Integrated Genetic Algorithm with Hill Climbing for Bandwidth Minimization Problem

Andrew Lim[1], Brian Rodrigues[2], and Fei Xiao[3]

[1] Department of Industrial Engineering and Engineering Management
Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong
iealim@ust.hk
[2] School of Business, Singapore Management University, 469 Bukit Timah Road
Singapore 259756
br@smu.edu.sg
[3] Department of Computer Science, National University of Singapore
3 Science Drive 2, Singapore 117543
xiaofei@comp.nus.edu.sg

Abstract. In this paper, we propose an integrated Genetic Algorithm with Hill Climbing to solve the matrix bandwidth minimization problem, which is to reduce bandwidth by permuting rows and columns resulting in the nonzero elements residing in a band as close as possible to the diagonal. Experiments show that this approach achieves the best solution quality when compared with the GPS [1] algorithm, Tabu Search [3], and the GRASP with Path Relinking methods [4], while being faster than the latter two newly-developed heuristics.

1 Introduction

For $A = \{a_{ij}\}$ a symmetric matrix, the matrix bandwidth minimization problem is to find a permutation of rows and columns of the matrix A so as to bring all the non-zero elements of A to reside in a band that is as close as possible to the main diagonal, that is to $Minimize\{max\{|i-j| : a_{ij} \neq 0\}\}$. In

The bandwidth minimization problem has been found to be relevant to a wide range of applications. For example, in solving large linear systems, Gaussian elimination can be performed in $O(nb^2)$ time on matrices of bandwidth b, which is much faster than the normal $O(n^3)$ algorithm if $b << n$. In this work, we propose a Genetic Algorithm (GA) integrated with Hill Climbing to solve the bandwidth minimization problem. Genetic Algorithms [2], have been widely used in solving combinatorial optimization problems. The Genetic Algorithm, while having been shown to be especially good in global search, is not well suited for tuned search. In developing a solution for the bandwidth minimization problem, we have therefore combined a Genetic Algorithm with a Hill Climbing Algorithm. Computational Results show that this integrated algorithm performs well. Compared with the classical GPS algorithm, and the newly-developed techniques of Tabu Search and GRASP with Path Relinking, it provides the best solution quality while being faster than the latter two heuristics.

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2 Computational Experiments

we compare our GA_HC with best known heuristic methods on two sets of instances from Harwell-Boeing Matrix Collection (`http://math.nist.gov/MatrixMarket/data/Harwell-Boeing`).

Table 1. Performance comparison according to problem size

	GPS	TS	GRASP_PR	GA_HC
33 instances with n=30,..,199				
$B_f(G)$	31.42	23.33	22.52	22.48
Deviation	39.77%	3.78%	0.18%	0.00%
CPU seconds	0.003	2.36	4.21	2.54
80 instances with n=200,...,1000				
$B_f(G)$	156.38	100.78	99.43	97.02
Deviation	61.18%	3.88%	2.49%	0.00%
CPU seconds	0.11	121.66	323.19	85.22

We can find in Table 1 that the best solution in quality is obtained for the first test set by the GA_HC which is about 40 percent better than the classical GPS algorithm. On the second test set, the GA_HC also obtains the best solution in quality and is faster than the TS and GRASP_PR. Overall, our GA_HC obtains the best solution in quality comparing with the best known algorithms. And it is faster than the newly developed TS and GRASP_PR algorithm.

3 Conclusions

We have proposed a combined GA with Hill Climbing for solving the bandwidth minimization problem. Extensive experiments have shown that our new GA with Hill Climbing obtains best solutions for the bandwidth minimization problem in reasonable time.

References

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