Industrial Hypercomputation

Mike Stannett
Department of Computer Science, University of Sheffield
Regent Court, 211 Portobello Street, Sheffield S1 4DP, United Kingdom
M.Stannett@dcs.shef.ac.uk

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According to the Church-Turing Thesis, it is impossible to devise a 'recipe-driven' model of computing that is more powerful than the Turing machine – but it says nothing about other forms of computing.

Hypercomputation is the study of real or theoretical systems which display 'super-Turing' potential, systems that can do provably more than today's machines. The study of these systems is truly interdisciplinary, involving fields as diverse as computer science [13], mathematics [18], quantum theory [5], biology [7], neural networks [12], general relativity [4], the theory of intelligence [6] and philosophy [2]. Surprisingly for such an unusual topic, the field has also reached public notice with recent articles in the popular computing press [1].

Mathematicians love to explore the limits of their theories – even Turing suggested that human intelligence requires more than the standard version of computing described by the Turing machine [16, 10]. Subsequent work has done much to characterise the computational difficulty of mathematical concepts; in addition to the development of the Arithmetical Hierarchy [9], key undecidability results range from Church and Turing's work on the Halting Problem [3, 15] to Matiyasevitch's celebrated proof of the undecidability of Hilbert's Tenth Problem [8].

More recently, researchers have begun to ask whether *physical* systems exist which display hypercomputational traits [14] – could we one day *design and build* a hypercomputer? The answer to this question depends on precisely what you think physics itself looks like. Computers are physical devices, and what they can do depends on whether they're operating in a Newtonian universe, or near the the vicinity of a space-time singularity.

The most familiar non-standard model is the *quantum computer*. These are super-Turing in a very real sense – for some problems, they can outperform even the fastest Turing machine [11]. Less well-known are hypercomputers that exploit general relativistic features. In *Malament-Hogarth spacetimes*, for example, one can set up a thought experiment in which a computer falling into a singularity experiences infinite time, yet its entire existence seems to last only finitely long to an external observer [4]. Suppose we want to find out if some problem will eventually halt; all we have to do is set it running on a PC that's falling into the singularity. From our point of view, the program's entire execution takes place in finite time, and we can arrange matters so that a message is sent by the PC to an agreed location if and only if the program eventually halts. In other words, we can solve the Halting Problem.

But we don't need exotic physics to solve the Halting Problem – even Newtonian physics is powerful enough to allow hypercomputation. One of the more surprising findings of recent years is Xia's observation that a physical body under the influence of gravity can be propelled to 'infinity' in finite time [19]. If we arrange for a Turing machine to follow such a trajectory, and to execute one instruction for every mile of its journey, we again manage to complete an infinite number of program instructions in finite time; so watching the machine's execution through a telescope allows us to determine the Halting Problem.

So why isn't the world rushing to build hypercomputers? The problem is one of technical feasibility. In the first place, we don't know for certain what physics is really like, and even the examples described above have their fair share of teething problems (where do you buy a space-time singularity, for example?). Moreover, the various examples people have suggested to date tend to refer to universes that are exclusively quantum mechanical, or exclusively relativistic, or exclusively Newtonian – but the 'real world' combines features from all of these basic underlying models. So the race is on to describe a physically sensible hypercomputer that matches the world around us; only then will it be time to worry about making hypercomputers, first on an experimental, and eventually on an industrial, scale.

References

[1] M. Bedford. Computing the Uncomputable. *Computer Shopper*, 216:220–223, February 2006.

[2] S. Bringsjord and K. Arkoudas. The modal argument for hypercomputing minds. *Theoretical Computer Science*, 317:167–190, 2004.

[3] A. Church. An Unsolvable Problem of Elementary Number Theory. *Amer. J. Math.*, 58:345–363, 1936.

[4] M. Hogarth. Does General Relativity Allow an Observer to View an Eternity in a Finite Time. *Foundations of Physics Letters*, 5:73–81, 1992.

[5] T.D. Kieu. Hypercomputation with quantum adiabatic processes. *Theoret. Computer Science*, 317:93–104, 2004.

[6] P. Kugel. Towards a theory of intelligence. *Theoretical Computer Science*, 317:13–30, 2004.

[7] B.J. MacLennan. Natural computation and non-Turing models of computation. *Theoretical Computer Science*, 317:115–145, 2004.

[8] Y. Matiyasevich. *Hilbert's Tenth Problem.* MIT Press, Cambridge, MA, 1993.

[9] Y.N. Moschovakis. *Descriptive Set Theory.* North Holland, 1980.

[10] M.H.A. Newman. Alan Mathison Turing 1912 - 1954. *Biographical Memoirs of the Fellows of the Royal Society*, 1:253–263, November 1955.

[11] P. Shor. Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer. In S. Goldwasser, editor, *Proc. 35th Annual Symp. on Foundations of Computer Science*, page 124. IEEE Computer Society Press, Los Alamitos, CA, 1994.

[12] H.T. Siegelmann. *Neural Networks and Analog Computation.* Birkhäuser, 1999.

[13] M. Stannett. Hypercomputational Models. In C. Teuscher, editor, *Alan Turing: Life and Legacy of a Great Thinker*, pages 135–158. Springer-Verlag, Berlin, 2004.

[14] M. Stannett. The Case for Hypercomputation. *J. Applied Mathematics and Computation*, 2006. To appear.

[15] A.M. Turing. On Computable Numbers, with an Application to the Entscheidungsproblem. *Proc. LMS, Series 2*, 42:230–265, 1936-1937.

[16] A.M. Turing. *Systems of Logic based on Ordinals.* PhD thesis, Princeton University, 1938. Published as [17].

[17] A.M. Turing. Systems of Logic based on Ordinals. *Proc. LMS, Series 2*, 45:161–228, 1939. Originally produced as [16].

[18] B. Wells. Is There a Nonrecursive Decidable Equational Theory? *Minds and Machines*, 12:303–326, 2002.

[19] Z. Xia. The existence of non-collision singularities in the n-body problem. *Annals of Maths*, 135(3):411–468, 1992.