RoboStar Technology
Modelling Uncertainty in RoboChart using Probability

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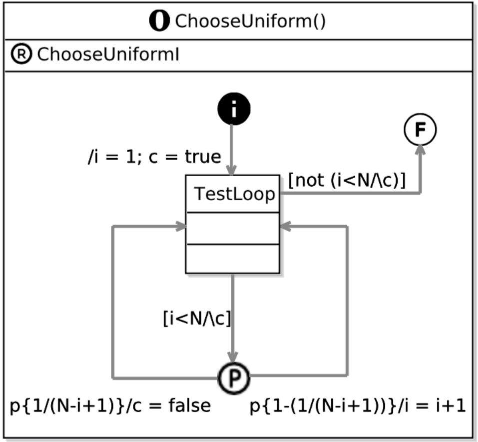
Overview

- Modelling uncertainty in robotic applications.
- Example: pose estimation.
- Fitting models to data.
- Example: least-squares regression.
- Example: random sample consensus.
- Model checking in Prism: small number of data points!
- Theorem proving in Isabelle: arbitrary data set.

A Probabilistic Algorithm in RoboChart



$$1 - \frac{1}{N-i+1} = \frac{N-i}{N-i+1}$$

Make the choice on the k'th iteration.

$$\frac{N-1}{N} \times \frac{N-2}{N-1} \times \cdots \times \frac{N-k+1}{N-k+2} \times \frac{1}{N-k+1}$$

$$= \frac{1}{N}$$

Modelling Uncertainty in Robotic Applications

Six sources of uncertainty: (we deal with two here)

1. Unpredictable physical world. ✗
2. Sensors physical laws and noise. ✓
3. Actuators control noise and deterioration. ✗
4. Model errors abstract physics and environment. ✗
5. Control algorithms accuracy vs real time. ✓
6. Human factors introduce uncertain behaviours. ✗

Example: Pose estimation algorithms.

- Localisation, navigation problems: robot's position, orientation, velocity, ...
- Algorithms: Kalman filters, Bayesian filters, particle filters, ...

Ransac: Random Sample Consensus

- Iterative method to estimate mathematical model parameters.
- Observed data contains outliers.
- Must be filtered out: no influence on estimated parameters.
- Probabilistic parameter estimates.
- Algorithm due to Fischler & Bolles (SRI) 1981.
- Ransac solves Location Determination Problem (LDP).
- Given image of landmarks with known locations determine viewpoint.
- How many landmarks are required?
- Automatic solution of LDP under difficult viewing conditions.
- No known program verification of Ransac.

Ransac Algorithm: Context

- Widely used in model parameter estimation problems.
- Popular method for modelling sensor data.
- Used in some vision-based SLAM algorithms.
- (Simultaneous Localisation and Mapping.)
- Ransac provides efficient solution for image matching.
- Ransac is easily implemented and is robust.
- Standard Ransac sometimes suffers from low performance.
- Solution may not be reached when algorithm terminates.
- Relationship between number of iterations and probability of no outliers.

Typical Model Fitting Method: Least Squares Regression

- Model: $y = mx + b$.
- Observed value: (x_i, y_i), vertical residual: $y_i - (mx_i + b)$.
- Normalise: $(y_i - (mx_i + b))^2$: positive values, exaggerated outliers.
- Objective: minimise error ε.
- Where ε is the sum of normalised residuals:

$$\varepsilon = \sum_{i=1}^{N} (y_i - (mx_i + b))^2$$

- Formula describes up-open parabola.
- Minimum = parabolic vertex.

Least Squares Regression

Model parameters m (slope) and b (intercept).

Step 1: For each (x, y) point calculate x^2 and xy.

Step 2: Sum all x, y, x^2 and xy.

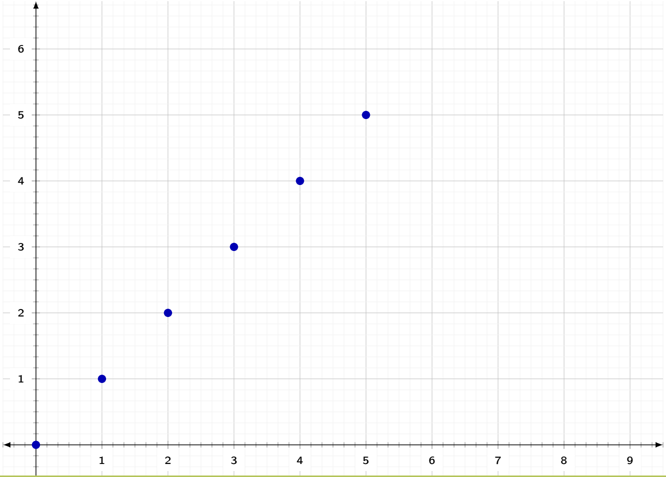
Step 3: Calculate slope m:

$$m = \frac{N\sum(xy) - \sum x \sum y}{N\sum(x^2) - (\sum x)^2}$$

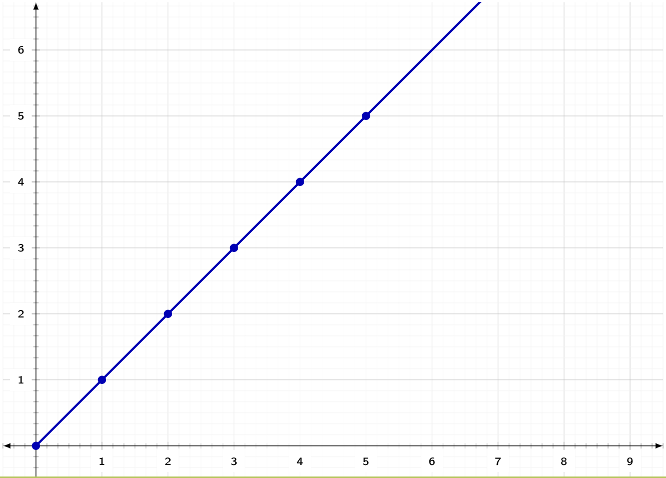
Step 4: Calculate intercept b:

$$b = \frac{\sum y - m\sum x}{N}$$

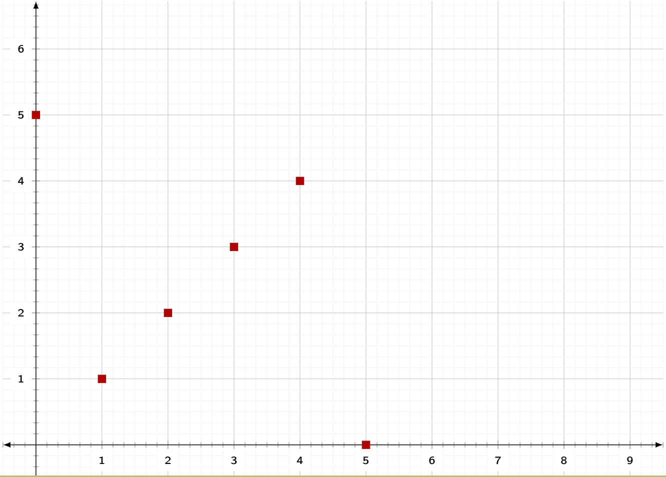
Examples of Least Squares Regression



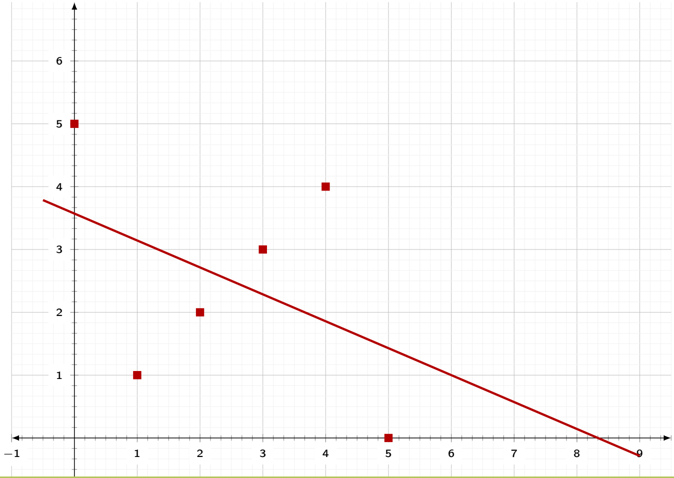
Examples of Least Squares Regression



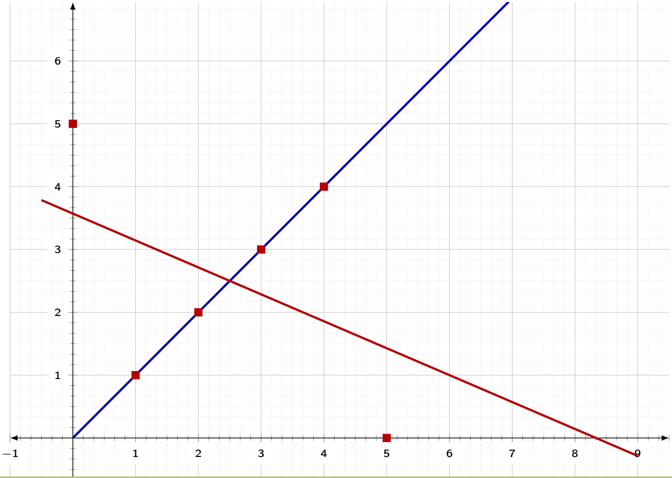
Examples of Least Squares Regression with Outliers



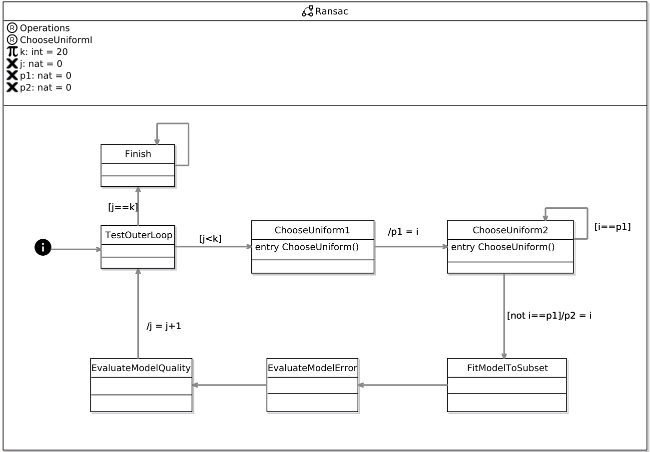
Examples of Least Squares Regression with Outliers

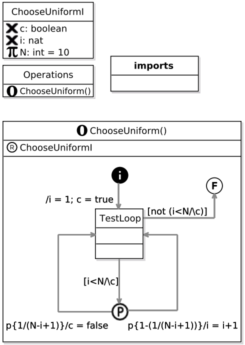


Examples of Least Squares Regression with Outliers

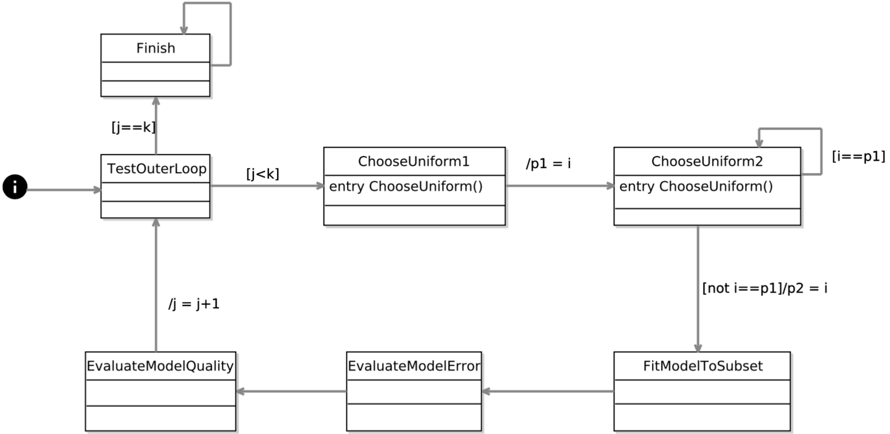


Ransac Algorithm





Ransac Algorithm



Model Checking Ransac

- Probabilistic model checking using Prism.
- Works comfortably for 6–8 points!
- Statistical model checking for more points.
- Discrete event simulation.
- RoboChart more abstract than Prism.
- High-level support for algorithms.
- RoboChart: control flow + structured, mathematical types.
- Automated translation from RoboChart to Prism.
- Integration of Prism tool into RoboTool.
- Formal data refinement: Ransac algorithm to reactive module.

Results

- Fast compilation versus slow model checking: 400 lines of RM.
- Slow compilation versus fast model checking:
- 300 lines of RM (200 lines formulas, 100loc).
- Property: how many iterations for 95% confidence no outliers?
- Automatically translate RoboChart to Prism.
- Run Prism model checker on sample data.
- Repeat over modestly large number of example.
- 14 iterations: consistent with theoretical analysis.
- Important for real-time guarantees for robot control.

Theoretical Analysis

$$k = \frac{\log(1-p)}{\log(1-(I/N)^d)}$$

k : number of iterations
p : desired probability of success
d : model estimation quorum
I : inliers
N: data points

- Ratio I/N is estimated: probability point is inlying.
- Assume d points for quorum are independent.
- $(I/N)^d$: probability that all d points are inliers.

Theoretical Analysis

- $1 - (I/N)^d$: probability at least one outlier:
  - Bad model could be estimated from this data.
- $(1 - (I/N)^d)^k$: probability algorithm never selects d inliers.
- This gives us $1 - p = (1 - (I/N)^d)^k$, and so

$$k = \frac{\log(1 - p)}{\log(1 - (I/N)^d)}$$

- Calculating

$$\begin{aligned} k &= \log(1 - 0{\cdot}95) / \log(1 - (4/6)^4) \\ &= 13{\cdot}61313558044104 4 \end{aligned}$$

From Model Checking to Theorem Proving

- Model checking describes bounded instance of RoboChart system.
- Advantage: you can automatically check some properties.
- Your model has to have small enough number of states.
- Class of formulas you can express may be limited.
- Moves effort from proof to modelling.
- Theorem prover works on potentially unbounded state space, even infinite.
- Express arbitrary properties, but proof automation can be disappointing.
- Isabelle/UTP: a mature and trustworthy theorem prover.
- Mechanised verification of RoboCharts: research objective

  Sound automated theorem prover for diagrammatic descriptions of reactive, timed, probabilistic controllers for RAS.

Conclusion

Take-home message for roboticists:

1. Uncertainty is essential to RAS.
2. Probabilism is an approach to modelling uncertainty.
3. RoboChart supports probabilism.
4. Translate to Prism and Isabelle for analysis.
5. Obtain probabilistic guarantees of behaviour in the face of uncertainty.