.g., local search in a multiobjective EA?
- **Preference articulation:**
  How to focus the search on interesting regions?
- **Data structures:**
  How to support, e.g., fast dominance checks?

Constraint Handling & Multiple Objectives

	penalty functions	**constraints as objectives**	**modified dominance**
	Add penalty term to fitness	Introduce additional objective(s)	extend to infeasible solutions
overall constraint violation	*[Michalewicz 1992]*	*[Wright, Loosemore 2001]*	*[Deb 2001]*
constraints treated separately	?	*[Coello 2000]*	*[Fonseca, Fleming 1998]*

Archiving / Environmental Selection



deterministic truncation

archive = only nondominated solutions

Additional selection criteria:

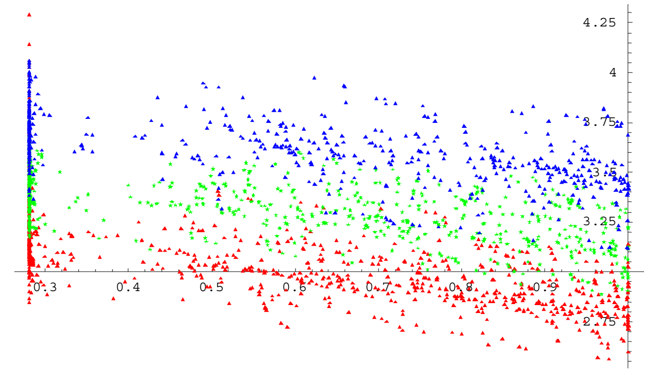
- Density information / other preferences
- Time
- Chance

Outline

1. **Introduction:** Why multiple objectives make a difference
2. **Basic Principles:** Terms you need to know
3. **Algorithm Design:** Do it yourself
4. **Performance Assessment:** Once upon a time
5. **Applications Domains:** Where EMO is useful
6. **Further Information:** What else

Once Upon a Time...

... multiobjective EAs were mainly compared visually:



ZDT6 benchmark problem: IBEA, SPEA2, NSGA-II

Performance Assessment: Approaches

❶ **Theoretically (by analysis):** difficult

- Limit behavior (unlimited run-time resources)
- Running time analysis

❷ **Empirically (by simulation):** standard

Problems: randomness, multiple objectives

Issues: quality measures, statistical testing, visualization, benchmark problems, parameter settings, …

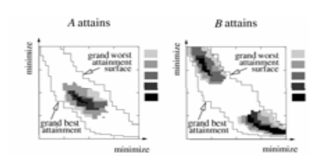
Two Approaches for Empirical Studies

Attainment function approach:

- Applies statistical tests directly to the samples of approximation sets
- Gives detailed information about how and where performance differences occur

Quality indicator approach:

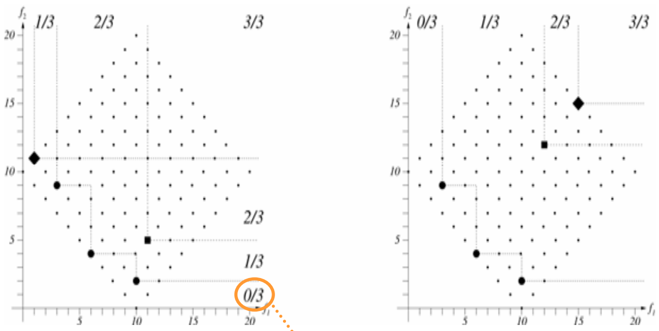
- First, reduces each approximation set to a single value of quality
- Applies statistical tests to the samples of quality values



Indicator	A	B
Hypervolume indicator	6.3431	7.1924
ϵ-indicator	1.2090	0.12722
R_2 indicator	0.2434	0.1643
R_3 indicator	0.6454	0.3475

Empirical Attainment Functions

three runs of two multiobjective optimizers



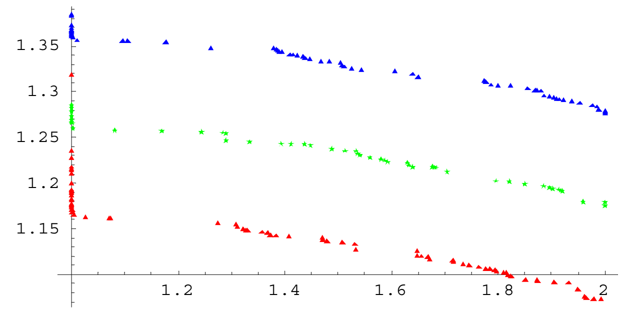
frequency of attaining regions

© Eckart Zitzler
ETH Zürich

Attainment Plots

GECCO 2005
Tutorial on EMO

50% attainment surface for IBEA, SPEA2, NSGA2 (ZDT6)

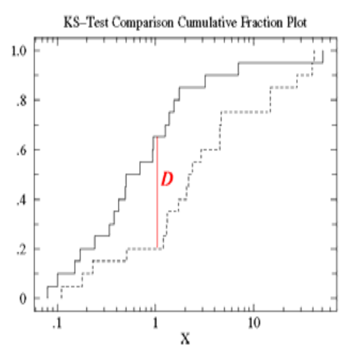


© Eckart Zitzler
ETH Zürich

Attainment Function Analysis

GECCO 2005
Tutorial on EMO

- A Kolmogorov-Smirnov test examines the maximum difference between two cumulative distribution functions
- A KS-like test can be used to probe differences between the empirical attainment functions of a pair of optimizers, *A* and *B*
- The null hypothesis is that the attainment functions of *A* and *B* are identical
- The alternative hypothesis is that the distributions differ somewhere



[Fonseca et al. 2001]

© Eckart Zitzler
ETH Zürich

Statistical Assessment

GECCO 2005
Tutorial on EMO

ZDT6

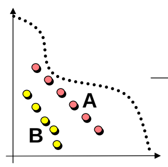
- IBEA – NSGA-II
  - significant difference (p=0)
- IBEA – SPEA2
  - significant difference (p=0)
- SPEA2 – NSGA-II
  - significant difference (p=0)

Knapsack

- IBEA – NSGA-II
  - no significant difference
- IBEA – SPEA2
  - no significant difference
- SPEA2 – NSGA-II
  - no significant difference

Quality Indicator Approach

Goal: compare two Pareto set approximations A and B



→

	A	B
hypervolume	*432.34*	*420.13*
distance	*0.3308*	*0.4532*
diversity	*0.3637*	*0.3463*
spread	*0.3622*	*0.3601*
cardinality	*6*	*5*

→ "**A** better"

Comparison method C = quality measure(s) + Boolean function

$A, B \xrightarrow{\text{quality measure}} \mathbb{R}^n \xrightarrow{\text{Boolean function}} \text{statement}$

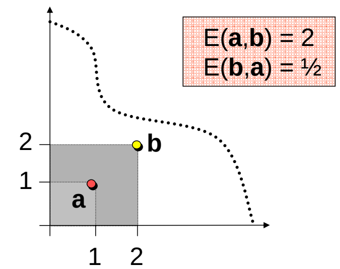
reduction **interpretation**

© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

Example: ε-Quality Indicator

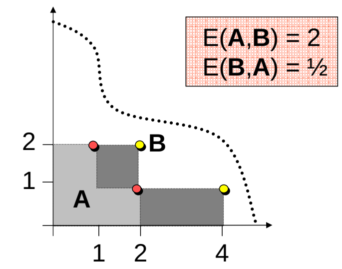
Two solutions:

$E(\mathbf{a},\mathbf{b}) =$
$\max_{1 \le i \le n} \min_{\varepsilon} \varepsilon \cdot f_i(\mathbf{a}) \ge f_i(\mathbf{b})$



Two approximations:

$E(\mathbf{A},\mathbf{B}) =$
$\max_{\mathbf{b} \in \mathbf{B}} \min_{\mathbf{a} \in \mathbf{A}} E(\mathbf{a},\mathbf{b})$



Unary quality indicator: I(A) = E(A,R) where R is a reference set

[Zitzler et al. 2003]

© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

Power of Unary Quality Indicators

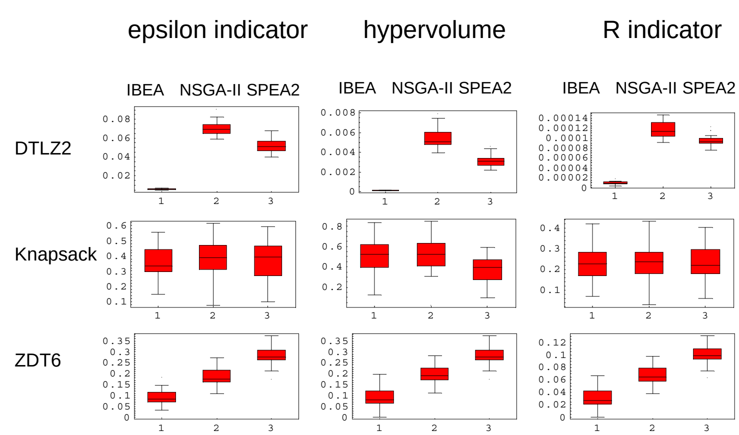
Important: compliance with dominance relations *[Zitzler et al. 2003]*

indicator	name / reference	Boolean function	compatibility	completeness
I_{HC}	enclosing hypercube indicator / Section III-B.1	$I_2^{HC}(A) < I_1^{HC}(B)$	$\succ\succ$	-
I_O	objective vector indicator / Section III-B.1	$I_i^O(A) < I_i^O(B)$	$\succ\succ$	-
I_H	hypervolume indicator / [7]	$I_H(A) > I_H(B)$	$\not\triangleright$	$\triangleright$
I_W	average best weight combination / [19]	$I_W(A) < I_W(B)$	$\not\triangleright$	$\succ\succ$
I_D	distance from reference set / [20]	$I_D(A) < I_D(B)$	$\not\triangleright$	$\succ\succ$
$I_{\epsilon 1}$	unary ϵ-indicator / Section III-B.2	$I_{\epsilon 1}(A) < I_{\epsilon 1}(B)$	$\not\triangleright$	$\succ\succ$
I_{PF}	fraction of Pareto-optimal front covered / [22]	$I_{PF}(A) > I_{PF}(B)$	$\not\triangleright$	-
I_P	number of Pareto points contained / Section III-B.2	$I_P(A) > I_P(B)$	$\not\triangleright$	-
I_{ER}	error ratio / [13]	$I_{ER}(A) > 0$	$\not\succ$	-
I_{CD}	chi-square-like deviation indicator / [14]	$I_{CD}(A) < I_{CD}(B)$	-	-
I_S	spacing / [23]	$I_S(A) < I_S(B)$	-	-
I_{ONVG}	overall nondominated vector generation / [13]	$I_{ONVG}(A) > I_{ONVG}(B)$	-	-
I_{GD}	generational distance / [13]	$I_{GD}(A) < I_{GD}(B)$	-	-
I_{ME}	maximum Pareto front error / [13]	$I_{ME}(A) < I_{ME}(B)$	-	-
I_{MS}	maximum spread / [21]	$I_{MS}(A) > I_{MS}(B)$	-	-
I_{MD}	minimum distance between two solutions / [24]	$I_{MD}(A) > I_{MD}(B)$	-	-
I_{CE}	coverage error / [24]	$I_{CE}(A) < I_{CE}(B)$	-	-
I_{DU}	deviation from uniform distribution / [25]	$I_{DU}(A) < I_{DU}(B)$	-	-
I_{OS}	Pareto spread / [26]	$I_{OS}(A) > I_{OS}(B)$	-	-
I_A	accuracy / [26]	$I_A(A) > I_A(B)$	-	-
I_{NDC}	number of distinct choices / [26]	$I_{NDC}(A) > I_{NDC}(B)$	-	-
I_{CL}	cluster / [26]	$I_{CL}(A) < I_{CL}(B)$	-	-

strictly better   not weakly better   not better   weakly better

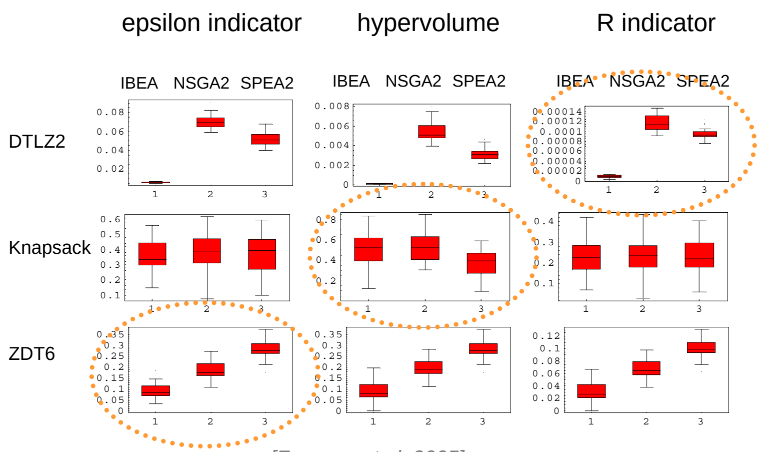
© Eckart Zitzler
ETH Zürich
GECCO 2005
Tutorial on EMO

Example: Box Plots



[Fonseca et al. 2005]

Example: Box Plots



[Fonseca et al. 2005]

Statistical Assessment (Kruskal Test)

ZDT6
Epsilon

is better than

	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		~0
SPEA2	1	1	

Overall p-value = 6.22079e-17.
Null hypothesis rejected (alpha 0.05)

DTLZ2
R

is better than

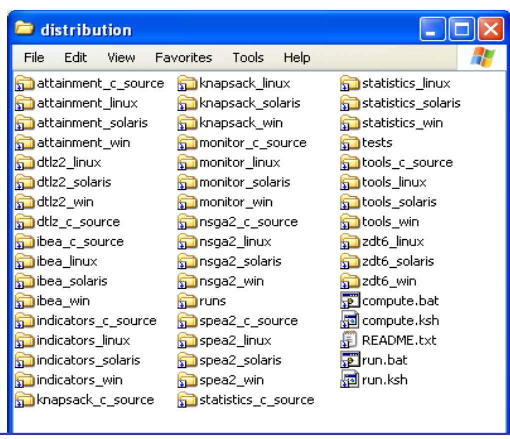
	IBEA	NSGA2	SPEA2
IBEA		~0	~0
NSGA2	1		1
SPEA2	1	~0	

Overall p-value = 7.86834e-17.
Null hypothesis rejected (alpha 0.05)

Knapsack/Hypervolume: H0 = No significance of any differences

Performance Assessment Tools

- Reference set calculation
- Attainment function calculation
- Indicators
- Statistical testing procedures



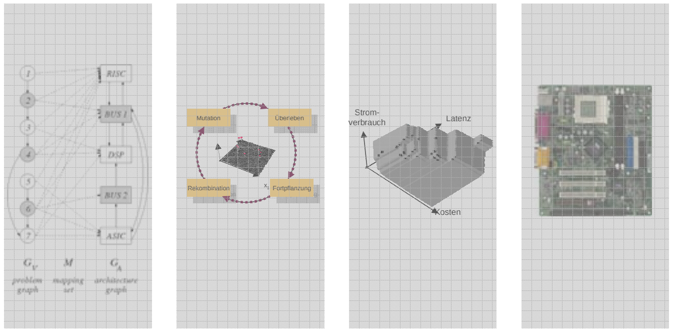
http://www.tik.ee.ethz.ch/pisa

Outline

1. **Introduction:** Why multiple objectives make a difference
2. **Basic Principles:** Terms you need to know
3. **Algorithm Design:** Do it yourself
4. **Performance Assessment:** Once upon a time
5. **Applications Domains:** Where EMO is useful
6. **Further Information:** What else

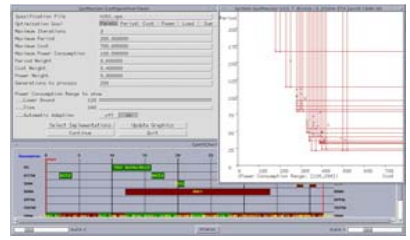
Design Space Exploration

Specification → Optimization → Evaluation → Implementation



Examples: computer design, biological experiment design, etc.

Example Applications



Architecture exploration:

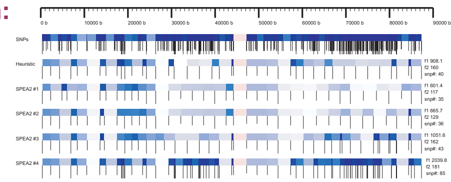
- min. cost
- max. performance
- min. power consumption

[Eisenring, Thiele, Zitzler 2000]

Genetic marker selection:

- min. cost
- max. sensitivity

[Hubley, Zitzler, Roach 2003]



Application: Genetic Programming

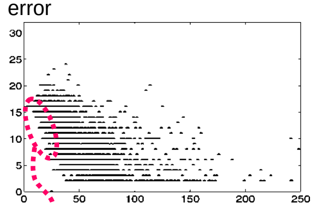
Problem: Trees grow rapidly

- Premature convergence
- Overfitting of training data

Common approaches:

- Constraint
  (tree size limitation)
- Penalty term
  (parsimony pressure)
- Objective ranking
  (size post-optimization)
- Structure-based (ADF, etc.)

Multiobjective approach:
Optimize both error and size

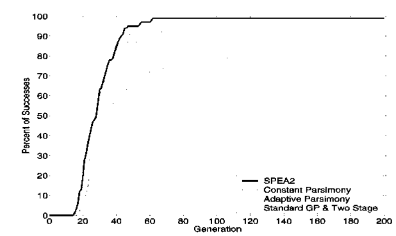


Keep and optimize small trees
(potential building blocks)

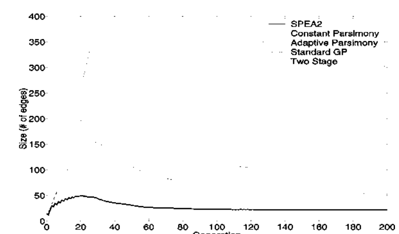
Multiobjective GP: Results

Multiobjective approach (SPEA2) can find

- a correct solution with higher probability
- a correct solution slightly faster



- more compact (correct) solutions



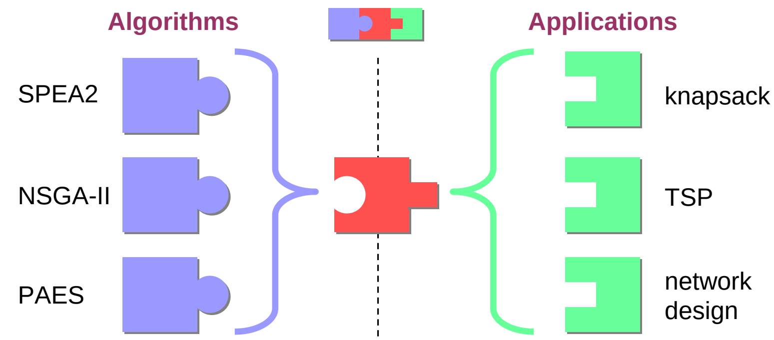
than alternative approaches on even-parity problem.

[Bleuler et al. 2001]

Outline

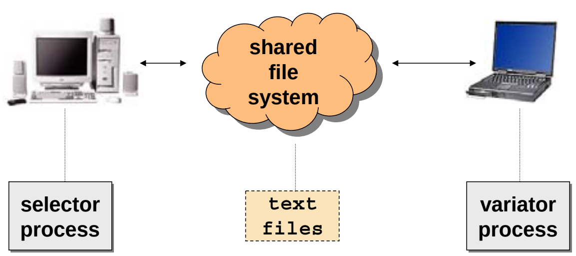
1. **Introduction:** Why multiple objectives make a difference
2. **Basic Principles:** Terms you need to know
3. **Algorithm Design:** Do it yourself
4. **Performance Assessment:** Once upon a time
5. **Applications Domains:** Where EMO is useful
6. **Further Information:** What else

The Concept of PISA



text-based
Platform and programming language independent **I**nterface
for **S**earch **A**lgorithms *[Bleuler et al.: 2003]*

PISA: Implementation

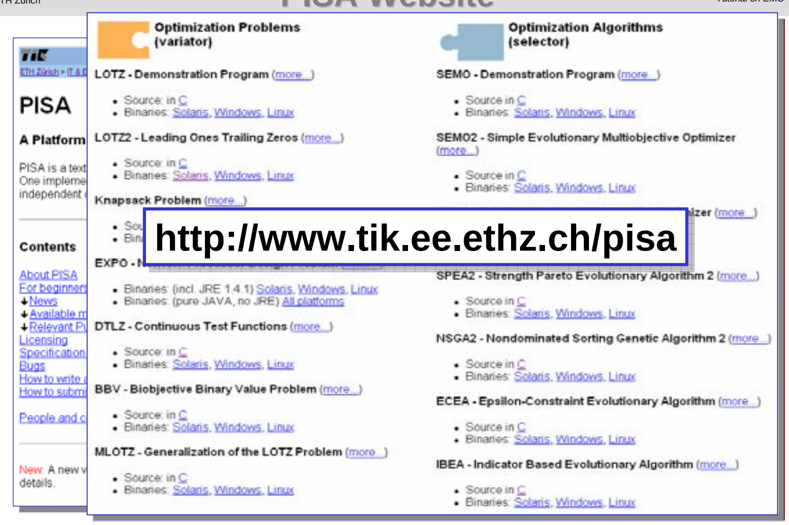


application independent:
- mating / environmental selection
- individuals are described by IDs and objective vectors

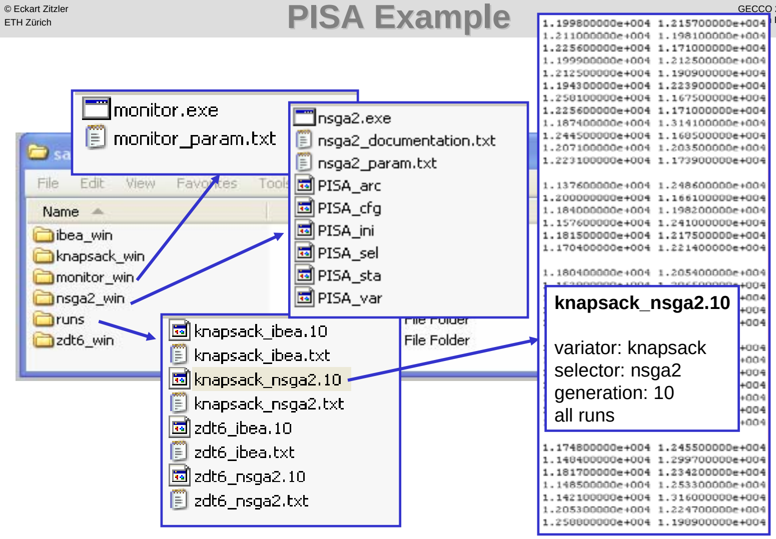
handshake protocol:
- state / action
- individual IDs
- objective vectors
- parameters

application dependent:
- variation operators
- stores and manages individuals

PISA Website



PISA Example



The EMO Community

Links:

- EMO mailing list:
  http://w3.ualg.pt/lists/emo-list/
- EMO bibliography:
  http://www.lania.mx/~ccoello/EMOO/

Events:

- Conference on Evolutionary Multi-Criterion Optimization (EMO 2007 to be held in Japan)

Books:

- ***Multi-Objective Optimization using Evolutionary Algorithms***
  Kalyanmoy Deb, Wiley, 2001
- ***Evolutionary Algorithms for Solving Multi Evolutionary Algorithms for Solving Multi-Objective Problems Objective Problems***, Carlos A. Coello Coello, David A. Van Veldhuizen & Gary B. Lamont, Kluwer, 2002

References

- Bleuler, S., Brack, M.,Thiele, L., Zitzler, E. (2001). Multiobjective Genetic Programming: Reducing Bloat Using SPEA2. CEC-2001, pp. 536 - 543.
- Bleuler, S., Laumanns, M., Thiele, L., Zitzler, E.: PISA - A Platform and Programming Language Independent Interface for Search Algorithms. EMO 2003, pp. 494 - 508.
- Coello, C. (2000). Treating Constraints as Objectives for Single-Objective Evolutionary Optimization, Engineering Optimization, 32(3):275-308.
- Deb, K. (2001). Multi -Objective Optimization using Evolutionary Algorithms.Wiley.
- Deb, K., Pratap, A., Agarwal, S., Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, Volume 6, Issue 2, pp.182 – 197.
- Eisenring, M., Thiele, L., Zitzler, E. (2000). Handling Conflicting Criteria in Embedded System Design. IEEE Design & Test of Computers, Vol. 17, No. 2, pp. 51-59.
- Fonseca, C. M., Fleming, P. J. (1993). Genetic algorithms for multiobjective optimization: Formulation, discussion and generalization. In S. Forrest (Ed.), ICGA Proceedings, pp. 416–423.
- Fonseca, C. M., Fleming, P. J. (1998). Multiobjective optimization and multiple constraint handling with evolutionary algorithms—part i: A unified formulation. IEEE Transactions on Systems, Man, and Cybernetics 28(1), pp. 26–37.
- Fonseca, C., Knowles, J., Thiele, L., Zitzler, E. (2005). A Tutorial on the Performance Assessment of Stochastic Multiobjective Optimizers. EMO 2005.
- Grunert da Fonseca, V., Fonseca, C., Hall, A. (2001). Inferential Performance Assessment of Stochastic Optimisers and the Attainment Function. EMO 2001, pp. 213-225.
- Hubley, R., Zitzler, E., Roach, J. (2003). Evolutionary algorithms for the selection of single nucleotide polymorphisms. BMC Bioinformatics, Vol. 4, No. 30.
- Laumanns, M., Thiele, L., Deb, K., Zitzler, E. (2002). Combining Convergence and Diversity in Evolutionary Multi-objective Optimization. Evolutionary Computation, Vol. 10, No. 3, pp. 263—282.
- Laumanns, M., Thiele, L., Zitzler, E. (2005). An Efficient Metaheuristic Based on the Epsilon-Constraint Method. European Journal of Operational Research, to appear.
- Michalewicz, Z. (1992). Genetic Algorithms + Data Structures = Evolution Programs. Springer.
- Schaffer, J. D. (1985). Multiple objective optimization with vector evaluated genetic algorithms. In J. J. Grefenstette (Ed.), ICGA Proceedings, pp. 93–100.
- Silverman, B. W. (1986). Density estimation for statistics and data analysis, Chapman and Hall, London.
- Wright, J., Loosemore, H. (2001). An Infeasibility Objective for Use in Constrained Pareto Optimization. EMO 2001, pp. 256-268.
- Zitzler, E., Künzli, S. (2004). Indicator-Based Selection in Multiobjective Search. PPSN VIII, pp. 832-842.
- Zitzler, E., Laumanns, M,, Thiele, L. (2002). SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization. Evolutionary Methods for Design, Optimisation, and Control, CIMNE, Barcelona, Spain, pages 95-100.
- Zitzler, E., Thiele, L., Laumanns, M., Fonseca, C., Grunert da Fonseca, V.: Performance Assessment of Multiobjective Optimizers: An Analysis and Review. IEEE Transactions on Evolutionary Computation, Vol. 7, No. 2, pages 117-132.