Searching for maximal entanglement in multipartite quantum systems

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Searching for maximal entanglement in multipartite quantum systems

- Outlining the problem
- Searching for Entanglement
  - An appropriate search algorithm
  - Choosing an entanglement measure
- Results: 4 qubit entanglement
The Problem

- Entanglement in multipartite systems.
  - Lack of known results
- Can we try a computational approach?
  - Searching for highly entangled states.
  - Essentially a function optimisation problem.
Hill Climbing Search

• The Search Space
  - Set of all mixed quantum states of n qubits.
  - It turns out that the search space has a global maximum

• Hill Climbing Search
  - Evaluation Function: maximising an entanglement measure.
  - Move function:
    ◦ Coefficients of pure states in density matrix construction.
    ◦ Probabilities used in density matrix construction.
    ◦ Construct new density matrix and reapply entanglement measure.
  - Maximise the entanglement measure to find highly entangled states.
Measuring Entanglement

- Not easy to find a computationally tractable entanglement measure on multipartite systems!
- Some measures on 2 qubits but still non-trivial to calculate.
- **Peres-Horodecki Criterion** – positive partial transpose
  - 2 qubits: necessary and sufficient condition for separability.
  - 3+ qubits: necessary condition.
- Negative eigenvalues of partial transpose
  → state contains some degree of entanglement
- Quantify this entanglement by taking the sum of the negative eigenvalues.
4 Qubit Entanglement

• This project examines 4 qubit entanglement.
• For 2 and 3 qubits we can use known results to obtain limits for maximal entanglement.
• Perhaps the search can suggest a maximal entanglement limit for 4 qubits.
4 Qubit Entanglement

\[ E = -5.74991 \]

\[ 0.38382+0.34249i|0001> + 0.34562+0.035282i|0100> + \\
0.066096+0.33982i|0110> + 0.29985+0.15992i|1000> + \\
0.34982+0.033982i|1010> + 0.07796+0.49974i|1111> \]
4 Qubit Entanglement

\[0.384 + 0.34357i|0001 \rangle + 0.34357|0100 \rangle + 0.34357i|0110 \rangle + 0.30315 + 0.16168i|1000 \rangle + 0.35368|1010 \rangle + 0.50526i|1111 \rangle\]

\[E = -5.8253\]

\[0.34899 + 0.18945i|0010 \rangle + 0.32904 + 0.38887i|0101 \rangle + 0.28417|0110 \rangle + 0.35896 + 0.36893i|1000 \rangle + 0.28916|1011 \rangle + 0.09971 + 0.3789i|1111 \rangle\]

\[E = -5.7792\]
Work in Progress

• Search currently begins at a random point in the search space
  – Starting the search at one of the highly entangled states.
• 5 qubits?
• Exponential size of the quantum state space makes the search computationally expensive
  – Manipulating very large matrices.