Harmony-Seeking Computations

A Science of Non-Classical Dynamics Based On The Progressive Evolution Of The Larger Whole

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In this paper, I am trying to lay out a new form of computation, which focuses on the harmony reached in a system. This type of computation in some way resembles certain recent results in chaos theory and complexity theory. However, the orientation of harmony-seeking computation is toward a kind of computation which finds harmonious configurations, and so helps to create things, above all, in real world situations: buildings, towns, agriculture, and ecology.

I try to show that this way of thinking about computation is closer to intuition and personal feeling than the processes we typically describe as "computations." It is also more useful, potentially, in a great variety of tasks we face in building and taking care of the surface of the Earth, and quite different in character since it is value-oriented, not value-free.

Examples are taken from art, architecture, biology, physics, astrophysics, drawing, crystallography, meteorology, dynamics of living systems, and ecology.

CONTENTS

Preface

This paper is based on results first presented in the four-volume work *The Nature of Order*.[1] The essential results, providing the underpinning for the current paper, are the following:

There is a structure, visible in any given part of the world, which we may call *the wholeness*. The wholeness is an abstract mathematical structure, existing in space. It captures what we may loosely consider as the global character of a given configuration, in itself and in relation to the world around it. The wholeness is a structure which exists at many levels of scale, and covers the interrelationships of the configurations at different scales. The primary entities of which the structure is built are centers, centers which become activated in the space as a result of the configuration as a whole. Centers have different levels of strength or coherence. The coherence of a configuration is caused by relationships among other centers. In particular, there are fifteen types of relationships among centers which increase or intensify the strength of any given center. These fifteen properties are listed below, and define the way that configurations within a configuration help each other. Within this scheme, unfolding of new configurations is a natural process, and can be understood and followed. We thus have a basis for making computations about unfolding. These are somewhat similar to the bifurcations that have been observed and analyzed in complex non-linear systems, but they are much richer and more complex than the theory of bifurcations can at present contemplate. Unfolding occurs as a result of structure-preserving (SP-) transformations. These SP-transformations are combinations and sequences of 15 possible spatial transformations based on the fifteen properties that determine how coherent centers may be built from one another. An advanced computational theory of these SP transformations does not yet exist, but it is my aim, in this paper, to show you how unfolding is built from these transformations, and how the outline of a new (computable) theory of unfolding can be established.

Please note that in this context the term "structure-preserving" relates to preserving the structure of *wholeness*. The term structure-preserving is sometimes used in mathematics to refer to transformations which preserve some particular structural aspect of a given system, but this particular aspect may be arbitrarily chosen. In my use of the term, it means that the given transformation preserves the *whole*, and is not arbitrary, but dependent on the observer's ability to *see* the whole.

II Fifteen Properties

Fifteen structural features that appear again and again and again in coherent systems are summarized very briefly.

1. STRONG CENTERS. Wholeness is composed of centers and centers arise from wholeness. A given wholeness is coherent to the extent that the centers which are in it are coherent. Centers are recursive in structure. Each center that exists acts to strengthen other centers, larger and smaller.

2. LEVELS OF SCALE. When a configuration contains centers, these centers will be associated with centers at a range of sizes that occur at well-marked levels of scale. The scale jumps between levels are surprisingly small: in coherent systems the centers of different sizes are often in size-ratios of 2 to 1, 3 to 1 and 4 to 1. If the jumps are larger – for instance 10 to 1 or 100 to 1, without intermediate levels, the coherence will tend to fall apart. This means that in coherent structures, the ladder or hierarchy of levels has evenly spaced rungs, and is continuous and smooth.

3. BOUNDARIES. Strong centers typically , though not always, have thick boundaries around them. These thick boundaries may exists in 1-, 2- or 3-D, and are themselves made of smaller centers which have the levels of scale relation to the larger centers being surrounded. These boundaries typically form a transition zone of interaction allowing physical, chemical, or biological processes to occur without contaminating the centers being surrounded, and the boundary is often on the order of one scale jump smaller than the thing it surrounds, thus may be equal to the diameter or half the diameter of the thing surrounded. Boundaries help form the field of force that creates and intensifies a center; they surround, enclose, separate and connect.

4. ALTERNATING REPETITION. When repetition of similar centers occurs in a coherent system, the centers typically alternate with a second system of centers, thus forming a double system of centers with a beat or rhythmic alternation, from the positive space between the repetitions. centers intensify other centers by repeating in a rhythm; when a second system of centers also repeats, in parallel, it intensifies the first system by providing a kind of counterpoint, or opposing beat.

5. POSITIVE SPACE. In coherent systems, there is no "background," or figure and ground. Instead, every bit of space is coherent, well shaped; and the space between coherent bits of space are also coherent and well-shaped. Thus every bit of space swells outward, is substantial in itself, and is never the leftover from an adjacent shape – like ripening corn, each kernel swelling until it meets the others, each one having its own positive shape caused by its growth as a cell from the inside. The positiveness of space is difficult to pin down exactly, but it is almost like a weak kind of convexity or quasi-convexity. In systems where the space is positive, the principal elements of space are nearly all quasi-convex, and the pieces of space between these elements are also quasi-convex.

6. GOOD SHAPE. This describes a particular, coherent quality of the particular shapes that occur in or around a coherent center. This kind of "good" shape is somewhat unusual, and is marked by the fact that the shape itself is made up from multiple coherent centers which together form the shape, and of other coherent centers which together form the shape of the space around the shape.

7. LOCAL SYMMETRIES. Strong centers often have strong symmetries: and local parts of space with strong symmetries will typically be strong centers. This feature binds together smaller centers within the whole, further creating coherence.

8. DEEP INTERLOCK AND AMBIGUITY. This occurs where coherent centers are "hooked" into their surroundings, making it difficult to disentangle the center from its surroundings. Often there are ambiguous zones which belong both to the center and to its surroundings, again making it difficult to disentangle the two.

9. CONTRAST. Every center relies to some degree on the contrast of discernible opposites, and on its differentiation from the ground where it occurs. It is

intensified when the ground, against which it is contrasted, is clarified and itself becomes made of centers: all this differentiation arises from the degree or sharpness of contrast that is attained. Note, though, that too-sharp a degree of contrast is offset by NOT-SEPARATENESS, below.

10. GRADIENTS. Centers are generated and strengthened by gradients of size, shape, or quality. Thus any quality among a system of centers which varies systematically, will produce a gradient, and this gradient, by pointing to a particular center, helps to build it and intensify its coherence.

11. ROUGHNESS. In coherent structures we usually find a rough arrangement and repetition of centers rather than exact repetition in shape, spacing and/or size. Thus apparently similar centers are different according to context, allowing each part to be adapted to the geometric constraints around it, thus modifying details of the repeating structure as it needs to be. Texture and imperfections are generated, and in part create the possibility of true uniqueness and life.

12. ECHOES. Within coherent configurations, there are often deep underlying similarities or family resemblances among the elements. These similarities are often characterized by typical angles, and typical curves, so that they generate what appear to be deeply related structures, sometimes so deep that everything seems to be related.

13. THE VOID. In the most profound centers that have perfect wholeness, there is often at the heart of the structure a void which is large, undifferentiated, like water, infinite in depth, surrounded by and contrasted with the clutter of the structure and fabric all around it.

14. SIMPLICITY AND INNER CALM. Essential to the completion of a coherent whole is a quality of simplicity. Every structural features that is unnecessary has been removed – so that the remaining structure has slowness, majesty, quietness. Everything superfluous has gone.

15. NON-SEPARATENESS. Connectedness; we experience a living whole as being at one with the world around it, not separate from it. This means that when not-separateness exists, visible strands of continuity of line, angle, shape, and form, connect the inside of a living center with the parts of the world beyond that center, so that it is, ultimately, impossible to draw a line separating the two.

These fifteen properties are thoroughly explained in Book 1: *The Phenomenon of Life*.[2] In Book 2: *The Process of Creating Life*, these structural features are shown to form a basis for the structure-preserving transformations that create life and coherence as configurations unfold.[3]

III INTRODUCTION

The central issue is adaptation. In many real world systems, both in nature, and in those places where human beings form communities with animals, plants, and other human beings, the central observable is a close-knit adaptation of the system elements, usually arising over time.

This close-knit adaptation has not yet been a major focus of scientific study, because it eludes simple algorithmic formulations. That is not because it is more complex, or too complex to be modeled. It happens, rather, because the elements of such adaptation are so extremely simple, and so rooted in common sense, that they nearly elude the algorithmic and algebraic formulations that we view, wrongly, as more sophisticated. For example, if a farmer places a row of fence posts, then runs a top rail, braces it here and there where it seems needed, allows it to relate in natural ways to declivities in the ground, or to the present of nearby trees, this is supremely ordinary, it is characterized entirely by common sense, and often, by the farmer's ability to pay attention to the situation of each post, each rock, each bit of soil, each slope -- and do it right. But this oh-so-simple process eludes algorithmic formulation, because algorithmic formulation is inappropriate, not tailored to this tasks, and almost certainly unhelpful in grasping what is really going on.

That is not to say that such a sensitive fence-building process is childish or unimportant. On the contrary, the character of this minute, step-by-step adaptation is vitally important, and we have been ignoring it at our peril. But we do not presently have a model which emulates this process. As a result, the ability to perform in the real world, according to such a process, has been worn away and destroyed by processes that are largely bureaucratic – often too bureaucratic -- but also, in their essence algorithmic. The planners, building officials, construction companies, engineers, who have redefined everyday processes during the last one hundred years, have been working in a broad context of algorithmic thinking and yes-no thinking. And they have – without explicitly intending to – destroyed a far more subtle process.

Until that subtle process is acknowledged, and re-defined in modern terms, it will not have the status it requires to play an effective role in modern society. The deep adaptation that nourishes the physical world requires this kind of adaptation. And this adaptation process, if we choose to think of it as computation, is a highly sophisticated computation, yet performed on real sticks and stones, and nevertheless potentially huge in the depth and subtlety of its results.

The progress of an evolving (unfolding) natural landscape, or the development of an embryo, have similar qualities. As cell division progresses, new cells take shape within the context of the surrounding cells, and, at the same time, adapt, so that they are shaped by these surroundings, and simultaneously play an active role in shaping their surroundings. Again this process has not been modeled. One reason for that, is that the prevailing assumption is that what is going on in the world is too often a nearly random aggregation of simple mechanical processes, with no special coordination or behavior as a whole. Within that view, there is little to be learnt from studying the process. It is just number crunching, without new insight.

But this view is, I believe, mistaken. The movement forward of the adapting cells, and the progressive adaptations that take place as each part rubs up to its neighbors, shapes them, and is shaped by them, coordinates the whole. Some profound coordination of the whole is occurring. I do not think this coordination is “merely” the effect of multiple random events and effects. There are strong reasons to think that this aggregation of chaotic events is, instead, a very highly organized larger structure-preserving process, in which the process in the large, does progressively pay attention to the whole, reflect the whole, and extend and make more beautiful – the whole. Possibly the phrase “structure-preserving” is not quite right. What I am referring to is a structure-confirming, -enhancing, -extending, -strengthening, -sensitive process. Mere “preserving” sounds rather too static and simplistic for the more active unfolding that’s actually going on.

That is what makes it worth studying. It is a type of computation, entirely unfamiliar to conventional mathematics, but a computation nevertheless, and one which reaches profound results. By observing this kind of computation going on, and then, hopefully understanding it well enough to simulate it, we may lead to a new era of man’s ability to think.

In this paper, I shall rely heavily on examples. That is partly because the subject of harmony-seeking computations is difficult, and one builds a sense of its feasibility by considering many kinds of examples from different spheres, and slowly grasping the general propositions which underlie all of them. It is also because harmony-seeking computations occur in nature, and can also occur in human creative processes. The constant awareness of these two very different spheres, and the process of comparing them, is what, above all, makes this kind of computation interesting.

IV HARMONY-SEEKING COMPUTATIONS

Relation Of A Given Computation To The Larger Whole Beyond It

As we shall see, in all the examples I shall give, the essential thing, an essential feature of the process described, is that the new configuration which is created, does something to participate in, strengthen, extend, and enlarge the force or “presence”, of some larger configuration that stands in the world around it. In many cases, this larger configuration in the world around it, is not itself amenable to change during the “computation,” but it does nevertheless enter in as a crucial factor.

If a lane exists, connecting two villages, and the road between them is then asphalted one day, the new wider and harder road is primarily affecting the configuration of these two villages, a tract of land much larger than the lane itself; Indeed, the new road may (according to its position in the region), also affect and strengthen an even larger part of the region’s economic network. We all recognize this as a fundamentally useful act.

A musical note that is played will be most effective, according to the degree that it develops a melodic line, a rhythm, or a harmony. If it does all three simultaneously, it will be most effective of all. Once again, we easily recognize the usefulness of this act.

These are simple cases. Other cases, as we shall see, involve creation of the most highly organized and complex wholes; These have, so far, eluded us in science. Even the exciting and useful theories of bifurcation, symmetry breaking, chaos, and generative algorithms, have not yet deeply plumbed the meaning, or the origin, of true complexity.

All this is obvious, indeed hardly more than the most ordinary common sense. Yet, obvious or not, the extraordinary fact remains that this kind of adaptive process (the farmer building his fence with respect for the land) does not currently have an acknowledged part to play in present day theories of algorithms, in developmental biology, in architecture, not even in system theory. It is just not part of the mental models in our tool bag that we currently address and use.

And once we take this seriously, and do indeed add knowledge and perception of this type of transformation to our kit of tools, it will be natural, then, to contemplate the possibility of a type of computation which examines possible moves forward, from a given state of a system, tests them, and evaluates the extent to which each of these moves participates in and adds to the larger wholes in the world around it: then chooses the most effective, the most harmony creating, the most successfully whole-creating!

Such a process of computing, if it can be attained, would be enormously powerful, and powerful in its implications.

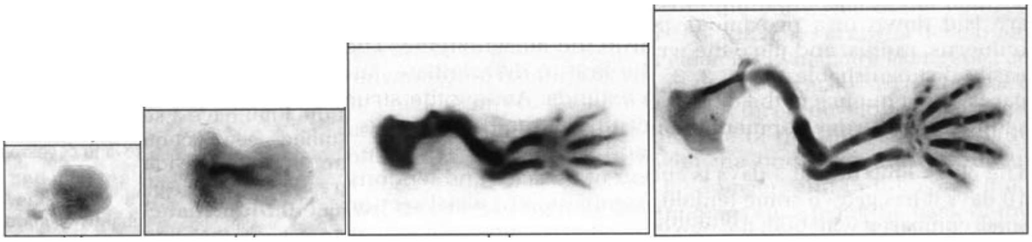
The Essence Of Harmony-Seeking Computation

The essence of the whole-seeking computation, lies in the following. It creates new configurations, unknown configurations, and *good* ones, by taking off from a known configuration, but without (necessarily) requiring the input of human creativity. The process *itself* is creative.

It works like this. Consider a given configuration $\mathscr{C}$. This configuration has certain features which are visible, and which, in the main define whatever whole, or wholeness we see in the configuration. But, in addition, in every configuration there are also traces, hints, of dim structures, not yet fully developed, but existing in a latent form, “between the lines” of the configuration. What happens in harmony-seeking computation, is that a process latches on to these latent structures, and enhances them, develops them. Sometimes what develops may be relatively small, with respect to the size of the entire configuration. At other times, very, very large structures may also be latent in a configuration. If the whole seeking computation identifies this latent larger whole, and strengthens it, so that what was before only barely visible, now becomes strong and easily visible, the configuration will seem, to an untrained eye, as having gone in a new direction all by itself. It is this process, that is the essence of all harmony-seeking computation.

Example 1: Embryogenesis

Consider an example of embryogenesis, a growing mouse foot. Here is how it grows in four days, from the 12th day to the 15th day. You see that each stage contains within it some structure that is defined, and some that is for the time being a vague and fuzzy mass of jelly, which anticipates the shape of the next step, which then consolidates and solidifies what was merely latent only hours before.



What are some of the transformations which constitute the SP-character of these moves? The form is governed by an axis from the attachment to the body, to the tip. In the second slide we see the emergence of a STRONG CENTER at the tip itself, forming a thick BOUNDARY (in one dimension) to the arm. This center is then accentuated by the appearance of a GRADIENT leading to the fingers, and this gradient is then embodied in the 3rd and 4th slides by LEVELS OF SCALE, CONTRAST, and LOCAL SYMMETRIES, and finally finding expression in the GOOD SHAPE of the whole.

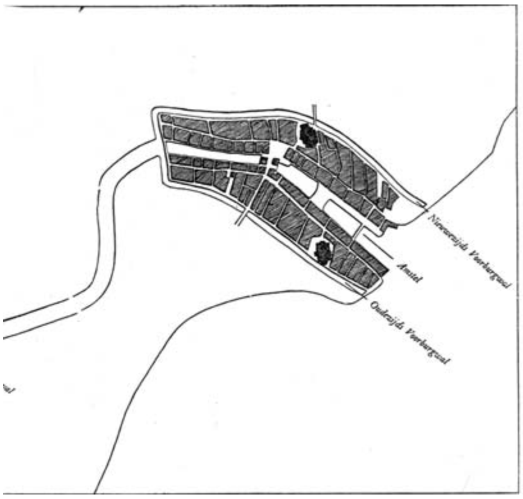
Example 2: A Bench Around A Tree

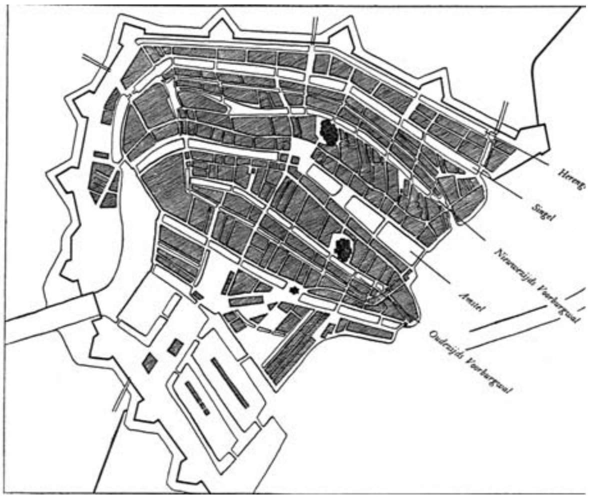
Consider, secondly, the example of a growing willow tree, and the act of the landowner who chooses to build a bench around the base of the trunk. The bench places a BOUNDARY around the tree. Compared with the size of the trunk, this is a tiny act, a tiny step. But the bench starts from the cylinder of the three trunks, puts a small ring-shaped structure around it (the seat). We may say that the possibility of this seat was inherent in the previous structure, latent there: and the bench builder simply made explicit and more solid, the structure that was present in a weak and latent form, at the base of the willow tree, already.

I mean this literally, not metaphorically. The ring-shaped structure which later finds embodiment and physical form in the seat, is already present before the seat is built, in the ring-shaped system of symmetries and sub-symmetries of the space around the tree, because they are induced by the presence of the trunk and its roughly cylindrical shape. This statement is the crux, and in this statement – generalized -- lies the mathematical kernel of what I have to say in this paper.

Example 3: Growth Of The City Of Amsterdam

Consider, next, the evolving wholeness of the city of Amsterdam. On page 9, upper left, is shown an early state of the city, a U-shaped wall, around a few city blocks, with the river Amstel down the middle. That is the structure which first existed. It in turn was formed by a simple and natural adherence to the position and shape of the mouth of the river Amstel. If you look for what is latent in the configuration, you see a horseshoe kind of structure, essentially hovering in the white part of the drawing, and present – but only dimly present. In the second state of the city (next page, top right), this larger horseshoe shape has been made real, not merely latent, realized by the surrounding concentric canals and streets, and canals have been built to drain the land, all following the natural line of the originally latent horseshoe.





In the third state of the city, late eighteenth century (below), the concentric structure of canals and streets has been intensified by adding further layers and filling in a much larger area in a way that supports and continues the structure of the second state. In the transformations applied here there has been particular emphasis on BOUNDARIES (in the canals, walkways and polders), on POSITIVE SPACE, LOCAL SYMMETRIES, GOOD SHAPE, DEEP INTERLOCK in the surrounding fortifications, and on ALTERNATING REPETITION throughout.

Please look back at the three examples, the mouse foot, the bench around the willow tree, and the growth of the city of Amsterdam with its canals and streets and wall. In each case, these steps <u>always</u> build on the structure that is there, do not destroy it or interfere with it, but rather enhance it and elaborate it and deepen it. As a result, what arises has wholeness, coherence, and beauty. *That is the trick, in a nutshell.* By continuously preserving the existing structure, a beautiful thing arises, naturally. Yet, because each whole is unique, and the

idiosyncracies lying latent in it, are also unique, the new whole that springs from this process is unpredictable, original, and creative.

But to perform this trick we must ask rather more precisely what it *means*, to take steps which intensify the latent structure that is present in a configuration. Can we define this idea precisely enough, so that we can start doing it on purpose?

Structure Of Wholeness

The idea that every given geometric configuration might have its own "wholeness" seems odd at first, and even teeters on the edge of soft talk about holistic healing in California hot tubs. However, during the last decades, it has become clear to me that there is a real and definable, mathematical structure underlying any configuration, which captures, or "is", the structure of the whole configuration, its global structure. Over the course of years I began to call this structure *the* wholeness. It is not uniquely defined, (mainly because the field is so difficult to describe in precise detail) but it is definite, and various attempts to represent this structure approximately all share something. There is a deeper, global "something" there.[4]

I first began to recognize this possibility in experiments undertaken at the Harvard Center for Cognitive Studies in the early 1960s. These experiments were inspired by the early 20th-century gestalt psychologists -- Wertheimer, Koehler, Koffka and others -- who wrote extensively about the perception and cognition of wholeness and of wholes in the world around us.[5] In the early 1960s, Huggins and I used various experimental techniques to find out how different perceivers see configurations.[6] The essence of the technique was to ask people to group different patterns according to their degrees of similarity. It became clear that there are broadly two kinds of perceivers, those who see patterns holistically, and those who see the patterns according to analytical classifications. The perceivers who saw analytically were in the majority, about 85%. The perceivers who saw holistically were in the minority – about 15%; these perceivers grouped patterns according to their overall figural character.

However, it was clear that the figural perception was somehow more real, while the analytical perception was arbitrary, and made according to arbitrary invented schemes. The figural quality, was inherent in the material. The ability to perceive figurally, was therefore of enormous importance.

I discovered, later, that it was possible to train people to perceive figurally; but it took a good deal of effort, to overcome what I believe is a culturally induced tendency to be analytical (perhaps understandable among Radcliffe students).

In broad terms, we may say that the global configuration is a structure of the main features of the configuration – in particular when it is seen as a whole. [7]

During the same series of experiments, I made a significant discovery. The coherence of the different configurations could be measured, by counting the number of locally symmetric sub-configurations it contains.[8] This measure gave results in very strong agreement with experimental results from cognition, memory, perception and so on, Thus a configuration and its structure do depend, in some considerable degree, on the nesting of LOCAL SYMMETRIES and on the structure of the system of locally symmetric subsets which occur in it.

What Are The Underlying Qualities Common to Different Examples of Harmony-Seeking Steps in Different Systems?

Example 4: Hayricks in a Field

Consider, now, a classic example of a traditional farmhouse, nestled in a hillside in just the right way, perhaps with a tree and barn and well, and making us marvel at its harmony with the land. The way the building fits, in such a case, is profound, not easy to analyze. Yet it was done by human beings, and it was done in a way that, for people of such a time, creating such harmony was a repeatable, replicable process.[9]

Hay ricks in Romania

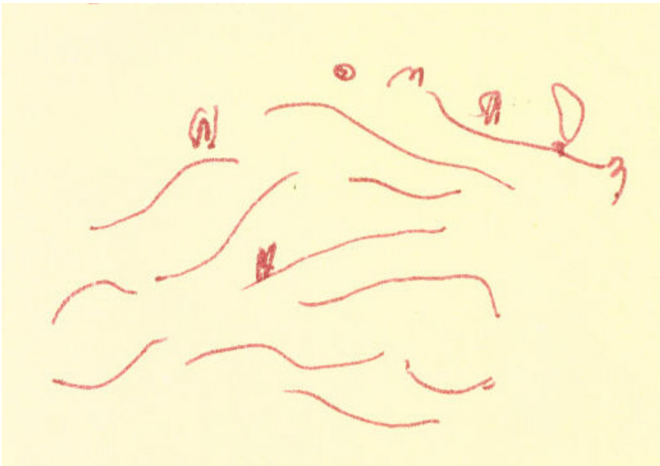
In this photograph above, even simpler than a well-placed farmhouse, two modest hayricks are placed in a rolling field. The placement is done in such a way as to complement the land, to enrich the land, it is humble, self effacing, there is no ego visible, only concern that the land and its harmony should be enlarged.

How is one to characterize what is happening here? What is it, that we see, and recognize as so supremely successful? In this paper, I shall try to describe, structurally, what is it that has taken place?

In the landscape where the hayricks are, the rolling quality of the land, with its dips and hollows, and the hedges and trees as they are placed, is respected. The two hayricks, are placed with considerable care, on slight outcrops of land, so that the hayrick is not only kept dry, and free from rot, but it actively enhances and emphasizes the slight bumps in the land. In addition, the actual structure of the hayricks (A wooden rick -- a tree of light timber, with hay piled up on it), mirrors and echoes the character of the grass land and of the surrounding vegetation.

Below, I show a sketch, which in very broad-brush terms, approximates the wholeness present in that place. The wholeness includes the sinuous curves, the kinds of shapes which are present in the field, the dips and hollows, the surrounding trees, the trees along the ridge, and the fallen down enclosing fence below.

In structural terms I can make a diagram of this explanation as follows:

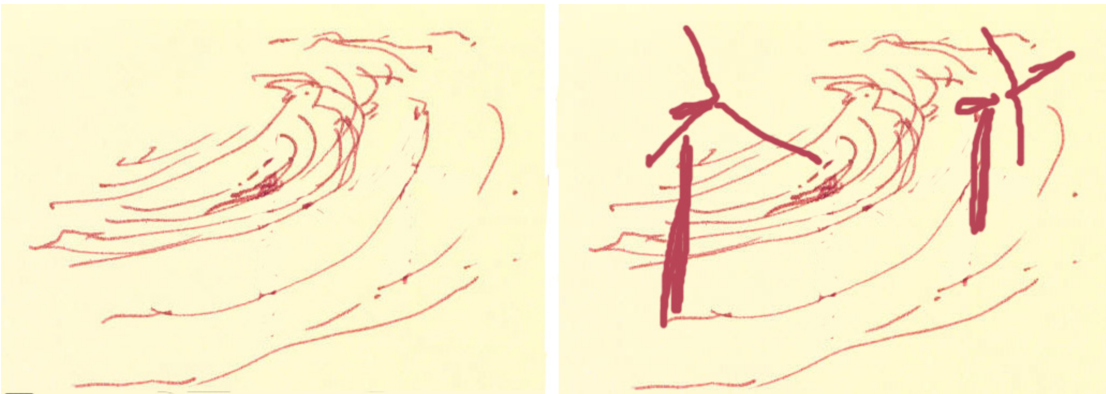


The people who built and placed these ricks, were, consciously or unconsciously, performing a harmony-seeking computation. There are ECHOES of shape and size between land and hayricks, ECHOES of a certain kind of curve, LOCAL SYMMETRIES in the ricks themselves, and the placing of the ricks emphasizes naturally occurring STRONG CENTERS that are generated by shelves and flattened places , bounded so that the hayricks nestle in the land, are subdued and congruent, and inside the structure which exists. The hayricks are kindly to the land, but they are placed with enormous care. They follow the wholeness. And if I were to move them slightly, to different positions, the placement and the whole then created, the ensemble, would be less profound and less harmonious.

Example 5: Giant Wind Turbines on the Danish Coast.

By comparison, let us look at what 20th- and 21st-century thinking often does instead. The thinking is rationalized (usually, by more or less algorithmic formulations of various kinds). However, when these computations about subtle and delicate wholes are rationalized according to current ways of thinking, they usually go wrong as far as respect for the larger underlying background structure is concerned. The subtlety and depth of the existing global structure in a landscape, is not so easily recognized by technological thinking. For example, in the attached photograph, the land has a global wholeness which is made of layered, nested plates, flat and created a two dimensional system of plates. However, the ecologist who placed these two giant turbines was evidently unconcerned about creating harmony with this structure.

Wind turbines on the flat north coast of Denmark



Please look at the photograph, placing your fingers over the two towers, so you can see the land as it was before they were placed there. It is hard to see it, because the towers are now so obtrusive, and so alien to the kind of structure which is there. When an ecologist becomes too concerned with one aspect of sustainability (the importance of wind energy, as in this case), and then plants 300 foot high windmills on an ocean front, meanwhile destroying the landscape – this is no longer an SP-transformation, but rather a well-meaning but ill-judged algorithmic computation of some inappropriate kind about cost benefit. But in this case, the wholeness and the structure of the place have been severely damaged. Sometimes, the place, its landscape and its internal adaptations -- often the result of centuries of patient work -- are irredeemably destroyed.

In the coast landscape where the Danish turbines have been placed, the structure that was there before is flat like a disc, a great flat disc, the size of the bay, flatted at the edge where the waves meet the sand and the grassland just beyond it. This enormous gentle discus-shape, perhaps half a mile across, is suddenly violated by the two vertical structures, sprouting turbine blades, which are dumped on it. Now the two massive vertical things, have no relation to the structure of the subtle disc that lay in the land before. The structure has been violated.

Let us compare what is happening in the hayrick example, and in the turbine example. In the case of the hayricks, the process of placing them has left the previously existing structure alone, indeed the process enhances it, strengthens it, increase its harmony. But in the case of the turbines, their shape and placement are at odds with the previously existing structure, they damage it, and it may not be too much to say that they destroy it.

Now what is that "IT" that is being left alone in the first case and that is being damaged and destroyed in the second case? It is the *wholeness*, that system of centers existing in the geometry of each place, which give it its character, shape, organization, rhythm, feeling, and global configuration and relation to the land around it. One may describe the relation of the action to the wholeness like this: In case 1, the injection is friendly, helpful: what is there is healed, complemented, and continued. In case 2, the injection is violent, disrespectful, and structurally at war with the land that was there before.

I think you can see from then foregoing description that my judgment about these two injections is not a romantic yearning for an idyllic past, but a structural judgment about something that is objectively present in one case, and missing in the other. One injection respects what is there, the other doesn't. And these structural judgments are about phenomena that are essentially mathematical in character. It is not only a question of feeling, but also a question of structural congruence or the lack of it, that exists between an existing structure and an injected structure which has been brought into it.

We very much need to understand more about this kind of structural respect.

V   EXAMPLES OF HARMONY-SEEKING COMPUTATIONS FROM DIFFERENT FIELDS

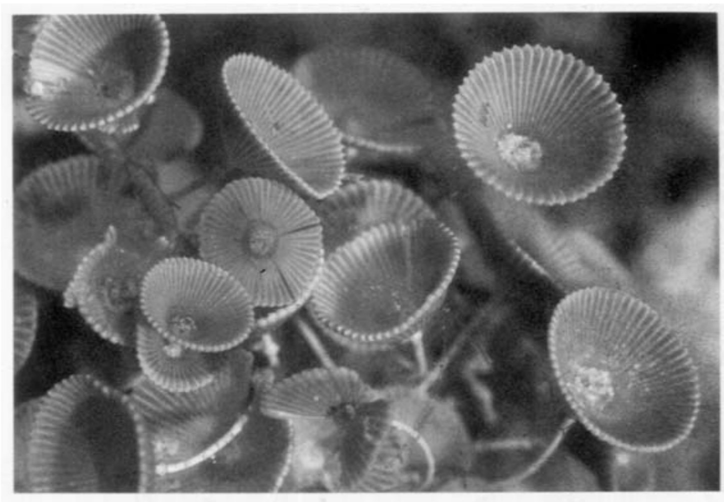
In order to understand the idea of harmony-seeking computations, it is useful to examine a wide series of examples of such computations, most of them occurring in the real world. We may also call them whole-seeking computations or healing computations. The following examples are taken from the fields of biology, painting, architecture, ecology, embryology, decorative arts, town planning, entomology, clothing fashion, mathematics, symmetry breaking, the game of GO, and astrophysics.

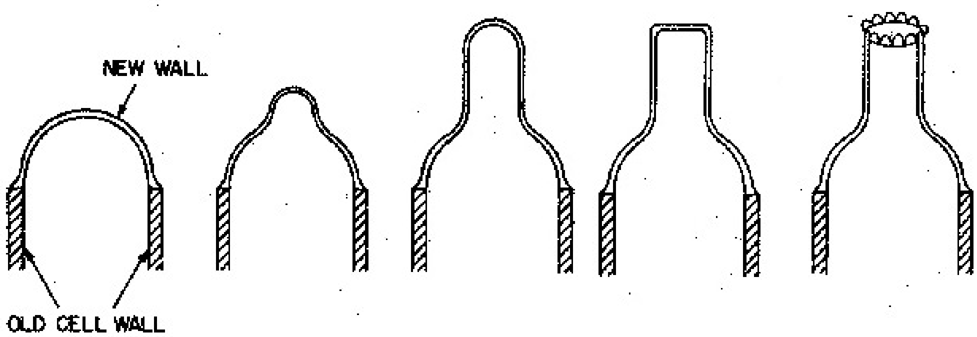
Example 6. Matisse making early brush strokes on a canvas

Here is one step in Matisse's process of painting. In the first state, the canvas has a background pattern, the stripes of a piece of cloth on the chair, a dark mass where the woman's throat and scarf are to be; staring at us, is the large white area in the middle which is the woman's face, as it is yet to be. But this white area is a vivid latent center, which, even though unpainted as yet, draws attention to itself, draws us in. It is the most salient structure in State 1. So, of course, Matisse goes to that place; we already see his brush hovering over it, in the first photo. And he then makes a mark, the face itself, dark, which differentiates it the latent center, gives it character, and continues, and develops, this important latent center in the evolving structure. CONTRAST and POSITIVE SPACE are the major transformations used by Matisse here, evident in the way he leaves the "things" – her hair and bodice – unpainted and white, and uses the dark paint to paint the "negative space" between them.

Example 7. Evolution of the whorled cap of *Acetabularia* – 6 steps

This example is taken from Brian Goodwin's well known work on the morphogenesis of Acetabularia, a single-celled alga which takes the form of a stem with a cupped whorl at the end. Goodwin studied the growth of this form, and investigated and explained the series of transformations which generated the form of the completed alga.[10]





The key morphogenetic sequence that Goodwin described, illustrated in his five-diagram sequence above, showing the stem bulging and then making the whorl. It goes as follows:

1 to 2. A hemisphere, formed by the bulging upward growth of the cell —turns into→ a hemisphere with a small centrally symmetric swelling on top (an ogee curve).
2 to 3.The hemisphere with a small centrally symmetric swelling on top —turns into→ an elongation of the swelling forming a neck.
3 to 4. The hemisphere with the elongated narrow neck —turns into→ a neck with a flattened top, upper end.
4 to 5. The flattened neck on top of the hemisphere, grows a ring of small projections (a whorl) that ultimately turn into the umbrella shape visible in the completed algae.

5 to 6. The whorl formed around the top of the cylinder grows outward —now turns into→ a full umbrella-like ring of small projections that ultimately turn into the umbrella shaped cap visible in the completed algae.

As Goodwin says, strongly and repeatedly in his writings on the *Acetabularia* example, this growth sequence is generated by natural physical phenomena. It has relatively little to do with DNA or genetic guidance. What is happening here is a naturally occurring and inevitable progression of morphological transformations that arise directly from the geometry and dynamics of the form itself, to generate the next geometry.[11]

It is highly significant that a prominent biologist has begun taking the emphasis of morphogenesis away from the influence of DNA, and has begun to see the morphogenesis as an autonomous process in the geometry itself.

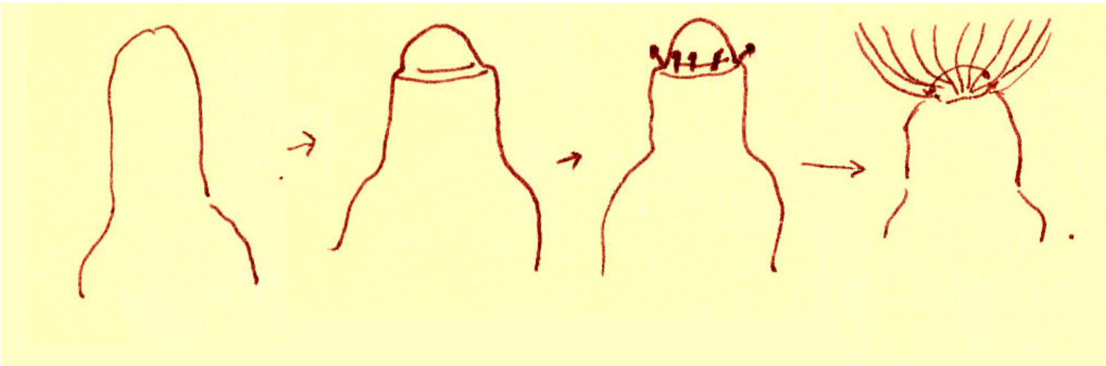
There is an interesting sequel to this story. When Brian first showed me the diagrammatic sequence illustrated above, I asked him if the progression from round-ended neck to flat-topped neck was correct. Why do you ask? he said. I told him that , looking at this purely from the point of view of structure-preserving transformations, I would have expected something different. It seemed to me very unlikely that a round topped structure could transform into a flat-topped structure. There is nothing like the flat top latent in the configuration.

There is, however, a ring *latent* in a hemispherical configuration, but further down the curve. If you have a rounded hill, or hemispherical end of a neck-like structure, there is, inherent in that configuration, a latent structure something like a shoulder, which is incipient near the top of the hill and just below it (see left-hand sketch below). In the right-hand photo below, we see such a structure on the prehistoric Wittenham Clumps near Oxford. It is, and evidently was, natural for the builders who made these mounds, a natural way to continue the smooth structure of a rounded hill. Somehow this point of inflexion in the curve of the hill is a natural development from the smooth curve.

If you are in doubt that it is "natural", consider the two possible transformations of a hemispherical hill. One, A, just flattens the top by squashing it into a flat surface. The other, B, introduces a band below the top, with two points of inflection in the curve, as shown below. If you ask yourself which of these two transformations leaves the hill alone more, and preserves the global structure of

the hill better, I think you will agree that it is B. It is in this sense, that B is more natural than A.

So I said to Brian, referring to his Acetabularia diagrams: If such a configuration were latent, and did then transform to create such a ring-like shoulder, then that *shoulder*, not the top, would be the most likely place where whorls and other irregularities might naturally form. So I asked him again, Are you sure that the real morphogenesis of *Acetabularia* doesn't go like that, rather than the way your diagram portrays it? Like this:



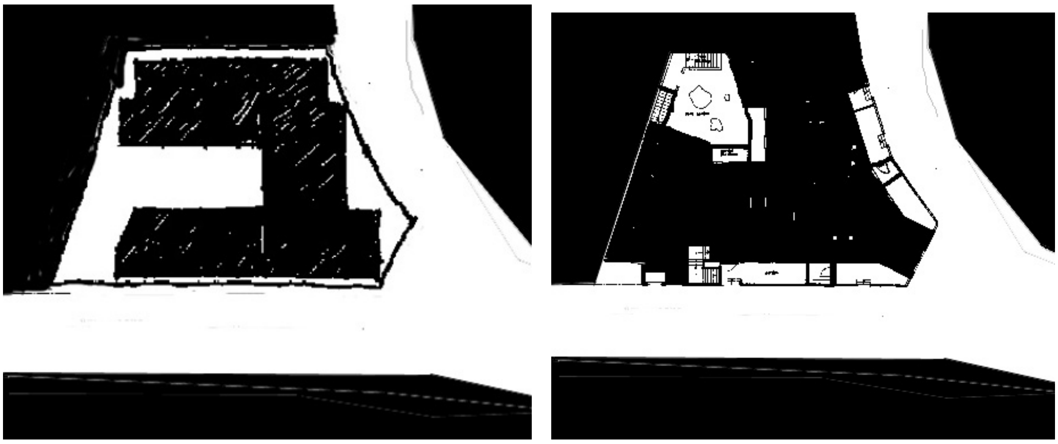
Brian laughed, and told me You are absolutely right: that *is* what actually happens. I just got the diagram wrong.

This story sheds interesting light on the way sophisticated harmony-seeking computations maybe able to help both observational biology, and theoretical biology, and structural modeling and simulation. We will come back to this later. The important point is, for now, that it is possible to make accurate statements about what structures are latent at a given phase of morphogenesis; and it is therefore possible to say, objectively, how things are likely to unfold.

Here is a view of the interiors of some *Acetabularia* caps, showing that within the whorl, there is indeed a remaining *hillock*, not a flat, just as my whole-seeking computation predicted. At the time I made the prediction, I had never seen enough detail of Acetabularia to know about this kind of cross-section. My guess arose solely from wholeness considerations, and from thought about what would be structure-preserving.

Photo showing cross section

Example 8. Two possible plans for a five–story apartment building in Tokyo



The left hand drawing shows a plan for this apartment building in Tokyo, done by a student. Though typical of many architectural projects in 1987 (the era of the project), the plan plainly lacks the wholeness-preserving quality of the right hand plan. The right hand plan shows the five-story apartment building which I subsequently built on the site.[12] The layout process for this design had about fifty structure-preserving steps, described in detail in the reference.[13]

The fact that the right hand plan (as built) complements and intensifies the wholeness of the site, is intuitively clear -- just looking at the ground plan alone. However, to emphasize what this really means, and to illustrate the deeper structures that are involved, I will describe some specific features of the global structure which existed in that place, and which were then solidified and complemented by the building that was placed there. Here are some of the structural features of the original site: The center formed by the Y-configuration of the fork; The curving nature of the two branches; The space between the forks, and its specific V-shape; The southern spot hit by the sun.

The plan for a new building, when injected into this site, does the following things which help to accentuate and strengthen these structural features of the place:

- The sharp end of the plan is given a “snub” nose, thus creating a place in front of it (it became the entry to a shop).
- The front gardens of ground floor apartments are placed to form a boundary between the building and the street.
- Since the boundary is of uniform width, the space of the street is maintained and continues its positive shape.
- The building is wrapped around the sunny south-facing spot on the site, making sunny space for all the apartment owners.
- On upper floors an inner layer of galleries and terraces form a boundary to this place.

As these various features are consolidated, they form a coherent structure, in which each feature supports and helps the others. Marked among this coherence, is the presence of positive space *throughout*, even in a geometrically complex configuration; the fact that there are levels of scale within the structure; the focus of major centers to the structure, each with its own strength and beauty;

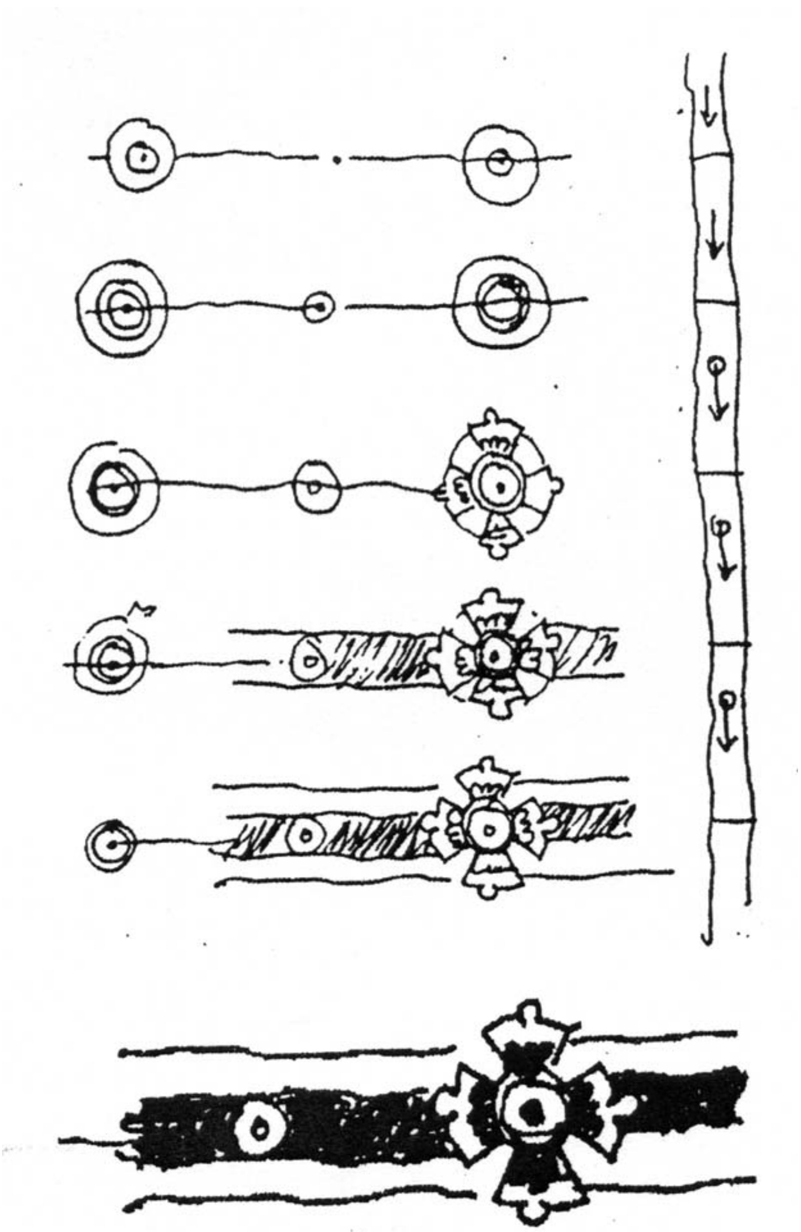
The process of building, and injecting, this structure into the configuration of the site, and designing and planning and building the entire five story-complex, I view *altogether* as a complex computation. It takes the initial configuration (the site) as its starting point, then transforms the configuration, by about 50 steps, finally ending with the configuration illustrated. Although the work of an architect of this kind would be conventionally viewed as an artistic act, hence as an arbitrary and "private" process, I believe this to be seriously in error. In the process illustrated here, each step is based on the configuration which is there at the moment before, and the latent structures which exist, and then makes one step forward, which specifically, and carefully, strengthens or supports, *and evolves naturally out of*, the structure which was there before.

Example 9. An ornament drawn by Hiro Nakano – 6 steps.

On the next page there is a much simpler example – just the evolution of a sketch – but following the same kind of process. This drawing starts with a row of evenly spaced dots. Then, it goes like this:

- Make a circle around alternate dots. (using ALTERNATING REPETITION)
- Intensify these new centers with an additional circle, and also make a much smaller circle around every other dot. (using STRONG CENTERS, ALTERNATING REPETITION)
- Take the bigger circles and subdivide each into eight sectors, four plain, and four that are fan-shaped alternating with the plain ones. Give each fan-shaped sector a special shape to emphasize it and give it more detail. (using GOOD SHAPE, LEVELS OF SCALE, ALTERNATING REPETITION, BOUNDARIES).
- Add a strong dark, shaded band, to connect the larger circles with the smaller ones they alternate with more strongly. (using CONTRAST, NOT SEPARATENESS, LOCAL SYMMMETRIES).
- Add two additional lines, making borders to frame the dark band, and thus creating a still stronger sense of unity. (using BOUNDARIES, NOT SEPARATENESS).
- Darken the central band much more, and blacken the flower shaped quadrants of the inner large circles. (using CONTRAST).

In the drawing on the next page, through this very simple sequence of six steps, we see the growth of a coherent and sophisticated structure, from extraordinarily simple moves. The names given in parentheses at the end of each step (above), are references to the fifteen properties and transformations, defined in *The Nature of Order*.[14] As we shall see later, these properties and transformations, play a fundamental role in the creation of all wholeness.



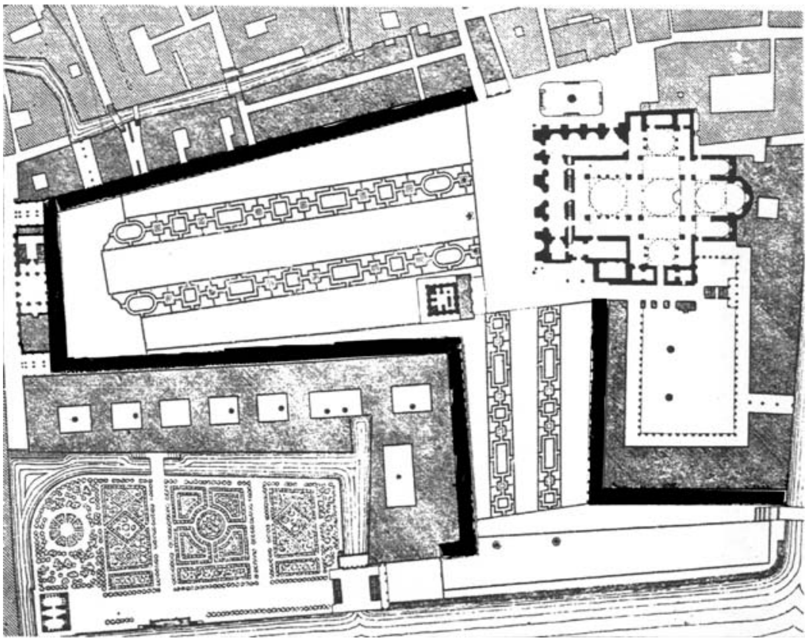
This example is instructive because it is complex enough to be interesting; and we get a vision, for the first time among these examples, that recursive use of these transformations on the emerging centers, and on the centers which the transformations themselves generate, is potentially a complete and very powerful process, perhaps one capable of generating all complex configurations in their totality.

However, there are still many years of work needed to generalize this harmony-seeking computation process in an effective and operational form for the broad range of problems we may wish to apply it to.

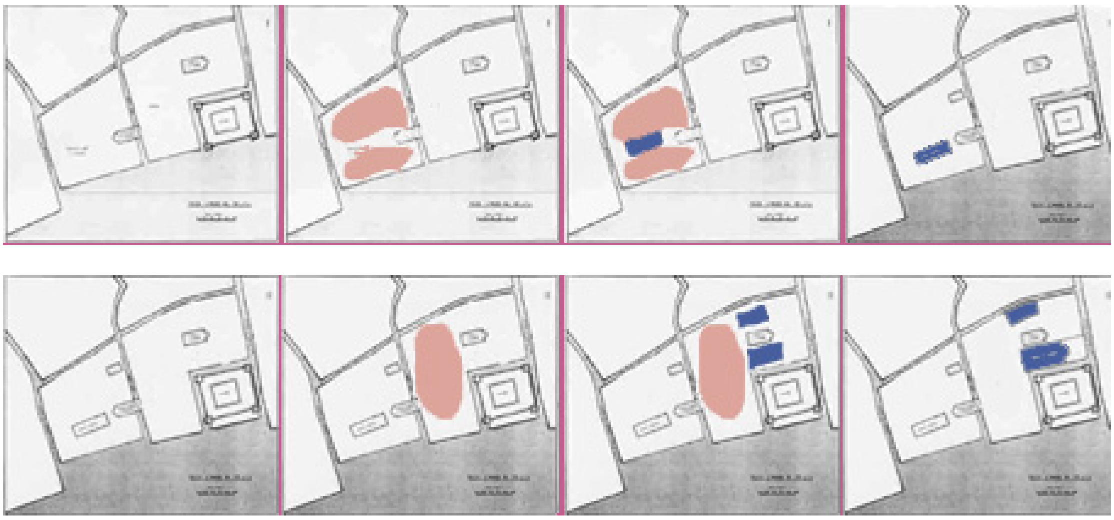
Example 10. Construction Of The Upham House – 200 Steps

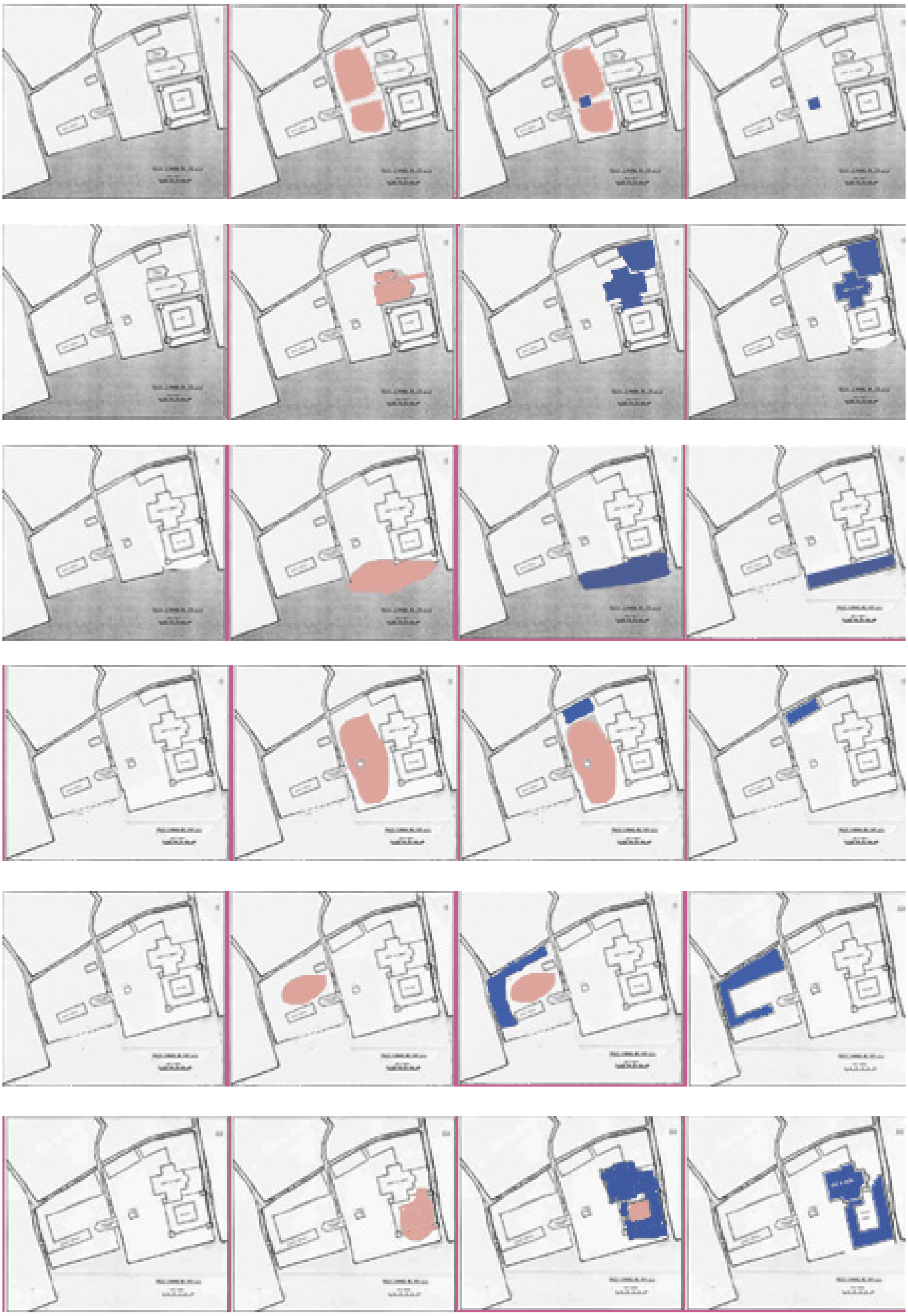
This house, built in Berkeley, California, demonstrates the effect of cyclical repetition of several hundred harmony-seeking computations. Starting with the site shown in the photograph above, two years later, the house shown below had been built on the site. The process is fully described in sixty pages of text in *The Nature of Order*, Book 2, *The Process of Creating Life*.[15]

Example 11. Historical evolution of St Mark's Square – 10 cycles



Above is the plan of St. Mark's square, one of the most famous and beautiful public places in the world. This marvelous and highly complex structure grew, steadily, over a period of about a thousand years. What happened there, may be best understood as a series of harmony-seeking computations, carried out at intervals, with intense and deliberate care. The series of growth cycles shown below is historically accurate. I cannot be certain that the nature of the activity undertaken at each stage was exactly what I have indicated; but I am fairly certain that it must have gone roughly as I suggest. The fact that it is possible to define a coherent and rather simple paradigm for each cycle of activity, and that in their essentials they are all the same, is remarkable, and strongly suggests that what happened was indeed, something along the lines of the underlying computation I have proposed.[16]





In every cycle, there are three entities at work.

1. There is the configuration in the large, which is roughly the size of the whole of the frame, although focus of attention is usually on an area about half the size of the illustrated frame – shifting from cycle to cycle.
2. There is the particular latent center, shown pink, which is the latent center that most calls for elaboration and development. The area of the latent center is usually slightly larger than the pink area, and the pink shows its focus.
3. And then there is the blue area, which shows one or more smaller centers which are built in order to strengthen and make more forceful, the latent center which is presently weak. In placing the blue centers, it is always done in such a way that it both strengthens the pink, previously latent area, and the large configuration which will be helped as a structure by the elaboration and articulation of the pink.

The procedure goes like this: Find the latent center which is most salient, seems most likely to strengthen the wholeness of the larger configuration. Act locally, in such a way that this latent center gets strengthened, and so that this strengthening helps, also, to strengthen the largest whole. Repeat this cycle ten times over a period of about 1000 years, (roughly once per century), and the result was St Mark's Square as we know it today. This rule, then, explains (or generates), the ten actual cycles of construction and improvement which occurred around St. Mark's in Venice from 600 AD to 1600 AD.

Example 12. Choosing A Tie That Works With A Suit

This is a very ordinary, daily event, at least for men.[17] Trying to find a tie that looks just right with the rest of what you are wearing? Although the example seems homely, like the examples given earlier, it is no less a question of finding the particular thing which is most harmonious, and which does most to support the wholeness of the context into which the tie is going to be injected.

This is a rather ordinary tie. However, it comes from a well-known house of distinguished men's clothes in Germany, and does have some interest.

The quiet harmony which is achieved comes, in my view, from three features above all:

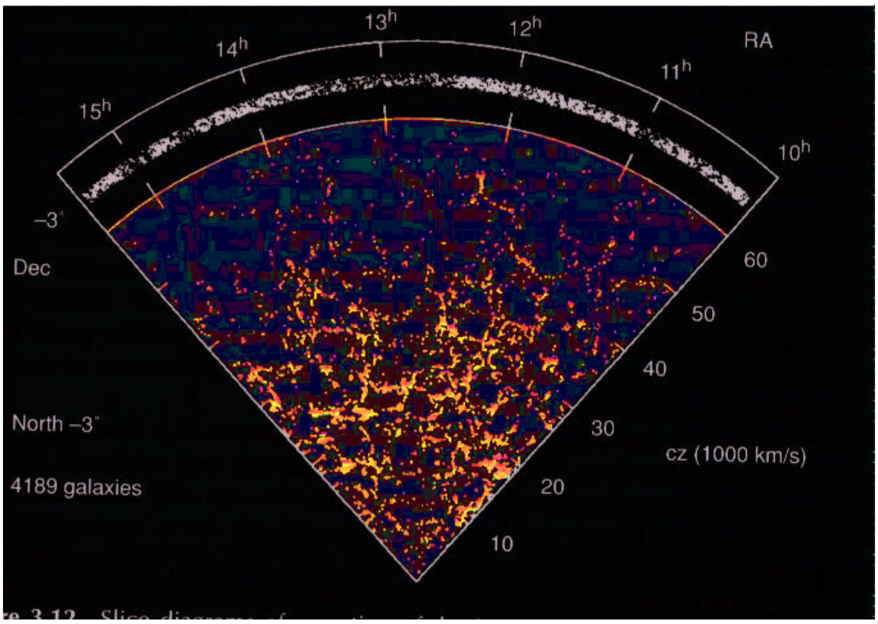
- First, the colors are directly related to the colors of the context: the red-brown of the jacket is reflected by a similar, quiet, brownish red in the tie. And the principal color of the small ornaments is not far from the flesh tone of the man's skin. So the colors are ECHOES of the adjacent surroundings. And the colors as a whole, have a kind of FAMILIES OF COLOR.[18]
- Second, the man, and his jacket are rather pale in feature and geometry: the tie changes the scale and the character, it has several smaller scales, thus introducing LEVELS OF SCALE where it is sorely needed, and the ornaments, which do this, themselves contain yet smaller LEVELS OF SCALE.
- Third, the CONTRAST of the red to the flesh tone and ornaments in the tie, reflects the ratio and contrast of the flat red of the man's jacket, and the smaller details of the man's face and fingers. Thus the tie ECHOES, in microcosm, the contrast of the context in which it sits.

I suppose the man who chose the tie, was, in some half-conscious way, feeling these three facts; or perhaps they came to him, singly, as a direct awareness of the way in which the whole is made stronger and more resonant, when this transformation generated by injection of the tie, occurs. It is hard to view such an ordinary act as being a computation. It is even harder when we place it in the company of other results in cell biology, physics, and astrophysics.

But that is of course my point, exactly. I do believe this kind of computation is going on, in art, in science, and in daily life, and we have missed it, so far, simply because we have not yet given it a recognizable and graspable name.

Example 13. Formation Of Giant Voids In The Universe: A Very Large Example of a Generated Wholeness

In recent years, attention of cosmologists has focused on the truly enormous structures of galaxies and galactic clusters called filaments. Subsequently, then, even greater attention has been focused on the voids which seem to be encircled by these filaments. The filaments are hundreds of millions of light years long, and tens of million of light years thick. The voids are often a hundred millions of light years in diameter.

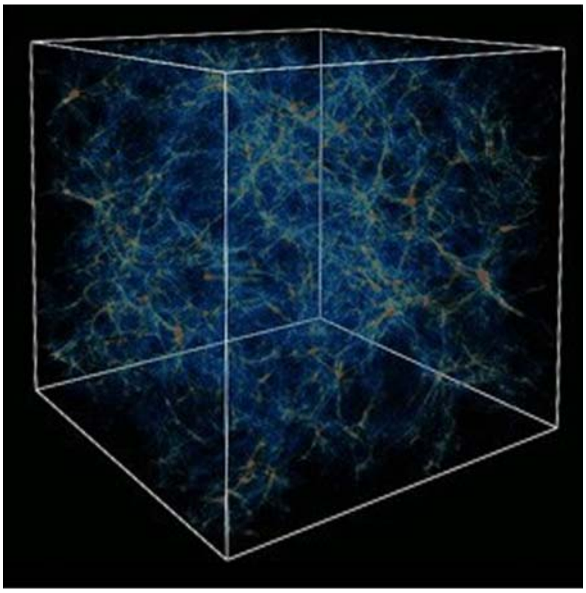


Shechtman et al, the Las Campanas Redshift Survey, showing the voids and filaments

The image above is one of the early pictures of the distribution of galaxies and galaxy clusters which clearly shows the voids and filaments.[19] From my point of view, one of the remarkable things about the voids is that they appear to be convex, and quasi spherical. To me that implies the presence of some active principle pushing outwards. In terms of the language of wholeness, the existence of THE VOID (a general structure existing at all scales, and not specific to astrophysics and cosmology), and of thick BOUNDARIES, and of the scale ratio given by LEVELS OF SCALE, is to be expected in all wholeness, since these are three of the fifteen main ways in which coherent structure typically occurs.

Further, the voids seem to be close packed, like soap bubbles, and like soap bubbles, where three bubbles meet, there is a linear film where the soap solution is concentrated. In just this way, the galaxies are spread out in linear filaments. The so called great wall is 500 Mpc long, 200 Mpc wide, and 15 light-years thick. (1500 light-years long, 600 light-years across (band) and 15 light-years thick. Filaments are typically 70 to 150 Mpc in length. (200 to 500 light-years). Voids are 10 to 150 Mpc (30 to 500 light-years in diameter).

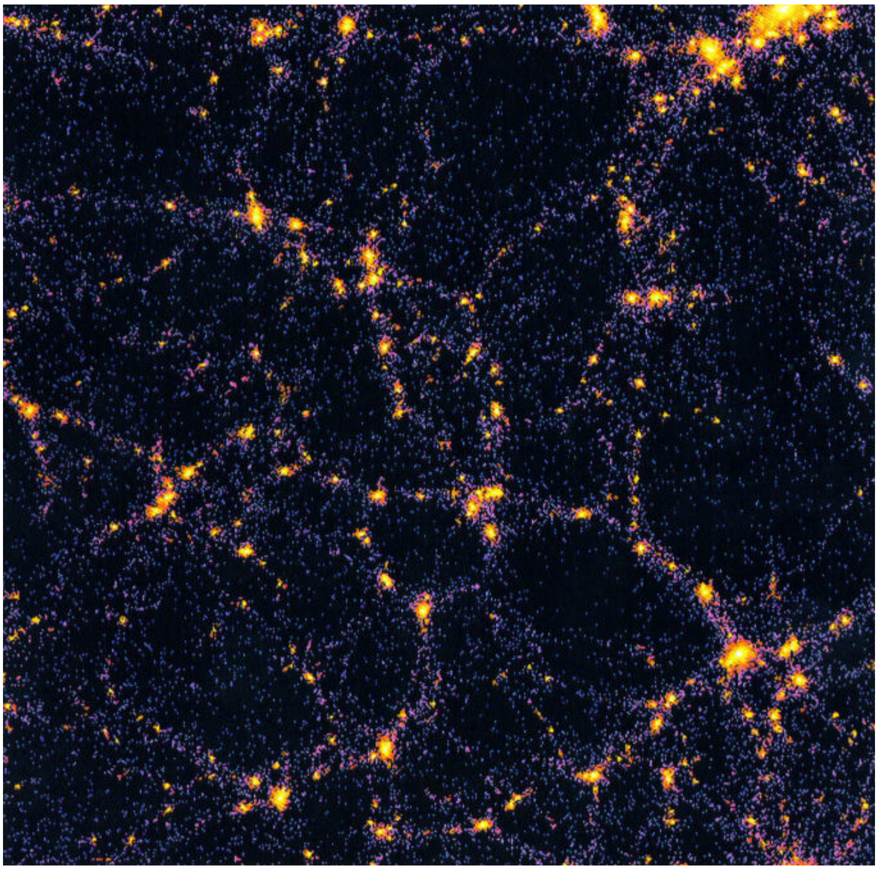
The artists 3D sketch below tries to show how these ring-like strands lie in 3D on the surface of the voids.



Artists conception of the 3D structure, showing roughly how the ring-like filaments lie in space, on the surface of the voids

The picture on the next page is a simulation by George Lake and Tom Quinn, showing the structure more clearly.[20] Looking at this structure, from the point of view of harmony-seeking computation, one of the structure's most unexpected features is the ratio (filament width to void diameter), about 1 to 10, or one order of magnitude. Why is this interesting? Because it so closely parallels our own cognitive and intuitive sense of the relative dimension of a *typical* ring's thickness and the diameters of the space that it surrounds. After all, the filaments could have been much thinner compared with the diameter of the voids, or could have been fatter compared with the voids. In the absence of any current theory which might give us a predictive model of the relative size (unless one fits the parameters just to get the result we observe), it is worth speculating whether the universe itself is making a wholeness-seeking computation, and that the result of this computation is the system of voids and filaments we see.

Next time you are in the bath, try spitting in the bathwater. Very often you will see that the spit moves in swirls on the surface of the water, and once again the roughly circular swirling ring-shapes that are formed have a diameter to thickness ratio in the range of about one order of magnitude. I do not believe the ring ratio merely fits a cognitive niche. Rather, I suspect there are, deep in geometry of space, reasons why ringlike structures with this kind of ratio occur. In current jargon, rings of this particular ratio might be viewed as attractors in some phase space. However, this is too easy. The tendency for such structures to occur might be shared over a very wide range of circumstances, and the voids and ring-filaments in the cosmos might form in shapes similar to the spit in the bath, not for dynamic reasons having to do with physics, but for mathematical reasons having to do with the structure of space itself.



The actual physical diameter of one of these voids, is unimaginably huge, about 100 million light years. It is in miles, 186,000 * 3600 * 365 * 24 * 100,000,000 = 5865696 * 10^6 * 10^8 ≈ 6 * 10^6 * 10^6 * 10^8 = 6 * 10^{20} miles. A jet plane flying at 600 mph, would take a 10^{14} years to cross this void – something like 5000 times the age of the universe itself. I say this only to emphasize the truly huge size of the voids that we are talking about, and in particular to drawn attention to the fact that if it is that huge, the ring thickness could be almost anything, and the ratio of ring thickness to ring diameter could have a large range of possible values.

There is no a-priori reason for the ratio (ring diameter/ring thickness) to be in the range of 10 to 1 (one order of magnitude). It could just as easily be 100 or 1000 or 1,000,000 – i.e. 2 or 3 or 6 orders of magnitude. Why is it on the order of one? Stated differently, what possible reason could there be that the rings around such a colossally enormous, almost impossibly huge-to-our-understanding, structure in the universe should have roughly the same geometric proportion as the gold wedding ring on a woman's finger or a rubber band. Is this not pushing coincidence a bit?

But if we accept that the harmony-seeking computation is based on transformations that are (for whatever reason) largely congruent with human cognition and mentality, then we may not be silly to consider such a process as an explanatory process even in the most outlandish structures.

This supposes then, that there may be mathematical reasons which generate circles or ring-like filaments whose thickness is just about one order of

magnitude smaller than their diameter. There is, at the moment, no astrophysical theory that would predict this result – at least none that I know of – but from the point of view of harmony-seeking computation that I am speaking about, there is a possibility that the cognitive simplicity of this particular ratio for a ring-structure, could suggest that it might *have* to be so for mathematical or quasi mathematical reasons.

Is this strange? Yes indeed, to our ears it is very strange. Is it more strange than Pauli's exclusion principle, when it was first put forward? I do not think so. Thus a purely geometric harmony-seeking computation, might explain at least one aspect of the structure better than any current physical theory. Does it seem unlikely? Yes, perhaps. But in the absence of better computations, I do not think it can be considered entirely useless. At the very least, some reason needs to be found why such processes, congruent with cognition, might occur in structures at the scale of the universe and at the scale of human beings, and the ring-ratios tend to be the same.

Three points about the voids. First, there is a ratio of void diameter to wall diameter, consistent with the LEVELS OF SCALE property. Not larger, not smaller. Second, we should try and find data about the evolution of voids (by examining different red shifts, hence different eras, to find out how the structure of voids has evolved and to try an find observational data about the ring thickness at different eras. Third, we should make a more precise prediction, based on the harmony-seeking computation, and test it against observations.

VI STRUCTURE-PRESERVING TRANSFORMATIONS: HELPING A LARGER WHOLE TO FORM

Viewing The Previous Examples as Computations

The examples on pages 4 to 30 all have the fact (most of the cases), or the possibility, of some kind of harmony-seeking at work. In all cases, there is some kind of structure-preserving process at work, mainly geometric in nature, sometime intellectually graspable, sometimes not so clear. Apparently, as a result of this structure-preserving process, at each step the generated configuration puts the starting configuration in a slightly more harmonious, more coherent, state than it was in before.

Most important, if we choose to do so, we may regard ALL these examples -- both physical processes, simulations, and actual paper and pencil exercises -- as computations, each of which takes its system forward, step by step, from an existing state W_1 to a state W_2, and then to W_3, . . . W_i , W_{i+1}, and so forth, and as it goes increasing structure and increasing harmony and wholeness in the system. These steps are carefully calculated, normal in many spheres of human experience and in many natural systems. Some of them are done by human instinct; some are done by animal instinct; some are done by adherence to a certain tribal sensibility; some are done by concern for ecology. Many occur naturally (without human intervention) as part of the behavior of some complex physical system.

In a generic sense they are all computations. But in nearly all these cases they are not easy to deal with as computations recognizable in the present way we understand calculations or algorithms. This does not make them trivial, it makes them fascinating. There are reasons to believe that a fundamental kind of process is at work in all these cases. If we can examine these computations, and begin to understand or extract the underlying way they work, all of them together, and if we succeed in getting the gist of this computation, we may find a way towards a powerful new way of computing that is guided by emerging harmony, and by a motion towards harmony.

I view these examples as *computations*, and state it that way now, because, after twenty years of work, it has slowly become likely, to me, that they all embody some kind of single process, or function, which has the capacity, starting with an arbitrary configuration, to reach a better and more harmonious configuration. Although the idea of an SP-transformation is, for the moment, still somewhat loosely defined, there is considerable similarity between the different examples in the way they work, and in what they accomplish. I conclude that it is reasonable then, to think of the examples as exemplars of the operation of a single type of mathematical function, or computation, at work.

However, to learn to see these examples as embodiments of a single type of computation, we must now work harder to try and be specific about what this computation might be, how it might work, and how, above all, we can put it to work, in cases where we seek better organization, better harmony, better deep adaptation and coherence for a configuration.

Experimental Confirmation

An important experimental result. Apart from the structural explanations which have been given already, my own cognitive experiments show strong agreement among observers as to comparative judgments regarding what steps are structure-preserving and what steps are not.

A step is a harmony-seeking step, if it preserves or extends or deepens structure. When asked to judge different possible steps, starting from a given configuration, in terms of the degree to which the steps are SP (structure-preserving), or not, people seem to agree – based on their intuitions, or cognition. This alone strongly suggests that the quality of “being SP” is to some degree objective, and gives a strong hint that the phenomenon reflects an underlying physical reality.

It is also true that if we ask people taking in part in such an experiment, to make (in their own chosen terms) any kind of structural diagram in which they try to capture the wholeness of the thing they are looking at, before judging which steps forward from it are structure-preserving and which are not, the level of agreement between observers in their ability to judge what is SP and what is not, goes up strongly. People who have performed such a private diagram-making process, even though their diagrams are not alike, will then agree more strongly with one another about what steps forward from that structure are SP. This occurs even though the making of a diagram, which describes the structure, is a private matter, and people are given no special instructions about how to do it, and the different diagrams people make are unlike. It seems to be enough, that after *attempting* to represent the structure, they can then judge more reliably what is structure-preserving and what is not. This, once more, is evidence that the phenomenon of SP is real and objective as a phenomenon, with objective structural content.

The SP-Postulate: Always Helping A Larger Whole To Form

How may we formulate, mathematically, the character of the harmony-seeking steps which occur in the examples?

In each case there is a whole, $\mathscr{W}$, and within the whole a latent center which is being modified, transformed, shaped, or reshaped, by a certain step. This latent center is the focus of the transformation, and the latent center sets the boundary of the geometrical and physical transformations that are then actively being undertaken. Let's call this focal latent center $\mathscr{L}$.
At the same time, there is a larger whole, often an order of magnitude bigger than $\mathscr{L}$. The transformation which is structure-preserving, preserves the structure of the $\mathscr{W}$, and to do so modifies the $\mathscr{L}$, and modifies it in relation to the whole context around it. Thus the output from this step is a modification geometrically within $\mathscr{L}$, but it is a function of both $\mathscr{L}$ and $\mathscr{W}$. In addition, there is a sense in which $\mathscr{L}$ is being *fitted* to $\mathscr{W}$, it is being made to fit $\mathscr{W}$, to be congruent with $\mathscr{W}$, adapted to $\mathscr{W}$, harmonious with $\mathscr{W}$. Further, in order to modify $\mathscr{L}$ in this way, new centers are going to be created within and around $\mathscr{L}$. We may refer to these new centers as $\mathscr{N}_i$, and since there may be several of them, we may think of them as $\mathscr{N}_1$, $\mathscr{N}_2$, $\mathscr{N}_3$, etc.
To undertake this transformation, in such a way that it is indeed a structure-preserving transformation, the various $\mathscr{N}_i$ are to be generated by fifteen generic types of transformation acting on $\mathscr{L}$. These are *the* fifteen principal center-creating and structure-preserving transformations listed on pages 2-4:

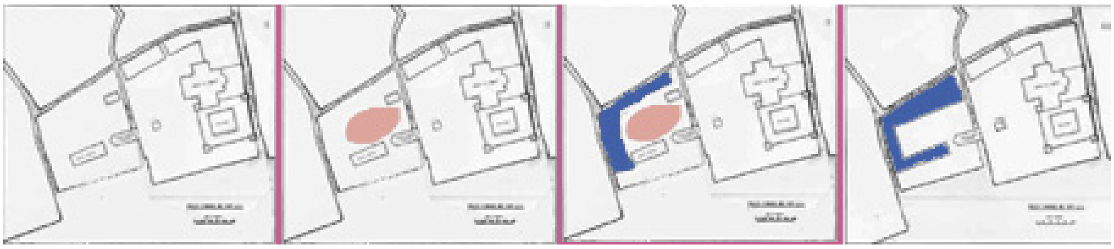
STRONG CENTERS
LEVELS OF SCALE
BOUNDARIES
ALTERNATING REPETITION
POSITIVE SPACE
GOOD SHAPE
LOCAL SYMMETRIES
DEEP INTERLOCK AND AMBIGUITY
CONTRAST
GRADIENTS
ROUGHNESS
ECHOES
THE VOID
SIMPLICITY AND INNER CALM
NOT SEPARATENESS

These transformations are enumerated and defined in considerable detail in chapter 2 of *The Nature of Order*, *Book 2 (The Process of Creating Life)*,[21] and in chapters 5 and 6 of *The Nature of Order, Book 1(The Phenomenon of Life).*[22]

The SP-Transformations Of St Mark's Square, Previously Discussed.

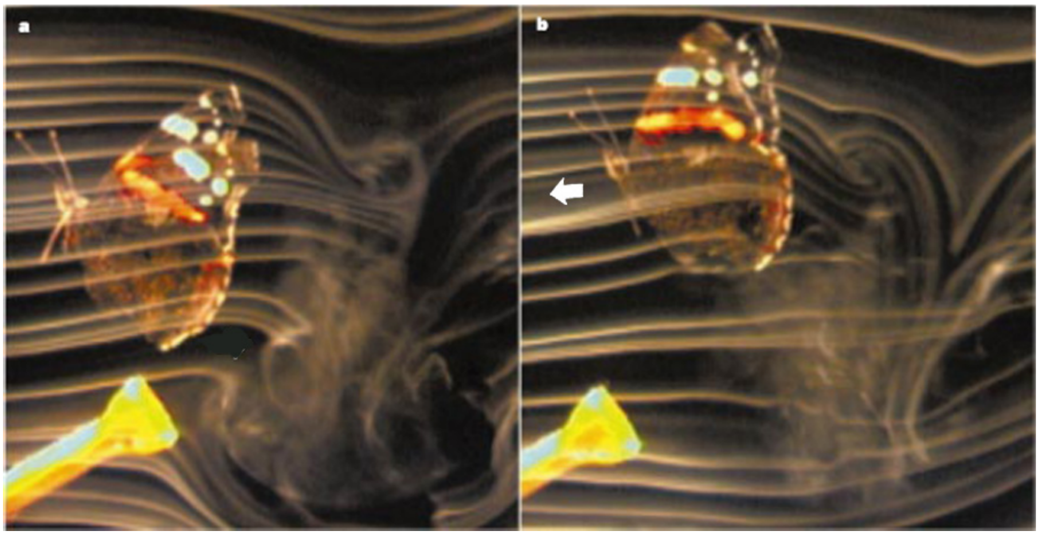
Please look back at pages 24-26. Here, in each cycle, the next building to be built is occurring in $\mathscr{L}$, and the larger context of the whole St. Mark's area is $\mathscr{W}$. However, there is now a subtlety. In a particular step, we know what $\mathscr{L}$ is, because we are looking back in time and see what the step was. But the people who actually did the step were not, at the time, so clear. They could not know what $\mathscr{L}$ was to be, until examination of the context and the larger whole revealed it to them. And there is a further subtlety. The context $\mathscr{W}$ is not something so vague and general as the whole St. Mark's area. It is, rather, a particular area within St. Mark's square, where a latent center has been identified as being in need of improvement, or presenting itself for elaboration and strengthening. This latent center, which plays a crucial role in the structure-preserving transformations, we are calling $\mathscr{L}$, and the detailed effects are created (as discussed in my text) by the fifteen transformations acting together. So it is actually the immediate local context of $\mathscr{L}$ that then gives rise to the step that transforms $\mathscr{L}$.

Alternatively we might say that it is each latent center $\mathscr{L}$, once identified, that is the entity to be transformed, and the transformation is to be done in such a way as to strengthen $\mathscr{L}$ in its ability to help and make cohesive the larger context $\mathscr{W}$. The particular bits of building needed to carry out the transformation are the various buildings and partial buildings $\mathscr{N}_i$ that are generated by the action of the transformations.



Here, for example, the red ellipse, is L, the latent center formed by the three buildings around it. To confirm and strengthen L, the blue building mass is built, thus forming a stronger rectangular space, by enclosure, and establishing continuity with the buildings on the right of the latent configuration.

Example 14. SP Transformations Performed By A Red Admiral Butterfly Flying In A Windstream



These two still pictures are clips from a remarkable movie made by Robert Srygley in Oxford.[23] The movie shows a flying red admiral butterfly, which surfs the windstream. If one examines the motion, it is clear that the butterfly is not flapping its wings like a bird, but surfs the windstream, moving with the laminar flow, taking advantage of the currents and their configuration and position. Essentially, the butterfly keeps adapting its motion and position in such a way that it makes a continuous series of structure-preserving moves with respect to the stream. In the movie you may see this motion.

In the particular case of the transition shown above, the butterfly moves in such a way as to align itself with the laminas of the flow (and ends up facing the direction shown by the white arrow). In this instance, the butterfly is $\mathscr{L}$, its wings, body position and so on are the $\mathscr{N}_i$, and the windstream is $\mathscr{W}$. The next movement and position and configuration of $\mathscr{L}$, depends on $\mathscr{W}$, and is an active effort for $\mathscr{L}$ to grow out of the structure of $\mathscr{W}$. The so-called "flight" of the butterfly, is a process in which the butterfly moves so as to be harmonious and aligned with the structure of the breeze, and the laminar flow of the breeze. To achieve this the $\mathscr{N}_i$ are continually re-configured to preserve and extend the structure in a harmonious way.

VII HARMONY-SEEKING RATHER THAN MERELY "EMERGENCE"

Coupled Local Atomic Events Generating Larger Wholes Through Interaction v.
A Whole-Based, Harmony-Seeking Process Which Works By Continually Strengthening Latent Centers.

It has nowadays become almost commonplace, to explain how geese fly in formation[24], how ants accomplish complex tasks together[25], or how slime mold (apparently an uncoordinated aggregate of cells), is able to move coherently as a whole[26]. In most cases, the explanation is, in some form or other, that the individual "cells" copy the behavior or action of their nearest neighbors; in many cases this ultra-simple rule of action explains considerably coherent movements of the larger body of cells.[27] In other more complicated cases, the atomic actions of individual cells are coupled: and the coupling helps more complex forms of behavior to occur in the group.[28] In either case, the resultant aggregate seems to be acting as a whole.

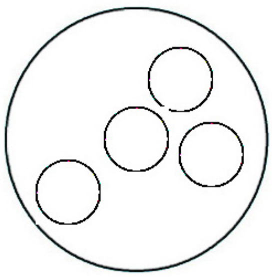
However, to be sure we are talking sense when we say these things, and to avoid exaggerating what we seem to have discovered under the rubric of emergence, it is important to analyze – with very great care – just what we mean by "acting as a whole," in these kinds of cases.

When health, or wholeness, or harmony exists in a part of the world, what is under discussion is always the relationship of a given system, to the larger world beyond that system. So the issue is not merely, whether a group of elements act together. What is important above all, is that when the elements are grouped together to form a system, the resulting system either does or does not act in such a way as to heal, or sustain, or improve, the coherence and health of the yet larger system around it --

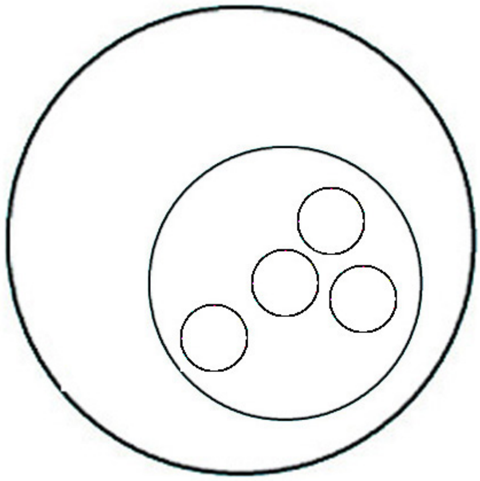
some part of the world *outside and beyond* the group – some system of which this group of elements is a part.

The **emergence** phenomenon is a **two**-fold relationship, between a set of elements and a group they form. The **harmony** phenomenon is a **three**-fold relationship, between a set of elements, the group they form, and the helpfulness of the ensuing group to the world beyond the group. Thus:

Emergence, a two level relationship



Harmony, a three level relationship



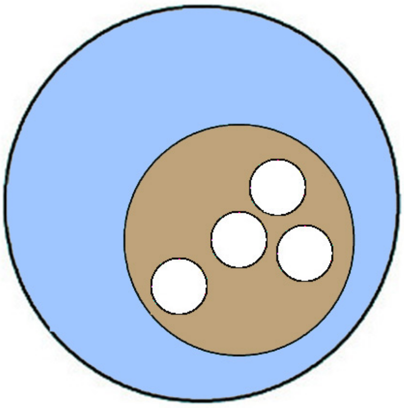
In the left hand case, we have a group of elements, and because of their interactive coupling, they act as a whole represented by the outer circle. In the right hand case we also have a group of coupled elements which act as a whole because of the coupling, but in addition, this whole acts to support or help some feature of the larger system (represented by the outermost circle).

The system on the left is interested only in itself. It does nothing to help the world beyond it, and does not contribute harmony or health to the world beyond it. The system on the right, is not only embedded in the bigger black ring: it *helps* that ring, and establishes harmony between the smaller ring of rings and the largest ring, so that the smaller ring is well adapted to the larger, and the large one benefits from the smaller.

In Detail, What Exactly Does It Mean For A System To Help The Larger System It Is Embedded In?

Let us return to the example of St Mark's Square. At each cycle the process identifies a latent center in the larger configuration. This latent center is an area or potential center which is weak, and which – if strengthened -- would improve the coherence of the whole. The area immediately around that latent center is healed or made more whole by the injection of the repaired latent center.

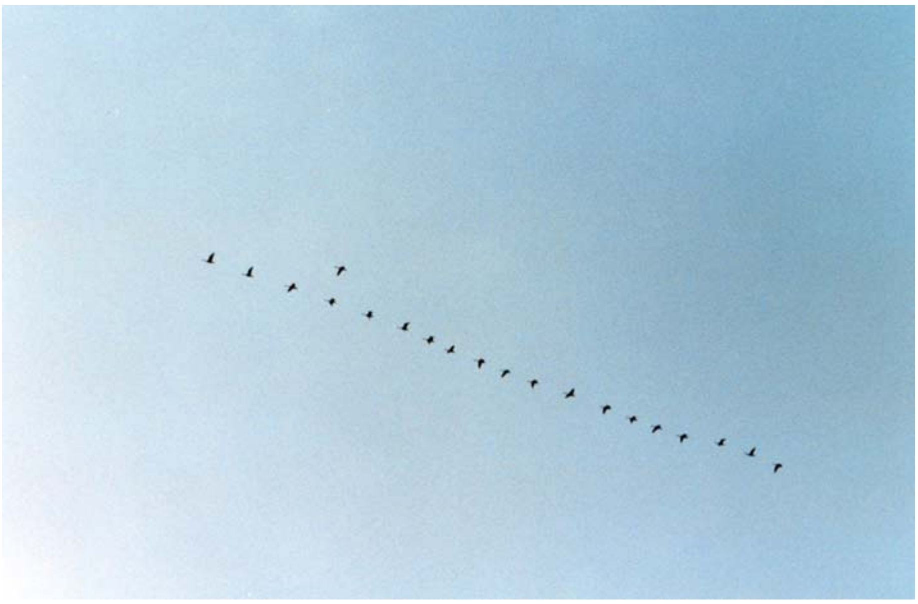
Abstractly we may express this concept through the following diagram. The red area is a latent center, a weak center which has the capacity to heal the blue area beyond it: and it is able to do so by creation of the smaller white centers -- they are built to create coherence in the red center. However, the key point is that the healing will not take place, unless the red center then also helps to heal the larger blue center beyond it.



This is not really obscure. But as a key point, it is sometimes lost in the current flurry of attention that "emergence" (so-called) nowadays draws unto itself.

Example 15. Flying Geese and the V-formation

Flying geese.[29] Many of the boid simulations due to Reynolds and others, essentially provide particle models in which the particles fly about, avoid each other, all go in the same direction, flock, play, and simulate some of the behavior of birds in a fairly realistic fashion.[30] These simulations do not do as well when it comes to generating the characteristic V-formations of migrating Canada geese. Much has been made of the fact that when in the V formation, the birds increase their flying range by about 70%, because they use less energy etc, etc, because of the vortex interactions, and that this is also true for aircraft flying in such formations.



That, of course, does not explain how the V-shape actually comes about in the case of migrating geese!

I have seen several boid simulations which demonstrate the flocking of the birds. Seemingly random motions quickly give way to groups of birds which fly together and in the same direction, while maintaining a typical spacing distance from the other birds. That does not in itself generate the V-formation. Those cases I have seen where the V-formation is claimed, or made to appear all, in my experience, have built in this configuration surreptitiously within the rules that govern the birds motion.

It has been said that birds can feel the vortex, when they fall out of formation – because they experience that it is harder work, thus creating a feedback loop that brings them back to the flock in an appropriate position.

The V-formation of Canada Geese

Let us consider what real geese may be doing. The three boids rules most frequently given are these:

- If you are about to crash into another bird, turn around.
- If you are far away from other birds, head towards the nearest bird.
- Otherwise, fly in the same direction as the bird next to you.[31]

This system of three rules generates flocks *very well*, in fact to an extent that seems amazing the first time you see it. But these rules, alone, do not create a persistent, stable, V-formation. To do that we need to add two further rules:

- Each bird will try to fly in the wake of the nearest bird (because they can sense the change in pressure and the easier ride in that position). The optimum positions are in the wake, but off-center from the axis -- not merely close to, or in the same direction as, the nearest bird.[32]

- The leader gets tired (because this is the only bird not getting the energy advantage), then drops away, and another bird takes its place as leader. There IS a rule of follow the leader (though boid enthusiasts deny it), but the leader is not an arbitrary "king." Instead the wake rule means that birds follow one another, without electing a leader; but at any one time there is a temporary leader who gets defined by the fact that (s)he is the only bird not behind another bird. Being in this position is tiring, and birds try to avoid it. So there is no permanently elected leader: but there is always a temporary leader, and that temporary leader keeps changing.

When we add these rules, two things happen. The dynamics do now generate a stable V; and the rules of action require that the birds compute or calculate in a way that makes a local center work to help a larger center, thus demonstrating the action of the basic rule I have mentioned earlier as fundamental to all harmony-seeking computations.

That can only be understood, and generated, by using a computation which looks at the growth of the ecological whole, and the emerging structure of the V-configuration as a whole. It is significant, I think, that the real situation cannot be modeled properly without a harmony-seeking transformation. That is because the real computation needed to generate the V, requires a computation which explicitly relates the individual to the whole.

The two key bits of additional information needed to simulate the real flights of geese in V-formation, are:

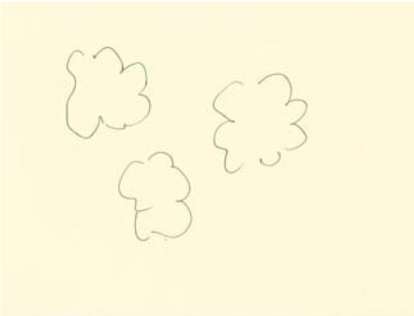
- An explicit statement to the effect that the individual goose does NOT merely act as a loose and uncoupled entity, generating structure with others through "emergence," but that the contribution of this one goose's flight, to the overall flight of the flock, does have to be made in a very specific way (flying in the wake of another bird) which helps the larger whole to work better.
- A temporary leader is willing to give up the desire to fly behind another bird, for as long as he chooses to hold out. For this one bird, rule 1 is temporarily abandoned.

Our currently overstated enthusiasm for "emergence" as evidenced in boid-like computer games and simulations, does little to unravel the real, and more subtle problems of the whole and its organization, which hinge (almost always) on the way that small parts work to help a larger whole, and the ways that the larger whole also shapes and modifies the action of the smaller parts.

The real issue is that the emergence which is being attributed to birds (when they are viewed as mechanisms), is not as marvelously dumb or mechanistic as some emergence enthusiasts perhaps like to think. The fact is, that to produce the V-formation, the birds *themselves* have to perform a harmony-seeking computation, in the way that they act to relate themselves to the larger whole. The fact that the birds themselves perform this harmony-seeking computation, is the essence of the situation!

Example 16. Clouds And The Positive Space Which Arises Between Them

In the following photographs we see various examples of cloudy skies. We are familiar with the shapes of clouds, and do not need to look at them, just now. But I want you to look at the shape of the blue sky between the clouds. I had never studied this blue sky carefully until recently, and then began to notice (with something of the eye of a painter), that the blue patches are nearly always well-shaped. These are extremely different from the childlike cotton wool cloud formations we perhaps carry in our minds as a picture of the way clouds (especially cumulus) are.



Cotton wool clouds: the clouds are objects and the sky is background

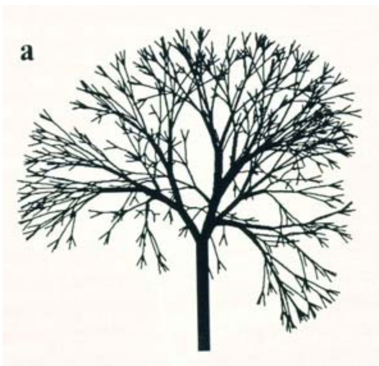
Look at the real example in the photograph below.

In every one of these cases, the blue sky is made of definite and positive shapes (POSITIVE SPACE). This notion is not entirely easy to make precise. For a painter it is straightforward. Every painter knows that a picture cannot be good unless all the spaces and components and fragments, even, have their own positive shape. But this positive space is quite tricky to formulate in mathematical terms. The idea of POSITIVE SPACE, is something like convexity. In mathematics, a convex body is one which has the property that for any straight line that connects two points inside the body, all the remaining points in between, along that line, also lie entirely inside the body. Positive space is space that is, in shape, coherent, it is formed of positive, somewhat convex lumps that have definite and recognizable shape, but it is less tightly constrained than mathematical convexity – hence *quasi-convex*.

In any case, there is no a-priori reason to expect such POSITIVE SPACE to appear in the blue sky between the clouds. Yet if you watch clouds blowing and changing minute by minute, they maintain the positive space of the blue, as well as the positive space of the white and gray clouds themselves, at every instant. How exactly might this be

A sketch of mine showing a real tree as it typically is,
showing the positive space generated between the tree's branches

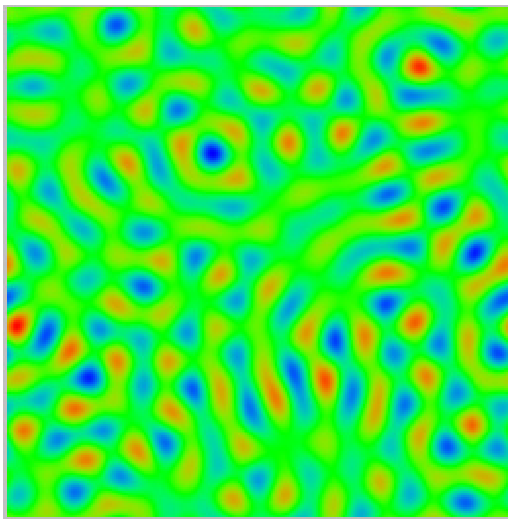
explained? A similar phenomenon occurs in the space between branches on a tree. This too, is typically positive space. Yet the space has no obvious energy to push and create its positiveness. Nor does it yet have an accepted mathematical formulation. All the L-system simulations that I have seen lack this positive character of the space between the branches. For example, see drawing on the right:[33]

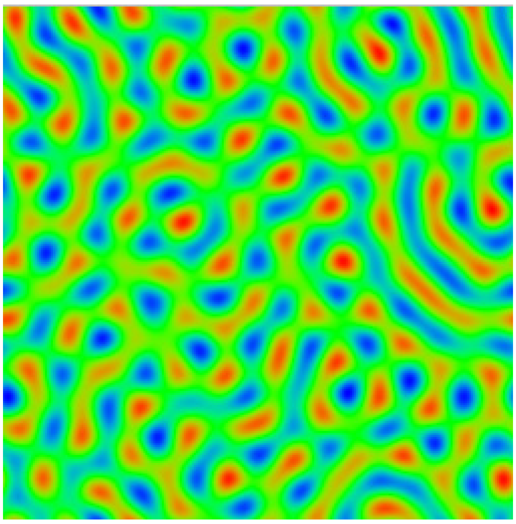


So how may we explain what nature is doing in these cases? Somehow the positiveness of space appears in the space between the clouds, but not for reasons obviously connected with the energetics of the system. Instead the system simply seems to have a disposition to have this positive space appear. What is causing it, and how does it work?

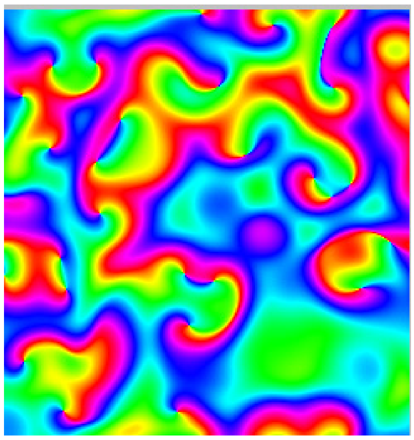
Of course what we see in two dimensions, is actually a three-dimensional phenomenon. The easiest way to imagine it, is to consider both the white bits, and the blue bits, as quasi-convex bodies. Somehow, as the clouds evolve, it is a co-evolving system in which this loose packing of differently sized blue and white quasi-convex bodies is maintained all the time – or nearly all the time. Such a changing dynamic packing of irregular sized cells could be a more complex 3D analog of Taylor vortices or Benard convection. On the next page I show two simulations from the laboratory of Professor Michael Cross at Caltech, demonstrating stages of Benard-like formations in a medium under the impetus of the Swift-Hohenberg equation, and one under the impetus of the complex Ginsburg-Landau equation.[34] They are certainly not like clouds, but far more regular. Still, it is not hard to see how the positive space I have illustrated in cloud systems, could come about as a result of more complex interactions stemming from these kinds of effects, and possibly driven, in addition, by some iterative rule similar to the POSITIVE SPACE transformation.

Although this is not (in this form) as far as I know, part of the presently accepted physics of the cloud system, it is exactly what one would expect from an accurate harmony-seeking computation, which has, as one of its most important structure-preserving transformations, the continuous maintenance of POSITIVE SPACE at every step.





Configurations arising from the Swift Hohenberg equation

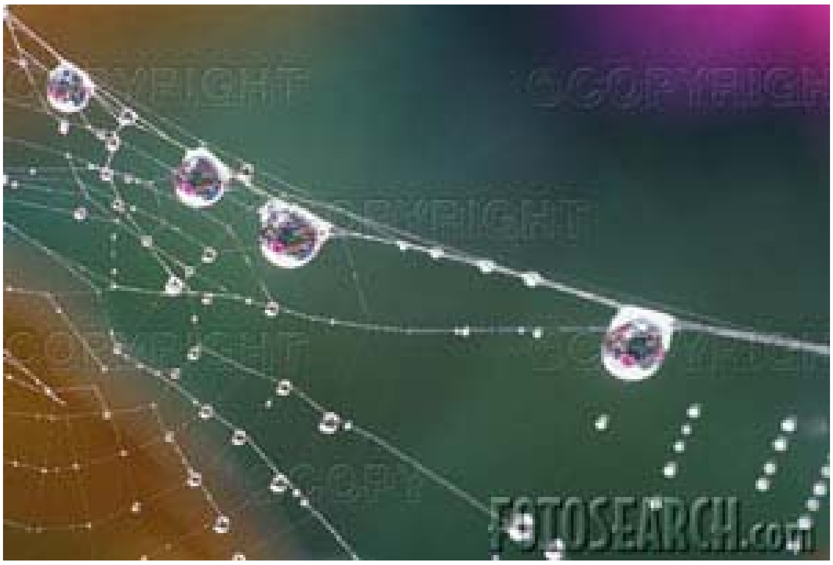


On the right: configuration arising from application of the complex Ginsburg Landau equation

VIII STRUCTURE-PRESERVING TRANSFORMATIONS AND SYMMETRY BREAKING

The Possibility that Structure-Preserving Transformations are Deep Generalizations of Symmetry-Breaking.

One of the most familiar analyses of the evolution of natural phenomena and configurations, is the one that Ian Stewart and Martin Golubitsky have popularized: The phenomenon of symmetry breaking in geometry, not only in systems of equations and variables.[35]



Example 17. Dewdrops On A Spider's Thread

Ian Stewart's explanation of symmetry breaking as an output from various natural phenomena, leading to interesting morphological results, is by now well known, and the regular spacing of dewdrops on a spider's web is one case he and Golubitsky have discussed extensively. Crudely put, the water coated on a spider's web thread, starts life as a uniformly coated cylinder of water (made roughly uniform in thickness by surface tension). When the surface tension starts to break up the continuity of the coating, it leads to a configuration which falls apart, but still repeats at least some of the symmetries present in the continuous cylinder, since there is no reason for all *those* symmetries to be removed as well.

This idea is very similar to the idea of a structure-preserving transformation. We have a structure—the infinite translational symmetries of the cylinder along the thread, and the rotational symmetries of the cylinder around the thread. As this system moves to a fragmented version (caused by the action of the surface tension, or by jiggling of the thread), the simplest end product is the configuration which destroys the least possible number of symmetries, or maintains as many of the remaining symmetries as possible. Golubitsky and Stewart's most recent work on symmetries in the equation systems of bifurcation theory, continue such ideas.[36] But of course, this one particular way of preserving the structure of what is there, is very limited indeed, compared with all the possible ways of preserving and enhancing structure.

In addition, even the symmetry-breaking interpretation of what is going on in the simple dewdrop example, is geometrically too limited. Each dewdrop takes on local spherical symmetries and axial symmetries normal to the thread, and symmetries parallel to the thread but not aligned with it, that are not present in the symmetry scheme of the infinite cylinder.

In my view, the symmetry-breaking idea is not yet, by itself, sufficiently profound to be useful as a general theory explaining real-world complex configurations, or to account for the harmony-seeking phenomenon I am describing in this paper. As I have said earlier, it has been demonstrated that "the" wholeness consists, in part, of the entire system of overlapping local symmetries at a wide variety of scales in a configuration.[37] We therefore need to have a view where somehow the underlying structure of *all* these symmetries, working together, is preserved. And further, the centers that are present in a given wholeness are not all LOCAL SYMMETRIES. Other centers are formed by GRADIENTS, ECHOES, BOUNDARIES, DEEP INTERLOCK, POSITIVE SPACE, NOT SEPARATRENESS, and so on. These other properties and that entire structure, *too*, have to be preserved when a harmony-seeking computation starts with a currently existing structure, and finds its way to a stronger structure that is latent in this overall configuration, and can be brought out by a few transformations.

In summary, we may say: A structure-preserving transformation is a more complex and much richer version of the phenomenon whose simplest cases have in recent years been called "symmetry breaking" or "symmetry reduction." This nomenclature is rather over simplified, and does not do justice to the real potential power complexity of the underlying phenomenon. An SP-transformation is a transformation which moves a complex configuration forward, retaining as much of its wholeness structure as possible. In so doing the configuration usually becomes richer and more complex in unforeseeable ways that benefit the larger whole.

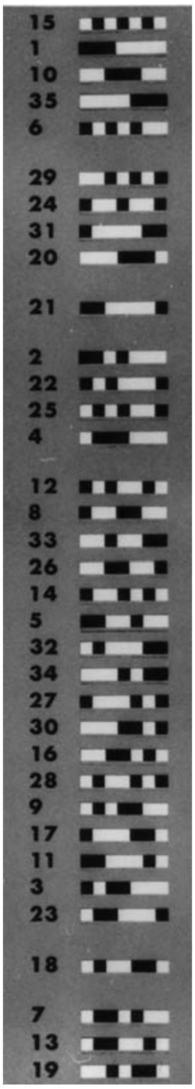
Local Symmetry Production

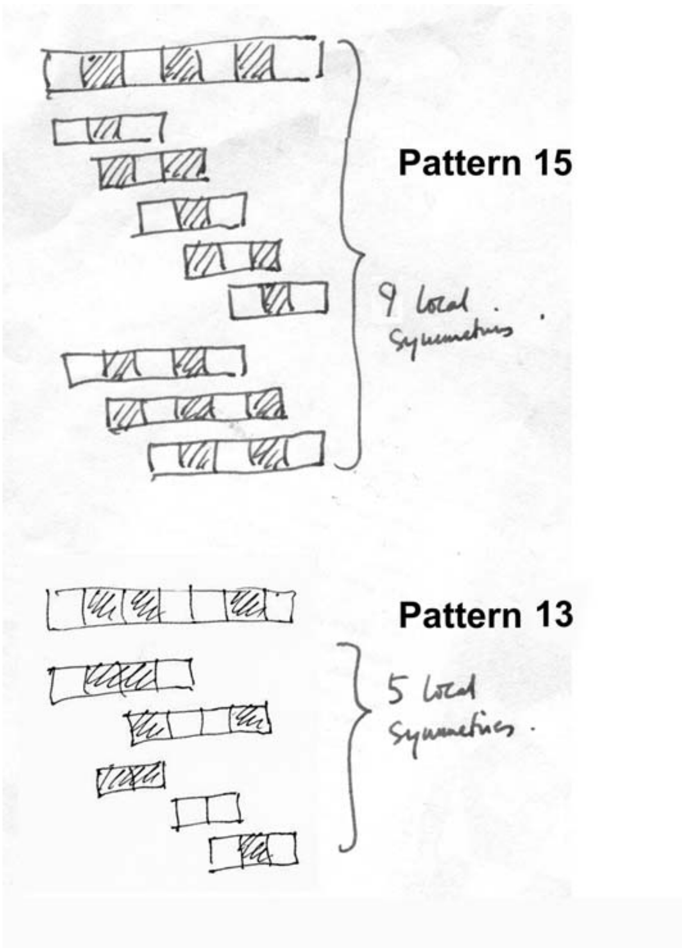
Indeed, even in these cases of symmetry reduction, what we actually see, if we look closely, is symmetry ***elaboration***. The infinite Euclidean 3D-space, does indeed lose the simple symmetries, but at the same time, new local symmetries appear, and are generated.

This is a better picture of how harmony-seeking computations work. The failure to see it, comes, I think, from the fact that in recent years the symmetry structure of the plane, or of the 3D-continuum, have been viewed too simply through the symmetries *of the whole* It has not been sufficiently clear that there are, hidden in the plane, or in the continuum, an infinite number of *symmetries of smaller local sets*, all over the place, and that there are, in the world, an infinity of systems of smaller and smaller symmetries which occur in these nested sets. What happens when the whole evolves under the harmony-seeking computations I envisage, is that many of these smaller symmetries are generated, or strengthened, thus giving a hierarchical nesting of local symmetries at different levels of scale.

Example 18. Black And White Strips

This is particularly easily visible in some cognitive experiments on black and white strips my colleagues and I did years ago.[38] The left hand illustration shows the rank order of coherence, of 35 such strips, as measured by a variety of independent cognitive and perceptual tasks. Those at the top are most coherent, cognitively; those at the bottom least coherent. After two years puzzling over the experimental results, I found that the rank order is predicted almost exactly, by counting the number of local symmetries in the pattern. The patterns at the top have 9 local symmetries, those at the bottom have 5, and the others lie in between (see accompanying diagram which enumerates the nine local symmetries (viewed as sets) in the pattern labeled **15**.





What is most significant is that the presence of these local symmetries in the pattern, and the number of them, cause what is seen as coherence. This structure is obviously not a product of symmetry *breaking*. It is an example of symmetry production. In innumerable cases, especially in organic development, local symmetries (limb-buds for example) are created, and this is one of the most important phenomena in the development process.

In order to understand this source of harmony, it is necessary, I believe, to see the symmetries as part of a system of overlapping nested sets, in space, each of which may take on local symmetries or not, within its own local frame.

One view of a harmony-seeking computation, in this context, is that it is a type of computation which injects as many overlapping local symmetries as possible, into a finite framework. Salingaros has shown that such compressed systems of local symmetries are present precisely in the acknowledged great buildings especially of ancient society.[39]

The Parthenon: highest in the count of local symmetry-density among the 25 famous buildings measured by Salingaros

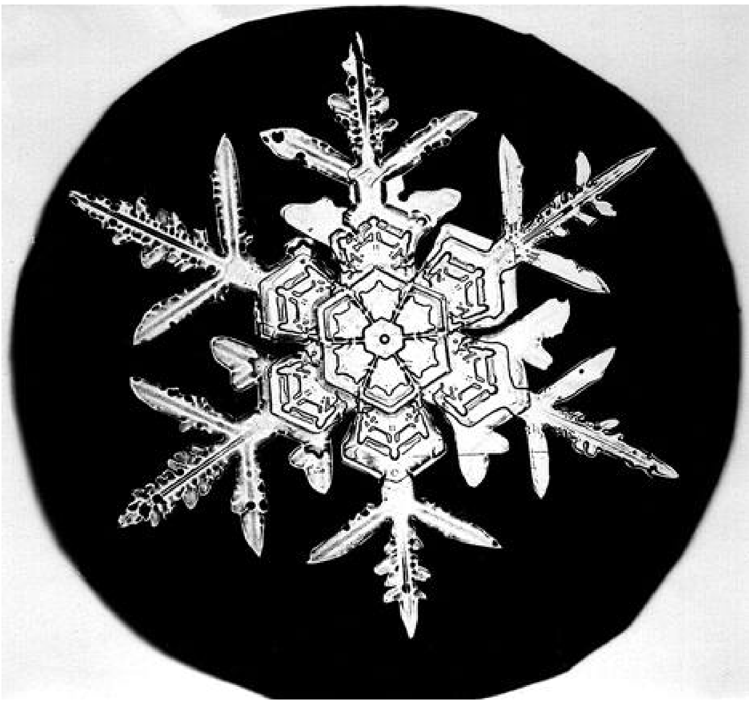
Example 19. Snow Crystals

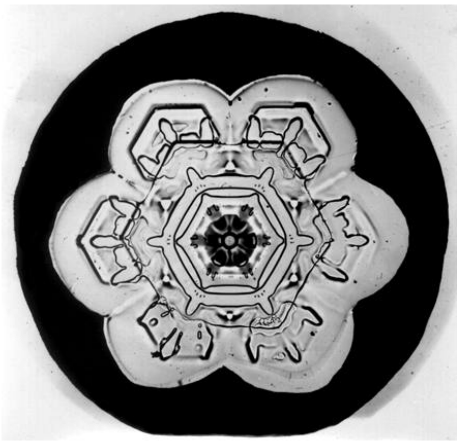
Another rich source of information about the deficiencies of algorithmic computations in morphological matters, lies in the attempts to simulate snow crystal formation. Snow crystal growth has been simulated with partial success by DLA methods, and by cellular automata methods.

But the results of these simulations pale when compared with the extraordinary variety seen in real crystals. Bentley photographed some 5,000 snow crystals during his lifetime, and these precise and exquisite photographs show us the kinds of structures which a successful computation must be able to create.[40] In particular, we see very extraordinary LOCAL SYMMETRIES, DEEP INTERLOCK, POSITIVE SPACE, and GOOD SHAPE, at levels which are common in works of art, but none of which show up strongly in the results of the DLA simulations or cellular automata simulations so far published.

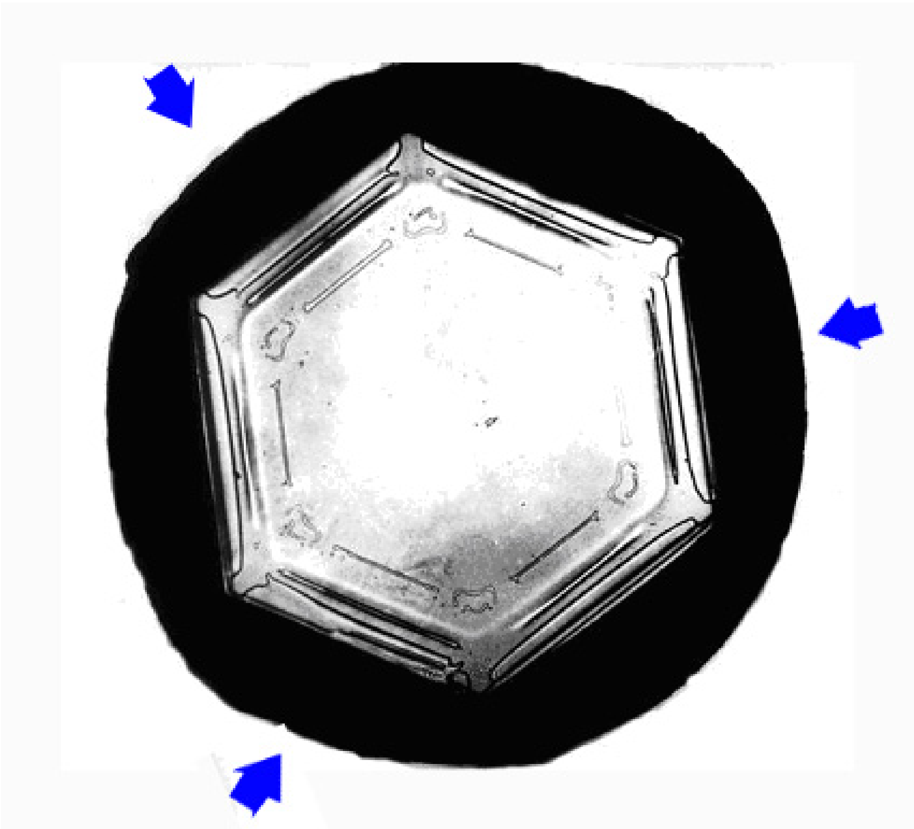
The currently prevailing theory of snow crystal growth, says that the crystal grows outward, from a small hexagonal plate which is the starting point. The six-fold symmetry occurs in the six arms, so it is said, because the conditions of temperature and spatial constraint are essentially the same along each of the six radial axes: hence the high level of morphological similarity from arm to arm. This idea has been put forward with great clarity by Libbrecht at Cal Tech.[41]

If you examine the crystal illustrated below, you see that there is an alternating pattern, three arms have one pattern, and the other three alternating with them, have another. This is easiest to see in the moth-like figures that occur on the three arms that are 120 degrees apart. This alternating three-fold symmetry requires some other explanation than simple growth outward under time-dependent spatially uniform temperature changes. Some larger SP-transformation is entering into the computation.

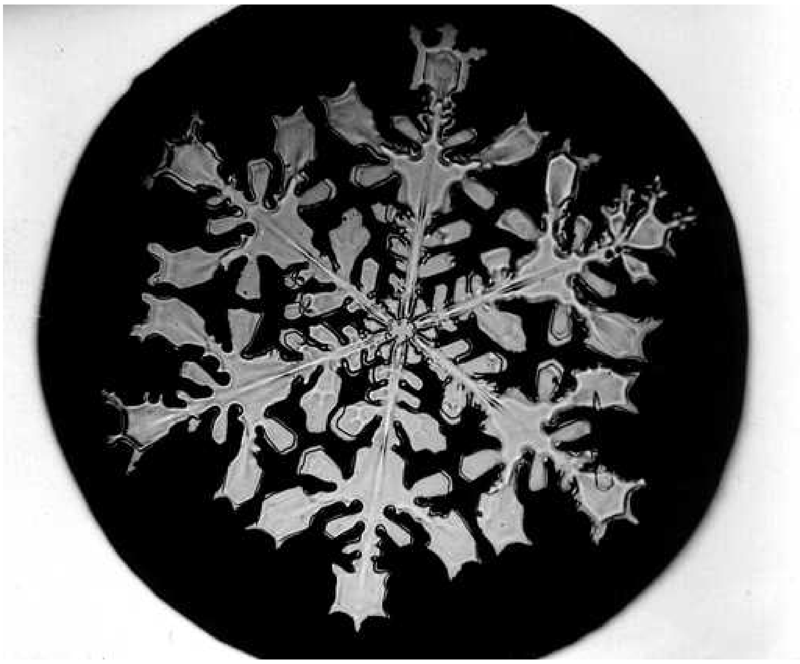


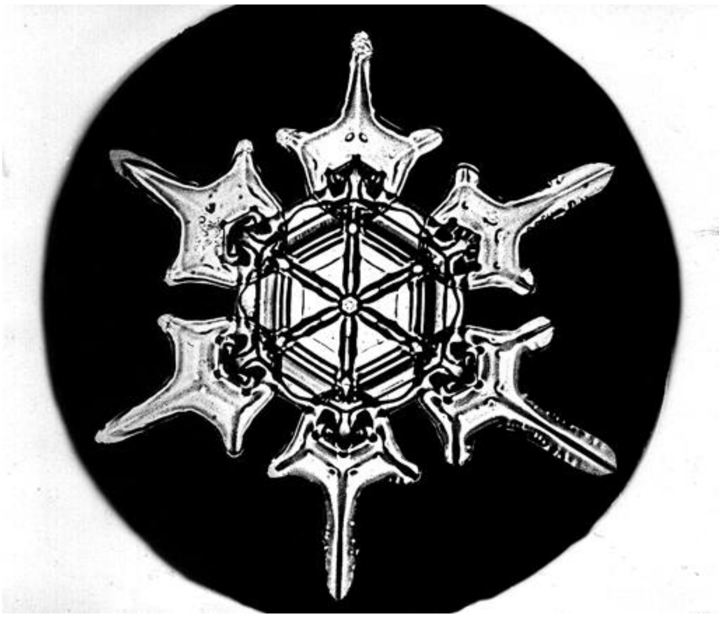


The next, very simple plate crystal shows another fascinating aberration. Again, we have relatively uniform hexagonal symmetry, centered on the six vertex axes as usual. However, if we look at the outer edges of the hexagon, in the positions marked by the blue arrows we see three sides which have a different symmetry -- a shape that has two ogive S-curves just inside the edge of the hexagon. This new symmetry is based on three axes that are edge-oriented, not vertex-oriented. Clearly this highly ordered regularity cannot be explained by similarities merely caused by uniform temperature conditions from axis to axis. The three axes of symmetry now coming forward are not a subsystem of the original six growth axes, but three new ones that are interlaced among them. I believe the phenomenon can be explained by strongly invoking the LOCAL SYMMETRIES transformation: but not as a part of the simple kind of growth mechanism described by Libbrecht.



In the next crystals there is a remarkable and very strong adherence to the formation of POSITIVE SPACE. The spaces between the ice are themselves coherent in shape (GOOD SHAPE), and well-formed. This occurs because of a balance between diffusion and aggregation, and has been well documented. But the important thing is that coherent structure (POSITIVE SPACE, DEEP INTERLOCK, LOCAL SYMMETRIES) emerges in the interplay of the ice and the voids between them.





Some explanations are given by Libbrecht.[42] Further information was obtained by Ukichiro Nakaya, who observed and photographed thousands of snow crystals, and also experimented by growing them under controlled conditions.[43] It seems to me certain that only some model based on a nested system of symmetries and axes of symmetry can ultimately provide an adequate explanation. Once achieved, this kind of system will then give us a model which is not only fractal (a far too specific term, which is good advertising and good shorthand, but not good mathematics since it is only one particular mathematical formulation, standing for literally thousands of variations none of which actually embody that particular formula). Such a model may be a step towards a capability of modeling the general character of highly ordered structure at many levels simultaneously.

IX SP TRANSFORMATIONS AND COMPLEXITY THEORY

A Small Modification Needed In The Work Of The Modern Masters and Complexity Theory

What I have said in the forgoing, can be simply summarized. Consider the modern masters: Mandelbrot, Goodwin, Stewart, Prusinkiewicz, Gordon, Nakaya, Reynolds. We must, and do, take off our hats to them. But if we examine the contributions made to complexity theory, by these modern masters, we see that while making enormous strides, and placing our discipline on its first legs, it must also be said that each one of them has fallen slightly short in one all-important respect. All of them, at least all that I know, have tried to explain complex emergence, as a product of coupled interactions among local events. Yet, when one examines in very painstaking detail, what is actually going on, and what is emerging, it turns out in every case that there is some (sometimes small -- but not small enough to be overlooked) aspect of the emerging whole that cannot be properly explained by this approach. Though they aspire to reach the whole that emerges, through analytical means, in fact it is just the whole – the real whole in the world that makes us marvel in the first place – that is propelled by a second process, which is whole-oriented. I have been at pains to show that this whole-seeking or harmony-seeking process is not teleological, not goal-seeking. It comes about, because of a new type of operation, which is performed on the structure that exists, and which then brings to fruition a larger, unexpected, and unanticipated new structure of wholeness in each case. The existence of such a computation, and its operation in virtually every creative process in nature and in art -- this is the real creativity of the universe at work.

I believe there is a realistic chance that this creative kind of emergence, where the whole inspires the emergent structure, and gives it direction can, at least in part, be encompassed by a computable formula, by a rational and attainable, though very new, mathematical formulation.

This hinges, I think, on the construction of a formalism in which nested systems of symmetries and centers, are acted on by the transformations I have described, and where the future (the (t+1)-trajectory in phase space) is determined by the latent structures already present at time t, where these latent structures, too, are couched and described in terms of symmetries and centers.

X ECOLOGY OF THE ORDINARY

Example 20. The Ecology Of The Ordinary

In natural systems we may **expect**, and in human-created urban systems we may **decree,** that when things are “OK”, harmony-seeking computations will be occurring, and must define the step-by-step unfolding.

Here are two almost purely natural systems. We see in the morphological character of the structures portrayed, that these are generated by harmony-seeking computations. What we see here are two structures, which are products of structure preserving transformations. One can trace a history of SP-transformations as the origin of what we see in the morphology.

Here are two mixed systems, including limited man-made portions. They are almost purely natural systems, but include parts undertaken by human processes which include human versions of SP transformations (the boat, the gardens, the farm-villa). We see in the morphological character of the mixed structures portrayed, that these are still generated by harmony-seeking computations. What we see here are two structures, which are products of structure-preserving transformations. One can trace a history of SP-transformations as the origin of what we see in the morphology.

Nature just beginning to be touched with human intervention: even in the boat, harmony-seeking computation is still visible

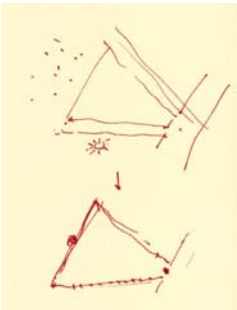
Nature made by human beings through cultivation – all a product of harmony seeking computations

Here we see two structures mainly dominated by humanness (not by nature in the raw), where harmony-seeking computations dominate the process and its resulting morphology. The situations seem primitive, because the technically advanced civilization we have become, has -- for the moment – reduced ability to perform these computations. We therefore see fewer and fewer harmonious results or examples of true harmony in our surroundings.

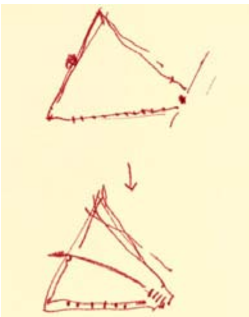
Example 21. Look around you, wherever you are today, in almost any city: Bradford, London, Birmingham, Atlanta, Stockholm, Tokyo, Moscow, Addis Ababa, Cape Town, Singapore, Lyon, Madrid, Santiago, Mumbai.

The destruction you see is caused by the absence of harmony seeking computations in the process that has produced these environments today.

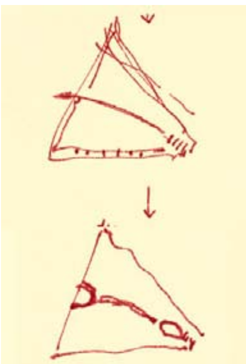
Example 22. Somerville. A Housing Project For 200 Apartments In Boston, In Which The Project Plan And Form Are to be Generated By Harmony-Seeking Computations.



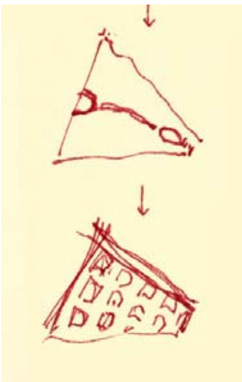
Cycle 1. The process starts with a triangular brown-field site in Somerville, 5.5 acres in area, between a railroad line, a bike path, and an existing neighborhood → The computation then identifies latent centers in the site: the bike path, along the south side, Warwick avenue along the west, and Lowell Street in the south east corner.



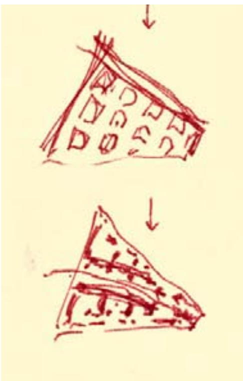
Cycle 2. latent centers in the site: the bike path, along the south side, Warwick avenue along the west, and Lowell Street in the south east corner. → The computation then identifies a connection and pedestrian precinct generates a curved line, more or less a median through the triangle of the site, but curving slightly and leading to stairs at the east end where there is a 20 foot rise to Lowell street.



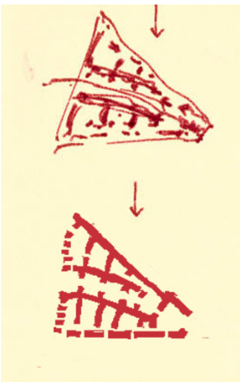
Cycle 3. Consideration of density suggests that the environment can become wholesome and enjoyable, only if entirely pedestrian, thus connecting this new area to the surround neighborhood and making it useful.



Cycle 4. The articulated pedestrian spine now has, as its most obvious latent centers, the areas on either side of it. To place 200 dwellings in this limited area, and to do it in such a way that people have pleasant gardens, for their own use, and for the neighbors who may like to enjoy these gardens, leads to a form of organization where houses are laid in long thin strips around useful open spaces. The gardens should, typically, have a diameter on the order of some 100 feet; the buildings in strips should have a depth of some 25 feet; the building height would then be 2 and 3 stories.



Cycle 5. The abstract and schematized grid-like array, is characterized by two parameters, courtyard diameter, and building thickness. However, its regularity is not essential or even good: indeed it needs to be adapted to the site boundaries in such a way as to generate coherent courtyards and pedestrian space, while leaving building volumes simple. This transformation requires the use of ROUGHNESS, LOCAL SYMMETRIES, DEEP INTERLOCK, NOT SEPARATENESS, GOOD SHAPE and INNER CALM. Application of these transformations in concert achieves the necessary computation at this stage.



Cycle 6. More careful adaptation. The schematic arrangement of the previous cycle, now gives way to a series of shapes which pay more detailed attention to each individual garden, as a shape in itself, using GOOD SHAPE, LOCAL SYMMETRIES, POSITIVE SPACE, THE VOID, and ALTERNATING REPETITION, so that the whole is coherent, and feels like one thing.

In addition, the boundary where this neighborhood meets and abuts other neighborhoods is modified by BOUNDARIES, DEEP INTERLOCK, and ALTERNATING REPETITION, so that it becomes a thickened semi permeable membrane, capable of allowing people who want to talk a stroll, to pass in and out comfortably, yet also maintaining a certain privacy for the interior of the neighborhood.

In the plan above we see the beginning and the framework, for a further harmony-seeking process. The plan shows 200 households on a five acre site, each house potentially unique, and where the pattern of gardens, walkways and road access have been laid down so as to protect the harmony of the adjacent neighborhood, and project the immediate environment for five hundred people.

The Uniqueness Of Each Region in the Generated Structure

One aspect of the generated structure for Somerville is highly significant. If we examine the structure which has been generated by the harmony-seeking computations that were used, we see that each part, though similar in broad structural character to others, is **unique**. That comes about because the application of the computation, to even slightly differing contexts, will inevitably produce morphologically different results.

For example, at the largest level of scale, each of the fifteen or so courtyard gardens has a slightly different shape and configuration. That occurs because the application of the principal transformations (POSITIVE SPACE, LOCAL SYMMETRIES, ECHOES and GRADIENTS, ALTERNATING REPETITION and CONTRAST), generates a different configuration for each one according to its starting point – that is, according to its context.

Different gardens in the project, each with its unique character and atmosphere

Still more exciting though, is that the same quality of uniqueness and subtle differentiation, continues to smaller and smaller scales. If we examine the drawings, we shall see that the detailed configuration of terraces, entrances, paths, lawns, stairs and archways, produces unique results in each part of the larger whole, and in each part of the individual gardens. This is not because of a shallow desire to make each thing different for its own sake (sometimes the driving force behind the more commercial post modern developments). It occurs because the effect of harmony seeking computations, on only slightly different starting conditions, is to generate entirely new and different configurations, but all members of a fairly simple family.

I do not think this uniqueness of every place is visible to such a considerable extent, in such simulations as the L-system generation for trees and plants.[44] L-systems create an approximation to harmonious structure of this kind; but it is not deep. The reason is, that the system and its adaptations (as it has presently been worked out) are too elementary. Harmony-seeking computations, by comparison, have the potential to do better: but the algorithms are far from perfect: but they have the potential to do better because they rely on fifteen kinds of transformations, acting in concert and in parallel: and that allows each place to develop its own peculiar nature in an appropriate and inevitable fashion.

XI CONCLUSIONS

A Single Conclusion From These Studies

There is a single all-important conclusion to be drawn from all these studies. Atomistic, bottom-up, computations cannot adequately describe what is really happening in the world, and – further – do not describe those especially important processes which *heal* the world, which bring order into configurations in the land, in nature, in buildings deeply adapted to the land, and also deeply adapted in their internal structure.

The issue is recursive. It is not only large-scale configurations that need to benefit from this insight. The idea that every configuration, at every level, is working to help the coherence of some larger configuration in which it is embedded, runs up and down the ladder of scales, and must do so in any living world. A successful computation will emulate this upwards- and downwards-reaching process. Purely bottom-up forms of calculation, not invoking this principle, will always remain too sterile to be real or profound.

What Is The Underlying Process Involved In These Harmony-Seeking Computations

I am not only proposing that we consider these many real world systems as computational processes. I am saying, that we need to find out how they work, and how, in particular, they work to allow the unfolding of structure, under the impact of wholeness. That will require a form of representation which is new. It has been sketched, in broad brushstrokes, in Books 1 and 2 of *The Nature Of Order*. We now need to work at finding ways of describing this kind of unfolding, in more well-defined mathematical and computational terms.

That will be a very long job. My colleagues and I have, in the last few years, gained intuitive and form insight into the nature of the harmony-seeking computations. But we are far from understanding them in detail. That enormous task must now be undertaken, hopefully by many dedicated scientists together.

I must emphasize that the phenomena I am talking about cannot successfully be grouped under what is loosely called ‘emergence.’ The emergence of wholes does not come about by autonomous processes which happen to aggregate themselves to form wholes. Rather they are processes which contain specific whole-seeking processes: these are the structure-preserving transformations I have described.

Structure-Preserving Transformations

In all these real-world examples, there is a common phenomenon. We may describe it by saying that the steps of the computation are SP (structure-preserving)-transformations which follow this scheme. Each SP-transformation operates on one wholeness, to produce another wholeness. It does it in such a way as to preserve or embellish or enhance the global structure of the first wholeness.

$$W_1 \xrightarrow{SP_1} W_2 \xrightarrow{SP_2} W_3 \xrightarrow{SP_3} W_4 \xrightarrow{SP_4}$$

The mathematical description of an SP-transformation is not yet fully known. However, there is abundant evidence to show that the concept of being SP, of being structure-preserving, is well-defined and objective, in the sense that different observers largely agree among different possible transformations of a given whole, which ones are more SP and which are less so.

Models Of The Wholeness In A Given Configuration?

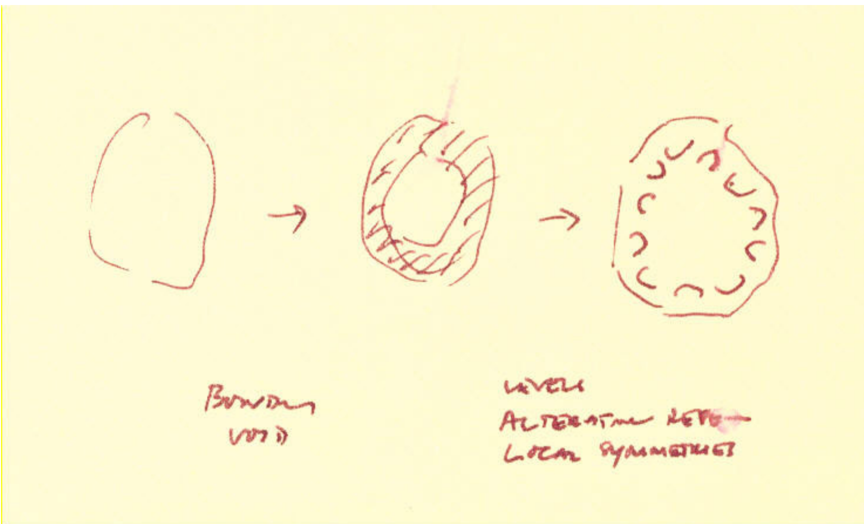
To establish the character of SP-transformations, we begin with five postulates about the structure of wholeness:

Postulate A1. In any configuration we see certain salient wholes, or centers. Each of these wholes is an identified, spatially contiguous subset of the configuration, which corresponds to something we see, or experience, as an "entity."
Postulate A2. The sub-configurations overlap spatially. They may be nested, or overlapping, or disjoint.
Postulate A3. Each sub-configuration has a measure associated with it. Let us thinking of this measure as the degree of coherence, or saliency within the larger whole.
Postulate A4. Certain centers may be very low saliency, almost invisible, but are nevertheless coherent configurations in their own right, which are created by others in the configuration. These will be referred to as latent centers.
Postulate A5. The wholeness is defined as the system of configurations, each one specifying its coherence, and each connected with other configurations which are part of it, or of which it is a part.

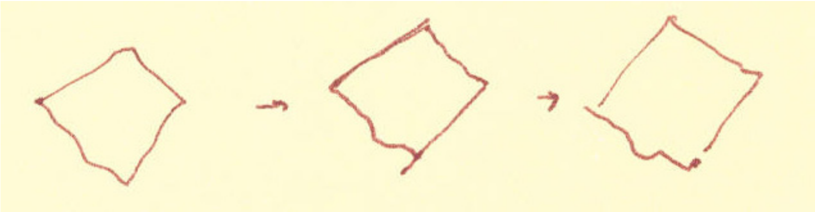
And three postulates about the definition of SP:

Postulate B1. A transformation is considered to be structure-preserving or SP, if it elaborates existing centers or latent centers in the present configuration, and does not introduce new centers that violate or 'cut across' existing centers.
Postulate B2. The best latent center to work on, is the latent center whose improvement or repair, will (probably) do the most to increase the coherence of the whole configuration.
Postulate B3. A successful SP transformation must always have morphological impact on the structure of some larger whole.

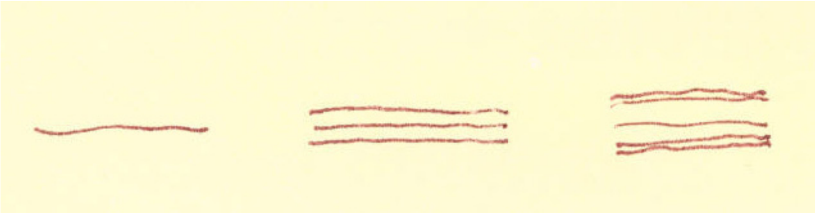
A Few Randomly Chosen Examples of Harmony-Seeking Computations



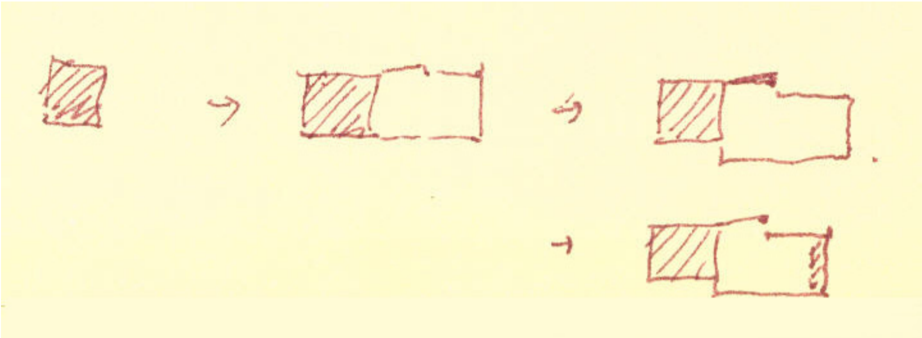
→ BOUNDARIES, THE VOID → LEVELS OF SCALE, ALTERNATING REPETITION, LOCAL SYMMETRIES



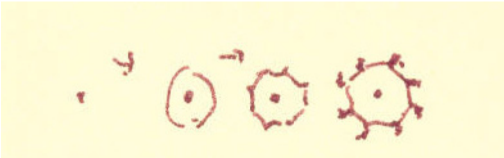
→ STRONG CENTERS, LEVELS OF SCALE → LOCAL SYMMETRIES →



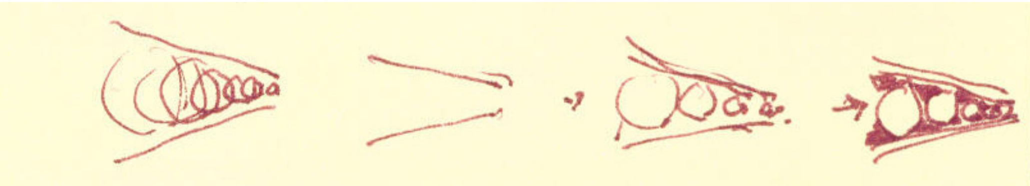
→ BOUNDARIES, THE VOID → GRADIENTS, LEVELS OF SCALE, ALTERNATING REPETITION, ECHOES



→POSITIVE SPACE → GRADIENTS, LEVELS OF SCALE → POSITIVE SPACE



→STRONG CENTER → LEVELS OF SCALE → DEEP INTERLOCK, LOCAL SYMMETRIES



→GRADIENTS → SIMPLICITY → POSITIVE SPACE → LOCAL SYMMETRIES → CONTRAST

The Paradigm Being Followed In Each SP-Transformation

Each of these examples is relatively simple: the same paradigmatic cycle is followed each time.

- The first frame is a configuration
- The second frame identifies the locus and extent of some latent center in that configuration – one which, if consolidated, would help the whole to become more coherent.
- The third frame establishes smaller centers to embody and solidify that latent center.

The steps taken in going from frame 1 to frame 2, and the steps taken in going from frame 2 to frame 3, are always some conglomerate of the fifteen fundamental SP-transformations defined in *The Nature of Order*.

We may also see how symmetry elaboration is happening at each step. At each step, the space is being differentiated, holistically, by an injection of local symmetries. It is possible, as I have said earlier, to view this injection of local symmetries as symmetry breaking. The symmetries which appear, in most cases, appear because other symmetries, often an infinity of other symmetries, are struck out, leaving the coherent form we see. Because of this symmetry breaking aspect, the injecting of local symmetries is structure-preserving, not structure-destroying. This is an extremely significant fact.
.
However, I find that it slightly obscures the core of what is going on to think of it, exclusively, in this way. Operationally, I find it more illuminating to focus on the fact that local symmetries of various kinds are being ***created***. But it is certainly salutary to remember that the kinds of local symmetry being created, when created well, do always enhance **the underlying structure of what was there before**.

Operationally, and emotionally, such a procedure is creating structure in a new way. It may be done, by an artist, with an intuitive grasp of the underlying latent structure, at each step in the unfolding of the whole. Or it may be done by an engineer. Socially speaking, this is a new kind of process, a new function for an artist, and a new challenge to engineers, architects and planners of all kinds. This new sort of work demands enormous concentration, and attention.

The intuitive act is nevertheless a computation, and we may be able to pin down what **kind** of computation it is. Then, if we can succeed in making a blind computation, even perhaps one day performed by a computer working in a new way, achieve similar holistic results, **that will be because the thing we recognize intuitively and emotionally as whole and coherent, is, mathematically, a particular recursively generated structure of symmetries and centers**: and it is this underlying structure which allows the human mind, and natural processes, both, to follow this path and to seek wholeness in the way they do. Most important, we may become conscious about this process, and consciously use this kind of computation to improve the coherence and harmony of our physical world.

Fact and Value

The example of the German business man's tie, and the example of the cosmological structure of voids and filaments, bring the potentially extraordinary nature of harmony-seeking computations into sharp focus.

In the parlance of 20^{th} century thought, it is very plain that choosing a tie, is a matter of personal taste; It has essentially nothing to do with fact. On the other hand, the dynamics which lie behind the distribution of matter in the universe is undoubtedly, a matter of fact, although the facts are still poorly understood, and much debated. Even though, of course, this task is at presently far from resolved, and the dynamics are not yet understood, it is an article of faith that at root it is a matter of FACT.

Equally, no matter how subtle my perception of the tie and its harmony might be, the idea that I treat its structure as a matter of fact, may also seem absurd to present day scientists, if they follow the canon of 20th century thought. It is an article of contemporary faith, that the goodness of the tie is a matter of TASTE, not fact. The harmony of the tie cannot be (according to 20th century thought) a matter of fact in any sense. The mental protocols of 20th century thinking have forbidden it.

Here we come to the profound change that has been lying in wait for 21st century science. It is a matter of historical record, that scientists of earlier eras – indeed, many of the great scientists of earlier eras -- had no difficulty whatever thinking of the great harmony that existed in the world. Pythagoras's phrase "the harmony of the spheres" was not an idle one. Newton, as a matter of record, considered the progress of the universe, in the large and in the small, to be entangled, inevitably, with a movement towards harmony, and with the greater harmony of the world as a necessary underpinning for the discoveries of science.[45] Leibniz, Kepler, thought the same.

Considering the tie as a matter of taste strongly limits our ability to understand its harmony. Considering the evolution of the universe as a mechanical product that can best be modeled through value-neutral means, also strongly limits our ability to understand the ensuing harmony, and in large-scale cases like this one creates a real possibility that we shall ultimately fail to understand the physics.

These two statements may seem hard nuts to swallow. But the possibility of harmony-seeking computations ventures, precisely, into this forbidden domain. It is challenging, undoubtedly, to succeed in defining and expanding this kind of concept sufficiently well, so that it can become an effective part of the way we think about the world. But it will play a small part in reopening a door that has been closed for far too long.

A New Science Of Harmony-Seeking Computation: When And Where?

A new science of harmony-seeking computation and SP-transformations can make amenable to computation, phenomena which are, for the moment, altogether beyond the reach of currently available computational methods. It will help to open doors to the global quality of harmony, figural goodness, ecological health and structural coherence as a computable feature of configurations,

One of first practical items on the agenda, is to provide well-defined and precise versions of the fifteen transformations in *The Nature Of Order*. Though easy to state, this is a remarkably difficult task for three reasons. The fifteen properties, though defined with some level of precision, remain somewhat elusive. Defining computable operations which can induce these properties in arbitrary configurations, is a challenging task. Further, it is difficult to define them as transformations, since this presupposes a language of configurations that is amenable to the transformations. And, finally, some of the transformations are easier than others to define operationally in sufficiently concrete terms. For example, LOCAL SYMMETRIES, BOUNDARIES and LEVELS

OF SCALE are relatively easy. POSITIVE SPACE and ECHOES are harder. SIMPLICITY AND INNER CALM and NOT SEPARATENESS are among the most difficult. However, I am fairly sure that the task of dealing with all fifteen transformations can be accomplished by a small team, in the next five years, and that should then open the door to a full fledged, though elementary, version of a first-draft science of harmony-seeking computations. First steps have already been taken.[46]

I hope the idea of harmony-seeking computation may then sit alongside other methods as a new tool in an armory of well-founded alternative computational techniques to be used when appropriate. It is likely to be appropriate whenever a computational task is defined more by issues of adaptation, health, wholeness, and wellness, with reference to the position some system in some still larger whole, or perhaps even by a desire for beauty, or life, or elegance.

All these might one day play a key role in very general kinds of computation. Science, architecture, biology, ecology, physics, and cosmology may all be the better for it.

Notes

1 Christopher Alexander, *The Nature of Order*, *in Four Volumes*, Center for Environmental Structure Publishing, Berkeley, California, 2002-2005.

2 *The Nature of Order*, *Book 1 The Phenomenon of Life,* pages 143-296.

3 *The Nature of Order*, *Book 2 The Process of Creating Life,* pages 65-84.

4 See the discussion of four Matisse self portraits, *The Phenomenon of Life*, page 97.

5 Max Wertheimer, Wolfgang Koehler, Kurt Koffka.. etc.

6 Christopher Alexander and A.W.F. Huggins, “On Changing The Way People See”, *Perceptual And Motor Skills*, Vol. 19, July, 1964, pp. 235-253.

7 Hochberg J. & McAlister E. (1953) A Quantitative Approach to Figural "Goodness." *Journal of Experimental Psychology* 46, 361-364.

8 Christopher Alexander and Susan Carey, "Subsymmetries", *Perception And Psychophysics*, Vol. 4 (2), February, 1968, pp. 73-77.

9 Photograph of hayricks in Romania by Radu B. Chindris

10 Gerry Webster and Brian Goodwin, *Form and Transformation*, Cambridge University Press, 1998, pages 193-216.

11 *Form and Transformation*, op cit. pages 209-230.

12 The Emoto Building, in Komagome, Tokyo, illustrated in *The Nature of Order*, Book 3, pages 166-73.

13 *Op.Cit.* The fifty steps are summarized on pages 167-71.

14 See *The Nature of Order*, *Book 1*, chapter 5, and *Book 2*, chapter 2.

15 Details of the construction and evolution of the Upham house are described in the Appendix of *The Nature of Order*, Book 2, *The Process of Creating Life*, pages 571-632

16 Historical data and additional analysis of the evolution of St. Mark’s Square are provided in Books 2 and 3 of *The Nature of Order:* Book 2, *The Process of Creating Life*, pages 251-55, and Book 3, *A Vision of a Living World*, pages 5-7.

17 Tie from Ahlborn Kravatten, Egestorfer Str. 28, 30890 Barsinghausen, Germany

18 This transformation (FAMILIES OF COLOR) is one of eleven transformations dealing with harmony in color. These color transformations are comparable to, and related to, the fifteen geometric transformations. They may be found in *The Nature of Order, Book 4*, *The Luminous Ground*, Chapter 7, Color and Inner Light, pages 157-240.

19 Steve Shechtman et al., *Astrophysical Journal*, **470**, 1996, page 172.

20 Simulation and photograph by George Lake and Tom Quinn, published in Greg Bothun, *Modern Cosmological Observations and Problems*, Taylor and Francis, 1998, figure 3.1.

21 Christopher Alexander, *The Process of Creating Life*, Berkeley, California, Center for Environmental Structure Publishing, 2003, pp. 18-84.

22 Christopher Alexander, *The Phenomenon of Life*, Berkeley, California, Center for Environmental Structure Publishing, 2002, pp. 143-296.

23 The experiments were performed by Dr Robert Srygley, and are described in Srygley, R.B. and Thomas, A.L.R., “Unconventional lift-generating mechanisms in free-flying butterflies,” *Nature* **420** (6916):660–664, 2002.

24 Cathryn J Polinsky, *Flight Simulation of Flocking Geese Using Particle Set Animation,* Swarthmore College, May 1999

25 Deborah Gordon, *Ants at Work: How an Insect Society is Organized,* New York, Free Press, 1999.

26 slime mold references.

[27] All reported in Stephen Johnson, *Emergence*, New York 2001.

[28] Debra Niehoff, *The Language of Life*, Joseph Henry Press, Washington DC, 2005

[29] Cathryn J Polinsky, *Flight Simulation of Flocking Geese Using Particle Set Animation,* Swarthmore College, May 1999

[30] Craig Reynolds, Flocks, Herds, and Schools: A Distributed Behavioral Model, the SIGGRAPH '87 boids paper.

[31] Mitchel Resnick et al, *StarLogo 2.1*, Media Lab, Massachusetts Institute of Technology, 2004.

[32] Lene K. Hjertager, Bjørn H. Hjertager, Niels G. Deen, Tron Solberg, Measurement of turbulent mixing in a confined wake flow using combined PIV and PLIF, Submitted to the *Can. J. of Chem. Eng*., 2003

[33] Simulation of a tree by H. Honda, Description Of The Form Of Trees By The Parameters Of The Treelike Body: Effects Of The Branching Angle And The Branch Length On The Shape Of The Tree Like Body, *Journal of Theoretical Biology*, 31: 331-338, 1971.

[34] Michael Cross, "Pattern Formation in non equilibrium systems", course notes from Cal Tech Physics 161b, 2000. See http://www.cmp.caltech.edu/~mcc/Patterns/index.html

[35] Ian Stewart and Martin Golubitsky, *Fearful Symmetry*, New York and London, 1988.

[36] Martin Golubitsky and Ian Stewart, *The Symmetry Perspective*, Basel, Birkhauser, 2001.

[37] Reference to definition of wholeness

[38] Christopher Alexander and Susan Carey, "Subsymmetries", *Perception And Psychophysics*, Vol. 4 (2), February, 1968, pp. 73-77.

[39] Salingaros, Nikos A. (1997). "Life and Complexity in Architecture From a Thermodynamic Analogy" in *Physics Essays,* vol. **10**, pp. 165-173

[40] Wilson A. Bentley, *Snow Crystals*, containing more than 2400 snow crystal images, was published by McGraw-Hill, 1931, but has long been out of print. A soft cover copy, identical in all respects, can be obtained today from Dover Publications, Inc.

[41] Kenneth G. Libbrecht, and Patricia Rasmussen, *The Snowflake,* Colin Baxter, 2004.

[42] For some simulations processes, see the website *SnowCrystals.com*, created by Kenneth G. Libbrecht, in the Department of Physics, California Institute of Technology.

[43] Ukichiro Nakaya, *Snow Crystals: Natural and Artificial*, Harvard University Press, 1954.

[44] Przemyslaw Prusinkiewicz and Aristid Lindenmayer, *The Algorithmic Beauty of Plants*, Springer Verlag, New York 1990.

[45] Michael White, *Isaac Newton: The Last Sorcerer*, London, Fourth Estate, 1997.

[46] See for example, Salingaros, Nikos A. (1997). "Life and Complexity in Architecture From a Thermodynamic Analogy" in *Physics Essays,* vol. **10**, pp. 165-173; Christopher Alexander and Stuart Cowan, "*The Field of Wholeness:A Mathematical Model*", in progress, 2006; Cees der Groot, *First Steps In a SiteLayout Program written In Squeak*, 2004; Salingaros, Nikos, A Scientific Basis for Creating Architectural Forms, *Journal of Architectural and Planning Research* **15** (1998), pages 283-293.