Computation and Growth

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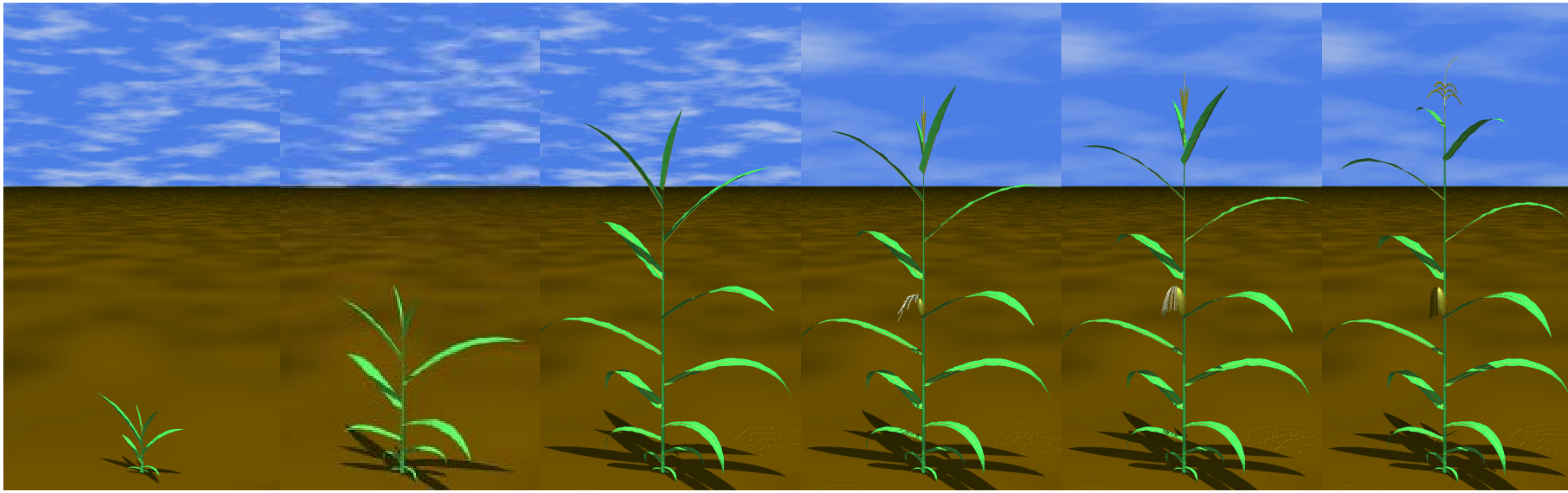
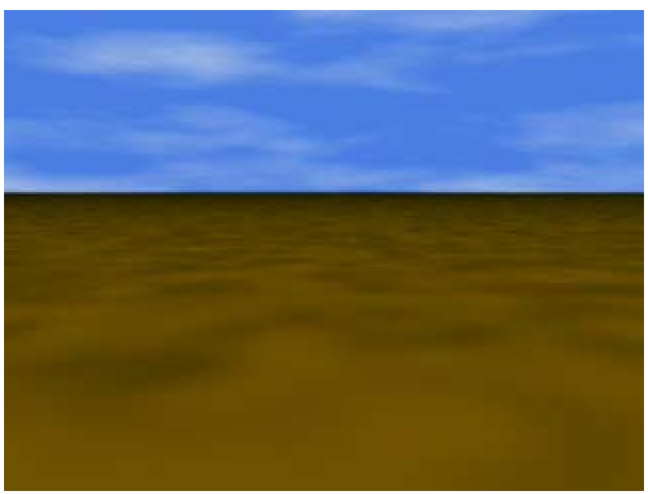
Susan Stepney. Nonclassical Computation: a dynamical systems perspective. In Rozenberg, Bäck, Kok, eds, *Handbook of Natural Computing*, Springer, 2009

corn (maize) plant: 40, 60, 80, 85, 90, 120 days

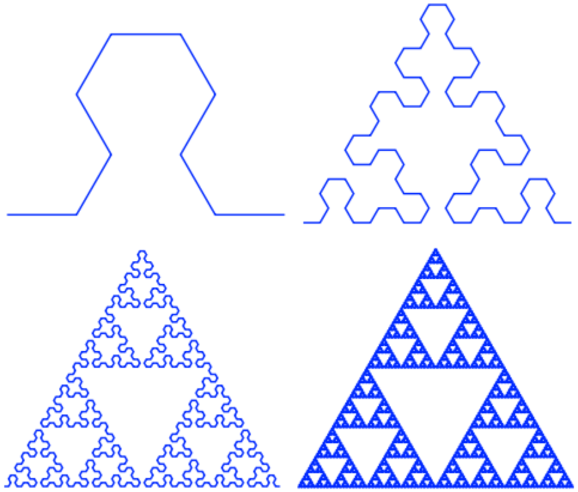


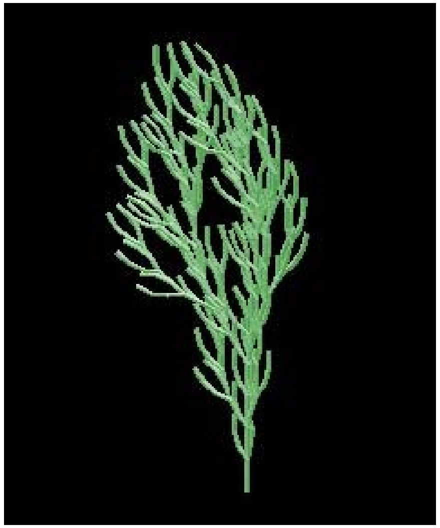


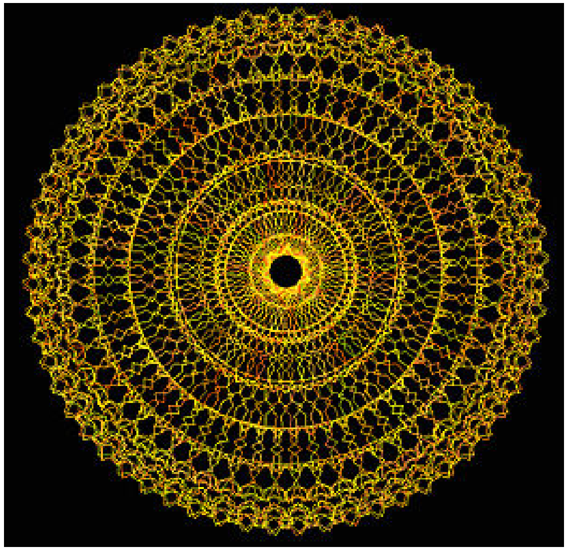
http://www.ifi.uzh.ch/~noser/Limages.html

growth grammars

- parallel rewriting systems
  - L-systems
    - growing plant-like structures
  - P-systems
    - growing nested membranes



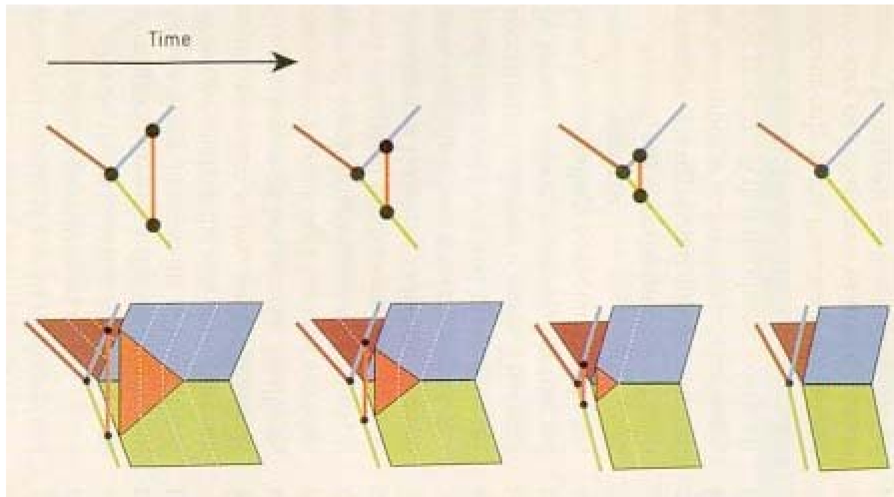




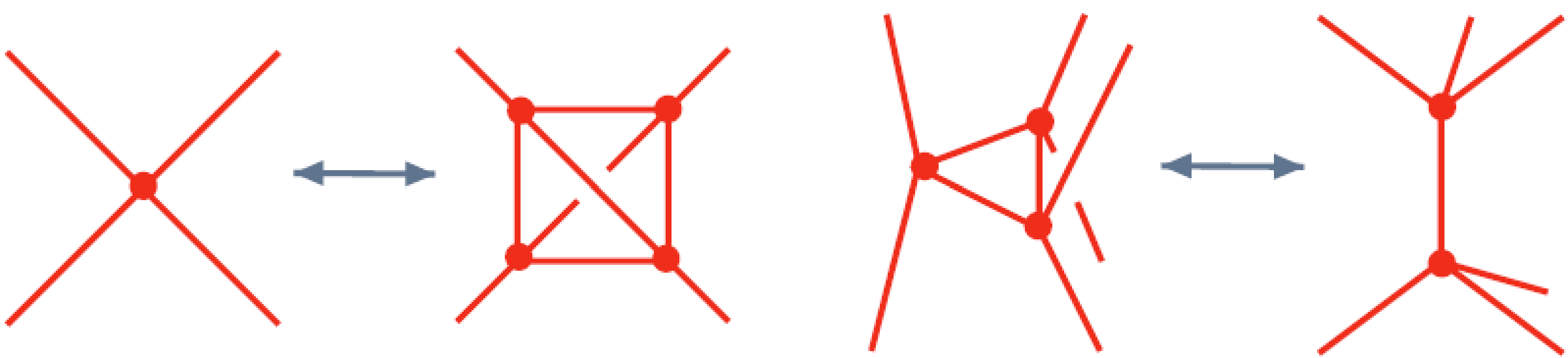
http://commons.wikimedia.org/wiki/L-system ; http://en.wikipedia.org/wiki/L-system

loop quantum gravity and spin foam

- growth and evolution of space itself



http://universe-review.ca/I02-30-lqg3.jpg



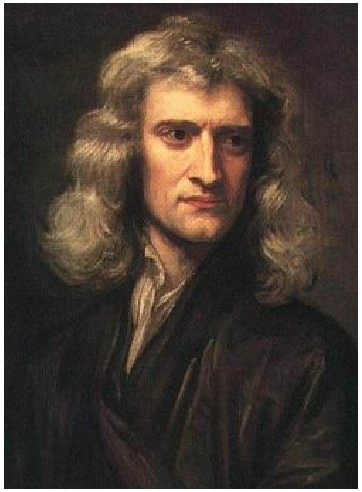
Freidel & Ryan. Spin foam models: the dynamics of quantum geometry. *Class. Quantum Grav.* **25**(11) 114004 (2008)

space : Newton *v.* Leibniz

- the 17th C view : Newton (1643–1727)
  - "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable."

- the *other* 17th C view : Leibniz (1646–1717)
  - Motion and position are real and detectable only in relation to other objects. Motion or position cannot be detected in relation to space itself, since space itself represents no object. Therefore empty space, a void, and so space itself is an unnecessary hypothesis.

http://www.friesian.com/space.htm



http://en.wikipedia.org/wiki/Isaac_Newton

http://en.wikipedia.org/wiki/Gottfried_Leibniz

physical space → state space

- array
  - pre-existing locations, to be populated with values
    - possibility of “empty” (uninitialised) locations, without values
    - need to distinguish “presence of nothing” from “absence”
      - “present, but empty, space” from “absence of space”
  - Newtonian data structure
    - cardinal locations: …, −2, −1, 0, 1, 2, 3, …
- list
  - locations and their values *co-created*
    - no possibility of “empty” (uninitialised) locations, without values
    - no need to distinguish “presence of nothing” from “absence”
      - “present, but empty, space” *cannot exist*
  - Leibnizian data structure
    - ordinal locations: 1st, 2nd, 3rd, …

co-construction of space

- *predetermined* state space and computation
  - Newtonian view of space
    - static data structures
  - predetermined degrees of freedom

- *co-constructed* state space and computation
  - Leibnizian view of space
    - dynamic data structures, growth grammars
  - "growth" of *new degrees of freedom* as computation proceeds
      - also encompasses "death", or collapse of degrees of freedom
    - not merely new "nodes" or more dimensions, but new *kinds* of dimensions, new possibilities

"Dynamical Systems with Dynamical Structure" $(DS)^2$

dynamical systems

- values of state variables evolving under a dynamical rule ("program") in a state space

with

dynamical structure

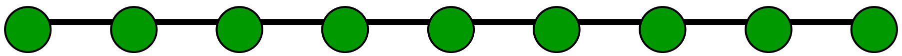
- also changing are:
  - number/types of state variables (dimensionality)
  - topology of connections (information flow)
  - dynamical rules of interactions ("program")

[Giavitto & Michel]

dynamical systems

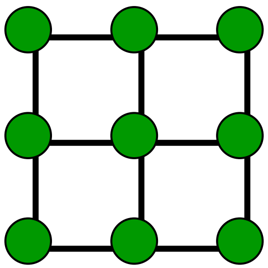
- state space
  - N state variables, each of type $\mathcal{X}$: $x_i \in \mathcal{X}$
    - N degrees of freedom
  - state vector in an N dimensional state space : $\mathbf{x} \in \mathcal{X}^N$
- dynamics
  - (deterministic) dynamics defined by $f : \mathcal{X}^N \rightarrow \mathcal{X}^N$
- trajectories
  - dynamics determines how the state vector $\mathbf{x}$ changes with time : $\mathbf{x}(t)$
  - qualitative behaviour of sets of trajectories
- attractors
  - dissipative (irreversible) systems : initial volumes of state space contract to attractors

topology v. dimensionality

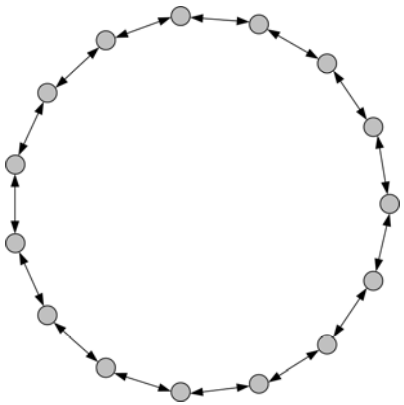


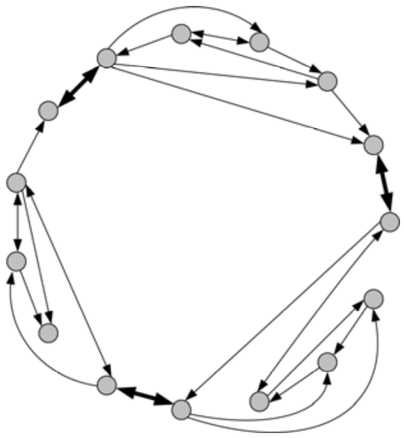
1D ?

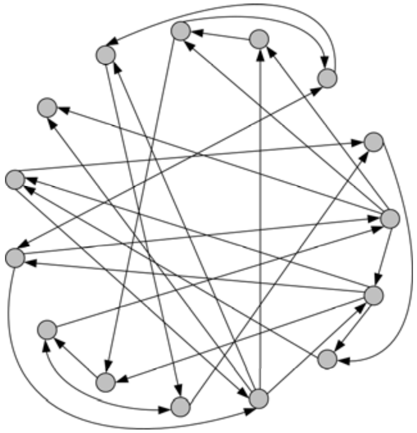
2D ?



- both 9D (state space dimensions)
- different topologies (information flow)

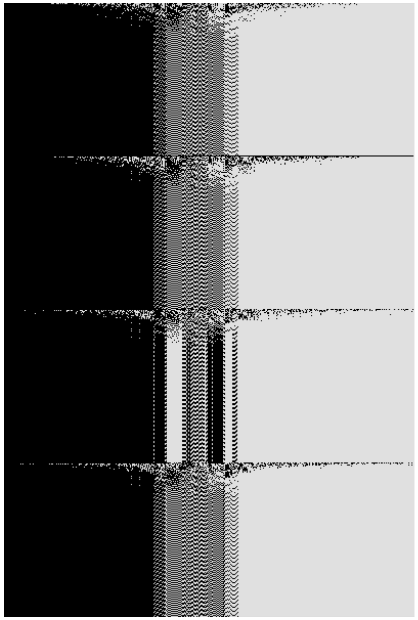


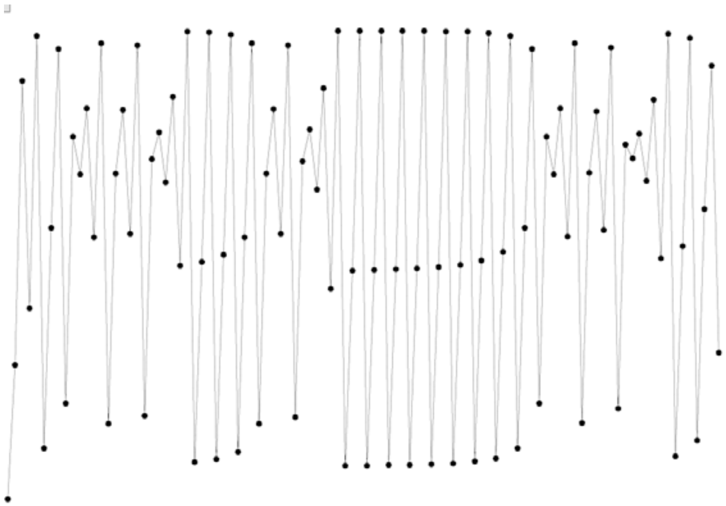


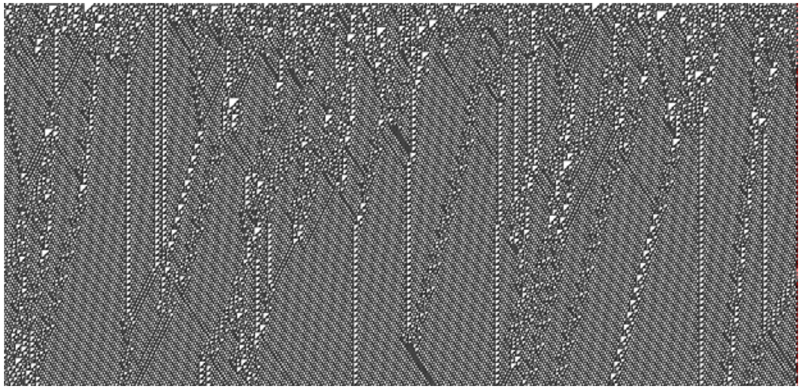


trajectories

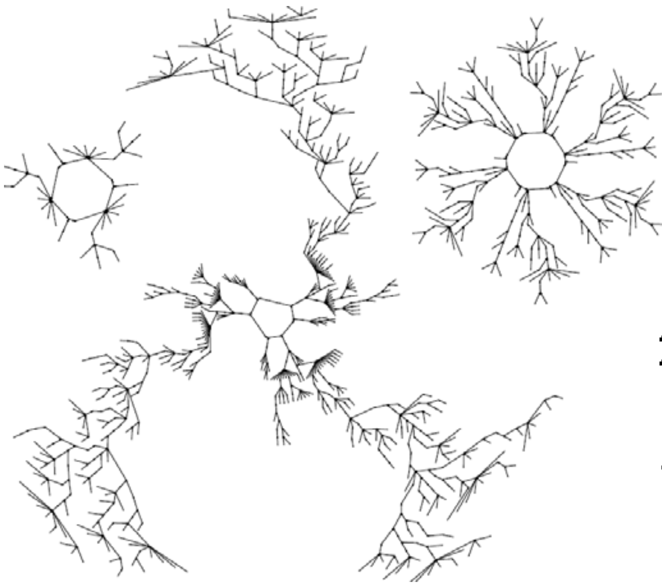
- paths through the (micro)states
  - cellular automata / RBNs
  - iterated maps
  - ODEs
  - ...

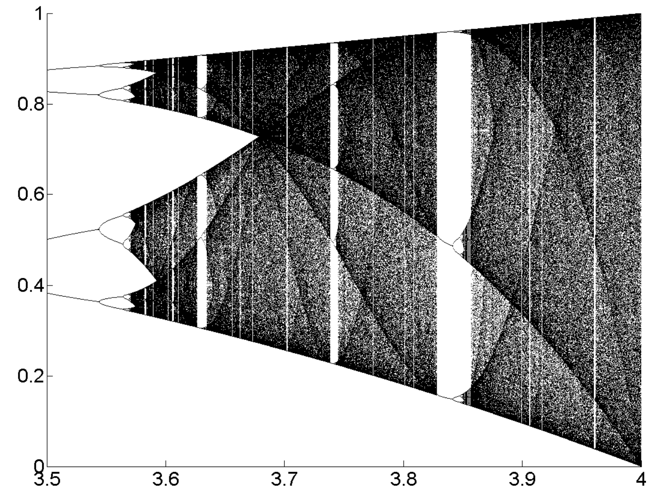


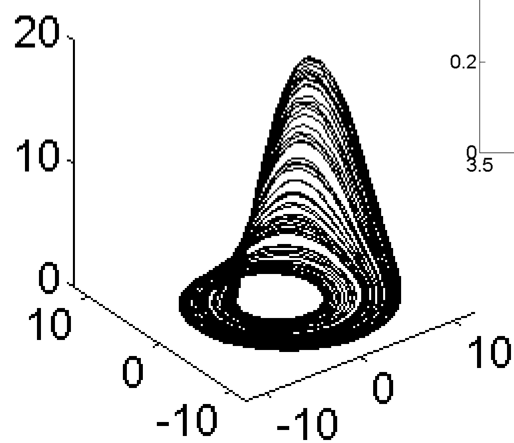


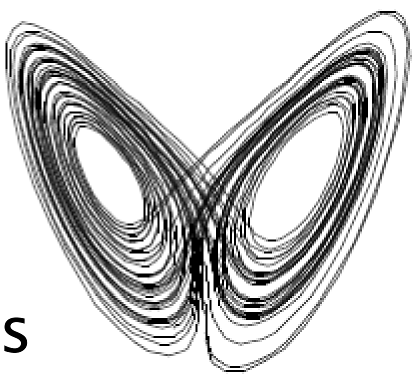


attractors









- attractors as macrostates of the dynamics
- computation : discovering the attractor
  - "halting (macro)state"

dynamical structure

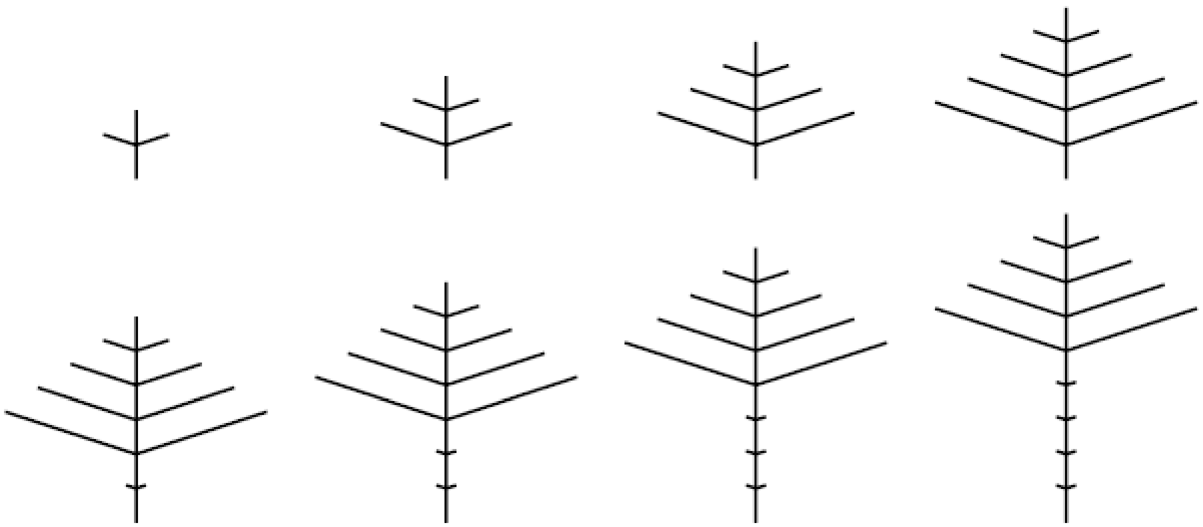
- dimensionality is a function of time : $\mathbf{x} \in \mathcal{X}^{N(t)}$
- the dynamics constructs the state space, and the state space constrains the dynamics
- in open systems (communicating with an environment), the system inputs "food" (which it "metabolises" to "grow": that is, it constructs dimensions), and excretes "waste" (collapses dimensions)

meta-dynamics

- dynamics : trajectories of the system through its state space
- meta-dynamics : dynamics of the state space itself
  - dimensionality changing
  - trajectories of the state space through the meta-space
- timescales
  - of dynamics relative to meta-dynamics
    - slowly changing dimensionality : can make approximations
      - eg, organism lifetime timescale *v* evolutionary timescale
    - similar timescales
      - L-systems growth models
- computation : discovering the meta-dynamics
  - eg, what new kinds of species evolve?

importance of death

- dimensionality **can go down as well as up**
  - state variables, degrees of freedom, can disappear
    - species go extinct, branches die back, ...

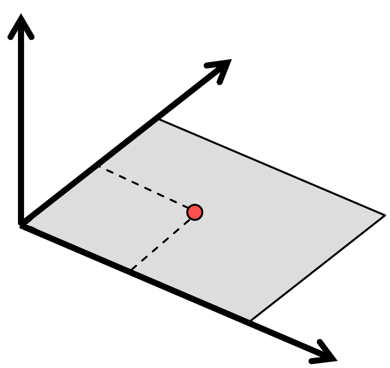


[Prusinkiewicz, Hammel, Hanan, Mech. 1996]

  - **scaffolding**, used to get to a particular state, then torn down

new dimensions, how?

- mathematical "trick"
  - have a "big enough" space, and "grow" into the currently "unused" dimensions
- but, how to distinguish "absence" (unused dimension) from the "presence of nothing" (used, but system at its origin)?
    - is it 2D (x,y), or 3D at z origin $(x,y,0)$?
  - another "trick": use ⊥ element
    - too many tricks?



- looking suspiciously "Newtonian"
  - pre-existing space, whether used or not

new dimensions, how?

- the trick hides the essence of the meta-dynamics

- why is the system confined to a particular sub-space?
- why does it change?
- what *type* of new dimension?
  - can this even be pre-specified?
- what are the new dynamical rules for the new dimensions?
  - adding a new equation for the new "species"

new dimensions, how?

- “curled” dimensions ??
  - “unfold” as needed
    - “fold up” again when no longer needed

- fractal dimensions ??
  - a new dimension gradually “fattens up” into a full dimension

- important that the new dimension and its new rules **emerge together**

wanted: a computational theory of growth

- need a whole new kind of dynamical systems theory
  - of **dynamical systems in meta-dynamical state spaces**
- necessary for understanding/building *emergent systems*
  - systems that co-construct their state spaces
- an unconventional view of computation, in terms of the dynamics of state space