

# the Mathematician's



- and the Return to Embodied Computation

S. Barry Cooper - *Non-Standard Computation Group*, University of York, June 3, 2011





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## Articles

### UBIQUITY SYMPOSIUM 'WHAT IS COMPUTATION?' COMPUTATION IS PROCESS

November 2010 | BY [DENNIS J. FRAILEY](#)



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*Various authors define forms of computation as specialized types of processes. As the scope of computation widens, the range of such specialties increases. Dennis J. Frailey posits that the essence of computation can be found in any form of process, hence the title and the thesis of this paper in the Ubiquity symposium discussion what is computation. --Editor*

The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century. Denning [2010], [in his opening statement](#), describes how *computation* was originally defined in the 1930s and how that definition has progressed through the ensuing decades. Church, Gödel, and Turing defined it in terms of mathematical functions, which they divided into the decidable (can be evaluated by algorithms) and the un-

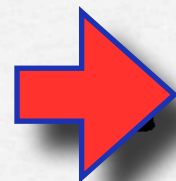




# “What Is Computation?”

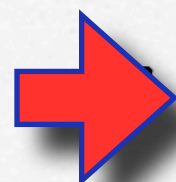
- ACM UBIQUITY SYMPOSIUM: Opening Statement by Peter J Denning:

- Many of us desire to be accepted as peers at the "table of science" and the "table of engineering"



Our current answers to this question are apparently not sufficiently compelling for us to be accepted at those tables

- ... there have been three significant developments that call for rethinking the basic reference model:



Interactive Computing; Natural Information Processes; Continuous Information Processes





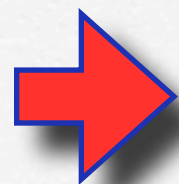


# “What Is Computation?”



- ACM UBIQUITY SYMPOSIUM: *Computation Is Process* by Dennis J Frailey:

- The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century
- Church, Gödel, and Turing defined it in terms of mathematical functions ... They were inclined to the view that only the algorithmic functions constituted computation

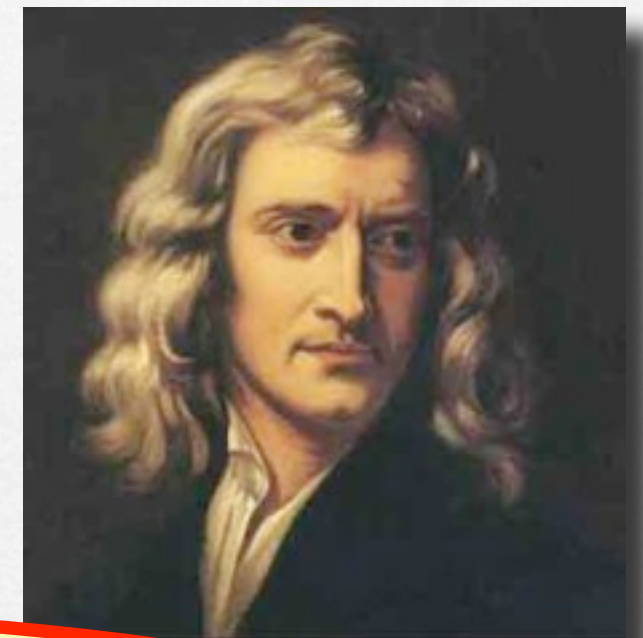


I'll call this the "mathematician's bias" because I believe it limits our thinking and prevent us from fully appreciating the power of computation

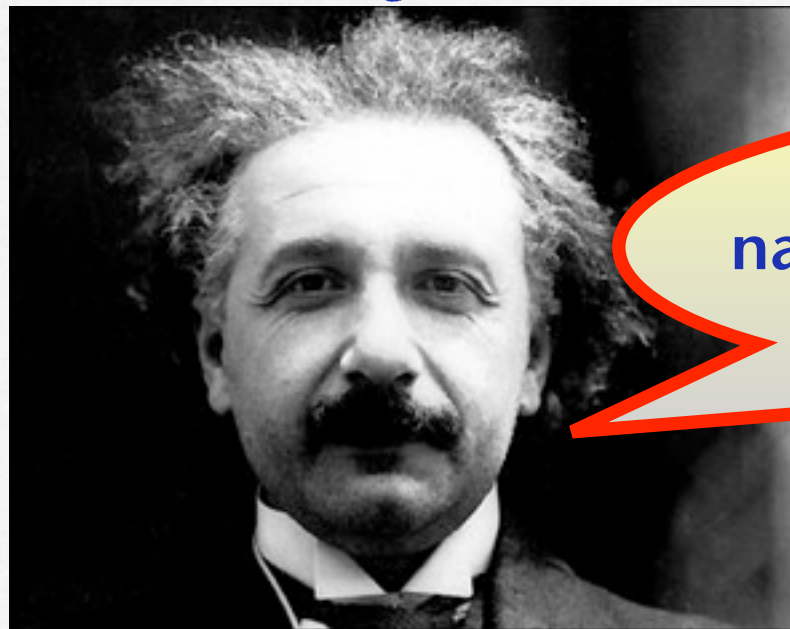




# In Pursuit of the Computable



- Newton onwards - mathematics rules science
  - we look for computable natural laws ...
  - theories which computably predict ...
  - try to capture truth via proofs ...



When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them

Albert Einstein: P.54, 'Out of My Later Years', 1950





# Computation Disembodied

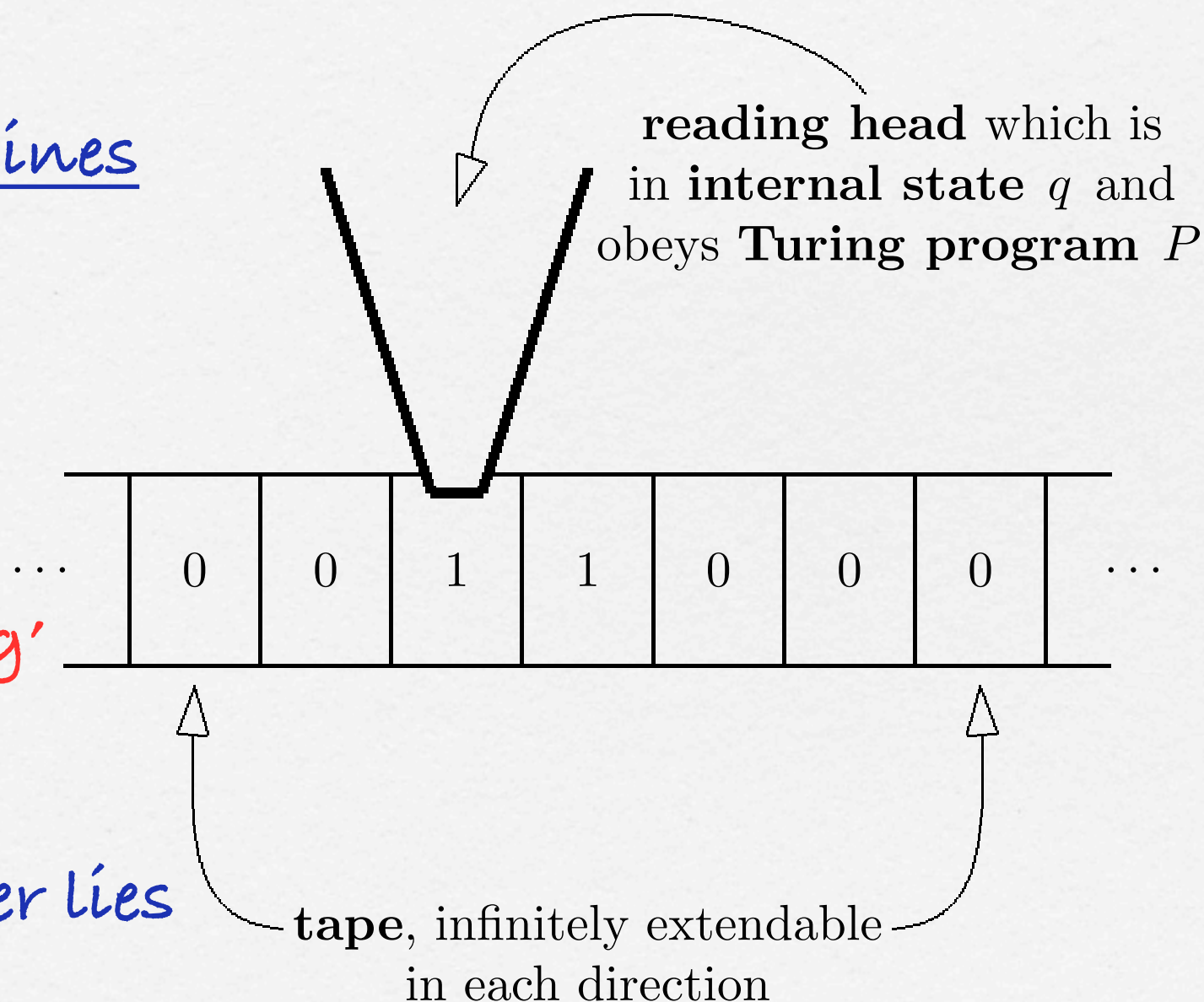
□ 1936 - Turing's machines

□ Hardware trivial

□ Actions simple

□ But compute 'anything'  
computable

□ All the computing power lies  
in the program ...



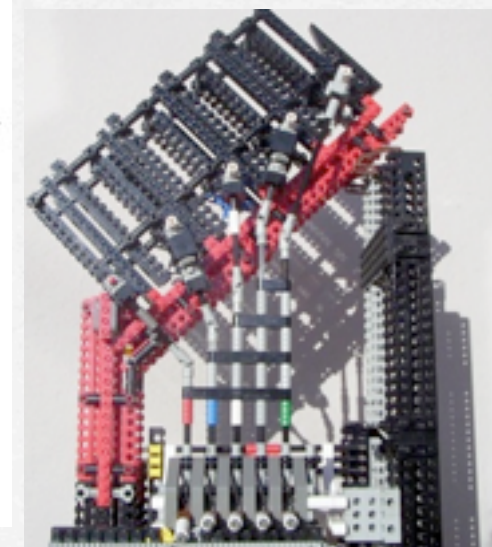
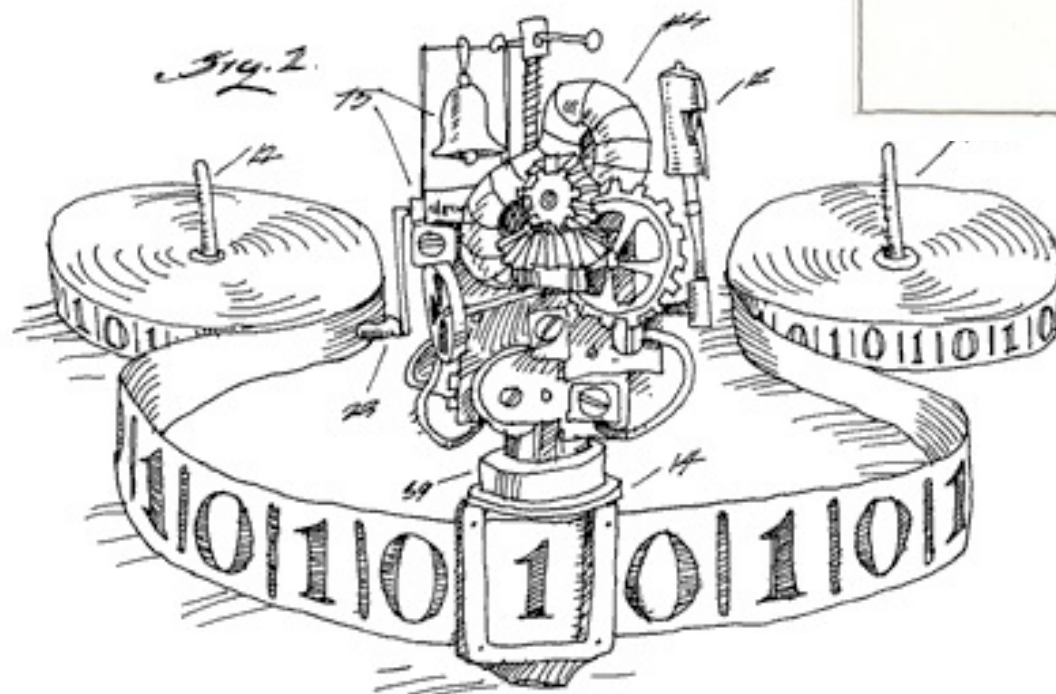
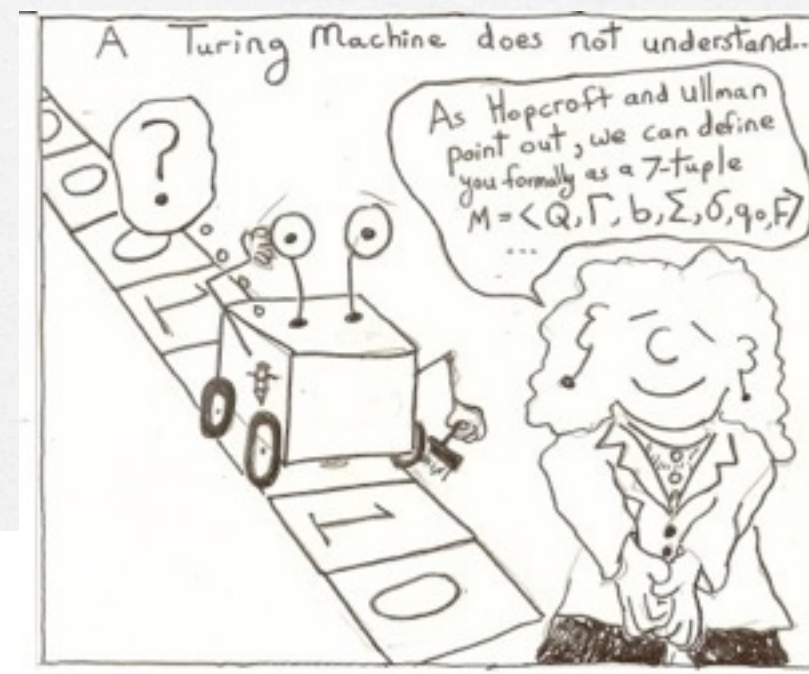
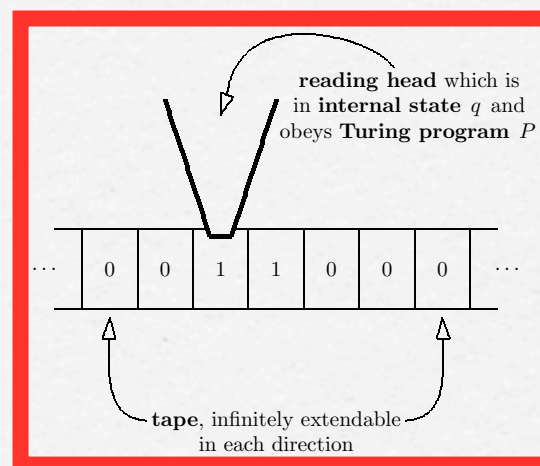




# Computation Disembodied

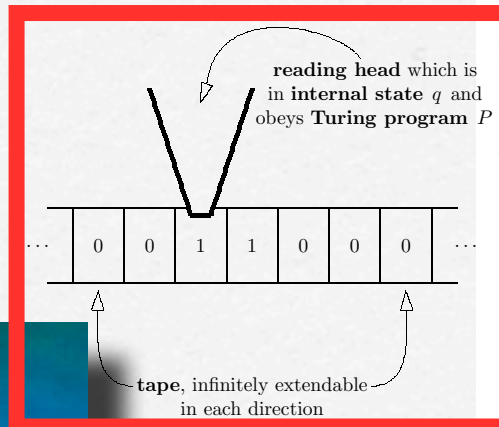


Turing solves the halting problem, only to discover that the REAL problem with his machine is what to do with all the tape.





# Computation Disembodied



GREAT  
DISCOVERIES

## THE MAN WHO KNEW TOO MUCH

*Alan Turing and  
the Invention  
of the Computer*

DAVID  
LEAVITT



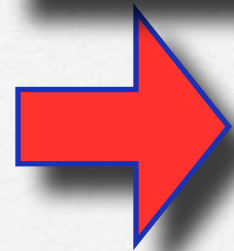
ALAN TURING, 1912 - 1954

© 2002 <http://www.jinwicked.com>





# Universality, and Programs as Data



Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

- An anticipation of the stored program computer
- A concept immediately understood by John von Neumann - as in his 1945 EDVAC report
- And 'program as data' key to the first computer ...





# A New Computing Paradigm ...

→ The omnipotent computer -

I am building a brain

→ Functionalism and AI - stress what a computer does as something realisable in different hardware -  
Hilary Putnam: "Minds and Machines", 1960

→ Virtual Machine (IBM, 1965) - software implementation of a programmable machine -  
JAVA, UNIX







# Program-Data Duality ...



Re 'what we compute',  
Turing took traditional mathematical objects, real numbers, functions etc. as the things to be computed.  
In subsequent work in Computer Science, the view of computation has broadened enormously. In the work on concurrent processes, the behaviour is the object of interest. There is indeed a lack of a clear-cut Church-Turing thesis in this wider sphere of computation - computation as interaction, as Robin Milner put it.

Samson  
Abramsky,  
private  
communication,  
March 2011





# Program-Data Duality ...



- “Formally, giving a program + data logically implies the output (leaving aside non-determinism or randomness), so why actually bother computing the result!”
- “... Can information increase in computation? Information theory and thermodynamics seem to tell us that it can't, yet intuitively, this is surely exactly why we compute - to get information we didn't have before.”





# Embodiment Problems Persist

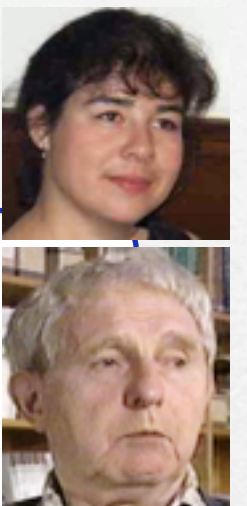
“One example of a problem that is not algorithmic is the following instruction from a recipe [quote Knuth, 1968]:

*‘toss lightly until the mixture is crumbly.’*

This problem is not algorithmic because it is impossible for a computer to know how long to mix: this may depend on conditions such as humidity that cannot be predicted with certainty ahead of time.

In the function-based mathematical worldview, all inputs must be specified at the start of the computation, preventing the kind of feedback that would be necessary to determine when it's time to stop mixing.”

*D. Goldín, P. Wegner [2005], The Church-Turing Thesis: Breaking the Myth.  
In CIE 2005: New Computational Paradigms: Papers presented at the conference in  
Amsterdam, June 8-12, 2005, LNCS 3526*







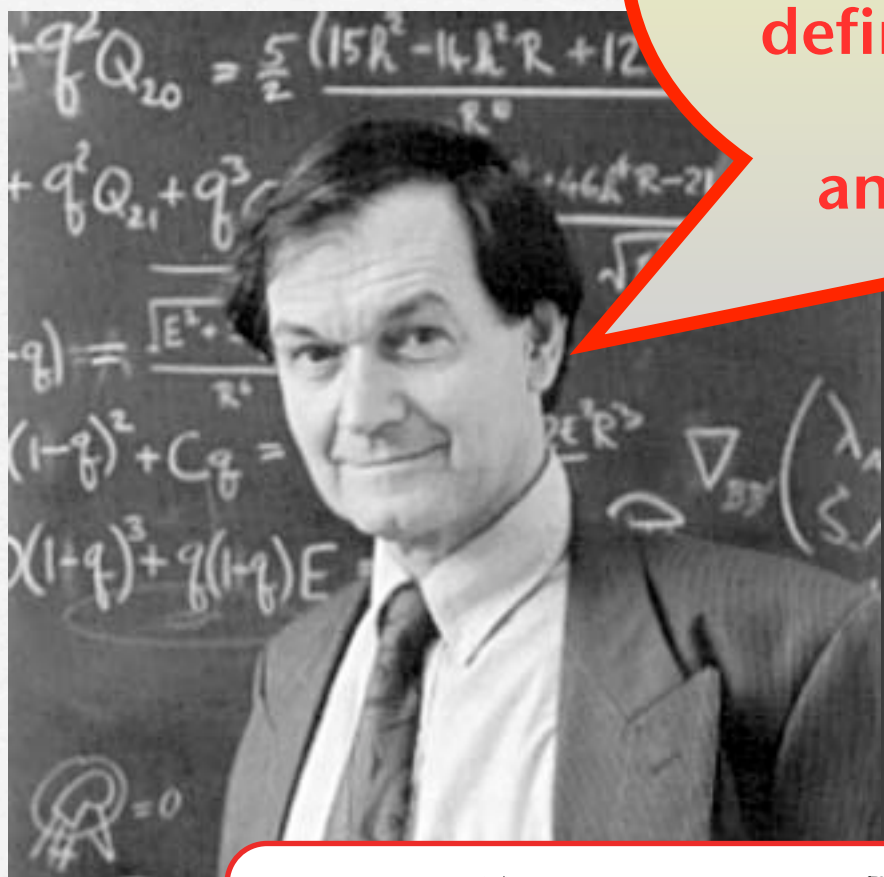
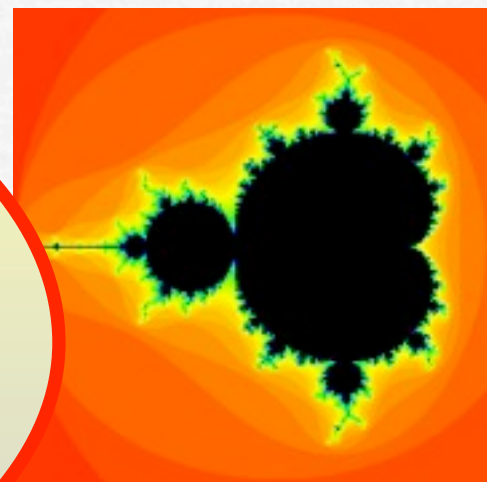
# A Gap in Classical Embodiment?

- A universal Turing machine can 'compute' an 'incomputable' real number ...
- ... which is not considered a computed outcome - as not available, or 'embodied', in further computations
- The quantifier involved is an unembodied chasm ...
- ... but can we observe definable outcomes being better embodied in more complex physical systems?



Now we witnessed ...  
a certain extraordinarily  
complicated looking set, namely the  
Mandelbrot set.

Although the rules which provide its  
definition are surprisingly simple, the set  
itself exhibits  
an endless variety of highly elaborate  
structures.

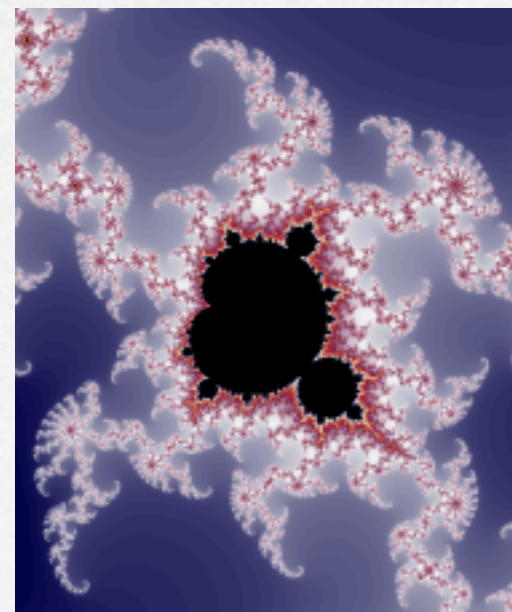
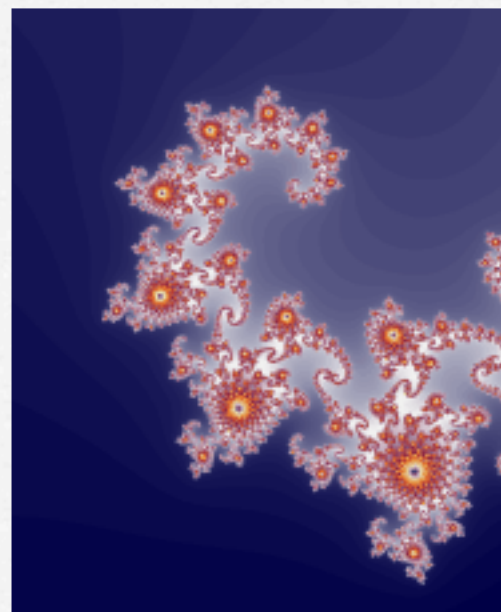


Roger Penrose

in "The Emperor's New mind", Oxford Univ. Press, 1994

OPEN PROBLEM:

Is the Mandelbrot  
set computable??



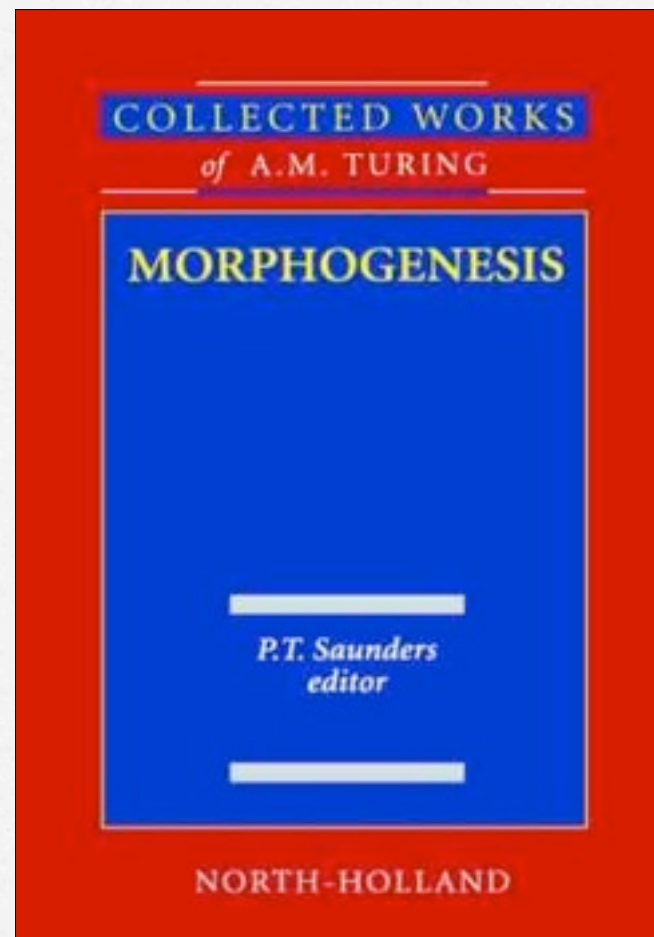




# Emergent Natural Patterns

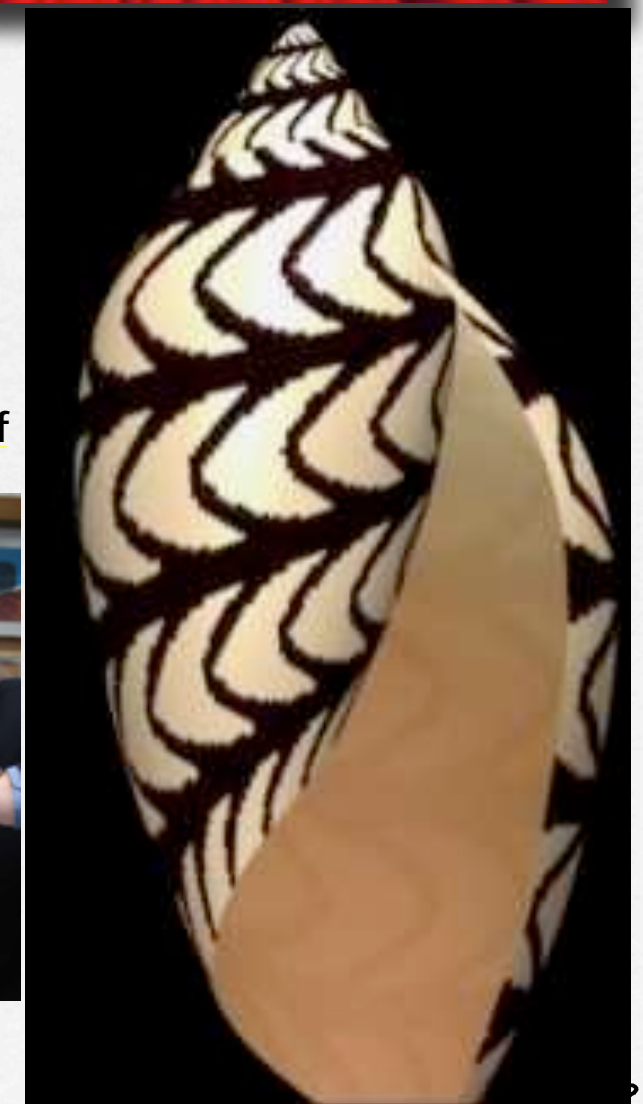


1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.



From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:

Przemyslaw Prusinkiewicz

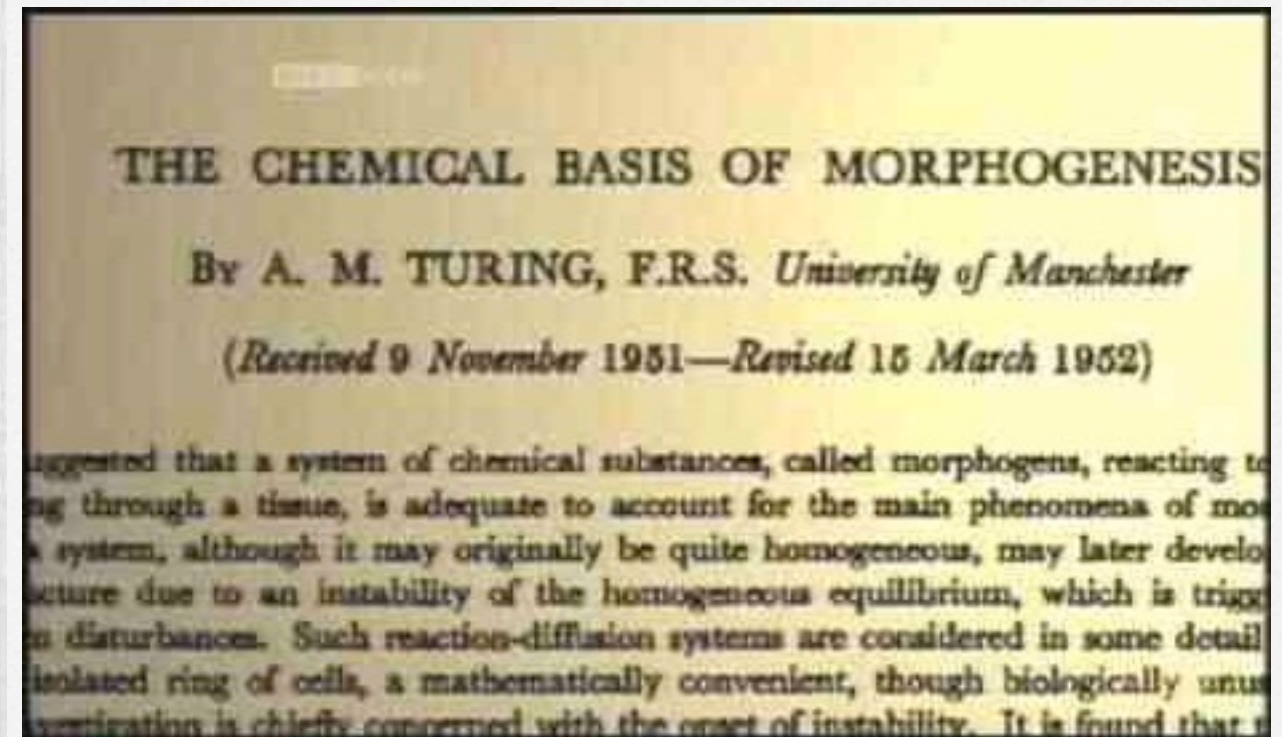


See <http://www.swintons.net/jonathan/turing.htm>





# Natural







# And - it's Definability Embodied

- Notice - It is often possible to define emergent properties in terms of the elementary actions
- While in mathematics, relations and even objects arise from descriptions - via notion of definability ...
- A potential source of incomputability ...
- Modelling how emergent phenomena not only generate descriptions, but derive form from them





**Simple Rules ...**

**... Connectivity ...**

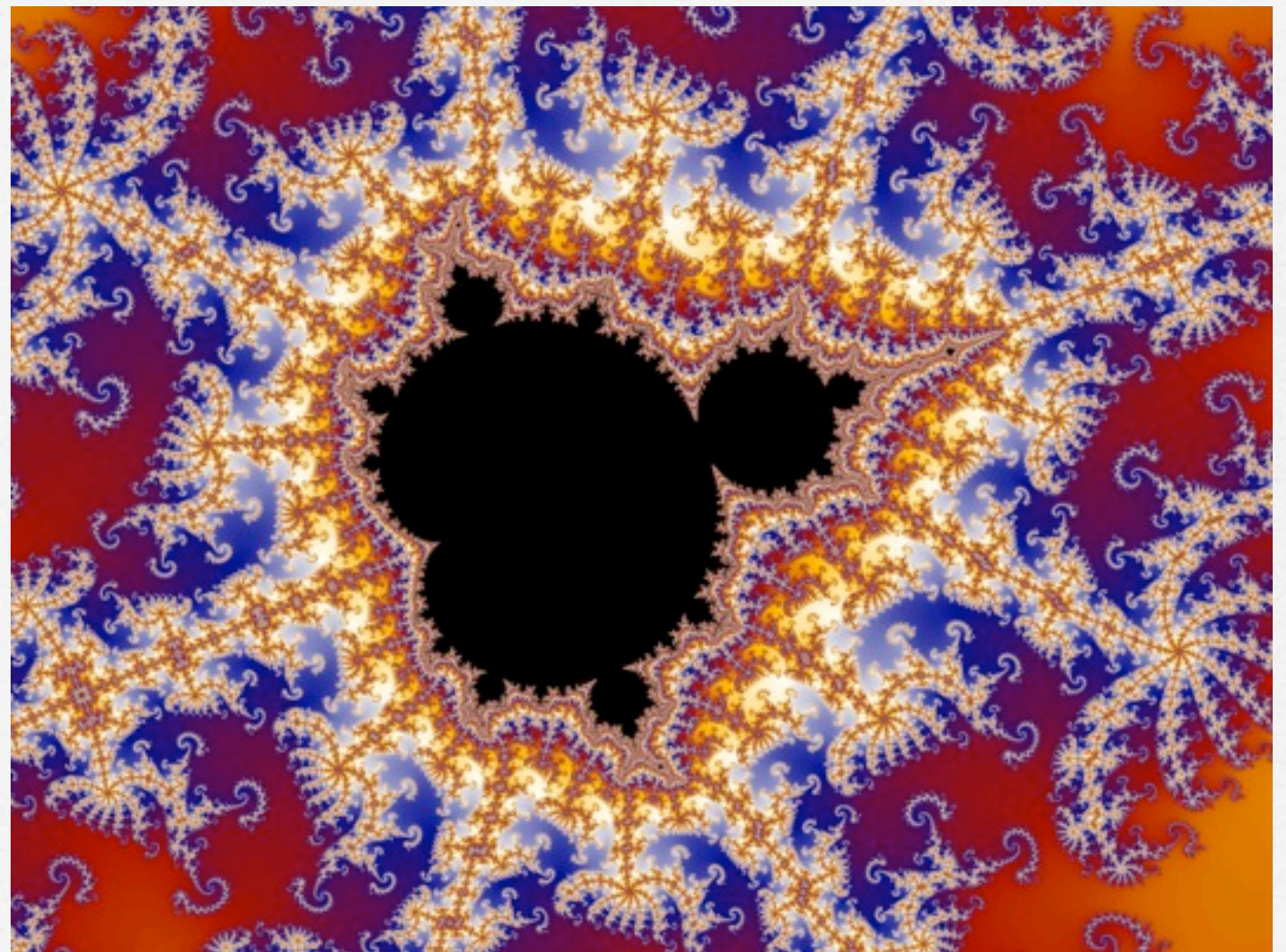
**... Emergent Forms ...**

**... Defined at the Edge  
of Computability**





# Morphogenesis - in same world as ...

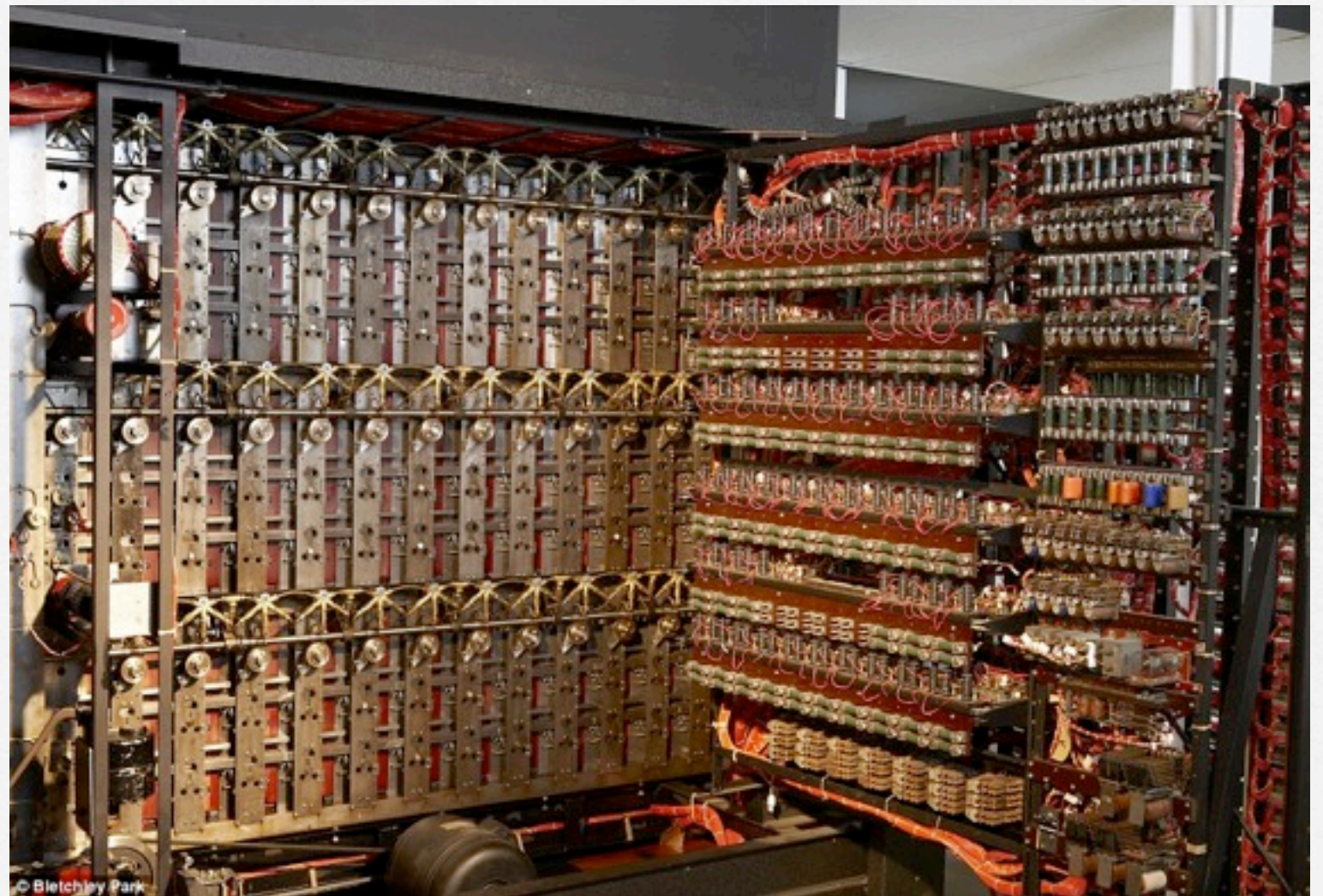
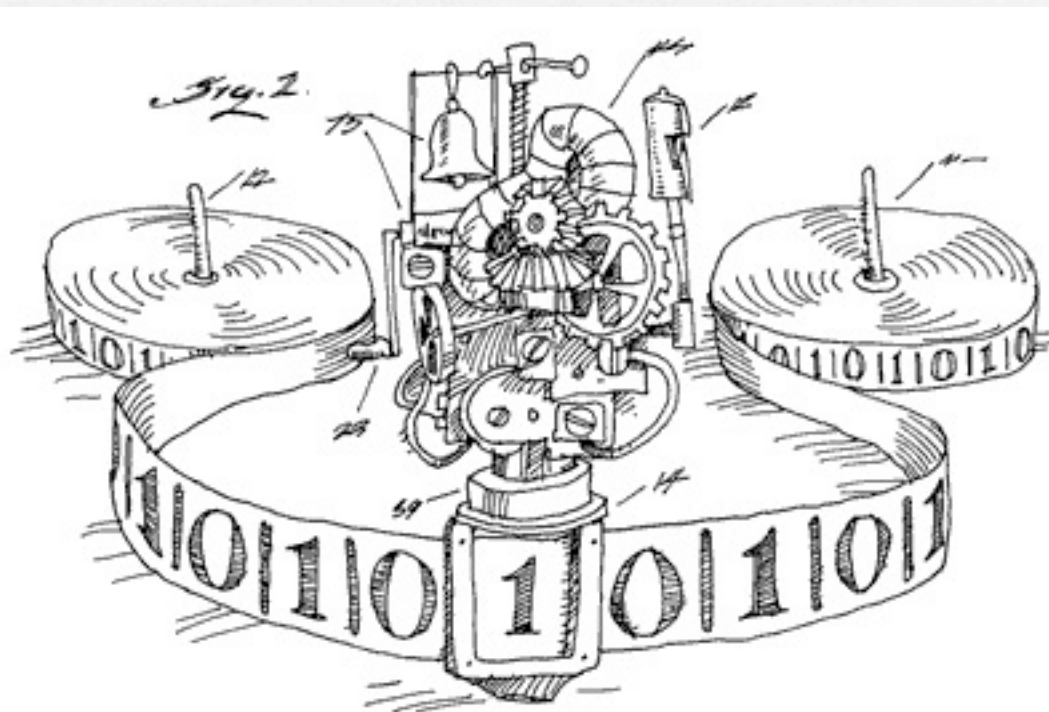


# ... the Mandelbrot set





# Morphogenesis - in same world as ...

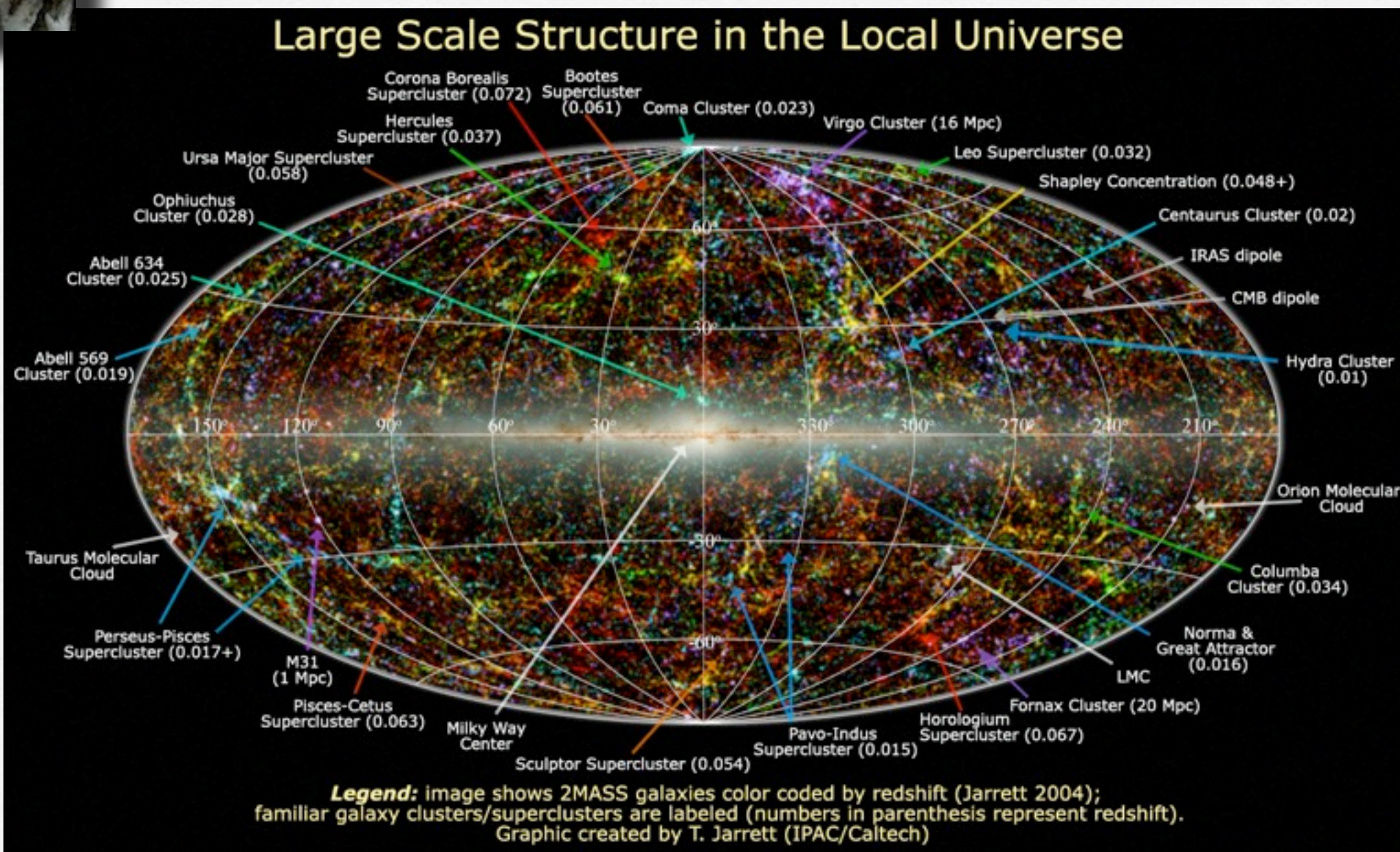
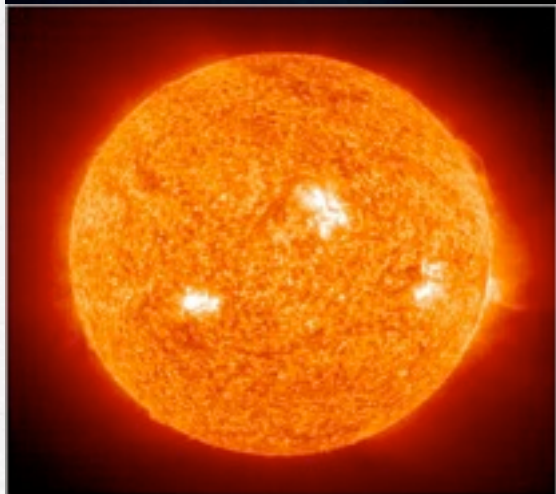


# ... the Halting Problem





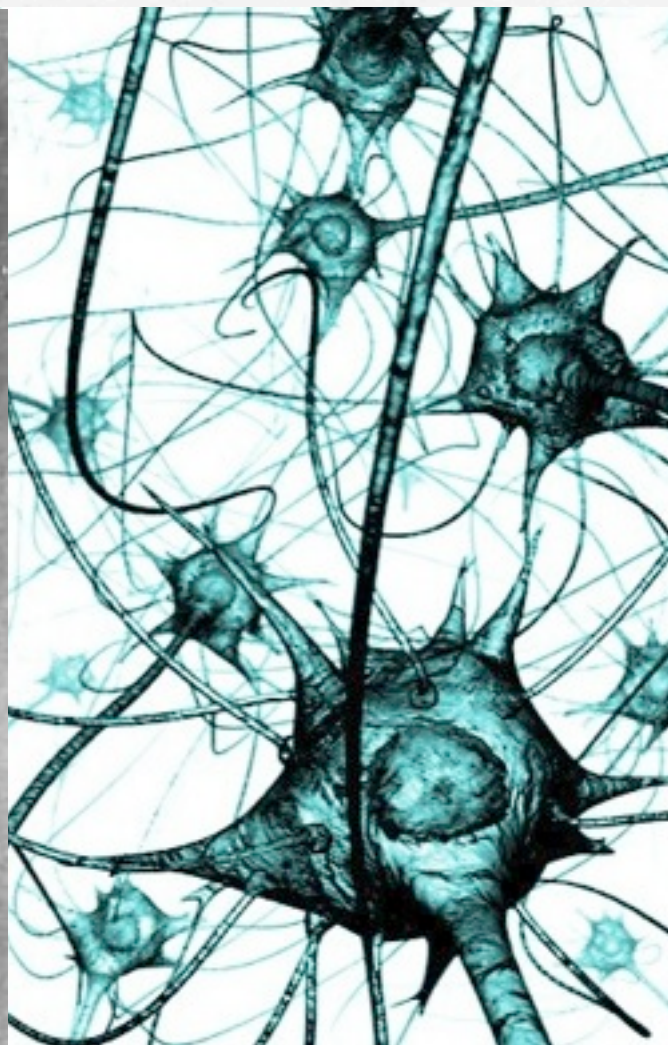
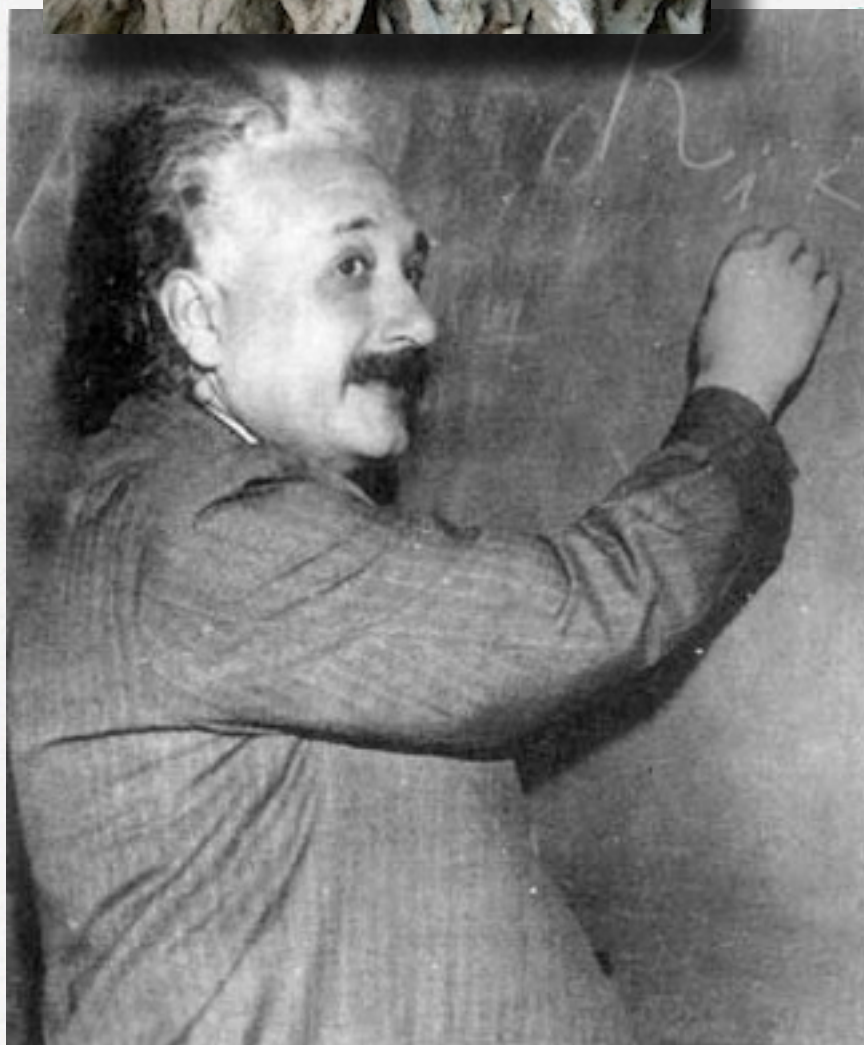
# Morphogenesis - in same world as ...







# Morphogenesis - in same world as ...



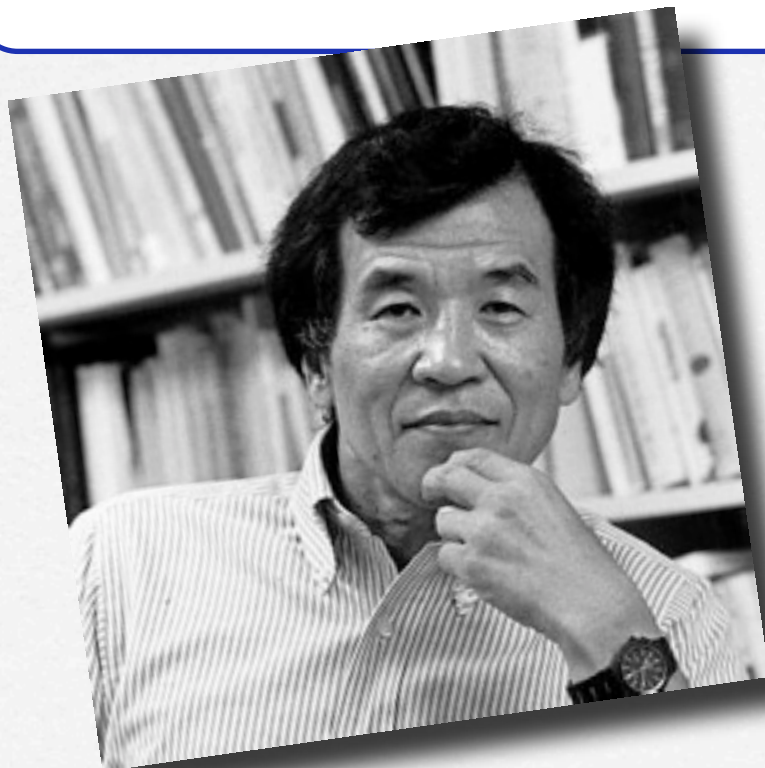
# ... mental creativity





# The Mind as Mathematics?

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

**"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."**

*Stanford Encyclopedia of Philosophy*



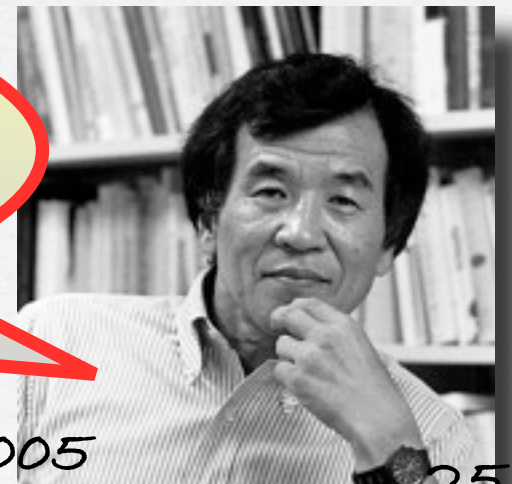


# The Mind as Mathematics?

- How can mentality have a causal role in a world that is fundamentally physical?
- And what about 'overdetermination' - the problem of phenomena having both mental and physical causes?

... the problem of mental causation is solvable only if mentality is physically reducible; however, phenomenal consciousness resists physical reduction, putting its causal efficacy in peril.

- Jaegwon Kim: Physicalism, or Something Near Enough, Princeton, 2005

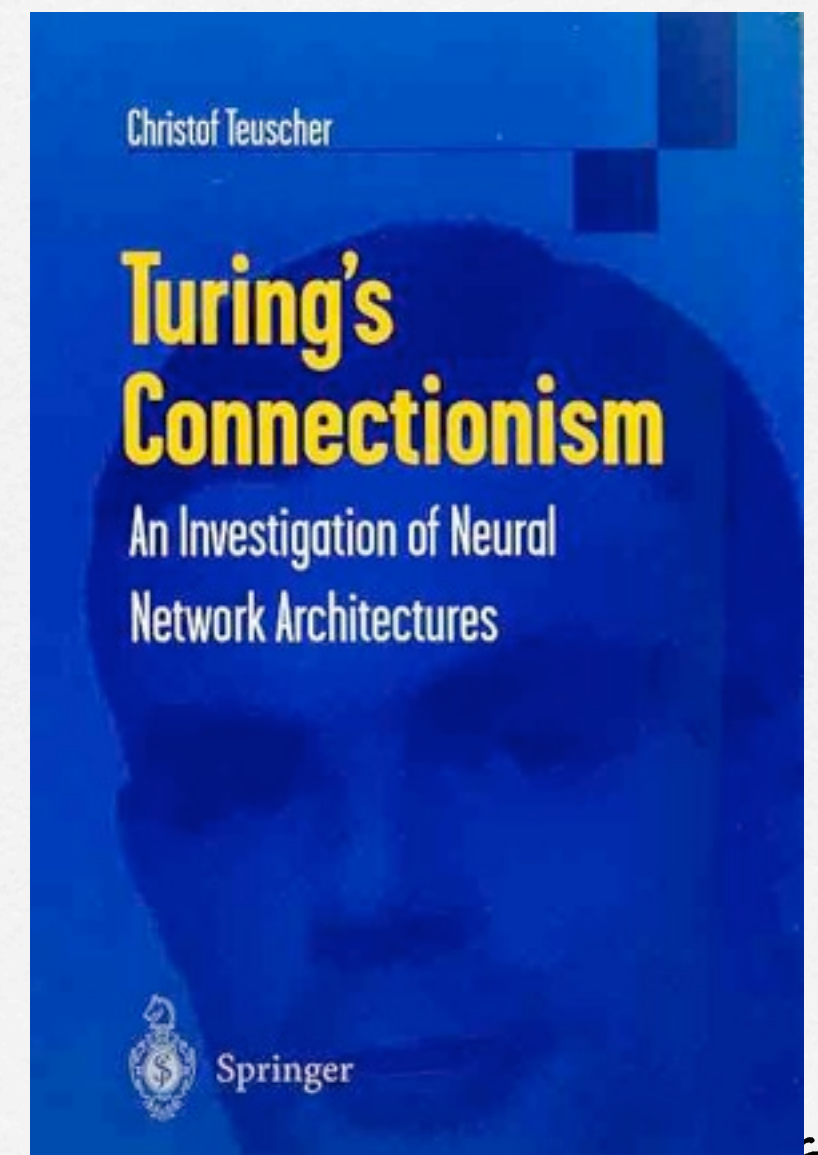
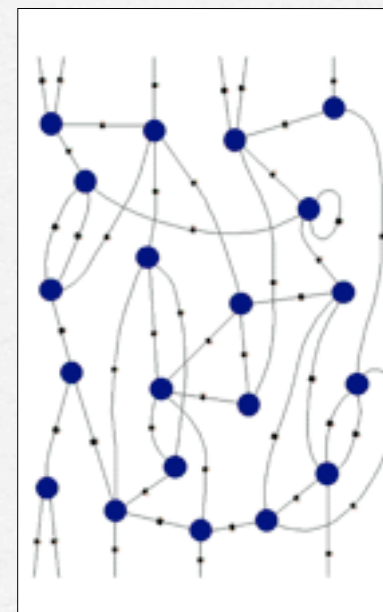
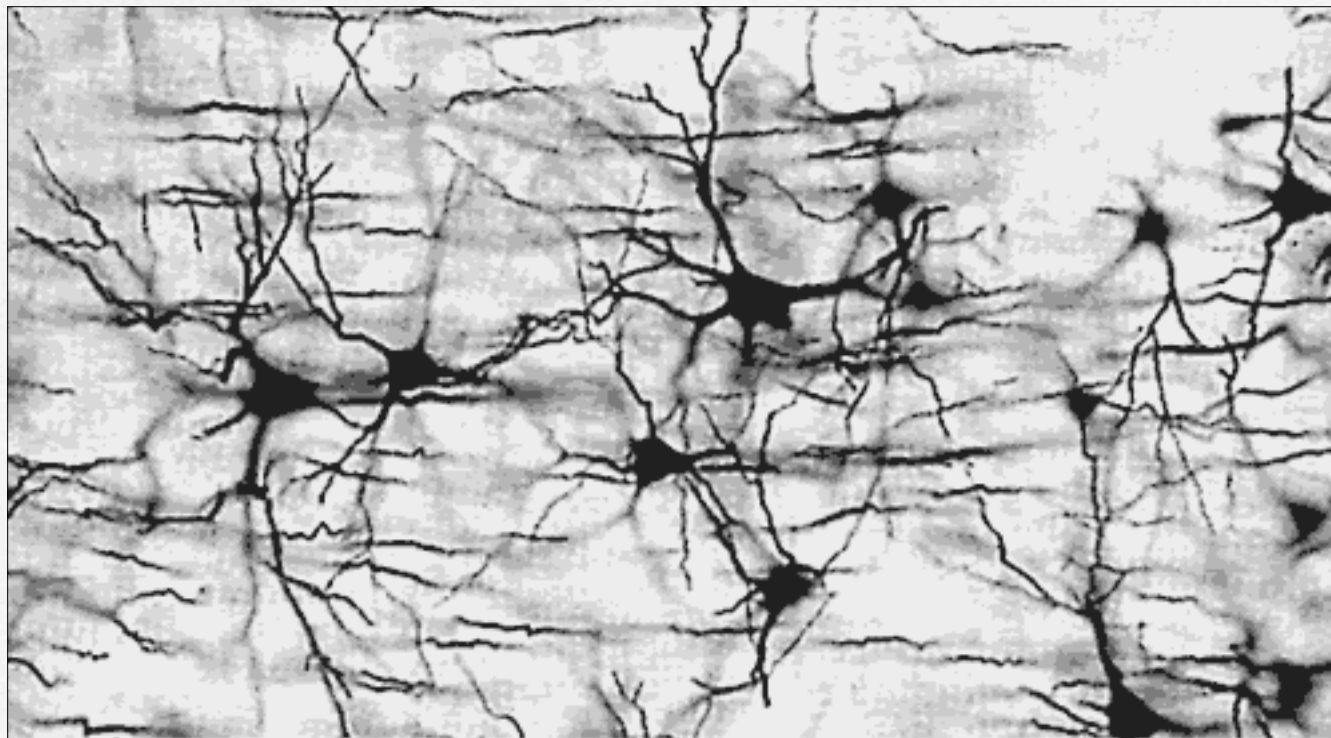






# The Mind as Mathematics?

- ❑ Turing, 1948: 'Unorganised machines' = neural nets
- ❑ Beginnings: Warren McCulloch and Walter Pitts, 1943







# The Mind as Mathematics?

There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: **they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.**



*Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),*

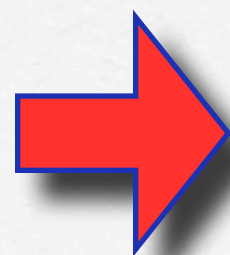
*On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74*





# The Mind as Mathematics?

- Connectionist models come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets
- But for Steven Pinker "... neural networks alone cannot do the job".



And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .





# The Mind as Mathematics?

We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.



Steven Pinker,

*How the Mind Works*, W. W. Norton, New York, 1997





# The Mind as Mathematics?

“As the brain forms images of an object - such as a face, a melody, a toothache, the memory of an event - and as the images of the object *affect* the state of the organism, yet another level of brain structure creates a swift nonverbal account of the events that are taking place in the varied brain regions activated as a consequence of the object-organism interaction. The mapping of the object-related consequences occurs in first-order neural maps representing the proto-self and object; the account of the *causal relationship* between object and organism can only be captured in second-order neural maps. ... one might say that the swift, second-order nonverbal account narrates a story: *that of the organism caught in the act of representing its own changing state as it goes about representing something else.*”

- António Damasio [1999], *The Feeling Of What Happens*, p.170





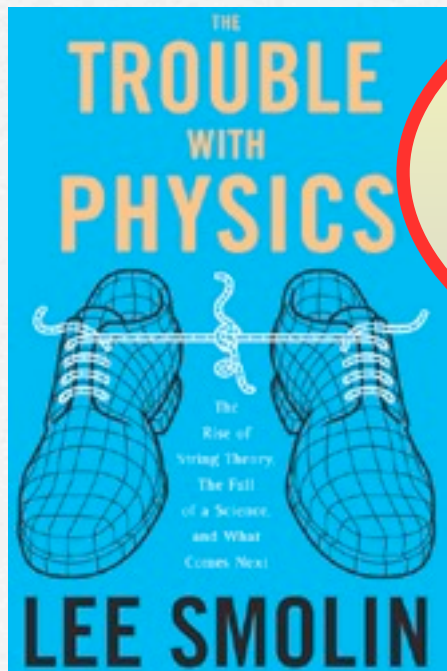


# ... Causality and Computation

- In modelling the physical universe -



... causality itself  
is fundamental



Lee Smolin, 'The Trouble With Physics', p.241





# Basic Causality and Global Formation

- Early champions of the role of causality - Roger Penrose, Rafael Sorkin, Fay Dowker, and Fotini Markopoulou

It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: **Causal relations can determine the spacetime geometry ...**

It's easy to talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. ... We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that **causality itself is fundamental** - and is thus meaningful even at a level where the notion of space has disappeared.

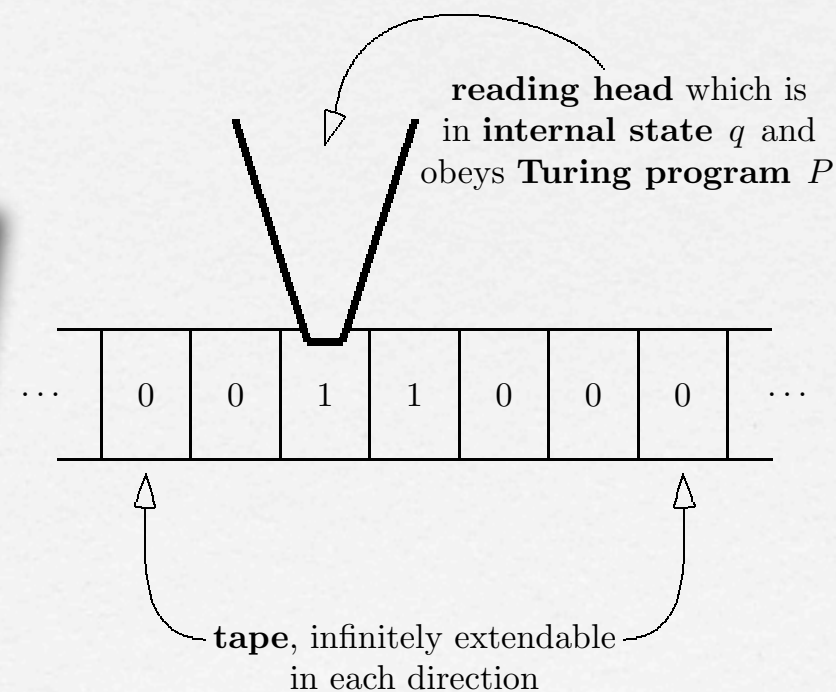
*Lee Smolin, The Trouble With Physics, p.241*





# Embodiment also about Information

- Turing, 1939 - Oracle Turing Machines ...
- Provide a model of how we compute using data given to us from unknown sources
- A model within which Newtonian computability etc comfortably fit ...





# Basic causality, Turing landscape and emergence ...

- can describe global relations in terms of local structure ...
- ... so capturing the *emergence* of large-scale formations



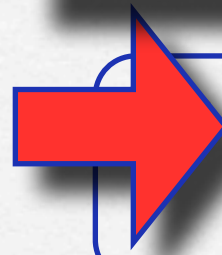
➔ Mathematically - formalise as *definability* over structure based on Turing functionals

➔ More generally - as *invariance under automorphisms*





# Hartley Rogers' programme ...



Fundamental problem: Characterise the Turing invariant relations

- Intuition: These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure
- Notice: The richness of Turing structure discovered so far becomes the raw material for a multitude of non-trivially definable relations







# Bi-interpretability

## Bi-interpretability Conjecture

(Harrington): **The Turing definable relations are exactly those with information content describable in second-order arithmetic**



- Where: Conjecture rules out there being non-trivial Turing automorphisms ...



- While: Partial results underpin observed certainties ...



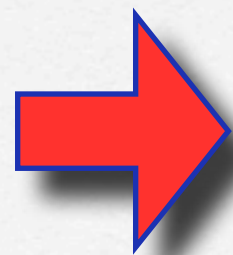
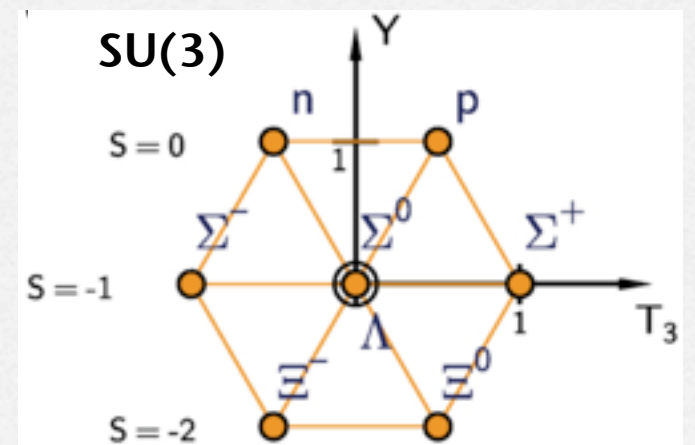


# Definability and Symmetries



Murray Gell-Mann

- Symmetries play a huge role in science ..
- ... expressing appropriate automorphisms
- ... or particular lapses in definability

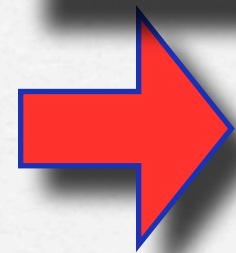


... so giving a clear route: from fundamental mathematical structures, and their automorphisms and breakdowns in definability - to far-reaching macro-symmetries in nature



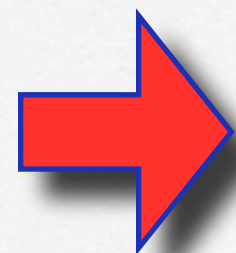


# A Quantum Embodiment ...



Processes for change of wave equation describing quantum state of a physical system:

- Deterministic continuous evolution via Schrödinger's equation - involves superpositions of basis states
- Probabilistic non-local discontinuous change due to measurement - observe a jump to a single basis state

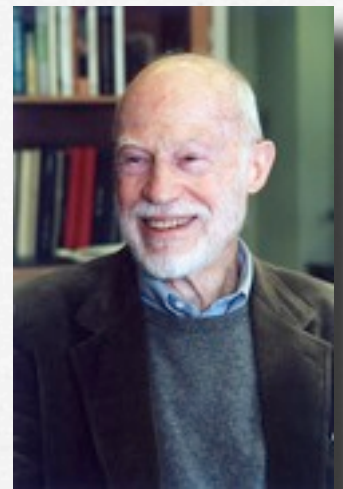


Putative mathematical explanation *in terms of collapse of distinct automorphic copies ... providing ...*





# ... an alternative to Many Worlds



Bryce DeWitt

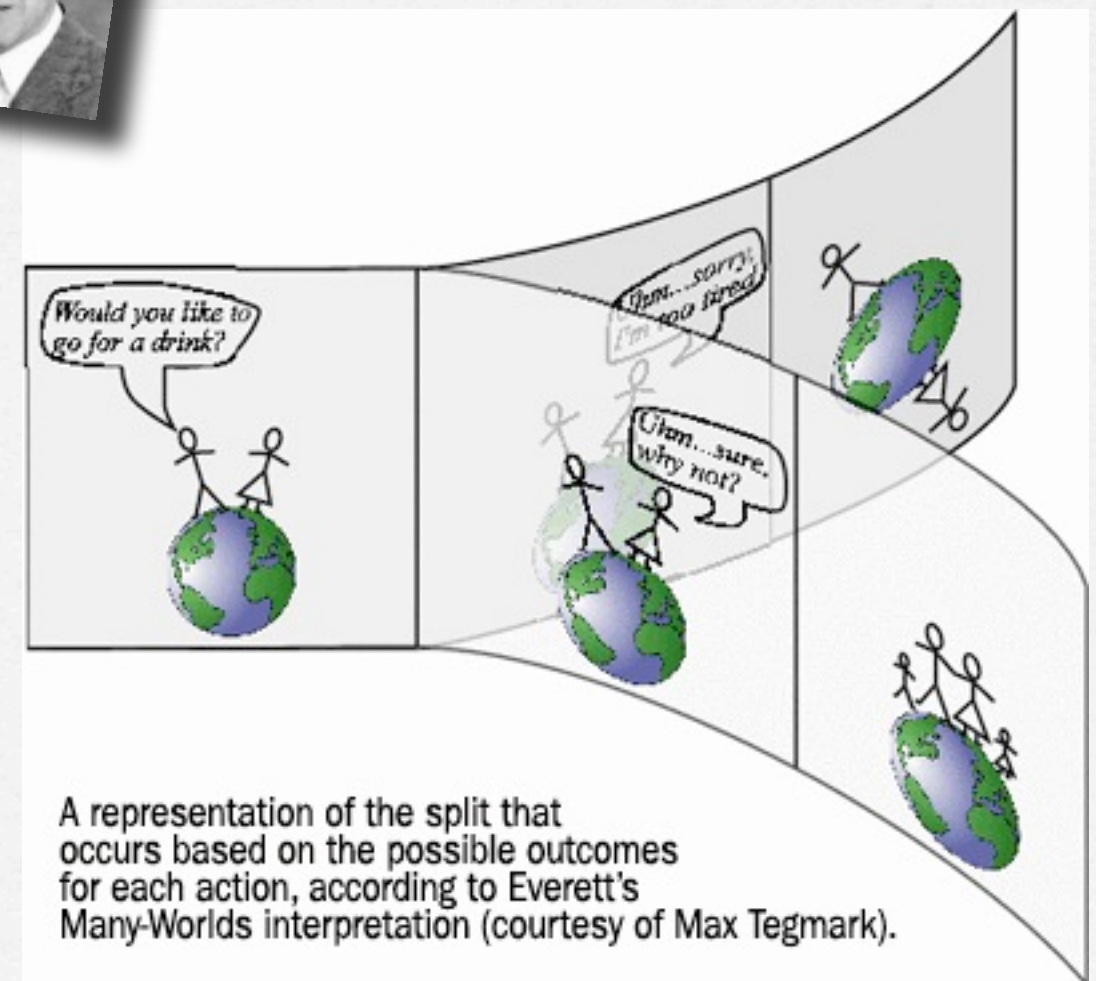


John Wheeler



**Hugh Everett III**

(Nov.11, 1930- July 19, 1982)







# And: Downward Causation Revisited

The “levels” involved are levels of organisation and integration, and the downward influence means that **the behavior of “lower” levels - that is, of the components of which the “higher-level” structure consists - is different than it would otherwise be, because of the influence of the new property that emerges in consequence of the higher-level organization.**



William Hasker, *'The Emergent Self'*, Cornell University Press, 1999, p.175





# A Computational Model Embodying

- ➔ With embodiment in terms of information ... natural laws based on algorithmic relations between reals
- ➔ Emergence described in terms of definability/invariance
- ➔ ... with failures of definable information content modelling mental phenomena, quantum ambiguity
- ➔ ... which generate new levels of computable structure
- ➔ ... and a fragmented scientific enterprise





‘Give them something  
to take home’

The Turing Universe  
is ‘about’ Embodied  
Computation



Gian-Carlo Rota,  
*‘Indiscrete  
Thoughts’*,  
Birkhäuser, 1996





Thank You!