A Tutorial on
Evolutionary Multi-Objective Optimization

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Overview of the Tutorial

- Multi-objective optimization
- Classical methods
- Evolutionary computing methods (EMO)
  - Differences
  - Non-elitist EMO
  - Elitist EMO
  - Constrained EMO
  - Applications of EMO
  - Salient research issues
- Conclusions

Multi-Objective Optimization

- We often face them

![Diagram showing the Pareto-optimal front with points A, B, and C. The axes are 10k Cost and Comfort, with points ranging from 40% to 90%.]

More Examples

- A cheaper but inconvenient flight
- A convenient but expensive flight

![Maps showing flight routes and costs.]

Prepared for GECCO 2004 (Seattle) by K, Deb (deb@iitk.ac.in)
Mathematical Programming Problem

Min/Max \((f_1(x), f_2(x), \ldots, f_M(x))\)

Subject to
\[ g_j(x) \geq 0 \]
\[ h_k(x) = 0 \]
\[ x^{(L)} \leq x \leq x^{(U)} \]

Minimize \(f_1(d, l) = \frac{x d^2}{2}\)
Minimize \(f_2(d, l) = \delta = \frac{64 P l^3}{3E \pi d^4}\)

subject to
\[ \sigma_{\text{max}} \leq S_y \]
\[ \delta \leq \delta_{\text{max}} \]

Pareto-Optimal Solutions

Non-dominated solutions: Among a set of solutions \(P\), the non-dominated set of solutions \(P'\) are those that are not dominated by any member of the set \(P\).

O(N log N) algorithms exist.

Pareto-Optimal solutions: When \(P = S\), the resulting \(P'\) is Pareto-optimal set.

A number of solutions are optimal.

Which Solutions are Optimal?

Relates to the concept of domination

\(x^{(1)}\) dominates \(x^{(2)}\) if
1. \(x^{(1)}\) is no worse than \(x^{(2)}\) in all objectives
2. \(x^{(1)}\) is strictly better than \(x^{(2)}\) in at least one objective
**Optimality Conditions**

**Fritz John Necessary Condition:**
Solution \( x^* \) satisfy
1. \( \sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0 \), and
2. \( u_j g_j(x^*) = 0 \) for all \( j = 1, 2, \ldots, J \).

Like single-objective optimization, local and global PO fronts exist:

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**Some Terminologies**

- **Ideal point,** \( z^* \): nonexistent, lower bound on Pareto-optimal set
- **Utopian point,** \( z^{**} \): nonexistent
- **Nadir point,** \( z^{\text{nad}} \): upper bound on Pareto-optimal set
- **Normalization:** 
  \[
  f_t^\text{norm} = \frac{f_t - z^*_t}{z^{\text{nad}}_t - z^*_t}
  \]

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**Differences with Single-Objective Optimization**

- One optimum versus multiple optima
- Requires search and decision-making
- Two spaces of interest, instead of one

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**Preference-Based Approach**

- Classical approaches follow it
Classical Approaches

- No Preference methods (heuristic-based)
- Posteriori methods (generating solutions)
- A priori methods (one preferred solution)
- Interactive methods (involving a decision-maker)

Weighted Sum Method

- Construct a weighted sum of objectives and optimize
  \[ F(x) = \sum_{m=1}^{M} w_m f_m(x). \]
- User supplies weight vector \( \mathbf{w} \)

α-Constraint Method

- Optimize one objective, constrain all other
  \[
  \text{Minimize } f_\mu(x), \\
  \text{subject to } f_m(x) \leq c_m, \; m \neq \mu;
  \]
- User supplies a \( \alpha \) vector
- Need to know relevant \( \alpha \) vectors
- Non-uniformity in Pareto-optimal solutions

Difficulties with Weighted Sum Method

- Need to know \( \mathbf{w} \)
- Non-uniformity in Pareto-optimal solutions
- Inability to find some Pareto-optimal solutions
**Difficulties with Most Classical Methods**

- Need to run a single-objective optimizer many times
- Expect a lot of problem knowledge
- Even then, good distribution is not guaranteed
- Multi-objective optimization as an application of single-objective optimization

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**A More Holistic Approach for Optimization**

- Decision-making becomes easier and less subjective
- Single-objective optimization is a degenerate case of multi-objective optimization
  - Step 1 finds a single solution
    - No need for Step 2
- Multi-modal optimization is a special case of multi-objective optimization

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**Two Goals in Ideal Multi-Objective Optimization**

1. Converge on the Pareto-optimal front
2. Maintain as diverse a distribution as possible
Why Use Evolutionary Algorithms?

- Population approach suits well to find multiple solutions
- Niche preservation methods can be exploited to find diverse solutions
- Implicit parallelism helps provide a parallel search
- Multiple applications of classical methods do not constitute a parallel search

History of Evolutionary Multi-Objective Optimization (EMO)

- Early penalty-based approaches
- VEGA (1984)
- Goldberg’s (1989) suggestion
- MOGA, NSGA, NPGA (1993-95) used Goldberg’s suggestion
- Elitist EMO (SPEA, NSGA-II, PAES, MOMGA etc.) (1998 Present)

What to Change in a Simple GA?

- Modify the fitness computation
- Emphasize non-dominated solutions for convergence
- Emphasize less-crowded solutions for diversity

Identifying the Non-dominated Set

Step 1 Set $i = 1$ and create an empty set $P'$. 
Step 2 For a solution $j \in P$ (but $j \neq i$), check if solution $j$ dominates solution $i$. If yes, go to Step 4.
Step 3 If more solutions are left in $P$, increment $j$ by one and go to Step 2; otherwise, set $P' = P' \cup \{i\}$.
Step 4 Increment $i$ by one. If $i \leq N$, go to Step 2; otherwise stop and declare $P'$ as the non-dominated set.
$O(MN^2)$ computational complexity
Finding the Non-dominated Set: An Efficient Approach

Kung et al.’s algorithm (1975)

Step 1 Sort the population in descending order of importance of $f_1$

Step 2, $\text{Front}(P)$ If $|P| = 1$, return $P$ as the output of $\text{Front}(P)$. Otherwise, $T = \text{Front}(p^{(1)}_{|P|})$ and $B = \text{Front}(p^{(|P|/2)}_{|P|})$. If the $i$th solution of $B$ is not dominated by any solution of $T$, create a merged set $M = T \cup \{i\}$. Return $M$ as the output of $\text{Front}(P)$.

$O(N \log N^M)$ for $M = 4$ and $O(N \log N)$ for $M = 2$ and 3

A Simple Non-Dominated Sorting Procedure

- Identify the best non-dominated set
- Discard them from population
- Identify the next-best non-dominated set
- Continue till all solutions are classified
- We discuss a $O(MN^2)$ algorithm later

Which are Less-Crowded Solutions?

- Crowding can be in decision variable space or in objective space

Non-Elitist EMOs

- Vector evaluated GA (VEGA) (Schaffer, 1984)
- Vector optimized EA (VOES) (Kursawe, 1990)
- Weight based GA (WBGA) (Hajela and Lin, 1993)
- Multiple objective GA (MOGA) (Fonseca and Fleming, 1993)
- Non-dominated sorting GA (NSGA) (Srinivas and Deb, 1994)
- Niched Pareto GA (NPGA) (Horn et al., 1994)
- Predator-prey ESO (Laumanns et al., 1998)
- Other methods: Distributed sharing GA, neighborhood constrained GA, Nash GA etc.
Vector-Evaluated GA (VEGA)

- Divide population into \( M \) equal blocks
- Each block is reproduced with one objective function
- Complete population participates in crossover and mutation
- Bias towards to individual best objective solutions
- A non-dominated selection: Non-dominated solutions are assigned more copies
- Mate selection: Two distant (in parameter space) solutions are mated
- Both necessary aspects missing in one algorithm

Multi-Objective GA (MOGA)

- Count the number of dominated solutions (say \( n \))
- Fitness: \( F = n + 1 \)
- A fitness ranking adjustment
- Niching in fitness space
- Rest all are similar to NSGA

Non-Dominated Sorting GA (NSGA)

<table>
<thead>
<tr>
<th>( f )</th>
<th>( f_1 )</th>
<th>( f_2 )</th>
<th>( f_{\text{max}} )</th>
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<tr>
<td>( \text{Front} )</td>
<td>\text{before}</td>
<td>\text{after}</td>
<td></td>
</tr>
<tr>
<td>1,10</td>
<td>2,20</td>
<td>4,20</td>
<td>2,10</td>
</tr>
<tr>
<td>6,70</td>
<td>7,40</td>
<td>4,09</td>
<td>1,69</td>
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<tr>
<td>4,20</td>
<td>7,08</td>
<td>4,64</td>
<td>2,30</td>
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<tr>
<td>2,00</td>
<td>4,00</td>
<td>5,00</td>
<td>1,00</td>
</tr>
<tr>
<td>3,75</td>
<td>4,00</td>
<td>6,00</td>
<td>1,60</td>
</tr>
<tr>
<td>3,00</td>
<td>6,00</td>
<td>25,00</td>
<td>3,30</td>
</tr>
</tbody>
</table>

- Niching in parameter space
- Non-dominated solutions are emphasized
- Diversity among them is maintained

Niched Pareto GA (NPGA)

- Solutions in a tournament are checked for domination with respect to a small subpopulation (\( t_{\text{dom}} \))
- If one dominated and other non-dominated, select second
- If both non-dominated or both dominated, choose the one with smaller niche count in the subpopulation
- Algorithm depends on \( t_{\text{dom}} \)
- Nevertheless, it has both necessary components
NPGA (cont.)

- Check for domination

Parameter Space

Population

Shortcomings of Non-Elitist EMOs

- Elite preservation is missing
- Elite preservation is important for proper convergence in SOEAs
- Same is true in EMOs
- Three tasks
  - Elite preservation
    - Progress towards the Pareto-optimal front
  - Maintain diversity among solutions

Elitist EMOs (cont.)

- Distance-based Pareto GA (DPGA) (Oszczka and Kundu, 1995)
- Thermodynamical GA (TDGA) (Kita et al., 1996)
- Strength Pareto EA (SPEA) (Zitzler and Thiele, 1998)
- Non-dominated sorting GA-II (NSGA-II) (Deb et al., 1999)
- Pareto-archived ES (PAES) (Knowles and Corne, 1999)
- Multi-objective Messy GA (MOMGA) (Veldhuizen and Lamont, 1999)
- Other methods: Pareto-converging GA, multi-objective micro-GA, elitist MOGA with coevolutionary sharing

Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

Elites are preserved

Non-dominated sorting

Crowding distance sorting

Rejected
NSGA-II (cont.)

Diversity is maintained: $O(MN \log N)$

Overall Complexity: $O(MN^2)$

An Illustration of NSGA-II

Six parents and six offspring

Parents after one iteration: (a, 3, 1, e, 5, b)

Strength Pareto EA (SPEA)

- Stores non-dominated solutions externally
- Pareto-dominance to assign fitness
  - External members: Assign number of dominated solutions in population (smaller, better)
  - Population members: Assign sum of fitness of external dominating members (smaller, better)
- Tournament selection and recombination applied to combined current and elite populations
- A clustering technique to maintain diversity in updated external population, when size increases a limit
**SPEA (cont.)**

- Fitness assignment and clustering methods

![Fitness Assignment Diagram]

- Clustering (d and p_max)

**Comparative Results: Convergence**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SCH</th>
<th>FON</th>
<th>POL</th>
<th>KUR</th>
</tr>
</thead>
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<tr>
<td>NSGA-II</td>
<td>0.003391</td>
<td>0.001931</td>
<td>0.015563</td>
<td>0.028964</td>
</tr>
<tr>
<td>SPEA</td>
<td>0.003403</td>
<td>0.125692</td>
<td>0.037812</td>
<td>0.045617</td>
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<tr>
<td>PAES</td>
<td>0.001313</td>
<td>0.151263</td>
<td>0.030864</td>
<td>0.057373</td>
</tr>
</tbody>
</table>

**Comparative Results: Diversity**

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>SCH</th>
<th>FON</th>
<th>POL</th>
<th>KUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSGA-II</td>
<td>0.477999</td>
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<tr>
<td>SPEA</td>
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<td>0.792352</td>
<td>0.977835</td>
<td>0.852990</td>
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<tr>
<td>PAES</td>
<td>1.063788</td>
<td>1.025272</td>
<td>1.020507</td>
<td>1.109858</td>
</tr>
</tbody>
</table>

**Pareto Archived ES (PAES)**

- An (1+1)-ES
- Parent $p_i$ and child $c_j$ are compared with an external archive $A_t$
- Child can enter the archive and can become a parent

![Pareto Archived ES Diagram]
**Constrained Handling**

- Penalty function approach
  \[ F_m = f_m + R_m \Omega(y). \]
- Explicit procedures to handle infeasible solutions
  - Jiménez’s approach
  - Ray-Tang-Seow’s approach
- Modified definition of domination
  - Fonseca and Fleming’s approach
  - Deb et al.’s approach

**Constrained NSGA-II Simulation Results**

\[
\begin{align*}
\text{(Min)} & \quad f_1(x) - z_1 \\
\text{(Min)} & \quad f_2(x) = \frac{1 + x_2^2}{1 + 2x_2^2} \\
x_2 + 9x_1 & \geq 6 \\
-x_2 + 9x_1 & \geq 1
\end{align*}
\]

**Constrain-Domination Principle**

A solution \( i \) constrained dominates a solution \( j \), if any is true:

1. Solution \( i \) is feasible and solution \( j \) is not.
2. Solutions \( i \) and \( j \) are both infeasible, but solution \( i \) has a smaller overall constraint violation.
3. Solutions \( i \) and \( j \) are feasible and solution \( i \) dominates solution \( j \).

**EMO Applications**

1. Identify different trade-off solutions for choosing one
2. Understanding insights about the problem
   - Reveal common properties among P×O solutions
   - Identify what causes trade-offs
   - Such information are valuable to users
   - May not exist other means of finding above
3. To aid in other optimization tasks
For a Better Decision-Making

- Spacecraft trajectory optimization (Coverston-Carroll et al., 2000) with JPL Pasadena
- Three objectives for inter-planetary trajectory design
  - Minimize time of flight
  - Maximize payload delivered at destination
  - Maximize heliocentric revolutions around the Sun
- NSGA invoked with SE TOP software for evaluation

Revealing Salient Insights: Truss Structure Design

Revealing Salient Insights: A Cantilever Plate Design

Earth Mars Rendezvous
**Trade-Off Solutions**

- Symmetry in solutions about mid-plane, discovery of stiffener

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**Revealing Salient Insights: Gear-box Design**

- A multi-spindle gear-box design
- 29 variables (integer, discrete, real-valued)
- 101 non-linear constraints

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**Outcome of an Analysis of Solutions**

- Module varies proportional to square-root of power ($m \propto \sqrt{P}$)
- Not known earlier

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**Revealing Salient Insights: Epoxy Polymerization**

- Three ingredients (NaOH, EP and AA0) added hourly
- 54 ODEs solved for a 7-hour simulation
- Maximize high chain length (Mn) and minimize polydispersity index (PDI)
- NaOH and AA0 varies in [0, 1] and EP in [0, 2]
- Total $3 \times 7$ or 21 variables
**Epoxy Polymerization (cont.)**

- A problem having a non-convex Pareto-optimal front
- Some patterns emerge among obtained solutions
- Need to check their chemical significance

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**Goal Programming and Others**

- Goal programming to find multiple solutions
  - Avoids fixing a weight vector (Deb, 2001)
- Genetic programming to reduce bloating: Program size as a second objective (Beuker et al., 2001)
- Reducing the chance of getting trapped in local optima (Knowles et al., 2001)
- Use secondary objectives for maintaining diversity (Abbass and Deb, 2003; Jensen, 2003)

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**EMO for Other Optimization Tasks**

- Constrained handling
  - Constraint violations as additional objectives (Sury, Radcliffe and Boyd, 1995; Coello (2000))
- Find partial front near zero-CV
- May provide a flexible search

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**Salient Research Tasks**

- Scalability of EMOs to handle more than two objectives
- Mathematically convergent algorithms with guaranteed spread of solutions
- Test problem design
- Performance metrics and comparative studies
- Other EMOs: Multi-modal EMOs, Dynamic EMOs
- Controlled elitism
- Developing practical EMOs: Hybridization, parallelization
- More application case studies
Scalability Issues

- Pareto-optimal region is a higher-dimensional surface
- Pareto-optimal front may be of smaller dimension

Some Results on Scalability of EMOs

- PESA, SPEA2, and NSGA-II compared up to 8 objectives (Khare, Yao, Deb, 2003)
- PESA best for convergence, but poor in diversity and running time (exponential)
- SPEA2 good for diversity, but poor in convergence and running time
- NSGA-II best for running time and good for diversity, but poor in convergence in higher objectives
- Very different outcome for large number of objectives

Scalability Issues (cont.)

- Complexity of niching procedures Who is one’s neighbor?
- Algorithms differ in maintaining diversity (NSGA-II vs. SPEA)

Convergence Issues

- Lukewarm interest till date
- NSGA-II, SPEA etc, have problem of convergence
  Pareto-optimal solutions can be lost to maintain a well-diverse set
- Rudolph and Agapie’s algorithm for guaranteed convergence
Convergence Issues (cont.)

- Shortcomings of Rudolph and Agapie's algorithm
  - No guarantee on spread of solutions
  - No time complexity measure
- Lamanns et al. (2001) suggest a remedy
  - α-dominance and diversity through hyper-box dominance
  - A new solution is compared with an archive in each iteration
  - α-dominance concept is practical

![Convergence Diagram](image)

Comparative Study on DTLZ Functions

<table>
<thead>
<tr>
<th>EMO</th>
<th>Convergence measure</th>
<th>Sparsity</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Std. Dev.</td>
<td>Average</td>
</tr>
<tr>
<td>DTLZ1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>NSGA II</td>
<td>0.0137106</td>
<td>0.000145</td>
<td>0.124474</td>
</tr>
<tr>
<td>C&amp;WNSGA II</td>
<td>0.0167465</td>
<td>0.000424</td>
<td>0.099478</td>
</tr>
<tr>
<td>PESA</td>
<td>0.0160282</td>
<td>0.002583</td>
<td>0.165878</td>
</tr>
<tr>
<td>SPEA2</td>
<td>0.0180222</td>
<td>0.000546</td>
<td>0.099889</td>
</tr>
<tr>
<td>α-MOEA</td>
<td>0.0148453</td>
<td>0.000244</td>
<td>0.099764</td>
</tr>
</tbody>
</table>

DTLZ2

| NSGA II | 0.014896 | 0.01028 | 0.139228 | 0.02891 |
| C&WNSGA II | 0.0252327 | 0.000996 | 0.195951 | 0.06013 |
| PESA | 0.0130232 | 0.00449 | 0.024258 | 0.23955 |
| SPEA2 | 0.0122429 | 0.00149 | 0.099771 | 0.00331 |
| α-MOEA | 0.012799 | 0.00123 | 0.0993707 | 0.000974 |

Test Problem DTLZ2

- EA and archive populations evolve
- One EA and one archive member are mated
- Archive update using α-dominance
- EA update using usual dominance

![Test Problem Diagram](image)
Finding a Partial Pareto-Optimal Set

- Using a DM's preference (not for a solution but for a region)
- Guided domination principle (Branke et al., 2000)
- Biased niching approach (Deb, 2002)
- Weighted domination approach (Parmee et al., 2000)

Distributed Computing: A Three-Objective Problem

- Spatial computing, not temporal

Distributed Computing of Pareto-Optimal Set

- Guided domination concept to search different parts of P-O region
- Usual island model with migration

Two-Objective Test Problems

- Pareto-optimal front is non-trollable and known
- ZDT problems:
  
  Min, \( f_1(x) = f_1(x_1) \)
  
  Min, \( f_2(x) = g(x_2) h(f_1, g) \)

- Choose \( f_1(), g() \) and \( h() \) to introduce various difficulties
Zitzler Deb Thiele’s Test Problems

ZDT1
\[ f_1(x) = z_1, \]
\[ g(x) = 1 + \frac{1}{N} \sum_{i=2}^{N} z_i, \]
\[ h(f_1, g) = 1 - \sqrt{f_1/g}. \]

ZDT2
\[ f_1(x) = z_1, \]
\[ g(x) = 1 + \frac{1}{N} \sum_{i=2}^{N} z_i, \]
\[ h(f_1, g) = 1 - (f_1/g)^2. \]

ZDT3
\[ f_1 = z_1, \]
\[ g = 1 + \frac{1}{N} \sum_{i=2}^{N} z_i, \]
\[ h = 1 - \sqrt{f_1/g} = (f_1/g) \sin(10\pi f_1). \]

ZDT4
\[ f_1 = z_1, \]
\[ g = 1 + \frac{1}{N} \sum_{i=2}^{N} z_i, \]
\[ h = 1 - \sqrt{f_1/g}. \]

Parameter Interactions

- More difficult problems using parameter interactions
- True variables \( y_i \) are linearly related to other auxiliary variables \( x_i \):
  \[ x = My \]
- Fitness computed using \( x \)
- All parameters must change to remain Pareto-optimal
Scalable Test Problems (Deb et al. 2001)

**Step 1** Define Pareto-optimal front mathematically

**Step 2** Build the objective search space using it

**Step 3** Map variable space to objective space

- Scalable DTLZ problems suggested

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Constraint Surface Approach

- Define a rectangular hyper-box
- Chop off regions using constraints
- Adv: Easy to construct
- Disadv: Difficult to define Pareto-optimal front

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Constrained Test Problem Generator

- Some test problems in Veldhuizen (1999)
- More controllable test problems are called for

Minimize $f_1(x) = x_1$
Minimize $f_2(x) = g(x) \left( 1 - \frac{f_1(x)}{g(x)} \right)$
Subject to $c(x) \equiv \cos(\theta)(f_2(x) - c) - \sin(\theta)f_1(x) \geq a \left[ \sin(b \pi (\sin(\theta)(f_2(x) - c) + \cos(\theta)f_1(x))) \right]^d$

Various Parameter Settings

$\theta = -0.2\pi, \quad b = 10, \quad c = 1, \quad e = 1.$

CTP2: $d = 6$ and $a = 0.2$  
CTP 7: $\theta = -0.05\pi, a = 40, b = 5, c = 1, d = 6, e = 0$
**Performance Metrics**

- A recent study by Zitzler et al. suggests at least $M$ metrics.
- Two essential metrics functionally:
  - Convergence measure
  - Diversity measure

**Metrics for Convergence**

- Error ratio:
  \[ ER = \frac{\sum_{i=1}^{\vert \mathcal{Q} \vert} e_i}{\vert \mathcal{Q} \vert} \]
- Set Coverage:
  \[ C(A, B) = \frac{\vert \{ b \in B \vert \exists a \in A : a \leq b \} \vert}{\vert B \vert} \]
- Generational distance:
  \[ GD = \left( \frac{\sum_{i=1}^{\vert \mathcal{Q} \vert} d_i^p}{\vert \mathcal{Q} \vert} \right)^{1/p} \]

**Metrics for Diversity**

- Spacing:
  \[ S = \sqrt{\frac{1}{\vert \mathcal{Q} \vert} \sum_{i=1}^{\vert \mathcal{Q} \vert} (d_i - \bar{d})^2} \]
- Spread:
  \[ \Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{\vert \mathcal{Q} \vert} |d_i - \bar{d}|}{\sum_{m=1}^{M} d_m + \vert \mathcal{Q} \vert \bar{d}} \]
- Chi-square like deviation measure

**Metrics for Diversity (cont.)**

- Distance from $P^*$
- Entropy measure
Metrics for Convergence and Diversity

- Hypervolume
- Attainment surface method

Running Metrics

- Like SGA, define metric that shows generation-wise variation
- Identify non-dominated set $F^{(t)}$ of each population $P^{(t)}$
- Comparison Set ($H$):
  - If exact P-O front is known, $H = P^*$
  - Else $H = \text{Non-dominated}(\bigcup_{t=0,1,\ldots} F^{(t)})$
- Convergence metric $C^{(t)}$: Average distance of each member of $F^{(t)}$ from $H$)
- Diversity metric $D^{(t)}$: Similar to entropy measure

Running Metrics on ZDT1

- Using Pareto-optimal solutions

Running Metrics (cont.)
**Scheduling EMOs**

- Objective space niching allows a straightforward application
- Most techniques use a local search
- Job-shop scheduling (Ishibuchi and Murata, 1998)

**Multi-Modal EMOs**

- Different solutions having identical objective values
- Multi-modal Pareto-optimal solutions: Design, Bioinformatics

**Multiple Gene Subsets for Leukemia Samples**

- Deb and Reddy (BioSystems, 2003)
- Multiple (26) four-gene combinations for 100% classification
- Discovery of some common genes

**Hybrid EMOs**

- Combine EAs with a local search method
  - Better convergence
  - Faster overall optimization
- Two hybrid approaches
  - Local search to update each solution in an EA population (Ishibuchi and Murata, 1998; Jaskiewicz, 1998)
  - First EA and then apply a local search (Deb and Goel, 2000)
### Posteriori Approach in an EMO

- Which objective to use in local search?

### An Idea for Local Search

- Extreme solutions are assigned extreme weights
- Linear relation between weight and fitness
- Many solutions can converge to same solution after local search

### Posteriori Versus Online Approaches

- Cantilever plate design
- Compared for identical evaluations
- Posteriori finds a better front

### Which Pareto-Optimal Solution to Choose?

- Needs to involve a decision-maker (DM)
- Interactive EMO is called for *Not much study yet*
- A few difficulties:
  - The act of a DM makes it a single-obj. problem
  - But, obj. is not known precisely and changes with iteration
  - EMO finds many solutions, but only one is desired
  - Is DM interested in evaluating more than one solution?
- EMO as a starter, then a classical approach
A Possible Interactive EMO

EMO: Find potentially good solutions robust, knee-like, etc.

Classical: Concentrate in an area based DM's preference

Conclusions

- Ideal multi-objective optimization is generic and pragmatic
- Evolutionary algorithms are ideal candidates
- Many efficient algorithms exist, more efficient ones are needed
- With some salient research studies, EMOs will revolutionize
  the act of optimization
- EAs have a definite edge in multi-objective optimization and
  should become more useful in practice in coming years

EMO Resources

Books
- C, A, C. Coello, D, A, VanVeldhuisen, and G, Lamont,
  Evolutionary Algorithms for Solving Multi-Objective Problems,
- K, Deb, Multi-objective optimization using evolutionary algorithms,
  problems)
- Paper Repository: http://www.lania.mx/~ccolloc/EMO/
  Conference Proceedings

  in Computer Science 1993), Heidelberg: Springer.

EMO Resources (cont.)

Conference Proceedings (cont.)
- Fonseca, C., Zitzler, E., Deb, K., Fleming, P, and Thiele, L, (Eds)
  (2003), Evolutionary Multi-Criterion Optimization (Lecture Notes in
  Computer Science 2632), Heidelberg: Springer.
- EMO-2005 in Mexico (http://www.cimat.mx/emo2005/)
  Mailing List
- emo-list@ualg.pt
- MCRIT-L@LISTSERV.UGA.EDU

Public-Domain Source Codes
- NSGA-II in C: http://www.iitk.ac.in/kangal/soft.htm
- SPEA2 and others: http://www.tik.ee.ethz.ch/pisa
  Java codes: University of Dortmund