Example of Shift-Reduce Parsing

A SUCCESSFUL PARSE

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	

NOTE: Reduction by $\underline{t} \rightarrow \underline{e}$ is also possible here

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	**Shift**
$ $\underline{t}$ * x	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	**Shift**
$ $\underline{t}$ * x	$	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{t}$ * $\underline{f}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\boxed{\underline{t} \quad \rightarrow \quad \underline{t} * \underline{f}} & & \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	**Shift**
$ $\underline{t}$ * x	$	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{t}$ * $\underline{f}$	$	**Reduce** by $\underline{t} \rightarrow \underline{t} * \underline{f}$
$ $\underline{t}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	**Shift**
$ $\underline{t}$ * x	$	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{t}$ * $\underline{f}$	$	**Reduce** by $\underline{t} \rightarrow \underline{t} * \underline{f}$
$ $\underline{t}$	$	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Shift**
$ $\underline{t}$ *	x $	**Shift**
$ $\underline{t}$ * x	$	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{t}$ * $\underline{f}$	$	**Reduce** by $\underline{t} \rightarrow \underline{t} * \underline{f}$
$ $\underline{t}$	$	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	$	**Accept**

A UNSUCCESSFUL PARSE

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	

NOTE: This time reduce by $\underline{t} \rightarrow \underline{e}$

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	**Shift**
$ $\underline{e}$ * x	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	**Shift**
$ $\underline{e}$ * x	$	**Reduce by** $\underline{f} \rightarrow x$
$ $\underline{e}$ * $\underline{f}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	**Shift**
$ $\underline{e}$ * x	$	**Reduce by** $\underline{f} \rightarrow x$
$ $\underline{e}$ * $\underline{f}$	$	**Reduce by** $\underline{t} \rightarrow \underline{f}$
$ $\underline{e}$ * $\underline{t}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	**Shift**
$ $\underline{e}$ * x	$	**Reduce by** $\underline{f} \rightarrow x$
$ $\underline{e}$ * $\underline{f}$	$	**Reduce by** $\underline{t} \rightarrow \underline{f}$
$ $\underline{e}$ * $\underline{t}$	$	**Reduce by** $\underline{e} \rightarrow \underline{t}$
$ $\underline{e}$ * $\underline{e}$	$	

Grammar:

$$
\begin{array}{lcl}
\underline{e} & \rightarrow & \underline{e} + \underline{t} \\
 & | & \underline{t} \\
 & & \\
\underline{t} & \rightarrow & \underline{t} * \underline{f} \\
 & | & \underline{f} \\
 & & \\
\underline{f} & \rightarrow & (\underline{e}) \\
 & | & x
\end{array}
$$

Parsing steps:

Stack	Input	Action
$	x * x $	**Shift**
$ x	* x $	**Reduce** by $\underline{f} \rightarrow x$
$ $\underline{f}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{f}$
$ $\underline{t}$	* x $	**Reduce** by $\underline{t} \rightarrow \underline{e}$
$ $\underline{e}$	* x $	**Shift**
$ $\underline{e}$ *	x $	**Shift**
$ $\underline{e}$ * x	$	**Reduce by** $\underline{f} \rightarrow x$
$ $\underline{e}$ * $\underline{f}$	$	**Reduce by** $\underline{t} \rightarrow \underline{f}$
$ $\underline{e}$ * $\underline{t}$	$	**Reduce by** $\underline{e} \rightarrow \underline{t}$
$ $\underline{e}$ * $\underline{e}$	$	**Fail**