Example of Shift-Reduce Parsing
A SUCCESSFUL PARSE
Grammar:

\[ e \rightarrow e + t \]

\[ t \rightarrow t * f \]

\[ f \rightarrow (e) \]

Parsing steps:

<table>
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<tr>
<th>Stack</th>
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<tbody>
<tr>
<td>$</td>
<td>x * x $</td>
<td></td>
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### Grammar:

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<tr>
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<th>→</th>
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<tbody>
<tr>
<td>e</td>
<td>→</td>
<td>e + t</td>
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<td>l</td>
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<td>t</td>
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<tr>
<td>t</td>
<td>→</td>
<td>t * f</td>
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<td>f</td>
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<tr>
<td>f</td>
<td>→</td>
<td>( e )</td>
</tr>
<tr>
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Grammar:

\[
e \rightarrow e + t \\
  \vdash  t \\
\]

\[
t \rightarrow t * f \\
  \vdash  f \\
\]

\[
f \rightarrow (e) \\
  \vdash  x \\
\]

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Grammar:

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\begin{align*}
  e &\rightarrow e + t \\
  t &\rightarrow t * f \\
  f &\rightarrow (e) \\
\end{align*}
\]

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Where $f \rightarrow x$, $t \rightarrow f$, and $t \rightarrow (e)$ are reduction rules.
Grammar:

\[ e \rightarrow e + t \]
\[ t \rightarrow t * f \]
\[ f \rightarrow (e) \]

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**NOTE:** Reduction by $t \rightarrow e$ is also possible here
Grammar:

\[
\begin{align*}
e & \rightarrow e + t \\
& \mid t \\

\rule{0pt}{2ex}
t & \rightarrow t * f \\
& \mid f \\

\rule{0pt}{2ex}
f & \rightarrow (e) \\
& \mid x
\end{align*}
\]

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<tr>
<td>$ t *</td>
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\begin{align*}
  e & \rightarrow e + t \\
  t & \rightarrow t * f \\
  f & \rightarrow ( e ) \\
  t & \rightarrow t * f
\end{align*}
\]

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A UNSUCCESSFUL PARSE
Grammar:

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e & \rightarrow e + t \\
  & \mid t \\
 t & \rightarrow t * f \\
  & \mid f \\
f & \rightarrow ( e ) \\
  & \mid x
\end{align*}
\]

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**NOTE:** This time reduce by $ t \rightarrow e $
Grammar:

- $e \rightarrow e + t$
- $t \rightarrow t * f$
- $f \rightarrow (e)$

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Input

Shift
Reduce by $f \rightarrow x$
Reduce by $t \rightarrow f$
Reduce by $t \rightarrow e$
Shift
Shift
$
Grammar:

\[
\begin{align*}
  e \quad &\rightarrow \quad e + t \\
  t \quad &\rightarrow \quad t * f \\
  f \quad &\rightarrow \quad (e)
\end{align*}
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e & \rightarrow \ e + \ t \\
\quad & \mid \quad \ t \\
t & \rightarrow \ t * \ f \\
\quad & \mid \quad \ f \\
f & \rightarrow \ ( \ e ) \\
\quad & \mid \quad \ x \\
\end{align*}
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