

Unifying Theories of Programming in `ProofPower-Z`

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1 UTP Relations Theory

1.1 Creating the Theory

SML

```
open_theory "utp-z-library";
force_delete_theory "utp-rel" handle Fail _ => ();
new_theory "utp-rel";
open_theory "utp-rel";
set_flags[("z-type-check-only",false),("z-use-axioms",true),
          ("standard-z-paras",false),("standard-z-terms",false)];

(* Setting PP to accept the refinement symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["refinedby"],
      Value "⊑",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ()
;

(* Setting PP to accept the equivalence symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["equivalentto"],
      Value "≡",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ()
;
```

1.2 Definitions

1.2.1 Names and Alphabets

\mathbb{Z}
| $[NAME]$

\mathbb{Z}
| $any_name : NAME$

\mathbb{Z}
| $ALPHABET \triangleq \mathbb{P} NAME$

\mathbb{Z}
| $dash : NAME \rightarrow NAME$

\mathbb{Z}
| $one, two : NAME \rightarrow NAME$
|
|

| $ran\ one \cap ran\ two = \emptyset$

\mathbb{Z}
| $dashed : ALPHABET$
|
|

| $dashed = ran\ dash$

\mathbb{Z}
| $undashed : ALPHABET$
|
|

| $undashed = \{n : NAME \mid n \notin ran\ dash\}$

\mathbb{Z}
| $in_a : ALPHABET \rightarrow ALPHABET$
|
|

| $\forall a:ALPHABET \bullet in_a \ a = a \cap undashed$

^z
| **out_a** : $ALPHABET \rightarrow ALPHABET$

| $\forall a:ALPHABET \bullet out_a \ a = a \cap dashed$

^z
| **homogeneous** : $\mathbb{P} \ ALPHABET$

| $homogeneous = \{a : ALPHABET$
| $\mid (\forall n : undashed \bullet n \in a \Leftrightarrow dash \ n \in a)\}$

^z
| **composable** : $\mathbb{P} \ (ALPHABET \times ALPHABET)$

| $composable = \{a1, a2 : ALPHABET$
| $\mid (\forall n : undashed \bullet n \in a2 \Leftrightarrow dash \ n \in a1)\}$

1.2.2 Bindings

^z
| **VALUE** ::= $Int(\mathbb{Z}) \mid Bool(\mathbb{B})$
| $Channel(NAME)$
| $Seq(seq \ VALUE) \mid Set(\mathbb{P} \ VALUE)$
| $Pair(VALUE \times VALUE)$
| $Sync$

^z
| **INT_VAL** $\hat{=}$ $\{n:\mathbb{Z} \bullet Int(n)\}$

^z
| **BOOL_VAL** $\hat{=}$ $\{Bool(true), Bool(false)\}$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{CHANNEL_VAL} \triangleq \{n:\mathbf{NAME} \bullet \mathbf{Channel}(n)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{SEQ_VAL} \triangleq \{s:\mathbf{seq} \ \mathbf{VALUE} \bullet \mathbf{Seq}(s)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{SET_VAL} \triangleq \{s:\mathbb{P} \ \mathbf{VALUE} \bullet \mathbf{Set}(s)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{PAIR_VAL} \triangleq \{v1,v2:\mathbf{VALUE} \bullet \mathbf{Pair}(v1,v2)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{EVENT_VAL} \triangleq \{c:\mathbf{CHANNEL_VAL}; v:\mathbf{VALUE} \bullet \mathbf{Pair}(c,v)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{SEQ_EVENT_VAL} \triangleq \{s:\mathbf{seq} \ \mathbf{EVENT_VAL} \bullet \mathbf{Seq}(s)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{SET_EVENT_VAL} \triangleq \{s:\mathbb{P} \ \mathbf{EVENT_VAL} \bullet \mathbf{Set}(s)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{SET_SEQ_EVENT_VAL} \triangleq \{s:\mathbb{P} \ \mathbf{SEQ_EVENT_VAL} \bullet \mathbf{Set}(s)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{PAIR_SEQ_EVENT_VAL} \triangleq \{s1,s2:\mathbf{SEQ_EVENT_VAL} \bullet \mathbf{Pair}(s1,s2)\} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad \mathbf{UNARY_F} \triangleq \mathbf{VALUE} \leftrightarrow \mathbf{VALUE} \end{array}$$

$$\begin{array}{l} \mathbb{Z} \\ | \quad (\forall f:\mathbf{UNARY_F}; v:\mathbf{VALUE} \bullet f \ v \in \mathbf{VALUE}) \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{any_un_fun} : \text{UNARY_F} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{BINARY_F} \triangleq (\text{VALUE} \times \text{VALUE}) \rightarrow \text{VALUE} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | (\forall f : \text{BINARY_F}; vv : \text{VALUE} \times \text{VALUE} \bullet f \text{ } vv \in \text{VALUE}) \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{any_bin_fun} : \text{BINARY_F} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{RELATION} \triangleq \text{VALUE} \leftrightarrow \text{VALUE} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{any_rel} : \text{RELATION} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{EXPRESSION} ::= \text{Val}(\text{VALUE}) \\ | \quad \quad \quad | \text{Var}(\text{NAME}) \\ | \quad \quad \quad | \text{Fun}_1(\text{UNARY_F} \times \text{EXPRESSION}) \\ | \quad \quad \quad | \text{Fun}_2(\text{BINARY_F} \times \text{EXPRESSION} \times \text{EXPRESSION}) \\ | \quad \quad \quad | \text{Rel}(\text{RELATION} \times \text{EXPRESSION} \times \text{EXPRESSION}) \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{BINDING} \triangleq \text{NAME} \rightarrow \text{VALUE} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{BINDINGS} \triangleq \mathbb{P} \text{ BINDING} \end{array}$$

$$\begin{array}{l} \text{Z} \\ | \text{FV} : \text{EXPRESSION} \rightarrow \mathbb{P} \text{ NAME} \\ \hline \end{array}$$

$$\begin{array}{|l}
\forall v:VALUE; n:NAME; exp1,exp2:EXPRESSION; \\
f1:UNARY_F; f2:BINAR\!Y_F; r:RELATION \\
\bullet FV(Val(v)) = \emptyset \\
\wedge FV(Var(n)) = \{n\} \\
\wedge FV(Fun_1(f1,exp1)) = FV(exp1) \\
\wedge FV(Fun_2(f2,exp1,exp2)) = FV(exp1) \cup FV(exp2) \\
\wedge FV(Rel(r,exp1,exp2)) = FV(exp1) \cup FV(exp2)
\end{array}$$

z

$$\begin{array}{|l}
\mathbf{WF_BINDING_EXPRESSION}_R \hat{=} \\
\{b:BINDING; e:EXPRESSION \mid FV(e) \subseteq dom\ b\}
\end{array}$$

z

$$\begin{array}{|l}
\mathbf{Eval} : WF_BINDING_EXPRESSION_R \rightarrow VALUE \\
\hline
\forall b:BINDING; v:VALUE; n:NAME; exp1,exp2:EXPRESSION; \\
f1:UNARY_F; f2:BINAR\!Y_F; r:RELATION \\
\bullet Eval(b, Val(v)) = v \\
\wedge (((b, Var(n)) \in WF_BINDING_EXPRESSION_R) \Rightarrow \\
Eval(b, Var(n)) = b\ n) \\
\wedge (((b, exp1) \in WF_BINDING_EXPRESSION_R) \Rightarrow \\
Eval(b, Fun_1(f1, exp1)) = f1(Eval(b, exp1))) \\
\wedge (((b, exp1) \in WF_BINDING_EXPRESSION_R \\
\wedge (b, exp2) \in WF_BINDING_EXPRESSION_R) \Rightarrow \\
Eval(b, Fun_2(f2, exp1, exp2)) = \\
f2(Eval(b, exp1), Eval(b, exp2))) \\
\wedge (((b, exp1) \in WF_BINDING_EXPRESSION_R \\
\wedge (b, exp2) \in WF_BINDING_EXPRESSION_R) \Rightarrow \\
Eval(b, Rel(r, exp1, exp2)) = \\
Bool((Eval(b, exp1), Eval(b, exp2)) \in r))
\end{array}$$

1.2.3 UTP Relations Predicates

z

$$\begin{array}{|l}
\mathbf{REL_PREDICATE} \hat{=}
\end{array}$$

$$\{a:ALPHABET; bs:BINDINGS \mid (\forall b:bs \bullet dom\ b = a)\}$$

z

$$\mathbf{UnrestVar} : REL_PREDICATE \rightarrow \mathbb{P}\ NAME$$

$$\begin{aligned} \forall u:REL_PREDICATE \\ \bullet\ UnrestVar\ u = \{n:u.1 \mid (\forall b:u.2; v:VALUE \\ \bullet\ b \oplus \{n \mapsto v\} \in u.2)\} \end{aligned}$$

z

$$\mathbf{unrestTypedVar} : (REL_PREDICATE \times NAME \times \mathbb{P}\ VALUE) \rightarrow \mathbb{B}$$

$$\begin{aligned} \forall r:REL_PREDICATE; n:NAME; T:\mathbb{P}\ VALUE \bullet \\ unrestTypedVar(r,n,T) \Leftrightarrow (\forall b:r.2; v:T \bullet b \oplus \{n \mapsto v\} \in r.2) \end{aligned}$$

z

$$\mathbf{REL_CONDITION} : \mathbb{P}\ REL_PREDICATE$$

$$\begin{aligned} REL_CONDITION = \{r : REL_PREDICATE \\ \mid (r.1 \setminus undashed) \subseteq UnrestVar(r)\} \end{aligned}$$

z

$$\mathbf{dashCond} : REL_CONDITION \rightarrow REL_PREDICATE$$

$$\begin{aligned} \forall u:REL_CONDITION \bullet \\ dashCond(u) = (\{n:u.1 \mid n \in undashed \bullet dash\ n\} \cup u.1, \\ \{b:u.2 \\ \bullet \\ \{n:dom\ b \cap undashed \bullet dash\ n \mapsto b\ n\} \\ \cup \\ \{n:dash\ (dom\ b \cap dashed)\} \end{aligned}$$

z

$\mathbf{U} : ((NAME \rightarrow NAME) \times ALPHABET) \rightarrow REL_PREDICATE$

$\forall f:NAME \rightarrow NAME ; a':ALPHABET$

$| a' \subseteq dashed$

$\bullet (\exists a:ALPHABET$

$| a \subseteq undashed$

$\wedge a' = dash \llbracket a \rrbracket$

$\bullet U(f, a') =$

$(\{n:NAME \mid n \in a \bullet dash(f \ n)\} \cup a,$

$\{ b:BINDING$

$| dom \ b = \{n:NAME \mid n \in a \bullet dash(f \ n)\}$

$\cup a$

$\wedge (\forall n:NAME \mid n \in a \bullet$

$b(dash(f \ n)) = b(n)) \}$

$)$

$)$

1.2.4 Syntactic Restrictions

z

$\mathbf{WF_Equals}_R \triangleq$

$\{a:ALPHABET; n:NAME; e:EXPRESSION \mid n \in a \wedge FV(e) \subseteq a\}$

z

$\mathbf{WF_ALPHABET_EXPRESSION} \triangleq$

$\{a:ALPHABET; e1, e2:EXPRESSION \mid (FV(e1) \cup FV(e2)) \subseteq a\}$

z

$\mathbf{WF_Cond}_R \triangleq$

$\{u1, b, u2:REL_PREDICATE \mid (b.1 \subseteq u1.1 \wedge u1.1 = u2.1)\}$

z

$\mathbf{WF_Semi}_R \triangleq$

$$\left| \{u1, u2:REL_PREDICATE \mid (u1.1, u2.1) \in composable\} \right.$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_Skip}_R \triangleq \{a:ALPHABET \mid a \in homogeneous\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_Assign}_R \triangleq \right. \\ \left| \quad \{a:ALPHABET; ns:seq\ NAME; exps:seq\ EXPRESSION \right. \\ \left| \quad \mid (\forall n:ran\ ns \bullet n \in a \wedge n \in undashed) \right. \\ \left| \quad \wedge (\forall e:ran\ exps \bullet FV(e) \subseteq a \wedge FV(e) \subseteq undashed) \right. \\ \left| \quad \wedge \#ns = \#exps \neq 0 \right. \\ \left| \quad \wedge a \in homogeneous\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_Subst}_R \triangleq \right. \\ \left| \quad \{u:REL_PREDICATE; e:EXPRESSION; n:NAME \right. \\ \left| \quad \mid FV(e) \subseteq u.1 \wedge n \in u.1\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_Var}_{R_EndR} \triangleq \right. \\ \left| \quad \{a:ALPHABET; n:NAME \mid a \in homogeneous \wedge n \in undashed \right. \\ \left| \quad \wedge \{n, dash(n)\} \subseteq a\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_VarT}_{R_EndTR} \triangleq \right. \\ \left| \quad \{a:ALPHABET; n:NAME; T:SET_VAL \mid (a, n) \in WF_Var_{R_EndR}\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_Extend_rest}_R \triangleq \right. \\ \left| \quad \{u : REL_PREDICATE; a : ALPHABET \right. \\ \left| \quad \mid a \subseteq undashed \wedge (a \cup \{n:a \bullet dash\ n\}) \cap u.1 = \emptyset\} \right. \end{array}$$

$$\begin{array}{l} \text{Z} \\ \left| \mathbf{WF_REL_PREDICATE_PAIR} \triangleq \right. \end{array}$$

$$\left| \begin{array}{l} \{u1, u2: REL_PREDICATE \mid u1.1 = u2.1\} \end{array} \right|$$

$$\begin{array}{l} \mathbf{ZF_Glb}_{\mathbf{R_LubR}} \triangleq \{a: ALPHABET; us: \mathbb{P} \text{ } REL_PREDICATE \\ \mid (\forall u_us: us \bullet u_us.1 = a)\} \end{array}$$

1.2.5 Operations on Values

$$\left| \begin{array}{l} rel _ \leq_{\mathbf{R}} _ \end{array} \right|$$

$$\begin{array}{l} _ \leq_{\mathbf{R}} _ : VALUE \leftrightarrow VALUE \\ \hline (- \leq_{\mathbf{R}} _) = \{s1, s2: SEQ_VAL \\ \mid ((Seq^\sim)s1) \text{ prefix}_Z ((Seq^\sim)s2)\} \end{array}$$

$$\left| \begin{array}{l} rel _ <_{\mathbf{R}} _ \end{array} \right|$$

$$\begin{array}{l} _ <_{\mathbf{R}} _ : VALUE \leftrightarrow VALUE \\ \hline (- <_{\mathbf{R}} _) = \{s1, s2: SEQ_VAL \\ \mid ((Seq^\sim)s1) \text{ prefix}_Z ((Seq^\sim)s2) \\ \wedge ((Seq^\sim)s1) \neq ((Seq^\sim)s2)\} \end{array}$$

$$\left| \begin{array}{l} rel _ \in_{\mathbf{R}} _ \end{array} \right|$$

$$\begin{array}{l} _ \in_{\mathbf{R}} _ : VALUE \leftrightarrow VALUE \\ \hline (- \in_{\mathbf{R}} _) = \{e: VALUE; s: SET_VAL \mid e \in ((Set^\sim)s)\} \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{rel} - \not\subseteq_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \not\subseteq_{\mathbf{R}} - : \text{VALUE} \leftrightarrow \text{VALUE} \\ \hline (- \not\subseteq_{\mathbf{R}} -) = \{e:\text{VALUE}; s:\text{SET_VAL} \mid e \notin ((\text{Set}^\sim)s)\} \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{rel} - \subseteq_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \subseteq_{\mathbf{R}} - : \text{VALUE} \leftrightarrow \text{VALUE} \\ \hline (- \subseteq_{\mathbf{R}} -) = \{s1,s2:\text{SET_VAL} \mid ((\text{Set}^\sim)s1) \subseteq ((\text{Set}^\sim)s2)\} \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \mathbf{MkSingleton}: \text{VALUE} \rightarrow \text{VALUE} \\ \hline \forall v:\text{VALUE} \bullet \text{MkSingleton}(v) = \text{Seq}(\langle v \rangle) \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \mathbf{MkPair}: (\text{VALUE} \times \text{VALUE}) \rightarrow \text{VALUE} \\ \hline \forall v1,v2:\text{VALUE} \bullet \text{MkPair}(v1,v2) = \text{Pair}(v1,v2) \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{fun} - \mathbf{SeqDif}_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \mathbf{SeqDif}_{\mathbf{R}} - : (\text{VALUE} \times \text{VALUE}) \rightarrow \text{VALUE} \\ \hline \end{array}$$

$$\begin{array}{|l}
\forall s1,s2:VALUE \mid \{s1,s2\} \subseteq SEQ_VAL \\
\mid \wedge ((Seq^\sim) s2) \text{ prefix}_Z ((Seq^\sim) s1) \\
\mid \bullet s1 \text{ SeqDif}_R s2 = \\
\mid \quad Seq(((Seq^\sim)s1) -_Z ((Seq^\sim)s2))
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid fun - \frown_R -
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid - \frown_R - : (VALUE \times VALUE) \leftrightarrow VALUE \\
\mid \hline
\forall s1,s2:VALUE \\
\mid \mid \{s1,s2\} \subseteq SEQ_VAL \\
\mid \bullet s1 \frown_R s2 = Seq(((Seq^\sim)s1) \frown ((Seq^\sim)s2))
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid fun - \lrcorner_R -
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid - \lrcorner_R - : (VALUE \times VALUE) \leftrightarrow VALUE \\
\mid \hline
\forall s,st:VALUE \\
\mid \mid s \in SEQ_VAL \wedge st \in SET_VAL \\
\mid \bullet s \lrcorner_R st = Seq(((Seq^\sim)s) \lrcorner ((Set^\sim)st))
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid fun - \cup_R -
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mid - \cup_R - : (VALUE \times VALUE) \leftrightarrow VALUE \\
\mid \hline
\forall s1,s2:VALUE \\
\mid \mid \{s1,s2\} \subseteq SET_VAL
\end{array}$$

$$\bullet s1 \cup_R s2 = Set(((Set^\sim)s1) \cup ((Set^\sim)s2))$$

$$\begin{array}{c} \text{Z} \\ \hline fun _ \cap_R _ \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline _ \cap_R _ : (VALUE \times VALUE) \leftrightarrow VALUE \\ \hline \forall s1,s2:VALUE \\ \quad | \{s1,s2\} \subseteq SET_VAL \\ \bullet s1 \cap_R s2 = Set(((Set^\sim)s1) \cap ((Set^\sim)s2)) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline fun _ \setminus_R _ \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline _ \setminus_R _ : (VALUE \times VALUE) \leftrightarrow VALUE \\ \hline \forall s1,s2:VALUE \\ \quad | \{s1,s2\} \subseteq SET_VAL \\ \bullet s1 \setminus_R s2 = Set(((Set^\sim)s1) \setminus ((Set^\sim)s2)) \end{array}$$

1.2.6 Relations Constructs

$$\begin{array}{c} \text{Z} \\ \hline \mathbf{True}_R : ALPHABET \rightarrow REL_PREDICATE \\ \hline \forall a:ALPHABET \bullet True_R a = (a, \{b : BINDING \mid dom\ b = a\}) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \mathbf{False}_R : ALPHABET \rightarrow REL_PREDICATE \\ \hline \forall a:ALPHABET \bullet False_R a = (a, \emptyset) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \text{fun } - \equiv_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline - \equiv_{\mathbf{R}} - : (\text{REL_PREDICATE} \times \text{REL_PREDICATE}) \rightarrow \text{REL_PREDICATE} \\ \hline \forall u1, u2 : \text{REL_PREDICATE} \\ \bullet (u1.1 = u2.1 \wedge u1.2 = u2.2 \Rightarrow u1 \equiv_{\mathbf{R}} u2 = \text{True}_{\mathbf{R}} u1.1) \\ \wedge (u1.1 \neq u2.1 \vee u1.2 \neq u2.2 \Rightarrow u1 \equiv_{\mathbf{R}} u2 = \text{False}_{\mathbf{R}} u1.1) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline =_{\mathbf{R}} : \text{WF_Equals}_{\mathbf{R}} \rightarrow \text{REL_PREDICATE} \\ \hline \forall a_n_e : \text{WF_Equals}_{\mathbf{R}} \\ \bullet =_{\mathbf{R}} (a_n_e) = \\ (a_n_e.1, \{b : \text{BINDING} \\ \quad | \text{dom } b = a_n_e.1 \\ \quad \wedge b(a_n_e.2) = \\ \quad \quad \text{Eval}(b, a_n_e.3)\}) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline =_{+\mathbf{R}} : \text{WF_ALPHABET_EXPRESSION} \rightarrow \text{REL_PREDICATE} \\ \hline \forall a_e_e : \text{WF_ALPHABET_EXPRESSION} \\ \bullet =_{+\mathbf{R}} (a_e_e) = \\ (a_e_e.1, \{b : \text{BINDING} \\ \quad | \text{dom } b = a_e_e.1 \\ \quad \wedge \text{Eval}(b, a_e_e.2) = \\ \quad \quad \text{Eval}(b, a_e_e.3)\}) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \neg_{\mathbf{R}} : \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE} \\ \hline \end{array}$$

$$\begin{array}{|l} \forall u:REL_PREDICATE \bullet \neg_R u = (u.1, (True_R u.1).2 \setminus u.2) \end{array}$$

$$\begin{array}{|l} \text{fun } - \oplus_R - \end{array}$$

$$\begin{array}{|l} - \oplus_R -: REL_PREDICATE \times ALPHABET \rightarrow REL_PREDICATE \\ \hline \forall u:REL_PREDICATE; a : ALPHABET \\ \bullet u \oplus_R a = (u.1 \cup a, \\ \{b:BINDING \mid (u.1 \triangleleft b) \in u.2 \\ \wedge dom\ b = u.1 \cup a\}) \end{array}$$

$$\begin{array}{|l} \text{fun } - \wedge_R - \end{array}$$

$$\begin{array}{|l} - \wedge_R -: REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE \\ \hline \forall u1, u2:REL_PREDICATE \bullet \\ u1 \wedge_R u2 = (u1.1 \cup u2.1, \\ (u1 \oplus_R u2.1).2 \cap (u2 \oplus_R u1.1).2) \end{array}$$

$$\begin{array}{|l} \text{fun } - \vee_R - \end{array}$$

$$\begin{array}{|l} - \vee_R -: REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE \\ \hline \forall u1, u2:REL_PREDICATE \bullet \\ u1 \vee_R u2 = (u1.1 \cup u2.1, \\ (u1 \oplus_R u2.1).2 \cup (u2 \oplus_R u1.1).2) \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{fun } - \Rightarrow_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \Rightarrow_{\mathbf{R}} \text{.} : \text{REL_PREDICATE} \times \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE} \\ \hline \forall u1, u2 : \text{REL_PREDICATE} \bullet u1 \Rightarrow_{\mathbf{R}} u2 = (\neg_{\mathbf{R}} u1) \vee_{\mathbf{R}} u2 \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{fun } - \Leftrightarrow_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \Leftrightarrow_{\mathbf{R}} \text{.} : \text{REL_PREDICATE} \times \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE} \\ \hline \forall u1, u2 : \text{REL_PREDICATE} \bullet \\ u1 \Leftrightarrow_{\mathbf{R}} u2 = (u1 \Rightarrow_{\mathbf{R}} u2) \wedge_{\mathbf{R}} (u2 \Rightarrow_{\mathbf{R}} u1) \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{fun } - \triangleleft_{\mathbf{R}} - \triangleright_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - \triangleleft_{\mathbf{R}} - \triangleright_{\mathbf{R}} \text{.} : \text{WF_Cond}_{\mathbf{R}} \rightarrow \text{REL_PREDICATE} \\ \hline \forall u1_b_u2 : \text{WF_Cond}_{\mathbf{R}} \\ \bullet u1_b_u2.1 \triangleleft_{\mathbf{R}} u1_b_u2.2 \triangleright_{\mathbf{R}} u1_b_u2.3 = \\ (u1_b_u2.2 \wedge_{\mathbf{R}} u1_b_u2.1) \\ \vee_{\mathbf{R}} ((\neg_{\mathbf{R}} u1_b_u2.2) \wedge_{\mathbf{R}} u1_b_u2.3) \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline \text{fun } - ;_{\mathbf{R}} - \end{array}$$

$$\begin{array}{c} \mathbf{Z} \\ \hline - ;_{\mathbf{R}} \text{.} : \text{WF_Semi}_{\mathbf{R}} \rightarrow \text{REL_PREDICATE} \end{array}$$

$$\begin{array}{|l}
\hline
\forall u1_u2: WF_Semi_R \\
\bullet u1_u2.1 ;_R u1_u2.2 = \\
\quad (in_a u1_u2.1.1 \cup out_a u1_u2.2.1, \\
\quad \{b1:u1_u2.1.2; b2:u1_u2.2.2 \\
\quad | (\forall n:dom b2 \\
\quad \quad | n \in undashed \\
\quad \quad \bullet b2(n)=b1 \ (dash \ n)) \\
\bullet (undashed \triangleleft b1) \cup (dashed \triangleleft b2)\}) \\
\hline
\end{array}$$

$$\begin{array}{|l}
^Z \\
\hline
\mathbf{\Pi}_R: WF_Skip_R \rightarrow REL_PREDICATE \\
\hline
\end{array}$$

$$\begin{array}{|l}
\forall a: WF_Skip_R \\
\bullet \Pi_R a = (a, \{b : BINDING \\
\quad | dom b = a \\
\quad \wedge (\forall n:a \mid n \in undashed \\
\quad \quad \bullet b(n) = b(dash(n)))\}) \\
\hline
\end{array}$$

$$\begin{array}{|l}
^Z \\
\hline
\mathbf{Assign}_R : WF_Assign_R \rightarrow REL_PREDICATE \\
\hline
\end{array}$$

$$\begin{array}{|l}
\forall aa: WF_Assign_R \\
\bullet \\
\quad (\#aa.2 = 1 \\
\quad \quad \wedge (\exists n:NAME \mid n = head(aa.2) \bullet \\
\quad \quad \quad Assign_R(aa) = \\
\quad \quad \quad =_R(aa.1, dash(n), head aa.3) \\
\quad \quad \quad \wedge_R \Pi_R (aa.1 \setminus \{n, dash(n)\}))) \\
\vee (\#aa.2 > 1 \\
\quad \quad \wedge (\exists n:NAME \mid n = head(aa.2) \bullet \\
\quad \quad \quad Assign_R(aa) = \\
\quad \quad \quad =_R(aa.1, dash(n), head aa.3) \\
\hline
\end{array}$$

$\forall a:ALPHABET; u:REL_PREDICATE$
 $\bullet \forall_R (a, u) = \neg_R (\exists_R (a, \neg_R u))$

Z
 $\text{ / }_R : WF_Subst_R \rightarrow REL_PREDICATE$

$\forall u_e_n:WF_Subst_R$
 $\bullet \text{ / }_R u_e_n =$
 $(u_e_n.1.1, \{b:BINDING \mid dom\ b = u_e_n.1.1$
 $\wedge b \oplus \{u_e_n.3 \mapsto Eval(b, u_e_n.2)\}$
 $\in u_e_n.1.2\})$

Z
 $\text{ fun } - \text{ intchoice}_R -$

Z
 $- \text{ intchoice}_R -: WF_REL_PREDICATE_PAIR \rightarrow REL_PREDICATE$

$\forall pair_u:WF_REL_PREDICATE_PAIR$
 $\bullet pair_u.1 \text{ intchoice}_R pair_u.2 = pair_u.1 \vee_R pair_u.2$

Z
 $\text{ fun } - \text{ conj}_R -$

Z
 $- \text{ conj}_R -: WF_REL_PREDICATE_PAIR \rightarrow REL_PREDICATE$

$\forall pair_u:WF_REL_PREDICATE_PAIR$
 $\bullet pair_u.1 \text{ conj}_R pair_u.2 = pair_u.1 \wedge_R pair_u.2$

Z
 $\text{ var}_R: WF_Var_{R_EndR} \rightarrow REL_PREDICATE$

$$\frac{}{\forall a_n: WF_Var_{R_EndR} \bullet var_R a_n = \exists_{-R} (\{a_n.2\}, \Pi_R a_n.1)}$$

$$\frac{}{\begin{array}{l} \text{Z} \\ \text{end}_R: WF_Var_{R_EndR} \rightarrow REL_PREDICATE \end{array}} \frac{}{\forall a_n: WF_Var_{R_EndR} \bullet end_R a_n = \exists_{-R} (\{dash a_n.2\}, \Pi_R a_n.1)}$$

$$\frac{}{\text{Z}} \text{fun } - \vdash_R -$$

$$\frac{}{\text{Z}} \frac{}{- \vdash_R -: WF_Extend_rest_R \rightarrow REL_PREDICATE} \frac{}{\forall u_a: WF_Extend_rest_R \bullet u_a.1 \vdash_R u_a.2 = u_a.1 \wedge_R \Pi_R(u_a.2 \cup \{n:u_a.2 \bullet dash n\})}$$

1.2.7 Refinement

$$\frac{}{\text{Z}} \text{fun } \langle_R - \rangle_R$$

$$\frac{}{\text{Z}} \frac{}{\langle_R - \rangle_R: REL_PREDICATE \rightarrow REL_PREDICATE} \frac{}{\forall u: REL_PREDICATE \bullet \langle_R u \rangle_R = \forall_{-R} (u.1, u)}$$

$$\frac{}{\text{Z}} \text{fun } - \sqsubseteq_R -$$

$$\begin{array}{|l} \text{Z} \\ \hline - \sqsubseteq_R \text{ :- } WF_REL_PREDICATE_PAIR \rightarrow REL_PREDICATE \\ \hline \forall \text{ pair_}u\text{:}WF_REL_PREDICATE_PAIR \\ \bullet \text{ pair_}u.1 \sqsubseteq_R \text{ pair_}u.2 = \\ \langle_R (\text{pair_}u.2 \Rightarrow_R \text{ pair_}u.1) \rangle_R \end{array}$$

1.2.8 The Complete Lattice

$$\begin{array}{|l} \text{Z} \\ \hline \mathbf{monotonic} : \mathbb{P} (REL_PREDICATE \rightarrow REL_PREDICATE) \\ \hline \text{monotonic} = \{f\text{:}REL_PREDICATE \rightarrow REL_PREDICATE \\ \mid (\forall u1, u2 : REL_PREDICATE \\ \bullet (u1 \sqsubseteq_R u2) \Rightarrow_R (f(u1) \sqsubseteq_R f(u2)) = \\ True_R \emptyset) \} \end{array}$$

$$\begin{array}{|l} \text{Z} \\ \hline \mathbf{Bot}_R : ALPHABET \rightarrow REL_PREDICATE \\ \hline \forall a\text{:}ALPHABET \bullet Bot_R a = True_R a \end{array}$$

$$\begin{array}{|l} \text{Z} \\ \hline \mathbf{Top}_R : ALPHABET \rightarrow REL_PREDICATE \\ \hline \forall a\text{:}ALPHABET \bullet Top_R a = False_R a \end{array}$$

Greatest lower bound .

$$\begin{array}{|l} \text{Z} \\ \hline \bigcap_R : WF_Glb_{R_LubR} \rightarrow REL_PREDICATE \\ \hline \forall a_us\text{:} WF_Glb_{R_LubR}; u\text{:}REL_PREDICATE \\ \bullet (u \sqsubseteq_R (\bigcap_R a_us) = True_R \emptyset) \Leftrightarrow \\ (u.1 = a_us.1 \\ \wedge (\forall u1\text{:}a_us.2 \bullet (u \sqsubseteq_R u1) = True_R \emptyset)) \end{array}$$

Least upper bound .

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{\bigcup}_R : WF_Glb_{R_LubR} \rightarrow REL_PREDICATE \\
\hline
\forall a_us: WF_Glb_{R_LubR}; u:REL_PREDICATE \\
\bullet ((\mathbf{\bigcup}_R a_us \sqsubseteq_R u) = True_R \emptyset) \Leftrightarrow \\
(u.1 = a_us.1 \\
\wedge (\forall u1:a_us.2 \bullet (u1 \sqsubseteq_R u) = True_R \emptyset))
\end{array}$$

Recursion .

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{REL_FUNCTION} \triangleq \\
\{f:REL_PREDICATE \rightarrow REL_PREDICATE \\
| (\exists a:ALPHABET \bullet \\
(\forall u_dom:dom\ f; u_ran:ran\ f \\
\bullet a = u_dom.1 = u_ran.1)))\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{\mu}_R : REL_FUNCTION \rightarrow REL_PREDICATE \\
\hline
\forall f: REL_FUNCTION \\
\bullet (\exists a:ALPHABET \\
| (\forall u_dom:dom\ f \bullet a = u_dom.1) \\
\bullet \mu_R f = \bigcap_R(a, \{u:REL_PREDICATE \\
| a = u.1 \\
\wedge f(u) \sqsubseteq_R u = True_R \emptyset\}))
\end{array}$$

In order to get a simpler definition of the greatest fixed point, we make the following calculation.

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{\nu}_R : REL_FUNCTION \rightarrow REL_PREDICATE \\
\hline
\end{array}$$

$\forall f: REL_FUNCTION$
 $\bullet (\exists a: ALPHABET$
 $\quad | (\forall u_dom: dom\ f \bullet a = u_dom.1)$
 $\bullet \nu_R f = \cup_R(a, \{u: REL_PREDICATE$
 $\quad | a = u.1$
 $\quad \wedge u \sqsubseteq_R f(u) = True_R \emptyset\}))$

\mathcal{Z}
 $\mathbf{varT}_R: WF_VarT_{R_EndTR} \rightarrow REL_PREDICATE$

$\forall a_n_t: WF_VarT_{R_EndTR}$
 $\bullet varT_R\ a_n_t = \cap_R(a_n_t.1 \setminus \{a_n_t.2\},$
 $\quad \{v: VALUE$
 $\quad | v \in ((Set^\sim)a_n_t.3)$
 $\bullet var_R\ (a_n_t.1, a_n_t.2) ;_R$
 $\quad Assign_R(a_n_t.1,$
 $\quad \langle a_n_t.2 \rangle,$
 $\quad \langle Val(v) \rangle \})$

\mathcal{Z}
 $\mathbf{endT}_R: WF_VarT_{R_EndTR} \rightarrow REL_PREDICATE$

$\forall a_n_t: WF_VarT_{R_EndTR}$
 $\bullet endT_R\ a_n_t = \cap_R(a_n_t.1 \setminus \{dash\ (a_n_t.2)\},$
 $\quad \{v: VALUE$
 $\quad | v \in ((Set^\sim)a_n_t.3)$
 $\bullet Assign_R(a_n_t.1, \langle a_n_t.2 \rangle,$
 $\quad \langle Val(v) \rangle) ;_R$
 $\quad end_R\ (a_n_t.1, a_n_t.2)\})$

2 UTP Theory for the Observational Variable *okay*

2.1 Creating the Theory

SML

```
open_theory "utp-rel";
force_delete_theory "utp-okay" handle Fail _ => ();
new_theory "utp-okay";
open_theory "utp-okay";
set_flags[("z-type-check-only",false),("z-use-axioms",true),
          ("standard-z-paras",false),("standard-z-terms",false)];
```

2.2 Definitions

Z

okay : NAME

okay ∈ undashed

Z

$(\forall b: \text{BINDING} \mid \text{okay} \in \text{dom } b \bullet b \text{ okay} \in \text{BOOL_VAL})$
 $\wedge (\forall b: \text{BINDING} \mid \text{dash okay} \in \text{dom } b \bullet b (\text{dash okay}) \in \text{BOOL_VAL})$

Z

ALPHABET_OKAY $\triangleq \{\text{okay}, \text{dash okay}\}$

Z

unrestOKAY, unrestOKAY' : REL_PREDICATE $\rightarrow \mathbb{B}$

$\forall r: \text{REL_PREDICATE} \bullet$
 $\text{unrestOKAY}(r) = \text{unrestTypedVar}(r, \text{okay}, \text{BOOL_VAL})$
 $\wedge \text{unrestOKAY}'(r) = \text{unrestTypedVar}(r, \text{dash okay}, \text{BOOL_VAL})$

$$\begin{array}{c}
z \\
| \\
\textbf{OKAY} : REL_PREDICATE \\
\hline
OKAY = =_R(ALPHABET_OKAY, okay, Val(Bool(true)))
\end{array}$$

$$\begin{array}{c}
z \\
| \\
\textbf{OKAY}' : REL_PREDICATE \\
\hline
OKAY' = =_R(ALPHABET_OKAY, dash\ okay, Val(Bool(true)))
\end{array}$$

2.2.1 Substitutions

$$\begin{array}{c}
z \\
| \\
fun - \sigma_f
\end{array}$$

$$\begin{array}{c}
z \\
| \\
- \sigma_f : REL_PREDICATE \leftrightarrow REL_PREDICATE \\
\hline
\forall d:REL_PREDICATE \\
| \quad dash\ okay \in d.1 \\
| \quad \bullet \quad d\ \sigma_f = /_R(d, Val(Bool(false)), dash\ okay)
\end{array}$$

$$\begin{array}{c}
z \\
| \\
fun - \sigma_t
\end{array}$$

$$\begin{array}{c}
z \\
| \\
- \sigma_t : REL_PREDICATE \rightarrow REL_PREDICATE \\
\hline
\forall d:REL_PREDICATE \\
| \quad dash\ okay \in d.1 \\
| \quad \bullet \quad d\ \sigma_t = /_R(d, Val(Bool(true)), dash\ okay)
\end{array}$$

3 UTP Designs Theory

3.1 Creating the Theory

SML

```

open_theory "utp-okay";
force_delete_theory "utp-des" handle Fail _ => ();
new_theory "utp-des";
open_theory "utp-des";
set_flags[("z-type-check-only",false),("z-use-axioms",true),
          ("standard-z-paras",false),("standard-z-terms",false)];
(* Setting PP to accept the refinement symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["refinedby"],
      Value "⊆",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ();
;

```

3.2 Definitions

Z

ALPHABET-DES $\triangleq \{a:ALPHABET \mid ALPHABET-OKAY \subseteq a\}$

Z

DES-PREDICATE $\triangleq \{u:REL-PREDICATE \mid u.1 \in ALPHABET-DES\}$

3.2.1 Syntactic Restrictions

Z

WF-DES-PREDICATE-PAIR \triangleq
 $\{d1,d2:DES-PREDICATE \mid d1.1 = d2.1\}$

Z

WF-DESIGN-PAIR $\triangleq \{d1,d2:WF-DES-PREDICATE-PAIR$

$$\mid d1.1.1 = d2.1.1\}$$

$$\stackrel{z}{\mid} \mathbf{WF_Assign}_D \triangleq \{aa:WF_Assign_R \mid aa.1 \in ALPHABET_DES\}$$

$$\stackrel{z}{\mid} \mathbf{WF_Skip}_D \triangleq \{a:ALPHABET_DES \mid a \in WF_Skip_R\}$$

$$\stackrel{z}{\mid} \mathbf{DES_FUNCTION} \triangleq \{f:WF_DES_PREDICATE_PAIR \leftrightarrow DES_PREDICATE \mid (\exists a:ALPHABET \bullet (\forall pair:dom\ f; u_ran:ran\ f \bullet a = pair.1.1 = u_ran.1)))\}$$

3.2.2 Designs Constructs

$$\stackrel{z}{\mid} fun\ _ \vdash_D _$$

$$\stackrel{z}{\mid} _ \vdash_D _ : WF_DES_PREDICATE_PAIR \rightarrow REL_PREDICATE$$

$$\stackrel{z}{\mid} \forall d:WF_DES_PREDICATE_PAIR$$

$$\bullet d.1 \vdash_D d.2 =$$

$$(\text{=}_R(d.1.1, okay, Val(Bool(true))))$$

$$\wedge_R d.1) \Rightarrow_R$$

$$(\text{=}_R(d.1.1, dash(okay), Val(Bool(true))))$$

$$\wedge_R d.2)$$

$$\stackrel{z}{\mid} \mathbf{DESIGN} \triangleq ran\ (_ \vdash_D _)$$

$$\stackrel{z}{\mid} \mathbf{\Pi}_D : WF_Skip_D \rightarrow REL_PREDICATE$$

$$\frac{}{\forall a:WF_Skip_D \bullet \Pi_D a = True_R a \vdash_D (\Pi_R a)}$$

$$\frac{}{\mathbf{Assign}_D : WF_Assign_D \rightarrow REL_PREDICATE}$$

$$\frac{}{\forall aa:WF_Assign_D \bullet Assign_D(aa) = True_R aa.1 \vdash_D (Assign_R(aa))}$$

$$\frac{}{\mathbf{Top}_D : ALPHABET_DES \rightarrow DES_PREDICATE}$$

$$\frac{}{\forall a:ALPHABET_DES \bullet Top_D(a) = =_R(a, okay, Val(Bool(false)))}$$

$$\frac{}{\mathbf{Bot}_D : ALPHABET_DES \rightarrow DES_PREDICATE}$$

$$\frac{}{\forall a:ALPHABET_DES \bullet Bot_D(a) = True_R a}$$

3.3 Healthiness Conditions

$$\frac{}{\mathbf{H1} : REL_PREDICATE \rightarrow REL_PREDICATE}$$

$$\frac{}{\forall d:REL_PREDICATE \bullet H1\ d = (=_R(\{okay\}, okay, Val(Bool(true)))) \Rightarrow_R d}$$

$$\frac{}{\mathbf{H1_healthy} : \mathbb{P}\ REL_PREDICATE}$$

$$\frac{}{H1_healthy = \{d:REL_PREDICATE \mid H1\ d = d\}}$$

z

H2_healthy : \mathbb{P} *REL-PREDICATE*

H2_healthy =
 $\{d:\text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1 \wedge$
 $\langle_R (/_R (d, \text{Val}(\text{Bool}(\text{false})), \text{dash okay}))$
 $\Rightarrow_R (/_R (d, \text{Val}(\text{Bool}(\text{true})), \text{dash okay})) \rangle_R$
 $= \text{True}_R \emptyset\}$

z

J : *ALPHABET* \leftrightarrow *DES-PREDICATE*

$\forall a':\text{ALPHABET}$
 $\mid a' \subseteq \text{dashed} \wedge \text{dash okay} \in a'$
 $\bullet (\exists a:\text{ALPHABET}$
 $\mid a \subseteq \text{undashed}$
 $\wedge a' = \text{dash } \langle a \rangle$
 $\bullet J \ a' = ((=_R(a \cup a', \text{okay}, \text{Val}(\text{Bool}(\text{true})))) \Rightarrow_R$
 $(=_R(a \cup a', \text{dash okay},$
 $\text{Val}(\text{Bool}(\text{true})))))$
 $\wedge_R (\Pi_R ((a \cup a') \setminus \text{ALPHABET_OKAY})))$

z

H2 : *REL-PREDICATE* \leftrightarrow *REL-PREDICATE*

$\forall d:\text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1$
 $\bullet H2 \ d = (d ;_R (J \ (\text{out_a } d.1)))$

z

H2-J_healthy : \mathbb{P} *REL-PREDICATE*

$$| H2_J_healthy = \{d:REL_PREDICATE \mid dash_okay \in d.1 \wedge H2\ d = d\}$$

^z

$$| \mathbf{H3} : REL_PREDICATE \rightarrow REL_PREDICATE$$

$$| \forall d:REL_PREDICATE; a:ALPHABET$$

$$| \quad | a \in WF_Skip_R$$

$$| \quad \wedge (d, (\Pi_R\ a)) \in WF_Semi_R$$

$$| \quad \bullet H3\ d = (d\ ;_R\ (\Pi_R\ a))$$

^z

$$| \mathbf{H3_healthy} : \mathbb{P}\ REL_PREDICATE$$

$$| H3_healthy = \{d:REL_PREDICATE \mid H3\ d = d\}$$

^z

$$| \mathbf{H4_healthy} : \mathbb{P}\ REL_PREDICATE$$

$$| H4_healthy =$$

$$| \quad \{d:REL_PREDICATE \mid$$

$$| \quad (\forall a:ALPHABET \mid (d, True_R\ a) \in WF_Semi_R$$

$$| \quad \bullet True_R\ a\ ;_R\ d = True_R\ a)\}$$

4 UTP Theory for the Observational Variables *wait*, *trace*, and *ref*

4.1 Creating the Theory

SML

```
| open_theory "utp-rel";  
| force_delete_theory "utp-wtr" handle Fail _ => ();  
| new_theory "utp-wtr";  
| open_theory "utp-wtr";  
| set_flags[("z-type-check-only",false),("z-use-axioms",true),  
|           ("standard-z-paras",false),("standard-z-terms",false)];
```

4.2 Definitions

Z

| ***wait*** : NAME

| *wait* ∈ undashed

Z

| $(\forall b: \text{BINDING} \mid \text{wait} \in \text{dom } b \bullet b \text{ wait} \in \text{BOOL_VAL})$
| $\wedge (\forall b: \text{BINDING} \mid \text{dash wait} \in \text{dom } b \bullet b (\text{dash wait}) \in \text{BOOL_VAL})$

Z

| ***ALPHABET_WAIT*** $\triangleq \{\text{wait}, \text{dash wait}\}$

Z

| ***WAIT*** : REL_PREDICATE

| $\text{WAIT} = =_R(\{\text{wait}\}, \text{wait}, \text{Val}(\text{Bool}(\text{true})))$

Z

| ***WAIT'*** : REL_PREDICATE

	$WAIT' = =_R(\{dash\ wait\}, dash\ wait, Val(Bool(true)))$
z	$tr : NAME$
	$tr \in undashed$
z	$(\forall b: BINDING \mid tr \in dom\ b \bullet b\ tr \in SEQ_EVENT_VAL)$ $\wedge (\forall b: BINDING \mid dash\ tr \in dom\ b \bullet b\ (dash\ tr) \in SEQ_EVENT_VAL)$
z	$ALPHABET_TR \triangleq \{tr, dash\ tr\}$
z	$ref : NAME$
	$ref \in undashed$
z	$(\forall b: BINDING \mid ref \in dom\ b \bullet b\ ref \in SET_EVENT_VAL)$ $\wedge (\forall b: BINDING \mid dash\ ref \in dom\ b \bullet b\ (dash\ ref) \in SET_EVENT_VAL)$
z	$ALPHABET_REF \triangleq \{ref, dash\ ref\}$
z	$unrestWAIT, unrestWAIT' : REL_PREDICATE \rightarrow \mathbb{B}$
	$\forall r: REL_PREDICATE \bullet$ $unrestWAIT(r) = unrestTypedVar(r, wait, BOOL_VAL)$ $\wedge unrestWAIT'(r) = unrestTypedVar(r, dash\ wait, BOOL_VAL)$

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{unrestTR}, \mathbf{unrestTR}' : REL_PREDICATE \rightarrow \mathbb{B} \\
| \quad \hline
| \quad \forall r:REL_PREDICATE \bullet \\
| \quad \quad \mathbf{unrestTR}(r) = \mathbf{unrestTypedVar}(r, tr, SEQ_EVENT_VAL) \\
| \quad \quad \wedge \mathbf{unrestTR}'(r) = \mathbf{unrestTypedVar}(r, \text{dash } tr, SEQ_EVENT_VAL)
\end{array}$$

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{unrestREF}, \mathbf{unrestREF}' : REL_PREDICATE \rightarrow \mathbb{B} \\
| \quad \hline
| \quad \forall r:REL_PREDICATE \bullet \\
| \quad \quad \mathbf{unrestREF}(r) = \mathbf{unrestTypedVar}(r, ref, SET_EVENT_VAL) \\
| \quad \quad \wedge \mathbf{unrestREF}'(r) = \\
| \quad \quad \quad \mathbf{unrestTypedVar}(r, \text{dash } ref, SET_EVENT_VAL)
\end{array}$$

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{wait} \neq tr \wedge \mathbf{wait} \neq ref
\end{array}$$

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{tr} \neq \mathbf{wait} \wedge \mathbf{tr} \neq \mathbf{ref}
\end{array}$$

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{ref} \neq \mathbf{tr} \wedge \mathbf{ref} \neq \mathbf{wait}
\end{array}$$

4.3 Substitution

$$\begin{array}{l}
\text{Z} \\
| \quad \mathbf{fun} _ \omega_f
\end{array}$$

$$\begin{array}{l}
\text{Z} \\
| \quad _ \omega_f : REL_PREDICATE \leftrightarrow REL_PREDICATE \\
| \quad \hline
| \quad \forall r:REL_PREDICATE \\
| \quad \quad | \quad \mathbf{wait} \in r.1
\end{array}$$

$$\begin{array}{|l} \bullet \ r \ \omega_f = /_R(r, Val(Bool(false)), \ wait) \end{array}$$

$$\begin{array}{|l} \text{z} \\ fun \ - \ \omega_t \end{array}$$

$$\begin{array}{|l} \text{z} \\ _ \ \omega_t : REL_PREDICATE \rightarrow REL_PREDICATE \\ \hline \forall \ r:REL_PREDICATE \\ \quad | \ \ wait \in \ r.1 \\ \bullet \ r \ \omega_t = /_R(r, Val(Bool(true)), \ wait) \end{array}$$

SML

```

| open_theory "utp-okay";
| force_delete_theory "utp-rea" handle Fail _ => ();
| new_theory "utp-rea";
| open_theory "utp-rea";
| new_parent "utp-wtr";
| set_flags[("z-type-check-only",false),("z-use-axioms",true),
|           ("standard-z-paras",false),("standard-z-terms",false)];

```

4.4 Definitions

Z

```

| ALPHABET_OWTR  $\hat{=}$ 
|   ALPHABET_OKAY  $\cup$  ALPHABET_WAIT  $\cup$  ALPHABET_TR
|    $\cup$  ALPHABET_REF

```

Z

```

| ALPHABET_REA  $\hat{=}$   $\{a:ALPHABET \mid ALPHABET_OWTR \subseteq a\}$ 

```

Z

```

| REA_PREDICATE  $\hat{=}$   $\{d:REL\_PREDICATE \mid d.1 \in ALPHABET\_REA\}$ 

```

Z

```

|  $okay \neq wait \wedge okay \neq tr \wedge wait \neq ref$ 

```

4.4.1 Syntactic Restrictions

Z

```

| WF_SkipREA  $\hat{=}$   $\{a:ALPHABET\_REA \mid a \in homogeneous\}$ 

```

4.4.2 Reactive Constructs

Z

```

|  $\Pi_{REA}$ :  $WF\_Skip_{REA} \rightarrow REA\_PREDICATE$ 
| _____

```

$$\begin{array}{|l}
\forall a: WF_Skip_{REA} \\
\bullet \Pi_{REA} a = \\
\quad ((=_R(a, okay, Val(Bool(false)))) \\
\quad \quad \wedge_R (=_{+R}(ALPHABET_OWTR, \\
\quad \quad \quad Rel((- \leq_R -), Var(tr), \\
\quad \quad \quad \quad Var(dash \ tr)), \\
\quad \quad \quad \quad Val(Bool(true))))) \\
\quad \vee_R ((=_R(a, dash \ okay, Val(Bool(true)))) \\
\quad \quad \wedge_R (=_R(a, dash \ tr, Var(tr))) \\
\quad \quad \wedge_R (=_R(a, dash \ wait, Var(wait))) \\
\quad \quad \wedge_R (=_R(a, dash \ ref, Var(ref))))
\end{array}$$

4.5 Healthiness Conditions

$$\begin{array}{|l}
^Z \\
\mathbf{R1} : REL_PREDICATE \rightarrow REL_PREDICATE
\end{array}$$

$$\begin{array}{|l}
\forall r: REL_PREDICATE \\
\bullet R1 \ r = r \wedge_R (=_{+R}(ALPHABET_OWTR, \\
\quad Rel((- \leq_R -), Var(tr), \\
\quad \quad Var(dash \ tr)), \\
\quad \quad Val(Bool(true))))
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mathbf{R1_healthy} : \mathbb{P} \ REL_PREDICATE
\end{array}$$

$$R1_healthy = \{r: REL_PREDICATE \mid r = R1 \ r\}$$

$$\begin{array}{|l}
^Z \\
\mathbf{R2} : REL_PREDICATE \leftrightarrow REL_PREDICATE
\end{array}$$

$$\begin{array}{|l}
\forall r: REL_PREDICATE \\
\mid ALPHABET_OWTR \subseteq r.1
\end{array}$$

$$\bullet R2 \ r = \quad /_R ((/_R(r, Val(Seq(\langle \rangle)), tr)), \\ Fun_2((- SeqDif_R -), \\ Var(dash \ tr), Var(tr), dash \ tr)$$

z

$$\mathbf{R2_healthy} : \mathbb{P} \ REL_PREDICATE$$

$$R2_healthy = \{r:REL_PREDICATE \mid r = R2 \ r\}$$

z

$$\mathbf{R3} : REA_PREDICATE \leftrightarrow REA_PREDICATE$$

$$\forall r:REA_PREDICATE \\ \mid r.1 \in WF_Skip_{REA} \\ \bullet R3 \ r = \quad (\Pi_{REA} \ r.1) \\ \quad \triangleleft_R (=_R(\{wait\}, wait, Val(Bool(true)))) \\ \quad \triangleright_R \ r$$

z

$$\mathbf{R3_healthy} : \mathbb{P} \ REA_PREDICATE$$

$$R3_healthy = \{r:REA_PREDICATE \mid r.1 \in WF_Skip_{REA} \wedge r = R3 \ r\}$$

z

$$\mathbf{R} : REA_PREDICATE \leftrightarrow REA_PREDICATE$$

$$\forall r:REA_PREDICATE \mid r.1 \in WF_Skip_{REA} \bullet R \ r = R1(R2(R3(r)))$$

z

$$\mathbf{R_healthy} : \mathbb{P} \ REA_PREDICATE$$

$$R_healthy = \{r:REA_PREDICATE \mid r.1 \in WF_Skip_{REA} \wedge r = R \ r\}$$

z

$$\mathbf{REA_PROCESS} \triangleq \{r:REA_PREDICATE \mid r \in R_healthy\}$$

5 UTP CSP Theory

5.1 Creating the Theory

SML

```
open_theory "utp-des";
force_delete_theory "utp-csp" handle Fail _ => ();
new_theory "utp-csp";
open_theory "utp-csp";
new_parent "utp-rea";
set_flags[("z-type-check-only",false),("z-use-axioms",true),
          ("standard-z-paras",false),("standard-z-terms",false)];

(* Setting PP to accept the refinement symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["refinedby"],
      Value "⊑",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ();
;

(* Setting PP to accept the equivalence symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["equivalentto"],
      Value "≡",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ();
;
```

5.2 Healthiness Conditions

^z

unrest***ALPHABET_CSP*** : *REL_PREDICATE* → \mathbb{B}

$\forall r:REL_PREDICATE \bullet$	$unrestALPHABET_CSP(r) \Leftrightarrow$ $(unrestOKAY(r) \wedge unrestWAIT(r)$ $\wedge unrestTR(r) \wedge unrestREF(r))$
\mathbb{Z}	$unrestALPHABET_CSP' : REL_PREDICATE \rightarrow \mathbb{B}$
$\forall r:REL_PREDICATE \bullet$	$unrestALPHABET_CSP'(r) \Leftrightarrow$ $(unrestOKAY'(r) \wedge unrestWAIT'(r)$ $\wedge unrestTR'(r) \wedge unrestREF'(r))$
\mathbb{Z}	$\mathbf{VAR_NAME} \triangleq \{n:NAME \mid n \notin ALPHABET_OWTR \wedge n \in undashed\}$
\mathbb{Z}	$\mathbf{CSP1} : REL_PREDICATE \rightarrow REL_PREDICATE$
$\forall r:REL_PREDICATE \bullet$	$CSP1 \ r = r \vee_R ($ $(=_R(ALPHABET_OWTR, okay, Val(Bool(false))))$ $\wedge_R (=_{+R}(ALPHABET_OWTR,$ $Rel((- \leq_R -), Var(tr),$ $Var(dash \ tr)),$ $Val(Bool(true)))))$
\mathbb{Z}	$\mathbf{CSP1_healthy} : \mathbb{P} \ REL_PREDICATE$
	$CSP1_healthy = \{r:REL_PREDICATE \mid r = CSP1 \ r\}$

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{CSP2} : REL_PREDICATE \leftrightarrow REL_PREDICATE \\
\hline
\forall r:REL_PREDICATE \\
| \{dash(okay), dash(wait), dash(tr), dash(ref)\} \subseteq r.1 \\
| \bullet \text{CSP2 } r = r ;_R J(out_a \ r.1)
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{CSP2_healthy} : \mathbb{P} \ REL_PREDICATE \\
\hline
\text{CSP2_healthy} = \\
| \{r:REL_PREDICATE \\
| \{dash(okay), dash(wait), dash(tr), dash(ref)\} \subseteq r.1 \\
| \wedge r = \text{CSP2 } r\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{CSP_PROCESS} \triangleq \{p:REA_PROCESS \mid ALPHABET_OWTR \subseteq p.1 \\
| \wedge p \in \text{CSP1_healthy} \\
| \wedge p \in \text{CSP2_healthy}\}
\end{array}$$

5.3 Definitions

5.3.1 Syntactic Restrictions

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{WF_CSP_PROCESS_PAIR} \triangleq \{p1, p2: \text{CSP_PROCESS} \mid p1.1 = p2.1\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{WF_PREFIXING} \triangleq \\
| \{a:ALPHABET_REA; n:VAR_NAME; e:EXPRESSION \\
| a \in WF_Skip_{REA} \wedge FV \ e \subseteq a\}
\end{array}$$

5.3.2 CSP Constructs

$$\begin{array}{|l}
\text{Z} \\
\hline
\mathbf{STOP} : WF_Skip_{REA} \rightarrow \text{CSP_PROCESS}
\end{array}$$

$\forall a:WF_Skip_{REA} \bullet$
 $STOP\ a = R(Assign_R(a, \langle wait \rangle, \langle Val(Bool(true)) \rangle))$

SML

val CSP_STOP_design_thm =
 $new_axiom\ ([\"CSP_STOP_design_thm\"],$
 $\quad \sqsubset \forall a:ALPHABET_REA$
 $\quad \mid a \in WF_Skip_{REA}$
 $\quad \bullet STOP\ a$
 $\quad = R\ (True_R\ a$
 $\quad \quad \vdash_D\ ((=_R(a, dash\ tr,$
 $\quad \quad \quad Var(tr)))$
 $\quad \quad \wedge_R\ (=_R(a,$
 $\quad \quad \quad dash\ wait,$
 $\quad \quad \quad Val(Bool(true))))))\ ^\top);$

Z

SKIP : $WF_Skip_{REA} \rightarrow CSP_PROCESS$

$\forall a:WF_Skip_{REA} \bullet$
 $SKIP\ a = R\ (\exists_R(\{ref\}, \Pi_{REA}\ a))$

SML

val CSP_SKIP_design_thm =
 $new_axiom\ ([\"CSP_SKIP_design_thm\"],$
 $\quad \sqsubset \forall a:ALPHABET_REA$
 $\quad \mid a \in WF_Skip_{REA}$
 $\quad \bullet SKIP\ a$
 $\quad = R(True_R\ a$
 $\quad \quad \vdash_D\ ((=_R(a, dash\ tr,$
 $\quad \quad \quad Var(tr)))$

$$\wedge_R (=_R(a, \text{dash wait}, \text{Val}(\text{Bool}(\text{false})))))) \quad \neg);$$

Z

$$\mathbf{CHAOS} : WF_Skip_{REA} \rightarrow CSP_PROCESS$$

$$\forall a:WF_Skip_{REA} \bullet \\ \mathbf{CHAOS} \ a = R(\text{True}_R \ a)$$

SML

$$\text{val } \mathbf{CSP_CHAOS_design_thm} = \\ \text{new_axiom } ([\text{"CSP_CHAOS_design_thm"}], \\ \begin{array}{l} \sqsubseteq \forall a:ALPHABET_REA \\ \quad | \ a \in WF_Skip_{REA} \\ \bullet \ \mathbf{CHAOS} \ a = R(\text{False}_R \ a \vdash_D \\ \quad \text{True}_R \ a) \neg); \end{array}$$

Z

$$\text{fun } _ \boxtimes_{CSP} _$$

Z

$$_ \boxtimes_{CSP} _ : WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS$$

$$\forall pp:WF_CSP_PROCESS_PAIR \\ \bullet \ pp.1 \boxtimes_{CSP} pp.2 \\ = CSP2((pp.1 \wedge_R pp.2) \triangleleft_R STOP \ (pp.1).1 \triangleright_R (pp.1 \vee_R pp.2))$$

SML

$$\text{val } \mathbf{CSP_}\boxtimes_{CSP}\mathbf{_design_thm_2} = \\ \text{new_axiom } ([\text{"CSP_}\boxtimes_{CSP}\mathbf{_design_thm_2"}], \\ \sqsubseteq \forall p1,p2:CSP_PROCESS$$

$$\begin{array}{l}
| (p1, p2) \in WF_CSP_PROCESS_PAIR \\
\bullet p1 \boxtimes_{CSP} p2 \\
= R(\\
(\neg_R (p1 \ \sigma_f \ \omega_f) \wedge_R \neg_R (p2 \ \sigma_f \ \omega_f)) \\
\vdash_D \\
((p1 \ \sigma_t \ \omega_f) \wedge_R (p2 \ \sigma_t \ \omega_f)) \\
\triangleleft_R ((=_{+R}(p1.1, Var(dash \ tr), Var(tr))) \\
\wedge_R (=_{+R}(p1.1, dash \ wait, Val(Bool(true))))) \\
\triangleright_R \\
((p1 \ \sigma_t \ \omega_f) \vee_R (p2 \ \sigma_t \ \omega_f)) \) \\
)\neg);
\end{array}$$

^z

$$\Phi: CSP_PROCESS \rightarrow CSP_PROCESS$$

$$\begin{array}{l}
\forall p: CSP_PROCESS \\
\bullet \Phi \ p = R(\ p \wedge_R (((=_{+R}(ALPHABET_OWTR, dash \ tr, \ Var(tr))) \\
\wedge_R (=_{+R}(ALPHABET_OWTR, dash \ wait, \\
Val(Bool(true))))) \\
\vee_R (=_{+R}(ALPHABET_OWTR, \\
Rel((- <_R -), Var(tr), \\
Var(dash \ tr)), \\
Val(Bool(true))))) \)
\end{array}$$

^z

$$fun \ \mathbf{do_A} \ -$$

^z

$$\mathbf{do_A} \ - : WF_PREFIXING \rightarrow CSP_PROCESS$$

$$\begin{array}{l}
\forall a_n_e: WF_PREFIXING \\
\bullet do_A \ a_n_e = \Phi(((=_{+R}(a_n_e.1, dash \ tr, \ Var(tr))) \\
\wedge_R (=_{+R}(a_n_e.1,
\end{array}$$

$$\begin{array}{c}
\text{Rel}((- \not\in_R -), \\
\text{Fun}_2(\text{MkPair}, \\
\text{Val}(\text{Channel}(a_n_e.2)), a_n_e.3), \\
\text{Var}(\text{dash ref}), \\
\text{Val}(\text{Bool}(\text{true})))) \\
\triangleleft_R \\
(=_R(a_n_e.1, \text{dash wait}, \text{Val}(\text{Bool}(\text{true})))) \\
\triangleright_R \\
(=_R(a_n_e.1, \text{dash tr}, \text{Fun}_2((- \frown_R -), \\
\text{Var}(\text{tr}), \\
\text{Fun}_1(\text{MkSingleton}, \\
\text{Fun}_2(\text{MkPair}, \\
\text{Val}(\text{Channel}(a_n_e.2)), a_n_e.3) \\
)))))
\end{array}$$

$$\begin{array}{c}
\text{Z} \\
\mid \text{fun } - \rightarrow_{\mathbf{CSP}} -
\end{array}$$

$$\begin{array}{c}
\text{Z} \\
\mid - \rightarrow_{\mathbf{CSP}} \text{ :- } (WF_PREFIXING \times CSP_PROCESS) \rightarrow \\
\text{CSP_PROCESS} \\
\hline
\forall a_n_e: WF_PREFIXING \\
\bullet a_n_e \rightarrow_{\mathbf{CSP}} SKIP \ a_n_e.1 = \\
\text{CSP2}((=_R(a_n_e.1, \text{dash okay}, \text{Val}(\text{Bool}(\text{true})))) \\
\wedge_R do_A(a_n_e)) \\
\wedge (\forall p: CSP_PROCESS \bullet a_n_e \rightarrow_{\mathbf{CSP}} p = (a_n_e \rightarrow_{\mathbf{CSP}} SKIP
\end{array}$$

$$\begin{array}{c}
\text{Z} \\
\mid \text{fun } \mathbf{do_C} \ -
\end{array}$$

$$\begin{array}{c}
\text{Z} \\
\mid \mathbf{do_C} \ - : WF_PREFIXING \rightarrow CSP_PROCESS
\end{array}$$

$\forall a_n_e: WF_PREFIXING$

- $do_C\ a_n_e = ((=_R(a_n_e.1, dash\ tr, Var(tr)))$
 $\wedge_R (=_{+R}(a_n_e.1,$
 $Rel((- \not\in_R -),$
 $Fun_2(MkPair,$
 $Val(Channel(a_n_e.2)),$
 $a_n_e.3),$
 $Var(dash\ ref)),$
 $Val(Bool(true))))))$
 \triangleleft_R
 $(=_R(a_n_e.1, dash\ wait, Val(Bool(true))))$
 \triangleright_R
 $(=_R(a_n_e.1,$
 $dash\ tr,$
 $Fun_2((- \frown_R -),$
 $Var(tr),$
 $Fun_1(MkSingleton,$
 $Fun_2(MkPair,$
 $Val(Channel(a_n_e.2)),$
 $a_n_e.3)$
 $))))$

SML

```
val CSP  $\rightarrow_{CSP}$  design_thm =
  new_axiom ([ "CSP  $\rightarrow_{CSP}$  design_thm" ],
    [  $\forall a\_n\_e: WF\_PREFIXING$ 
      •  $a\_n\_e \rightarrow_{CSP} SKIP\ a\_n\_e.1 = R( True_R\ a\_n\_e.1$ 
         $\vdash_D\ do\_C(a\_n\_e))^\top$  ] );
```

5.4 Further Healthiness Conditions

Z

$CSP3_healthy : \mathbb{P}\ CSP_PROCESS$

$$\begin{array}{|l}
\hline
CSP3_healthy = \{r:CSP_PROCESS \mid \\
\quad (=_R(r.1, wait, Val(Bool(false)))) \Rightarrow_R \\
\quad (r \equiv_R \exists_R(\{ref\}, r)) = True_R \ r.1\}
\end{array}$$

$$\begin{array}{|l}
^Z \\
\hline
\mathbf{CSP4_healthy} : \mathbb{P} \ CSP_PROCESS
\end{array}$$

$$\begin{array}{|l}
\hline
CSP4_healthy = \{r:CSP_PROCESS \mid \\
\quad (r, SKIP \ r.1) \in WF_Semi_R \wedge r ;_R SKIP \ r.1 = r\}
\end{array}$$

SML

6 UTP Circus Theory

6.1 Creating the Theory

SML

```

open_theory "utp-csp";
force_delete_theory "utp-circus" handle Fail _ => ();
new_theory "utp-circus";
open_theory "utp-circus";
set_flags[("z-type-check-only",false),("z-use-axioms",true),
          ("standard-z-paras",false),("standard-z-terms",false)];
(* Setting PP to accept the refinement symbol *)
val _ =
  ReaderWriterSupport.PrettyNames.add_new_symbols
    [(["refinedby"],
      Value "⊆",
      ReaderWriterSupport.PrettyNames.Simple)]
  handle Fail _ => ();

```

6.2 Definitions

\mathbb{Z}

$\mathbf{Z_VAR_NAME} \triangleq \{n:NAME \mid n \notin ALPHABET_OWTR\}$

\mathbb{Z}

$\mathbf{VAR_DECLS} \triangleq \{vars:seq \mathbf{Z_VAR_NAME}; types:seq \mathbf{EXPRESSION} \mid dom \, vars \in (\mathbb{F} \, _) \wedge dom \, types \in (\mathbb{F} \, _) \wedge \#(ran \, vars) = \#(ran \, types) > 0\}$

\mathbb{Z}

$\mathbf{CIRCUS_PREDICATE} : \mathbb{P} \, \mathbf{REA_PREDICATE}$

$\mathbf{CIRCUS_PREDICATE} = \{c : \mathbf{REA_PREDICATE} \mid unrestALPHABET_CSP(c)\}$

$$\begin{array}{|l}
\hline
\wedge \text{ unrestALPHABET_CSP}'(c) \} \\
\\
\text{Z} \\
\hline
\textbf{CIRCUS_CONDITION} : \mathbb{P} \text{ REA_PREDICATE} \\
\hline
\text{CIRCUS_CONDITION} = \{c : \text{REA_PREDICATE} \\
\quad | \text{ unrestALPHABET_CSP}(c) \\
\quad \wedge \text{ unrestALPHABET_CSP}'(c) \\
\quad \wedge c -_R \text{ ALPHABET_OWTR} \in \text{REL_CONDITION}\}
\end{array}$$

6.2.1 Syntactic Restrictions

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_Skip}_C \triangleq \{a : \text{WF_Skip}_R \mid \text{ALPHABET_OWTR} \subseteq a \}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_Guard}_C \triangleq \{g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \mid g.1 = a.1\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_if}_{C_fiC} \triangleq \\
\quad \{g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \mid g.1 = a.1\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_Semi}_C \triangleq \{a1, a2 : \text{CSP_PROCESS} \mid (a1, a2) \in \text{WF_Semi}_R \}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_Var}_C \triangleq \\
\quad \{n : \text{VAR_NAME}; t : \text{SET_VAL}; a : \text{CSP_PROCESS} \\
\quad | n \notin \text{ALPHABET_OWTR} \wedge n \in \text{undashed} \wedge \{n, \text{dash}(n)\} \subseteq a.1\}
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\hline
\textbf{WF_Assign}_C \triangleq \\
\quad \{a : \text{ALPHABET}; ns : \text{seq VAR_NAME}; exps : \text{seq EXPRESSION} \\
\quad | (\forall n : \text{ran } ns \bullet n \in a \wedge n \in \text{undashed})\}
\end{array}$$

$$\begin{array}{|l}
\wedge (\forall e: \text{ran } \text{exprs} \bullet FV(e) \subseteq a \wedge FV(e) \subseteq \text{undashed}) \\
\wedge \#ns = \#exprs \neq 0 \\
\wedge a \in \text{homogeneous} \\
\wedge ALPHABET_OWTR \subseteq a \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_SpecStatement}_C \triangleq \\
\{ a: ALPHABET; f: \mathbb{F} \text{ VAR_NAME}; preC: CIRCUS_CONDITION; \\
\quad postC: CIRCUS_PREDICATE \\
| f \subseteq a \wedge f \subseteq \text{undashed} \wedge a \in \text{homogeneous} \\
\wedge preC.1 = postC.1 = a \\
\wedge ALPHABET_OWTR \subseteq a \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_Condition}_C \triangleq \{ a: ALPHABET; g: CIRCUS_CONDITION \\
| a \in \text{homogeneous} \wedge a = g.1 \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_SchemaExp}_C \triangleq \\
\{ decls: VAR_DECLS; p: REL_PREDICATE \\
| (p.1 \setminus ALPHABET_OWTR) = \text{ran } (decls.1) \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_param}_C \triangleq \\
\{ x: VAR_NAME; T: EXPRESSION; a: CSP_PROCESS; e: EXPRESSION \\
| x \in a.1 \wedge FV(e) \subseteq a.1 \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_val}_C \triangleq \\
\{ x: VAR_NAME; t: SET_VAL; a: CSP_PROCESS; e: EXPRESSION \\
| x \notin FV(e) \wedge x \in a.1 \wedge FV(e) \subseteq a.1 \}
\end{array}$$

$$\begin{array}{|l}
\textbf{WF_res}_C \triangleq
\end{array}$$

$$\begin{array}{|l} \{ x:VAR_NAME; t:SET_VAL; a:CSP_PROCESS; y:VAR_NAME \\ | \{x,y\} \subseteq a.1 \} \end{array}$$

$$\begin{array}{|l} \mathbf{WF_vres}_C \triangleq \\ \{ x:VAR_NAME; t:SET_VAL; a:CSP_PROCESS; y:VAR_NAME \\ | x \neq y \wedge \{x,y\} \subseteq a.1 \} \end{array}$$

$$\begin{array}{|l} \mathbf{WF_PREFIXING}_C \triangleq \\ \{ a_n_e:WF_PREFIXING; A:CSP_PROCESS \mid a_n_e.1 = A.1 \} \end{array}$$

$$\begin{array}{|l} \mathbf{WF_PREFIXING}_{CinR} \triangleq \\ \{ a:ALPHABET_REA; c:VAR_NAME; x:VAR_NAME; \\ p:CIRCUS_CONDITION; \\ A: CSP_PROCESS \mid (a = A.1) \wedge (\{x,dash(x)\} \subseteq a) \} \end{array}$$

6.2.2 Circus Constructs

$$\begin{array}{|l} \mathbf{Stop} : WF_Skip_C \rightarrow CSP_PROCESS \\ \hline \forall a:WF_Skip_C \bullet \\ Stop \ a = R(True_R \ a \vdash_D (=_R(a, \ dash \ tr, Var(tr)) \\ \wedge_R =_R(a, \\ dash \ wait, \\ Val(Bool(true))) \)) \end{array}$$

$$\begin{array}{|l} \mathbf{Skip} : WF_Skip_C \rightarrow CSP_PROCESS \\ \hline \forall a:WF_Skip_C \bullet \\ Skip \ a = R(True_R \ a \vdash_D (=_R(a, \ dash \ tr, Var(tr)) \end{array}$$

$$\begin{array}{|l}
\wedge_R =_R(a, \\
\quad \text{dash wait}, \\
\quad \text{Val}(\text{Bool}(\text{false}))) \\
\wedge_R (\Pi_R (a \backslash \text{ALPHABET_OWTR})))
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\textbf{Chaos} : \text{WF_Skip}_C \rightarrow \text{CSP_PROCESS} \\
\hline
\forall a : \text{WF_Skip}_C \bullet \text{Chaos } a = R(\text{False}_R \ a \vdash_D \ \text{True}_R \ a)
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\text{fun } - \ ;_C -
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
- \ ;_C - : \text{WF_Semi}_C \rightarrow \text{CSP_PROCESS} \\
\hline
\forall u1_u2 : \text{WF_Semi}_C \\
\quad \bullet \ u1_u2.1 \ ;_C \ u1_u2.2 = u1_u2.1 \ ;_R \ u1_u2.2
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
\text{fun } - \ \&_C -
\end{array}$$

$$\begin{array}{|l}
\text{Z} \\
- \ \&_C - : \text{WF_Guard}_C \rightarrow \text{CSP_PROCESS} \\
\hline
\forall g : \text{CIRCUS_CONDITION}; \ a : \text{CSP_PROCESS} \\
\quad | \ (g, a) \in \text{WF_Guard}_C \\
\quad \bullet \ g \ \&_C \ a = \\
\quad R(\\
\quad \quad (g \Rightarrow_R \neg_R (a \ \omega_f \ \sigma_f)) \\
\quad \quad \vdash_D \\
\quad \quad (((g \wedge_R (a \ \omega_f \ \sigma_t))) \\
\quad \quad \quad \vee_R (\neg_R g \wedge_R =_R(a.1, \text{dash } tr, \text{Var}(tr)))
\end{array}$$

$$\wedge_R =_R (a.1, \textit{dash} \textit{ wait}, \\ \textit{Val}(\textit{Bool}(\textit{true})))$$
$$\mathbb{Z} \left| \begin{array}{c} fun - \boxtimes_C - \end{array} \right.$$
$$\begin{array}{l} Z \\ \mid \\ _ \boxtimes_C _ \vdash WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS \end{array}$$
$$\begin{aligned} & \forall aa: WF_CSP_PROCESS_PAIR \\ & \bullet aa.1 \boxtimes_C aa.2 = \\ & \quad R(\\ & \quad (\neg_R (aa.1 \ \omega_f \ \sigma_f) \wedge_R \neg_R (aa.2 \ \omega_f \ \sigma_f)) \\ & \quad \vdash_D \\ & \quad (((aa.1 \ \omega_f \ \sigma_t) \wedge_R (aa.2 \ \omega_f \ \sigma_t)) \\ & \quad \triangleleft_R ((=_{+R}(aa.1.1, Var(dash \ tr), Var(tr))) \\ & \quad \quad \wedge_R \\ & \quad \quad (=_{+R}(aa.1.1, dash \ wait, Val(Bool(true))))) \\ & \quad \triangleright_R \\ & \quad ((aa.1 \ \omega_f \ \sigma_t) \vee_R (aa.2 \ \omega_f \ \sigma_t)) \) \\ & \quad) \end{aligned}$$
$$\mathbb{Z} \mid fun - \cap_C -$$
$$\begin{array}{l} \text{Z} \\ \mid \\ _ \cap_{\mathcal{C}} _ : WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS \end{array}$$
$$\begin{array}{l} \forall aa:WF_CSP_PROCESS_PAIR \\ \bullet aa.1 \cap_C aa.2 = aa.1 \vee_R aa.2 \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \text{fun } - \rightarrow_C - \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline - \rightarrow_C -: WF_PREFIXING_C \rightarrow CSP_PROCESS \\ \hline \forall a_a: WF_PREFIXING_C \\ \bullet a_a.1 \rightarrow_C a_a.2 = R(True_R(a_a.1).1 \vdash_D (do_C a_a.1) \\ \wedge_R (\Pi_R ((a_a.1).1 \setminus ALPHABET_OWTR))) ;_C a_a.2 \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \text{fun } - \rightarrow_{CSync} - \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline - \rightarrow_{CSync} -: ((ALPHABET \times VAR_NAME) \times CSP_PROCESS) \\ \rightarrow CSP_PROCESS \\ \hline \forall a: ALPHABET; c: VAR_NAME; A: CSP_PROCESS \\ \bullet (a, c) \rightarrow_{CSync} A = (a, c, Val(Sync)) \rightarrow_C A \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \text{fun } - \rightarrow_{Cout} - \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline - \rightarrow_{Cout} -: WF_PREFIXING_C \rightarrow CSP_PROCESS \\ \hline \forall a_a: WF_PREFIXING_C \\ \bullet a_a.1 \rightarrow_{Cout} a_a.2 = a_a.1 \rightarrow_C a_a.2 \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \text{var}_C: WF_Var_C \rightarrow CSP_PROCESS \\ \hline \end{array}$$

$$\begin{array}{|l}
\forall n:VAR_NAME; t:SET_VAL; a:CSP_PROCESS \\
| (n,t,a) \in WF_Var_C \\
| \bullet var_C(n,t,a) = varT_R(a.1,n,t) ;_R a ;_R \\
| \quad \quad \quad endT_R(a.1,n,t)
\end{array}$$

$$\begin{array}{|l}
\textbf{NonRef}: (VAR_NAME \times VAR_NAME \times CIRCUS_CONDITION) \rightarrow \\
| SET_EVENT_VAL
\end{array}$$

$$\begin{array}{|l}
\forall c:VAR_NAME; x:VAR_NAME; p:CIRCUS_CONDITION \\
| \bullet NonRef(c,x,p) = Set(\{v:VALUE \\
| \quad \quad \quad /_R(p,Val(v),x) = True_R p.1 \\
| \bullet Pair(Channel(c),v)\})
\end{array}$$

$$\begin{array}{|l}
\textbf{Traces}: (VAR_NAME \times VAR_NAME \times CIRCUS_CONDITION) \rightarrow \\
| SET_SEQ_EVENT_VAL
\end{array}$$

$$\begin{array}{|l}
\forall c:VAR_NAME; x:VAR_NAME; p:CIRCUS_CONDITION \\
| \bullet Traces(c,x,p) = Set(\{v:VALUE \\
| \quad \quad \quad /_R(p,Val(v),x) = True_R p.1 \\
| \bullet Seq(\langle Pair(Channel(c),v) \rangle)\})
\end{array}$$

$$\begin{array}{|l}
\textbf{do_I}: (ALPHABET_REA \times VAR_NAME \times VAR_NAME \times CIRCUS_CONDITION) \\
| \rightarrow CSP_PROCESS
\end{array}$$

$$\begin{array}{|l}
\forall a:ALPHABET_REA; c:VAR_NAME; x:VAR_NAME; p:CIRCUS_CONDITION \\
| \bullet do_I(a,c,x,p) = \\
| \quad \quad \quad ((=_R(a,dash\ tr, Var(tr))) \\
| \quad \quad \quad \wedge_R (=_{+R}(a, \\
| \quad \quad \quad \quad \quad \quad Fun_2((- \cap_R -), \\
| \quad \quad \quad \quad \quad \quad Val(NonRef(c,x,p))),
\end{array}$$

	$ \begin{aligned} & \text{Var}(\text{dash } \text{ref})), \\ & \text{Val}(\text{Set}(\emptyset)))) \text{)} \\ & \triangleleft_R \\ & (=_R(a, \text{dash } \text{wait}, \text{Val}(\text{Bool}(\text{true}))) \text{)} \\ & \triangleright_R \\ & (=_{+R} (a, \\ & \quad \text{Rel } ((- \in_R -), \\ & \quad \quad \text{Fun}_2((- \text{SeqDif}_R -), \\ & \quad \quad \quad \text{Var}(\text{dash } \text{tr}), \\ & \quad \quad \quad \text{Var}(\text{tr})), \\ & \quad \quad \text{Val}(\text{Traces}(c, x, p)) \\ & \quad), \\ & \text{Val } (\text{Bool } \text{true})) \text{)} \end{aligned} $
z	$ \text{fun } - \rightarrow_{\mathbf{CinR}} - $
z	$ \begin{aligned} - \rightarrow_{\mathbf{CinR}} & \text{ : } ((\text{ALPHABET_REA} \times \text{VAR_NAME} \times \text{VAR_NAME} \times \\ & \text{CIRCUS_CONDITION}) \times \\ & \text{CSP_PROCESS}) \rightarrow \text{CSP_PROCESS} \end{aligned} $ <hr/> <p> $\forall a:\text{ALPHABET_REA}; c:\text{VAR_NAME}; x:\text{VAR_NAME};$ $p:\text{CIRCUS_CONDITION}; A:\text{CSP_PROCESS}$ </p> <ul style="list-style-type: none"> $(a, c, x, p) \rightarrow_{\mathbf{CinR}} A =$ $\text{var}_C(x, \text{Set}(\{v:\text{VALUE}\}),$ $\quad R($ $\quad \text{True}_R \text{ } A.1$ $\quad \vdash_D$ $\quad \text{do_I}(a, c, x, p) \wedge_R \Pi_R(A.1 \setminus \{x, \text{dash } x\})$ $\quad) ;_C$ $\quad A)$

$$\begin{array}{c} \text{Z} \\ \hline \text{fun } - \rightarrow_{\text{Cin}} - \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline - \rightarrow_{\text{Cin}} - : ((\text{ALPHABET_REA} \times \text{VAR_NAME} \times \text{VAR_NAME}) \times \text{CSP_PROCESS}) \\ \rightarrow \text{CSP_PROCESS} \\ \hline \forall a:\text{ALPHABET_REA}; c:\text{VAR_NAME}; x:\text{VAR_NAME}; A:\text{CSP_PROCESS} \\ \bullet (a, c, x) \rightarrow_{\text{Cin}} A = (a, c, x, \text{True}_R A.1) \rightarrow_{\text{CinR}} A \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \mathbf{MTrInter} : (\text{SEQ_EVENT_VAL} \times \text{SEQ_EVENT_VAL}) \rightarrow \\ \text{SET_SEQ_EVENT_VAL} \\ \hline \forall s1, s2 : \text{SEQ_EVENT_VAL} \bullet \\ \text{MTrInter}(s1, s2) = \\ \text{Set}(\{s : ((\text{Seq}^\sim)s1) \parallel_Z ((\text{Seq}^\sim)s2) \bullet \text{Seq}(s)\}) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \mathbf{MTrPar} : (\text{PAIR_SEQ_EVENT_VAL} \times \text{SET_EVENT_VAL}) \rightarrow \\ \text{SET_SEQ_EVENT_VAL} \\ \hline \forall ps:\text{PAIR_SEQ_EVENT_VAL} ; cs:\text{SET_EVENT_VAL} \bullet \\ \text{MTrPar}(ps, cs) = \\ \text{Set}(\{s : ((\text{Seq}^\sim)((\text{Pair}^\sim)ps).1) \\ \parallel_Z ((\text{Set}^\sim)cs) \parallel_Z \\ ((\text{Seq}^\sim)((\text{Pair}^\sim)ps).2) \bullet \text{Seq}(s)\}) \end{array}$$

$$\begin{array}{c} \text{Z} \\ \hline \mathbf{MTrParPred} : \text{SET_EVENT_VAL} \rightarrow \text{REL_PREDICATE} \\ \hline \forall cs:\text{SET_EVENT_VAL} \bullet \end{array}$$

$$\begin{aligned}
& MTrParPred \ cs = \\
& =_{+R} (\{tr, \text{dash } tr\}, \\
& \quad Rel \ ((- \in_R -), \\
& \quad \quad Fun_2((- \text{SeqDif}_R -), \text{Var}(\text{dash } tr), \\
& \quad \quad \quad \text{Var}(tr)), \\
& \quad Fun_2(MTrPar, \\
& \quad \quad Fun_2(MkPair, \\
& \quad \quad \quad Fun_2((- \text{SeqDif}_R -), \\
& \quad \quad \quad \quad \text{Var}(\text{one } tr), \\
& \quad \quad \quad \quad \text{Var}(tr)), \\
& \quad \quad \quad Fun_2((- \text{SeqDif}_R -), \\
& \quad \quad \quad \quad \text{Var}(\text{two } tr), \\
& \quad \quad \quad \quad \text{Var}(tr))), \\
& \quad \quad Val(cs))), \\
& \quad Val \ (Bool \ true))
\end{aligned}$$

^z

MSync: $SET_EVENT_VAL \rightarrow REL_PREDICATE$

$$\begin{aligned}
& \forall \ cs:SET_EVENT_VAL \bullet \\
& \quad MSync \ cs = \\
& \quad \quad (=_{+R}(\{one \ tr, two \ tr\}, \\
& \quad \quad \quad Fun_2((- \upharpoonright_R -), \\
& \quad \quad \quad \quad \text{Var}(\text{one } tr), \\
& \quad \quad \quad \quad \text{Val}(cs)), \\
& \quad \quad \quad Fun_2((- \upharpoonright_R -), \\
& \quad \quad \quad \quad \text{Var}(\text{two } tr), \\
& \quad \quad \quad \quad \text{Val}(cs))))
\end{aligned}$$

^z

BranchesWaiting: $REL_PREDICATE$

BranchesWaiting =

$$\begin{aligned}
&=_R(\{one\ wait\}, one\ wait, Val(Bool(true))) \\
&\vee_R =_R(\{two\ wait\}, two\ wait, Val(Bool(true)))
\end{aligned}$$

z

BranchesNotWaiting: *REL-PREDICATE*

$$\begin{aligned}
BranchesWaiting &= \\
&=_R(\{one\ wait\}, one\ wait, Val(Bool(false))) \\
&\wedge_R =_R(\{two\ wait\}, two\ wait, Val(Bool(false)))
\end{aligned}$$

z

MRefPar: *SET-EVENT-VAL* \rightarrow *REL-PREDICATE*

$$\begin{aligned}
&\forall cs:SET_EVENT_VAL \bullet \\
&\quad MRefPar(cs) = \\
&\quad =_{+R} (\{dash\ ref\}, \\
&\quad \quad Rel\ ((- \subseteq_R -), \\
&\quad \quad \quad Var(dash\ ref), \\
&\quad \quad \quad Fun_2\ ((- \cup_R -), \\
&\quad \quad \quad \quad Fun_2((- \cap_R -), \\
&\quad \quad \quad \quad \quad Fun_2((- \cup_R -), \\
&\quad \quad \quad \quad \quad \quad Var(one\ ref), \\
&\quad \quad \quad \quad \quad \quad \quad Var(two\ ref) \quad), \\
&\quad \quad \quad \quad \quad \quad \quad Val(cs)), \\
&\quad \quad \quad Fun_2((- \setminus_R -), \\
&\quad \quad \quad \quad Fun_2((- \cap_R -), \\
&\quad \quad \quad \quad \quad Var(one\ ref), \\
&\quad \quad \quad \quad \quad \quad Var(two\ ref) \\
&\quad \quad \quad \quad), \\
&\quad \quad \quad \quad Val(cs)) \quad) \quad), \\
&\quad Val\ (Bool\ true))
\end{aligned}$$

z

MSt: (*ALPHABET* \times *ALPHABET* \times *ALPHABET*) \rightarrow *REL-PREDICATE*

$\forall ns1, ns2, st:ALPHABET \bullet$

$st = \emptyset \Rightarrow MSt(st, ns1, ns2) = True_R \emptyset$

$\wedge st \neq \emptyset \Rightarrow$

$(\exists n:st \bullet$

$n \in ns1 \Rightarrow MSt(st, ns1, ns2) =$
 $(=_R(\{dash\ n, one\ n\},$
 $dash\ n,$
 $Var(one\ n)))$
 $\wedge_R MSt(st \setminus \{n\},$
 $ns1, ns2)$

$\wedge n \in ns2 \Rightarrow MSt(st, ns1, ns2) =$
 $(=_R(\{dash\ n, two\ n\},$
 $dash\ n,$
 $Var(two\ n)))$
 $\wedge_R MSt(st \setminus \{n\},$
 $ns1, ns2)$

$\wedge n \notin ns1 \cup ns2 \Rightarrow MSt(st, ns1, ns2) =$
 $(=_R(\{dash\ n, n\},$
 $dash\ n,$
 $Var(n)))$
 $\wedge_R MSt(st \setminus \{n\},$
 $ns1, ns2)$

$)$

^z

MWtRefStPar: $(SET_EVENT_VAL \times ALPHABET \times ALPHABET \times ALPHABET)$
 $\rightarrow REL_PREDICATE$

$\forall ns1, ns2, st:ALPHABET; cs:SET_EVENT_VAL \bullet$

$MWtRefStPar(cs, st, ns1, ns2) =$

$BranchesWaiting \wedge_R MRefPar(cs)$

\triangleleft_R

	$ \begin{aligned} & (=_R(\{dash\ wait\}, dash\ wait, Val(Bool(true)))) \\ & \triangleright_R \\ & BranchesNotWaiting \wedge_R MSt(st, ns1, ns2) \end{aligned} $
\mathbb{Z}	$ \begin{array}{l} \mathbf{MPar}: (SET_EVENT_VAL \times ALPHABET \times ALPHABET \times ALPHABET) \rightarrow \\ REL_PREDICATE \\ \hline \forall ns1, ns2, st: ALPHABET; cs: SET_EVENT_VAL \bullet \\ MPar(cs, st, ns1, ns2) = \\ MTrParPred(cs) \\ \wedge_R MSync(cs) \\ \wedge_R MWtRefStPar(cs, st, ns1, ns2) \end{array} $
\mathbb{Z}	$ \begin{array}{l} \mathbf{DivPar}: (CSP_PROCESS \times CSP_PROCESS \times SET_EVENT_VAL) \rightarrow \\ REL_PREDICATE \\ \hline \forall a1, a2: CSP_PROCESS; cs: SET_EVENT_VAL \\ \bullet DivPar(a1, a2, cs) = \\ \exists_R(\{dash(one\ tr), dash(two\ tr)\}, \\ ((a1\ \omega_f\ \sigma_f) ;_C \\ (=_R(a1.1, dash(one\ tr), Var(tr)))) \\ \wedge_R \\ ((a2\ \omega_f) ;_C \\ (=_R(a2.1, dash(two\ tr), Var(tr)))) \\ \wedge_R MSync(cs) \\) \end{array} $
\mathbb{Z}	$ fun\ -\ \llbracket C\ - \rrbracket_C\ - $
\mathbb{Z}	$ -\ \llbracket C\ - \rrbracket_C\ -: (CSP_PROCESS \times $

$$(ALPHABET \times SET_EVENT_VAL \times ALPHABET) \\ \times CSP_PROCESS) \rightarrow CSP_PROCESS$$

$$\begin{aligned} & \forall a1, a2: CSP_PROCESS; cs: SET_EVENT_VAL; ns1, ns2: ALPHABET \\ & \quad | (a1, a2) \in WF_CSP_PROCESS_PAIR \\ & \quad \quad \wedge ns1 \cap dashed = ns2 \cap dashed = \emptyset \\ & \quad \quad \wedge ns1 \cap ns2 = \emptyset \\ & \quad \bullet a1 \llbracket_C (ns1, cs, ns2) \rrbracket_C a2 = \\ & \quad R(\\ & \quad (\neg_R (DivPar(a1, a2, cs)) \wedge_R \neg_R (DivPar(a2, a1, cs))) \\ & \quad \vdash_D \\ & \quad ((((a1 \omega_f \sigma_t) ;_C U(one, out_a a1.1)) \\ & \quad \quad \wedge_R ((a2 \omega_f \sigma_t) ;_C U(two, out_a a2.1))) \\ & \quad \quad +_R (\{tr\} \cup (a1.1 \setminus (ALPHABET_OWTR \cup dashed)))) \\ & \quad ;_C (MPar(cs, a1.1 \setminus (ALPHABET_OWTR \cup dashed), ns1, ns2))) \\ & \quad) \end{aligned}$$

z

MTrInterPred: REL-PREDICATE

$$\begin{aligned} MTrInterPred = \\ & =_{+R} (\{tr, dash\ tr\}, \\ & \quad Rel ((- \in_R -), \\ & \quad \quad Fun_2((- SeqDif_R -), Var(dash\ tr), \\ & \quad \quad \quad Var(tr)), \\ & \quad Fun_2(MTrInter, \\ & \quad \quad Fun_2((- SeqDif_R -), \\ & \quad \quad \quad Var(one\ tr), \\ & \quad \quad \quad Var(tr)), \\ & \quad \quad Fun_2((- SeqDif_R -), \\ & \quad \quad \quad Var(two\ tr), \\ & \quad \quad \quad Var(tr)))), \\ & Val (Bool\ true)) \end{aligned}$$

z

MRefInter: *REL_PREDICATE*

MRefInter =
 $=_{+R} (\{dash\ ref\},$
 $Rel\ (\ - \subseteq_R \ -),$
 $Var(dash\ ref),$
 $Fun_2((- \cap_R \ -),$
 $Var(one\ ref),$
 $Var(two\ ref))),$
 $Val\ (Bool\ true))$

z

MWtRefStInter: $(ALPHABET \times ALPHABET \times ALPHABET) \rightarrow$
REL_PREDICATE

$\forall ns1, ns2, st: ALPHABET \bullet$
 $MWtRefStInter(st, ns1, ns2) =$
 $BranchesNotWaiting \wedge_R MRefInter$
 \triangleleft_R
 $(=_R(\{dash\ wait\}, dash\ wait, Val(Bool(true))))$
 \triangleright_R
 $BranchesWaiting \wedge_R MSt(st, ns1, ns2)$

z

MInter: $(ALPHABET \times ALPHABET \times ALPHABET) \rightarrow REL_PREDICATE$

$\forall ns1, ns2, st: ALPHABET \bullet$
 $MInter(st, ns1, ns2) =$
 $MTrInterPred$
 $\wedge_R MWtRefStInter(st, ns1, ns2)$

z

DivInter: $(CSP_PROCESS \times CSP_PROCESS) \rightarrow REL_PREDICATE$

$\forall a1, a2: CSP_PROCESS$

- $DivInter(a1, a2) =$
 $\exists_R(\{dash(one\ tr), dash(two\ tr)\},$
 $((a1\ \omega_f\ \sigma_f) ;_C$
 $(=_R(a1.1, dash(one\ tr), Var(tr))))$
 \wedge_R
 $((a2\ \omega_f) ;_C$
 $(=_R(a2.1, dash(two\ tr), Var(tr))))$
 $)$

z

$fun - \llbracket C - \rrbracket_C -$

z

$- \llbracket C - \rrbracket_C -: (CSP_PROCESS \times (ALPHABET \times ALPHABET)$
 $\times CSP_PROCESS) \rightarrow CSP_PROCESS$

$\forall a1, a2: CSP_PROCESS; ns1, ns2: ALPHABET$

- | $(a1, a2) \in WF_CSP_PROCESS_PAIR$
- $\wedge ns1 \cap ns2 = \emptyset$
- $a1 \llbracket C (ns1, ns2) \rrbracket_C a2 =$
 $R($
 $(\neg_R (DivInter(a1, a2)) \wedge_R \neg_R (DivInter(a2, a1)))$
 \vdash_D
 $(((((a1\ \omega_f\ \sigma_t) ;_C U(one, out_a\ a1.1))$
 $\wedge_R ((a2\ \omega_f\ \sigma_t);_C U(two, out_a\ a1.1))) +_R$
 $(\{tr\} \cup (a1.1 \setminus (ALPHABET_OWTR \cup dashed))))$
 $\wedge_R (\Pi_R (a1.1 \setminus ALPHABET_OWTR))) ;_C$
 $(MInter((a1.1 \setminus ALPHABET_OWTR) \setminus dashed, ns1, ns2))$
 $)$

Z	$fun \ - \ \backslash_C \ -$
Z	$\begin{array}{l} \hline - \ \backslash_C \ - : (CSP_PROCESS \times SET_EVENT_VAL) \rightarrow CSP_PROCESS \\ \hline \forall a:CSP_PROCESS; cs:SET_EVENT_VAL \bullet \\ \quad \exists s:VAR_NAME \\ \quad \ s \notin a.1 \\ \quad \bullet \ a \ \backslash_C \ cs = \\ \quad R(\exists_{-R}(\{s\}, \\ \quad \quad (/_R(/_R((a \oplus_R \{s\}), Var(s), dash \ tr), \\ \quad \quad \quad Fun_2((- \cup_R \ -), Val(cs), Var(ref)), \\ \quad \quad \quad \quad dash \ ref)) \\ \quad \quad \wedge_R \\ \quad \quad (=_{+R}(\{tr, dash \ tr, s\}, \\ \quad \quad \quad Fun_2((- SeqDif_R \ -), \\ \quad \quad \quad \quad Var(dash \ tr), \\ \quad \quad \quad \quad Var(tr)), \\ \quad \quad \quad Fun_2((- \backslash_R \ -), \\ \quad \quad \quad \quad Val(Set(EVENT_VAL)), \\ \quad \quad \quad \quad Val(cs)))) \) \) ;_C \ Skip(a.1) \end{array}$
Z	$\begin{array}{l} \hline \mathbf{CIRCUS_FUNCTION} \triangleq \\ \quad \{f:CSP_PROCESS \leftrightarrow CSP_PROCESS \\ \quad \ (\exists a:ALPHABET \bullet \\ \quad \quad (\forall u_dom:dom \ f; u_ran:ran \ f \\ \quad \quad \bullet \ a = u_dom.1 = u_ran.1))\} \end{array}$
Z	$\begin{array}{l} \hline \mu_C : CIRCUS_FUNCTION \rightarrow CSP_PROCESS \\ \hline \end{array}$

| $\forall f: \text{CIRCUS_FUNCTION} \bullet \mu_C f = \mu_R f$

z

| **Assign_C** : $WF_Assign_C \rightarrow CSP_PROCESS$

$\forall a:ALPHABET; ns: seq\ VAR_NAME; exps: seq\ EXPRESSION$
 | $(a, ns, exps) \in WF_Assign_C$
 • $Assign_C(a, ns, exps) =$
 $R(True_R(a)$
 $\quad \vdash_D$
 $\quad (Assign_R(a, ns, exps)$
 $\quad \quad \wedge_R (=_R(a, \text{dash } tr, Var(tr)))$
 $\quad \quad \wedge_R (=_R(a, \text{dash } wait, Val(Bool(false)))))$
 $\quad)$

z

| **SpecStatement_C** : $WF_SpecStatement_C \rightarrow CSP_PROCESS$

$\forall a:ALPHABET; f: \mathbb{F}\ VAR_NAME; preC: CIRCUS_CONDITION;$
 $postC: CIRCUS_PREDICATE$
 | $(a, f, preC, postC) \in WF_SpecStatement_C$
 • $SpecStatement_C(a, f, preC, postC) =$
 $R(preC$
 $\quad \vdash_D$
 $\quad (postC$
 $\quad \quad \wedge_R (=_R(a, \text{dash } tr, Var(tr)))$
 $\quad \quad \wedge_R (=_R(a, \text{dash } wait, Val(Bool(false))))$
 $\quad \quad \wedge_R (\Pi_R(a \setminus (ALPHABET_OWTR$
 $\quad \quad \quad \cup (f \cup \{n:f \bullet \text{dash } n\})))$
 $\quad \quad)$
 $\quad)$

z

| **fun** {_C - }_C

z
 $\{ _ \}_C : WF_Condition_C \rightarrow CSP_PROCESS$

$\forall a:ALPHABET; g: CIRCUS_CONDITION$
 $\mid (a,g) \in WF_Condition_C$
 $\bullet \{ _ \}_C(a,g) = SpecStatement_C(a,\{ \},g, True_R \ a)$

z
 $fun \langle _ \rangle_C$

z
 $\langle _ \rangle_C : WF_Condition_C \rightarrow CSP_PROCESS$

$\forall a:ALPHABET; g: CIRCUS_CONDITION$
 $\mid (a,g) \in WF_Condition_C$
 $\bullet \langle _ \rangle_C(a,g) = R(True_R(a) \vdash_D g)$

z
 $\mathbf{Typing} : (VAR_DECLS \times ALPHABET) \rightarrow REL_PREDICATE$

$\forall decls:VAR_DECLS; a:ALPHABET$
 $\mid ran \ (decls.1) \subseteq a$
 \bullet
 $(\#decls.1 = 1$
 $\quad \wedge (\exists n:Z_VAR_NAME; e:EXPRESSION$
 $\quad \mid n = head(decls.1) \wedge e = head(decls.2)$
 $\quad \bullet Typing(decls,a) =$
 $\quad \quad =_{+R} (a, Rel((- \in_R -),$
 $\quad \quad \quad Var(n),e),$
 $\quad \quad \quad Val (Bool \ true)))$
 $)$
 $\vee (\#decls.1 > 1$
 $\quad \wedge (\exists n:Z_VAR_NAME; e:EXPRESSION$

$$\begin{aligned}
& | n = \text{head}(\text{decls}.1) \wedge e = \text{head}(\text{decls}.2) \\
& \bullet \text{Typing}(\text{decls}, a) = \\
& \quad =_{+R} (a, \text{Rel}((- \in_R -), \\
& \quad \quad \text{Var}(n), e), \\
& \quad \quad \text{Val}(\text{Bool true})) \\
& \quad \wedge_R \text{Typing}((\text{tail decls}.1, \\
& \quad \quad \text{tail decls}.2), \\
& \quad \quad a)) \\
&)
\end{aligned}$$

^z

SchemaExp_C: $WF_SchemaExp_C \rightarrow CSP_PROCESS$

$$\begin{aligned}
& \forall \text{decls}:VAR_DECLS; p:REL_PREDICATE \\
& | (\text{decls}, p) \in WF_SchemaExp_C \\
& \bullet (\exists f:\mathbb{F} \ Z_VAR_NAME \mid f \subseteq \text{undashed} \\
& \quad \wedge \text{ran}(\text{decls}.1 \upharpoonright \text{dashed}) = \text{dash}(f)) \\
& \bullet \text{SchemaExp}_C(\text{decls}, p) = \\
& \quad \text{SpecStatement}_C(\\
& \quad \quad \text{ran}(\text{decls}.1) \cup ALPHABET_OWTR, \\
& \quad \quad f, \\
& \quad \quad \exists_{-R}(\text{ran}(\text{decls}.1) \setminus \text{undashed}, \\
& \quad \quad \quad \text{Typing}(\text{decls}, p.1) \wedge_R p), \\
& \quad \quad \text{Typing}(\text{decls}, p.1) \wedge_R p))
\end{aligned}$$

^z

GUARDED_ACTIONS \cong

$$\begin{aligned}
& \{ \text{conditions}:seq \ CIRCUS_CONDITION; \\
& \quad \text{actions}:seq \ CSP_PROCESS \\
& | \text{dom conditions} \in (\mathbb{F} \ -) \wedge \text{dom actions} \in (\mathbb{F} \ -) \\
& \quad \wedge \#(\text{ran conditions}) = \#(\text{ran actions}) > 0 \\
& \quad \wedge (\exists a:ALPHABET \bullet \\
& \quad \quad (\forall c:(\text{ran conditions}) \bullet c.1 = a)
\end{aligned}$$

$$\wedge (\forall A: (ran \ actions) \bullet A.1 = a))\}$$

z

TrueGuards : GUARDED_ACTIONS \rightarrow REL-PREDICATE

$\forall \ gactions:GUARDED_ACTIONS$

$$\begin{aligned} & \bullet \\ & (\#gactions.1 = 1 \\ & \quad \wedge (\exists \ g:CIRCUS_CONDITION; \ a:CSP_PROCESS \\ & \quad \quad | \ g = head(gactions.1) \\ & \quad \quad \wedge \ a = head(gactions.2) \\ & \quad \bullet \ TrueGuards(gactions) = g \) \\ &) \\ & \vee (\#gactions.1 > 1 \\ & \quad \wedge (\exists \ g:CIRCUS_CONDITION; \ a:CSP_PROCESS \\ & \quad \quad | \ g = head(gactions.1) \\ & \quad \quad \wedge \ a = head(gactions.2) \\ & \quad \bullet \ TrueGuards(gactions) = \\ & \quad \quad \quad g \vee_R \\ & \quad \quad \quad TrueGuards(tail \ gactions.1, \\ & \quad \quad \quad \quad \quad tail \ gactions.2) \) \\ &) \end{aligned}$$

z

NonDivActions : GUARDED_ACTIONS \rightarrow REL-PREDICATE

$\forall \ gactions:GUARDED_ACTIONS$

$$\begin{aligned} & \bullet \\ & (\#gactions.1 = 1 \\ & \quad \wedge (\exists \ g:CIRCUS_CONDITION; \ a:CSP_PROCESS \\ & \quad \quad | \ g = head(gactions.1) \\ & \quad \quad \wedge \ a = head(gactions.2) \\ & \quad \bullet \ NonDivActions(gactions) = \end{aligned}$$

$$\begin{array}{l}
\qquad\qquad\qquad g \Rightarrow_R (\neg_R (a \ \omega_f \ \sigma_f)) \) \\
) \\
\vee (\ \#gactions.1 > 1 \\
\qquad \wedge (\ \exists g:CIRCUS_CONDITION; a:CSP_PROCESS \\
\qquad\qquad | \ g = head(gactions.1) \\
\qquad\qquad \wedge \ a = head(gactions.2) \\
\qquad\qquad \bullet \ NonDivActions(gactions) = \\
\qquad\qquad\qquad (g \Rightarrow_R (\neg_R (a \ \omega_f \ \sigma_f))) \\
\qquad\qquad\qquad \wedge_R \\
\qquad\qquad\qquad NonDivActions(tail \ gactions.1, \\
\qquad\qquad\qquad\qquad\qquad tail \ gactions.2) \) \\
)
\end{array}$$

z

ExecActions : *GUARDED_ACTIONS* \rightarrow *REL_PREDICATE*

$$\begin{array}{l}
\forall gactions:GUARDED_ACTIONS \\
\bullet \\
(\ \#gactions.1 = 1 \\
\qquad \wedge (\ \exists g:CIRCUS_CONDITION; a:CSP_PROCESS \\
\qquad\qquad | \ g = head(gactions.1) \\
\qquad\qquad \wedge \ a = head(gactions.2) \\
\qquad\qquad \bullet \ ExecActions(gactions) = \\
\qquad\qquad\qquad g \wedge_R (a \ \omega_f \ \sigma_t) \) \\
) \\
\vee (\ \#gactions.1 > 1 \\
\qquad \wedge (\ \exists g:CIRCUS_CONDITION; a:CSP_PROCESS \\
\qquad\qquad | \ g = head(gactions.1) \\
\qquad\qquad \wedge \ a = head(gactions.2) \\
\qquad\qquad \bullet \ ExecActions(gactions) = \\
\qquad\qquad\qquad (g \wedge_R (a \ \omega_f \ \sigma_t)) \\
\qquad\qquad\qquad \vee_R \\
\qquad\qquad\qquad ExecActions(tail \ gactions.1,
\end{array}$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \text{tail } gactions.2) \text{)}$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \text{fun } \mathbf{if}_C - \mathbf{fi}_C$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \mathbf{if}_C - \mathbf{fi}_C : GUARDED_ACTIONS \rightarrow CSP_PROCESS$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \forall gactions : GUARDED_ACTIONS$$

- $$\bullet \text{ if}_C gactions \text{ fi}_C =$$

$$R($$

$$(TrueGuards(gactions) \wedge_R NonDivActions(gactions))$$

$$\vdash_D$$

$$ExecActions(gactions)$$

$$)$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \text{fun } \mathbf{ifb}_C - \mapsto_C - \mathbf{else}_C - \mapsto_C - \mathbf{fib}_C$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \mathbf{ifb}_C - \mapsto_C - \mathbf{else}_C - \mapsto_C - \mathbf{fib}_C:$$

$$(CIRCUS_CONDITION \times CSP_PROCESS \times$$

$$CIRCUS_CONDITION \times CSP_PROCESS) \leftrightarrow$$

$$CSP_PROCESS$$

$$\begin{array}{|l} \text{ } \\ \text{ } \end{array} \quad \forall g1, g2 : CIRCUS_CONDITION; a1, a2 : CSP_PROCESS$$

$$| (\langle g1, g2 \rangle, \langle a1, a2 \rangle) \in GUARDED_ACTIONS$$

- $$\bullet \text{ ifb}_C g1 \mapsto_C a1 \text{ else}_C g2 \mapsto_C a2 \text{ fib}_C =$$

$$R(((g1 \vee_R g2) \wedge_R$$

$$((g1 \Rightarrow_R (\neg_R (a1 \omega_f \sigma_f)))$$

$$\wedge_R (g2 \Rightarrow_R (\neg_R (a2 \omega_f \sigma_f)))))$$

$$\begin{array}{|l}
\vdash_D \\
((g1 \wedge_R (\neg_R (a1 \omega_f \sigma_t))) \\
\quad \vee_R (g2 \wedge_R (\neg_R (a2 \omega_f \sigma_t)))) \\
)
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mathbf{param}_C: WF_param_C \rightarrow CSP_PROCESS \\
\hline
\forall x:VAR_NAME; T:EXPRESSION; a:CSP_PROCESS; e:EXPRESSION \\
| (x, T, a, e) \in WF_param_C \\
\bullet param_C(x, T, a, e) = /_R(a, e, x)
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mathbf{val}_C: WF_val_C \rightarrow CSP_PROCESS \\
\hline
\forall x:VAR_NAME; t:SET_VAL; a:CSP_PROCESS; e:EXPRESSION \\
| (x, t, a, e) \in WF_val_C \\
\bullet val_C(x, t, a, e) = \\
\quad var_C(x, t, Assign_R(a.1, \langle x \rangle, \langle e \rangle)) ;_C a
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mathbf{res}_C: WF_res_C \rightarrow CSP_PROCESS \\
\hline
\forall x:VAR_NAME; t:SET_VAL; a:CSP_PROCESS; y:VAR_NAME \\
| (x, t, a, y) \in WF_res_C \\
\bullet res_C(x, t, a, y) = \\
\quad var_C(x, t, a ;_R Assign_R(a.1, \langle y \rangle, \langle Var(x) \rangle))
\end{array}$$

$$\begin{array}{|l}
^Z \\
\mathbf{vres}_C: WF_vres_C \rightarrow CSP_PROCESS \\
\hline
\end{array}$$

$$\begin{array}{|l}
\forall x:VAR_NAME; t:SET_VAL; a:CSP_PROCESS; y:VAR_NAME \\
| (x,t,a,y) \in WF_vres_C \\
| \bullet vres_C(x,t,a,y) = \\
| \quad var_C(x,t, \\
| \quad \quad Assign_R(a.1, \langle x \rangle, \langle Var(y) \rangle)) ;_R \\
| \quad \quad a ;_R \\
| \quad \quad Assign_R(a.1, \langle y \rangle, \langle Var(x) \rangle))
\end{array}$$

6.3 Healthiness Conditions

$$\begin{array}{|l}
^Z \\
| \mathbf{C1} : CSP_PROCESS \rightarrow CSP_PROCESS \\
| \hline
| \forall a:CSP_PROCESS \bullet C1 \ a = a ;_R Skip(a.1)
\end{array}$$

$$\begin{array}{|l}
^Z \\
| \mathbf{C1_healthy} : \mathbb{P} \ CSP_PROCESS \\
| \hline
| C1_healthy = \{a:CSP_PROCESS \mid a = C1 \ a\}
\end{array}$$

$$\begin{array}{|l}
^Z \\
| \mathbf{C2} : CSP_PROCESS \rightarrow CSP_PROCESS \\
| \hline
| \forall a:CSP_PROCESS \bullet C2 \ a = (a \llbracket_C (a.1, \emptyset) \rrbracket_C Skip(a.1))
\end{array}$$

$$\begin{array}{|l}
^Z \\
| \mathbf{C2_healthy} : \mathbb{P} \ CSP_PROCESS \\
| \hline
| C2_healthy = \{a:CSP_PROCESS \mid a = C2 \ a\}
\end{array}$$

$$\begin{array}{|l}
^Z \\
| \mathbf{C3} : CSP_PROCESS \rightarrow CSP_PROCESS \\
| \hline
\end{array}$$

$$\begin{array}{|l} \forall a: CSP_PROCESS \bullet \\ C3 \ a = R(((\neg_R (a \ \sigma_f \ \omega_f)) \ ;_R \ True_R(a.1)) \\ \quad \vdash_D \\ \quad (a \ \sigma_t \ \omega_f)) \end{array}$$

$$\begin{array}{|l} z \\ \mathbf{C3_healthy} : \mathbb{P} \ CSP_PROCESS \\ \hline C3_healthy = \{a: CSP_PROCESS \mid a = C3 \ a\} \end{array}$$

$$\begin{array}{|l} z \\ \mathbf{CIRCUS_ACTION} \triangleq \{c: CSP_PROCESS \mid c \in C1_healthy \\ \quad \wedge \ c \in C2_healthy \\ \quad \wedge \ c \in C3_healthy\} \end{array}$$

$$\begin{array}{|l} SML \\ z_output_theory\{out_file="utp-circus.th.doc", \\ \quad theory="utp-circus"\}; \end{array}$$

7 THE Z THEORY utp-z-library

7.1 Parents

*cache'/home/consiste/marcel/proofpower/pp-examples/circus/db/circus
z_library*

7.2 Global Variables

$$\begin{aligned}
 (- \text{prefix}_Z -)[X] & \quad (\mathbb{Z} \leftrightarrow X) \leftrightarrow \mathbb{Z} \leftrightarrow X \\
 (- -_Z -)[X] & \quad (\mathbb{Z} \leftrightarrow X) \times (\mathbb{Z} \leftrightarrow X) \leftrightarrow \mathbb{Z} \leftrightarrow X \\
 (- \llbracket_Z - \rrbracket)[X] & \quad (\mathbb{Z} \leftrightarrow X) \times (\mathbb{Z} \leftrightarrow X) \leftrightarrow \mathbb{P} (\mathbb{Z} \leftrightarrow X) \\
 (- \llbracket_Z - \rrbracket_Z -)[X] & \quad (\mathbb{Z} \leftrightarrow X) \times \mathbb{P} X \times (\mathbb{Z} \leftrightarrow X) \leftrightarrow \mathbb{P} (\mathbb{Z} \leftrightarrow X)
 \end{aligned}$$

7.3 Fixity

$$\text{fun } 0 \text{ rightassoc} \quad (- -_Z -) \quad (- \llbracket_Z - \rrbracket_Z -) \quad (- \llbracket_Z - \rrbracket -)$$

$$\text{rel} \quad (- \text{prefix}_Z -)$$

7.4 Axioms

$$\begin{aligned}
 - \text{prefix}_Z - & \quad \vdash [X]((- \text{prefix}_Z -)[X] \in \text{seq } X \leftrightarrow \text{seq } X \\
 & \quad \wedge (\forall s, t : \text{seq } X \\
 & \quad \bullet (s, t) \in (- \text{prefix}_Z -)[X] \Leftrightarrow s \subseteq t)) \\
 - -_Z - & \quad \vdash [X]((- -_Z -)[X] \in (\text{seq } X) \times (\text{seq } X) \leftrightarrow \text{seq } X \\
 & \quad \wedge \text{dom } (- -_Z -)[X] = \{s, t : \text{seq } X \mid t \text{ prefix}_Z s\} \\
 & \quad \wedge (\forall s, t : \text{seq } X \\
 & \quad \mid t \text{ prefix}_Z s \\
 & \quad \bullet \exists r : \text{seq } X \\
 & \quad \bullet s = t \cap r \wedge (- -_Z -)[X] (s, t) = r)) \\
 - \llbracket_Z - \rrbracket - & \quad \vdash [X]((- \llbracket_Z - \rrbracket -)[X] \in (\text{seq } X) \times (\text{seq } X) \rightarrow \mathbb{P} (\text{seq } X) \\
 & \quad \wedge (\forall s1, s2 : \text{seq } X
 \end{aligned}$$

$$\begin{aligned}
& \bullet s1 = \langle \rangle \\
& \Rightarrow (- \llbracket_Z - \rrbracket [X] (s1, s2) = \{s2\} \wedge s2 = \langle \rangle \\
& \Rightarrow (- \llbracket_Z - \rrbracket [X] (s1, s2) = \{s1\} \\
& \quad \wedge s1 \neq \langle \rangle \\
& \quad \wedge s2 \neq \langle \rangle \\
& \Rightarrow (\exists e1, e2 : X; tl1, tl2 : seq X \\
& \quad | s1 = \langle e1 \rangle \cap tl1 \wedge s2 = \langle e2 \rangle \cap tl2 \\
& \bullet (- \llbracket_Z - \rrbracket [X] (s1, s2) \\
& \quad = \{s : seq X \\
& \quad \quad | s \in (- \llbracket_Z - \rrbracket [X] (tl1, \langle e2 \rangle \cap tl2) \\
& \quad \quad \bullet \langle e1 \rangle \cap s\} \\
& \quad \cup \{s : seq X \\
& \quad \quad | s \in (- \llbracket_Z - \rrbracket [X] (\langle e1 \rangle \cap tl1, tl2) \\
& \quad \quad \bullet \langle e2 \rangle \cap s\}))
\end{aligned}$$

- $\llbracket_Z - \rrbracket_Z -$

$$\begin{aligned}
& \vdash [X]((- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad \in (seq X) \times \mathbb{P} X \times (seq X) \rightarrow \mathbb{P} (seq X) \\
& \wedge (\forall s1, s2 : seq X; cs : \mathbb{P} X \\
& \bullet s1 = \langle \rangle \wedge s2 = \langle \rangle \\
& \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) = \{\langle \rangle\} \\
& \quad \wedge s1 \neq \langle \rangle \\
& \quad \wedge s2 = \langle \rangle \\
& \Rightarrow (\exists e1 : X; tl1 : seq X \\
& \quad | s1 = \langle e1 \rangle \cap tl1 \\
& \bullet (e1 \in cs \\
& \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
& \quad \quad = \{\langle \rangle\}) \\
& \wedge (e1 \notin cs \\
& \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
& \quad \quad = \{s : seq X \\
& \quad \quad | s \\
& \quad \quad \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad \quad \quad (tl1, cs, \langle \rangle) \\
& \quad \bullet \langle e1 \rangle \cap s\})) \\
& \wedge s1 = \langle \rangle \\
& \wedge s2 \neq \langle \rangle
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow (\exists e2 : X; tl2 : seq\ X \\
&\quad | s2 = \langle e2 \rangle \frown tl2 \\
&\quad \bullet (e2 \in cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
&\quad \quad = \{\langle \rangle\}) \\
&\quad \wedge (e2 \notin cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
&\quad \quad = \{s : seq\ X \\
&\quad \quad \quad | s \\
&\quad \quad \quad \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
&\quad \quad \quad \quad (\langle \rangle, cs, tl2) \\
&\quad \quad \bullet \langle e2 \rangle \frown s\}) \\
&\quad \wedge s1 \neq \langle \rangle \\
&\quad \wedge s2 \neq \langle \rangle \\
&\Rightarrow (\exists e1, e2 : X; tl1, tl2 : seq\ X \\
&\quad | s1 = \langle e1 \rangle \frown tl1 \wedge s2 = \langle e2 \rangle \frown tl2 \\
&\quad \bullet (e1 = e2 \wedge e1 \in cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
&\quad \quad = \{s : seq\ X \\
&\quad \quad \quad | s \\
&\quad \quad \quad \in (- \llbracket_Z - \rrbracket_Z -)[X] (tl1, cs, tl2) \\
&\quad \quad \bullet \langle e1 \rangle \frown s\}) \\
&\quad \wedge (e1 = e2 \wedge e1 \notin cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
&\quad \quad = \{s : seq\ X \\
&\quad \quad \quad | s \\
&\quad \quad \quad \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
&\quad \quad \quad \quad (tl1, cs, \langle e2 \rangle \frown tl2) \\
&\quad \quad \quad \cup (- \llbracket_Z - \rrbracket_Z -)[X] \\
&\quad \quad \quad \quad (\langle e1 \rangle \frown tl1, cs, tl2) \\
&\quad \quad \bullet \langle e1 \rangle \frown s\}) \\
&\quad \wedge (e1 \neq e2 \wedge e1 \in cs \wedge e2 \in cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) = \{\langle \rangle\}) \\
&\quad \wedge (e1 \neq e2 \wedge e1 \in cs \wedge e2 \notin cs \\
&\quad \quad \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
&\quad \quad = \{s : seq\ X
\end{aligned}$$

$$\begin{aligned}
& \mid s \\
& \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad (\langle e1 \rangle \frown tl1, cs, tl2) \\
& \quad \bullet \langle e2 \rangle \frown s \}) \\
& \wedge (e1 \neq e2 \wedge e1 \notin cs \wedge e2 \in cs \\
& \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
& = \{s : seq\ X \\
& \mid s \\
& \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad (tl1, cs, \langle e2 \rangle \frown tl2) \\
& \quad \bullet \langle e1 \rangle \frown s \}) \\
& \wedge (e1 \neq e2 \wedge e1 \notin cs \wedge e2 \notin cs \\
& \Rightarrow (- \llbracket_Z - \rrbracket_Z -)[X] (s1, cs, s2) \\
& = \{s : seq\ X \\
& \mid s \\
& \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad (\langle e1 \rangle \frown tl1, cs, tl2) \\
& \quad \bullet \langle e2 \rangle \frown s \} \\
& \cup \{s : seq\ X \\
& \mid s \\
& \in (- \llbracket_Z - \rrbracket_Z -)[X] \\
& \quad (tl1, cs, \langle e2 \rangle \frown tl2) \\
& \quad \bullet \langle e1 \rangle \frown s \} \})
\end{aligned}$$

7.5 Theorems

z_head_singleton_thm

$$\vdash \forall x : \mathbb{U} \bullet head\ \langle x \rangle = x$$

z_head_singleton_∧_thm

$$\vdash \forall x : \mathbb{U}; xs : (seq\ -) \bullet head\ (\langle x \rangle \frown xs) = x$$

z_tail_singleton_thm

$$\vdash \forall x : \mathbb{U} \bullet tail\ \langle x \rangle = \langle \rangle$$

z_singleton_∧_app_thm

$$\vdash \forall x : \mathbb{U}; s : (seq\ -); i : dom\ s$$

$$\bullet (\langle x \rangle \frown s)\ (1 + i) = s\ i$$

$z_tail_singleton_ \wedge_thm$

$$\vdash \forall x : \mathbb{U}; xs : (seq _) \bullet tail (\langle x \rangle \wedge xs) = xs$$

$z_ \in_ seq_ \Leftrightarrow_ app_ eq_ thm$

$$\vdash \forall s : (seq _); m : dom\ s; x : \mathbb{U} \\ \bullet (m, x) \in s \Leftrightarrow s\ m = x$$

$z_ \wedge_ app_ eq_ thm$

$$\vdash \forall s, t : (seq _) \\ \bullet (\forall m : dom\ s \bullet (s \wedge t)\ m = s\ m) \\ \wedge (\forall n : dom\ t \bullet (s \wedge t)\ (n + \# s) = t\ n)$$

$z_singleton_ \wedge_ \in_ seq_1_ thm$

$$\vdash \forall s : (seq _); x : \mathbb{U} \bullet \langle x \rangle \wedge s \in (seq_1 _)$$

$z_head_seqd_thm$

$$\vdash \ulcorner \forall x\ l \bullet \ulcorner head\ \urcorner \$ \urcorner Z' \langle \rangle \urcorner (Cons\ x\ l) \urcorner \urcorner = x \urcorner$$

$z_tail_seqd_thm$

$$\vdash \ulcorner \forall x\ l \bullet \ulcorner tail\ \urcorner \$ \urcorner Z' \langle \rangle \urcorner (Cons\ x\ l) \urcorner \urcorner = \ulcorner \ulcorner \$ \urcorner Z' \langle \rangle \urcorner\ l \urcorner \urcorner \urcorner$$

$z_ \mapsto_ app_ \in_ rel_ thm$

$$\vdash \forall X : \mathbb{U}; Y : \mathbb{U}; f : X \mapsto Y; x : X \\ | x \in dom\ f \\ \bullet (x, f\ x) \in f$$

$z_ifun_dom_eq_ran_eq_thm$

$$\vdash \forall x, x', y, y' : \mathbb{N}; f : \mathbb{N} \mapsto \mathbb{N} \\ \bullet x = x' \wedge f\ x = y \wedge f\ x' = y' \Rightarrow y = y'$$

$z_ \mapsto_ dom_ \neg_ eq_ \Rightarrow_ ran_ \neg_ eq_ thm$

$$\vdash \forall x, x', y, y' : \mathbb{N}; f : \mathbb{N} \mapsto \mathbb{N} \\ \bullet \neg x = x' \wedge f\ x = y \wedge f\ x' = y' \Rightarrow \neg y = y'$$

$z_ \mapsto_ ran_ \neg_ eq_ \Rightarrow_ dom_ \neg_ eq_ thm$

$$\vdash \forall x, x', y, y' : \mathbb{N}; f : \mathbb{N} \mapsto \mathbb{N} \\ \bullet \neg y = y' \wedge f\ x = y \wedge f\ x' = y' \Rightarrow \neg x = x'$$

$\exists_ \wedge_ distribution_ thm$

$$\ulcorner \forall x \\ \bullet P1\ x \wedge \neg P2\ x \wedge \neg P3\ x \\ \vee (P2\ x \wedge \neg P1\ x \wedge \neg P3\ x) \\ \wedge P3\ x \\ \wedge \neg P1\ x \\ \wedge \neg P2\ x \urcorner \\ \vdash \ulcorner \exists x \bullet P1\ x \Rightarrow Q1\ x \urcorner$$

$$\begin{aligned}
& \wedge \lceil \exists x \bullet P2 \ x \Rightarrow Q2 \ x \rceil \\
& \wedge \lceil \exists x \bullet P3 \ x \Rightarrow Q3 \ x \rceil \\
& \Leftrightarrow \lceil \exists x \\
& \bullet (P1 \ x \Rightarrow Q1 \ x) \\
& \quad \wedge (P2 \ x \Rightarrow Q2 \ x) \\
& \quad \wedge (P3 \ x \Rightarrow Q3 \ x) \rceil \\
\mathbf{z_inocuous_ref_dash_quantification_thm} \\
& \vdash (\forall \text{ref}', cs : \mathbb{P} \ \mathbb{N} \\
& \quad \bullet \exists \text{ref1}, \text{ref2} : \mathbb{P} \ \mathbb{N} \\
& \quad \bullet \text{ref}' \\
& \quad = (\text{ref1} \cup \text{ref2}) \cap cs \cup (\text{ref1} \cap \text{ref2} \setminus cs)) \\
& \Leftrightarrow \text{true} \\
\mathbf{z_}\leftrightarrow_\triangleleft_\text{app_idem_thm} \\
& \vdash [X, \\
& \quad Y](\forall f : X \leftrightarrow Y; x : \mathbb{U}; a : \mathbb{U} \\
& \quad | f \in X \leftrightarrow Y \wedge x \in \text{dom } f \wedge x \in a \\
& \quad \bullet (a \triangleleft f) \ x = f \ x) \\
\mathbf{z_seq_subtype_thm} \\
& \vdash \forall S, T : \mathbb{U} \mid S \subseteq T \bullet s \in \text{seq } S \Rightarrow s \in \text{seq } T \\
\mathbf{z_prefix_Z_clauses_thm} \\
& \vdash \forall s : (\text{seq } -) \bullet \langle \rangle \text{prefix}_Z \ s \\
\mathbf{z_prefix_Z_id_thm} \\
& \vdash \forall s : (\text{seq } -) \bullet s \text{prefix}_Z \ s \\
\mathbf{z_seqdiff_Z_thm} \\
& \vdash \forall s1, s2 : (\text{seq } -) \\
& \quad | s2 \text{prefix}_Z \ s1 \\
& \quad \bullet s1 \text{--}_Z \ s2 \in (\text{seq } -) \\
\mathbf{z_seq_cases_thm1} \\
& \vdash [X](\forall s : \text{seq } X \\
& \quad \bullet s = \langle \rangle \vee (\exists s1 : \text{seq } X; x : X \bullet s = s1 \frown \langle x \rangle)) \\
\mathbf{z_size_seq_cases_thm} \\
& \vdash [X](\forall s : \text{seq } X \mid \# s = 0 \bullet s = \langle \rangle) \\
& \quad \wedge (\forall s : \text{seq } X; i : \mathbb{N} \\
& \quad | \# s = i + 1 \\
& \quad \bullet \exists s1 : \text{seq } X; x : X \bullet \# s1 = i \wedge s = s1 \frown \langle x \rangle)) \\
\mathbf{z_}\frown_\text{one_one_thm1} \\
& \vdash \forall s1, t1, s2, t2 : (\text{seq } -)
\end{aligned}$$

$$\begin{array}{l}
| \# s2 = \# t2 \wedge s1 \frown s2 = t1 \frown t2 \\
\bullet s1 = t1 \wedge s2 = t2
\end{array}$$

z_∧_one_one_thm2

$$\vdash \forall s, t, r : (seq _) \bullet s \frown t = s \frown r \Leftrightarrow t = r$$

z_∧_unit_thm $\vdash \forall s, t : (seq _) \bullet s = s \frown t \Leftrightarrow t = \langle \rangle$

z_seqdiff_z-zero_thm

$$\vdash \forall s : (seq _) \bullet s -_Z s = \langle \rangle$$

z_seqdiff_z-zero_thm1

$$\begin{array}{l}
\vdash \forall s1, s2 : (seq _) \\
| s2 \text{ prefix}_Z s1 \wedge s1 -_Z s2 = \langle \rangle \\
\bullet s1 = s2
\end{array}$$

8 THE Z THEORY utp-rel

8.1 Parents

utp-z-library

8.2 Global Variables

NAME	$\mathbb{P} \text{ NAME}$
any_name	NAME
ALPHABET	$\mathbb{P} (\mathbb{P} \text{ NAME})$
dash	$\text{NAME} \leftrightarrow \text{NAME}$
one	$\text{NAME} \leftrightarrow \text{NAME}$
two	$\text{NAME} \leftrightarrow \text{NAME}$
dashed	$\mathbb{P} \text{ NAME}$
undashed	$\mathbb{P} \text{ NAME}$
in_a	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME}$
out_a	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME}$
homogeneous	$\mathbb{P} (\mathbb{P} \text{ NAME})$
composable	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME}$
VALUE	$\mathbb{P} \text{ VALUE}$
Int	$\mathbb{Z} \leftrightarrow \text{VALUE}$
Bool	$\mathbb{B} \leftrightarrow \text{VALUE}$
Channel	$\text{NAME} \leftrightarrow \text{VALUE}$
Seq	$(\mathbb{Z} \leftrightarrow \text{VALUE}) \leftrightarrow \text{VALUE}$
Set	$\mathbb{P} \text{ VALUE} \leftrightarrow \text{VALUE}$
Pair	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
Sync	VALUE
INT_VAL	$\mathbb{P} \text{ VALUE}$
BOOL_VAL	$\mathbb{P} \text{ VALUE}$
CHANNEL_VAL	$\mathbb{P} \text{ VALUE}$
SEQ_VAL	$\mathbb{P} \text{ VALUE}$
SET_VAL	$\mathbb{P} \text{ VALUE}$
PAIR_VAL	$\mathbb{P} \text{ VALUE}$
EVENT_VAL	$\mathbb{P} \text{ VALUE}$
SEQ_EVENT_VAL	$\mathbb{P} \text{ VALUE}$
SET_EVENT_VAL	

	$\mathbb{P} \text{ VALUE}$
SET_SEQ_EVENT_VAL	
	$\mathbb{P} \text{ VALUE}$
PAIR_SEQ_EVENT_VAL	
	$\mathbb{P} \text{ VALUE}$
UNARY_F	$\mathbb{P} (\text{VALUE} \leftrightarrow \text{VALUE})$
any_un_fun	$\text{VALUE} \leftrightarrow \text{VALUE}$
BINARY_F	$\mathbb{P} (\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE})$
any_bin_fun	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
RELATION	$\mathbb{P} (\text{VALUE} \leftrightarrow \text{VALUE})$
any_rel	$\text{VALUE} \leftrightarrow \text{VALUE}$
EXPRESSION	$\mathbb{P} \text{ EXPRESSION}$
Val	$\text{VALUE} \leftrightarrow \text{EXPRESSION}$
Var	$\text{NAME} \leftrightarrow \text{EXPRESSION}$
Fun₁	$(\text{VALUE} \leftrightarrow \text{VALUE}) \times \text{EXPRESSION} \leftrightarrow \text{EXPRESSION}$
Fun₂	$(\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}) \times \text{EXPRESSION} \times \text{EXPRESSION}$ $\leftrightarrow \text{EXPRESSION}$
Rel	$(\text{VALUE} \leftrightarrow \text{VALUE}) \times \text{EXPRESSION} \times \text{EXPRESSION} \leftrightarrow \text{EXPRESSION}$
BINDING	$\mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
BINDINGS	$\mathbb{P} (\mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
FV	$\text{EXPRESSION} \leftrightarrow \mathbb{P} \text{ NAME}$
WF_BINDING_EXPRESSION_R	$(\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \text{EXPRESSION}$
Eval	$(\text{NAME} \leftrightarrow \text{VALUE}) \times \text{EXPRESSION} \leftrightarrow \text{VALUE}$
REL_PREDICATE	
	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
UnrestVar	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME}$
unrestTypedVar	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})) \times \text{NAME} \times \mathbb{P} \text{ VALUE} \leftrightarrow \mathbb{B}$
REL_CONDITION	
	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
dashCond	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
U	$(\text{NAME} \leftrightarrow \text{NAME}) \times \mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Equals_R	$\mathbb{P} (\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION})$
WF_ALPHABET_EXPRESSION	
	$\mathbb{P} (\mathbb{P} \text{ NAME} \times \text{EXPRESSION} \times \text{EXPRESSION})$
WF_Cond_R	\mathbb{P} $((\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$

	$\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
	$\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
WF_Semi_R	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Skip_R	$\mathbb{P} (\mathbb{P} \text{ NAME})$
WF_Assign_R	$\mathbb{P} (\mathbb{P} \text{ NAME} \times (\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION}))$
WF_Subst_R	$\mathbb{P} ((\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})) \times \text{EXPRESSION} \times \text{NAME})$
WF_Var_{R_EndR}	$\mathbb{P} \text{ NAME} \leftrightarrow \text{NAME}$
WF_VarT_{R_EndTR}	$\mathbb{P} (\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{VALUE})$
WF_Extend_rest_R	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME}$
WF_REL_PREDICATE_PAIR	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Glb_{R_LubR}	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
(- ≤_R -)	$\text{VALUE} \leftrightarrow \text{VALUE}$
(- <_R -)	$\text{VALUE} \leftrightarrow \text{VALUE}$
(- ∈_R -)	$\text{VALUE} \leftrightarrow \text{VALUE}$
(- ∉_R -)	$\text{VALUE} \leftrightarrow \text{VALUE}$
(- ⊆_R -)	$\text{VALUE} \leftrightarrow \text{VALUE}$
MkSingleton	$\text{VALUE} \leftrightarrow \text{VALUE}$
MkPair	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- SeqDif_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- ∩_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- ⊢_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- ∪_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- ∩_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
(- \setminus_R -)	$\text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$
True_R	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
False_R	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
(- ≡_R -)	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
	$\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
	$\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
=_R	$\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
=_{+R}	$\mathbb{P} \text{ NAME} \times \text{EXPRESSION} \times \text{EXPRESSION}$

	$\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$(- \text{ intchoice}_R -)$	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$(- \text{ conj}_R -)$	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
var_R	$\mathbb{P} \text{ NAME} \times \text{NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
end_R	$\mathbb{P} \text{ NAME} \times \text{NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$(- +_R -)$	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})) \times \mathbb{P} \text{ NAME}$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$(\langle_R - \rangle_R)$	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$(- \sqsubseteq_R -)$	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
monotonic	\mathbb{P} $(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
Bot_R	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
Top_R	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
\cap_R	$\mathbb{P} \text{ NAME} \times (\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
\cup_R	$\mathbb{P} \text{ NAME} \times (\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
REL_FUNCTION	\mathbb{P} $(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
μ_R	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
ν_R	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$\text{var}T_R$	$\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{VALUE} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
$\text{end}T_R$	$\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{VALUE} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

8.3 Fixity

fun 0 rightassoc

$$\begin{array}{lll}
 (- \text{ conj}_R -) & (- \cap_R -) & (- \sqsubseteq_R -) \\
 (- \text{ intchoice}_R -) & (- \Leftrightarrow_R -) & (- \equiv_R -)
 \end{array}$$

$$\begin{array}{lll}
(- \text{SeqDif}_R -) & (- \oplus_R -) & (- \frown_R -) \\
(- +_R -) & (- \wedge_R -) & (- \triangleleft_R - \triangleright_R -) \\
(- \neg_R -) & (- \vee_R -) & (- \downarrow_R -) \\
(- ;_R -) & (- \Rightarrow_R -) & (\langle _R - \rangle_R) \\
(- \setminus_R -) & (- \cup_R -) &
\end{array}$$

$$\text{rel} \quad (- <_R -) \quad (- \subseteq_R -) \quad (- \in_R -) \quad (- \notin_R -) \quad (- \leq_R -)$$

8.4 Axioms

$$\begin{array}{ll}
\text{any_name} & \vdash \text{any_name} \in \text{NAME} \wedge \text{true} \\
\text{dash} & \vdash \text{dash} \in \text{NAME} \multimap \text{NAME} \wedge \text{true} \\
\text{one} & \\
\text{two} & \vdash \{one, two\} \subseteq \text{NAME} \multimap \text{NAME} \wedge \text{ran one} \cap \text{ran two} = \emptyset \\
\text{dashed} & \vdash \text{dashed} \in \text{ALPHABET} \wedge \text{dashed} = \text{ran dash} \\
\text{undashed} & \vdash \text{undashed} \in \text{ALPHABET} \\
& \quad \wedge \text{undashed} = \{n : \text{NAME} \mid n \notin \text{ran dash}\} \\
\text{in_a} & \vdash \text{in_a} \in \text{ALPHABET} \rightarrow \text{ALPHABET} \\
& \quad \wedge (\forall a : \text{ALPHABET} \bullet \text{in_a } a = a \cap \text{undashed}) \\
\text{out_a} & \vdash \text{out_a} \in \text{ALPHABET} \rightarrow \text{ALPHABET} \\
& \quad \wedge (\forall a : \text{ALPHABET} \bullet \text{out_a } a = a \cap \text{dashed}) \\
\text{homogeneous} & \vdash \text{homogeneous} \in \mathbb{P} \text{ ALPHABET} \\
& \quad \wedge \text{homogeneous} \\
& \quad = \{a : \text{ALPHABET} \\
& \quad \quad \mid \forall n : \text{undashed} \bullet n \in a \Leftrightarrow \text{dash } n \in a\} \\
\text{composable} & \vdash \text{composable} \in \mathbb{P} (\text{ALPHABET} \times \text{ALPHABET}) \\
& \quad \wedge \text{composable} \\
& \quad = \{a1, a2 : \text{ALPHABET} \\
& \quad \quad \mid \forall n : \text{undashed} \bullet n \in a2 \Leftrightarrow \text{dash } n \in a1\} \\
\text{Int} & \\
\text{Bool} & \\
\text{Channel} & \\
\text{Seq} & \\
\text{Set} & \\
\text{Pair} & \\
\text{Sync} & \vdash (\text{Sync} \in \text{VALUE} \\
& \quad \wedge \text{Int} \in \mathbb{Z} \multimap \text{VALUE}
\end{array}$$

$$\begin{aligned}
& \wedge Bool \in \mathbb{B} \rightarrow VALUE \\
& \wedge Channel \in NAME \rightarrow VALUE \\
& \wedge Seq \in seq\ VALUE \rightarrow VALUE \\
& \wedge Set \in \mathbb{P}\ VALUE \rightarrow VALUE \\
& \wedge Pair \in VALUE \times VALUE \rightarrow VALUE) \\
& \wedge disjoint \langle ran\ Int, \\
& \quad ran\ Bool, \\
& \quad ran\ Channel, \\
& \quad ran\ Seq, \\
& \quad ran\ Set, \\
& \quad ran\ Pair, \\
& \quad \{Sync\} \rangle \\
& \wedge (\forall W : \mathbb{P}\ VALUE \\
& \quad | \{Sync\} \\
& \quad \cup (Int\ (\mathbb{Z}) \\
& \quad \cup (Bool\ (\mathbb{B}) \\
& \quad \cup (Channel\ (NAME) \\
& \quad \cup (Seq\ (seq\ W) \\
& \quad \cup (Set\ (\mathbb{P}\ W) \\
& \quad \cup Pair\ (W \times W)))))) \\
& \subseteq W \\
& \bullet VALUE \subseteq W)
\end{aligned}$$

Constraint 1 $\vdash \forall f : UNARY_F; v : VALUE \bullet f\ v \in VALUE$

any_un_fun $\vdash any_un_fun \in UNARY_F \wedge true$

Constraint 2 $\vdash \forall f : BINARY_F; vv : VALUE \times VALUE \bullet f\ vv \in VALUE$

any_bin_fun $\vdash any_bin_fun \in BINARY_F \wedge true$

any_rel $\vdash any_rel \in RELATION \wedge true$

Val

Var

Fun₁

Fun₂

Rel

$$\begin{aligned}
& \vdash (Val \in VALUE \rightarrow EXPRESSION \\
& \quad \wedge Var \in NAME \rightarrow EXPRESSION \\
& \quad \wedge Fun_1 \in UNARY_F \times EXPRESSION \rightarrow EXPRESSION \\
& \quad \wedge Fun_2 \\
& \quad \in BINARY_F \times EXPRESSION \times EXPRESSION \rightarrow EXPRESSION \\
& \quad \wedge Rel \\
& \quad \in RELATION \times EXPRESSION \times EXPRESSION)
\end{aligned}$$

$$\begin{aligned}
& \mapsto \text{EXPRESSION}) \\
& \wedge \text{disjoint } \langle \text{ran } \text{Val}, \\
& \quad \text{ran } \text{Var}, \\
& \quad \text{ran } \text{Fun}_1, \\
& \quad \text{ran } \text{Fun}_2, \\
& \quad \text{ran } \text{Rel} \rangle \\
& \wedge (\forall W : \mathbb{P} \text{ EXPRESSION} \\
& \quad | \text{Val } (\text{VALUE}) \\
& \quad \cup (\text{Var } (\text{NAME}) \\
& \quad \quad \cup (\text{Fun}_1 (\text{UNARY_F} \times W) \\
& \quad \quad \cup (\text{Fun}_2 (\text{BINARY_F} \times W \times W) \\
& \quad \quad \cup \text{Rel } (\text{RELATION} \times W \times W))) \\
& \quad \subseteq W \\
& \bullet \text{ EXPRESSION} \subseteq W) \\
\mathbf{FV} \quad & \vdash \text{FV} \in \text{EXPRESSION} \rightarrow \mathbb{P} \text{ NAME} \\
& \wedge (\forall v : \text{VALUE}; \\
& \quad n : \text{NAME}; \\
& \quad \text{exp1}, \text{exp2} : \text{EXPRESSION}; \\
& \quad f1 : \text{UNARY_F}; \\
& \quad f2 : \text{BINARY_F}; \\
& \quad r : \text{RELATION} \\
& \bullet \text{FV } (\text{Val } v) = \emptyset \\
& \quad \wedge \text{FV } (\text{Var } n) = \{n\} \\
& \quad \wedge \text{FV } (\text{Fun}_1 (f1, \text{exp1})) = \text{FV } \text{exp1} \\
& \quad \wedge \text{FV } (\text{Fun}_2 (f2, \text{exp1}, \text{exp2})) \\
& \quad \quad = \text{FV } \text{exp1} \cup \text{FV } \text{exp2} \\
& \quad \wedge \text{FV } (\text{Rel } (r, \text{exp1}, \text{exp2})) = \text{FV } \text{exp1} \cup \text{FV } \text{exp2}) \\
\mathbf{Eval} \quad & \vdash \text{Eval} \in \text{WF_BINDING_EXPRESSION}_R \rightarrow \text{VALUE} \\
& \wedge (\forall b : \text{BINDING}; \\
& \quad v : \text{VALUE}; \\
& \quad n : \text{NAME}; \\
& \quad \text{exp1}, \text{exp2} : \text{EXPRESSION}; \\
& \quad f1 : \text{UNARY_F}; \\
& \quad f2 : \text{BINARY_F}; \\
& \quad r : \text{RELATION} \\
& \bullet \text{Eval } (b, \text{Val } v) = v \\
& \quad \wedge ((b, \text{Var } n) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \quad \Rightarrow \text{Eval } (b, \text{Var } n) = b \ n)
\end{aligned}$$

$$\begin{aligned}
& \wedge ((b, \text{exp1}) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \Rightarrow \text{Eval } (b, \text{Fun}_1 (f1, \text{exp1})) \\
& \quad = f1 (\text{Eval } (b, \text{exp1}))) \\
& \wedge ((b, \text{exp1}) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \wedge (b, \text{exp2}) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \Rightarrow \text{Eval } (b, \text{Fun}_2 (f2, \text{exp1}, \text{exp2})) \\
& \quad = f2 (\text{Eval } (b, \text{exp1}), \text{Eval } (b, \text{exp2}))) \\
& \wedge ((b, \text{exp1}) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \wedge (b, \text{exp2}) \in \text{WF_BINDING_EXPRESSION}_R \\
& \quad \Rightarrow \text{Eval } (b, \text{Rel } (r, \text{exp1}, \text{exp2})) \\
& \quad = \text{Bool} \\
& \quad \quad ((\text{Eval } (b, \text{exp1}), \text{Eval } (b, \text{exp2})) \\
& \quad \quad \in r))) \\
\mathbf{UnrestVar} \quad & \vdash \text{UnrestVar} \in \text{REL_PREDICATE} \rightarrow \mathbb{P} \text{ NAME} \\
& \wedge (\forall u : \text{REL_PREDICATE} \\
& \quad \bullet \text{UnrestVar } u \\
& \quad = \{n : u.1 \\
& \quad \quad | \forall b : u.2; v : \text{VALUE} \bullet b \oplus \{n \mapsto v\} \in u.2\}) \\
\mathbf{unrestTypedVar} \quad & \vdash \text{unrestTypedVar} \in \text{REL_PREDICATE} \times \text{NAME} \times \mathbb{P} \text{ VALUE} \rightarrow \mathbb{B} \\
& \wedge (\forall r : \text{REL_PREDICATE}; n : \text{NAME}; T : \mathbb{P} \text{ VALUE} \\
& \quad \bullet \text{unrestTypedVar } (r, n, T) \\
& \quad \Leftrightarrow (\forall b : r.2; v : T \bullet b \oplus \{n \mapsto v\} \in r.2)) \\
\mathbf{REL_CONDITION} \quad & \vdash \text{REL_CONDITION} \in \mathbb{P} \text{ REL_PREDICATE} \\
& \wedge \text{REL_CONDITION} \\
& = \{r : \text{REL_PREDICATE} \\
& \quad | r.1 \setminus \text{undashed} \subseteq \text{UnrestVar } r\} \\
\mathbf{dashCond} \quad & \vdash \text{dashCond} \in \text{REL_CONDITION} \rightarrow \text{REL_PREDICATE} \\
& \wedge (\forall u : \text{REL_CONDITION} \\
& \quad \bullet \text{dashCond } u \\
& \quad = (\{n : u.1 \mid n \in \text{undashed} \bullet \text{dash } n\} \cup u.1, \\
& \quad \quad \{b : u.2 \\
& \quad \quad \bullet \{n : \text{dom } b \cap \text{undashed} \\
& \quad \quad \quad \bullet \text{dash } n \mapsto b \ n\} \\
& \quad \quad \quad \cup \{n : \text{dash } (\text{dom } b \cap \text{dashed}) \\
& \quad \quad \quad \bullet n \mapsto b (\text{dash } n)\}\})) \\
\mathbf{U} \quad & \vdash U \in (\text{NAME} \mapsto \text{NAME}) \times \text{ALPHABET} \mapsto \text{REL_PREDICATE}
\end{aligned}$$

$$\begin{aligned}
& \wedge (\forall f : NAME \rightarrow NAME; a' : ALPHABET \\
& \quad | a' \subseteq dashed \\
& \quad \bullet \exists a : ALPHABET \\
& \quad \quad | a \subseteq undashed \wedge a' = dash \langle a \rangle \\
& \quad \bullet U(f, a') \\
& \quad \quad = (\{n : NAME \mid n \in a \bullet dash(f\ n)\} \cup a, \\
& \quad \quad \{b : BINDING \\
& \quad \quad \quad | dom\ b \\
& \quad \quad \quad \quad = \{n : NAME \mid n \in a \bullet dash(f\ n)\} \cup a \\
& \quad \quad \quad \wedge (\forall n : NAME \\
& \quad \quad \quad \quad | n \in a \\
& \quad \quad \quad \quad \bullet b(dash(f\ n)) = b\ n\})) \\
- \leq_R - & \quad \vdash (- \leq_R -) \in VALUE \leftrightarrow VALUE \\
& \quad \wedge (- \leq_R -) \\
& \quad \quad = \{s1, s2 : SEQ_VAL \\
& \quad \quad \quad | (Seq \sim) s1\ prefix_Z (Seq \sim) s2\} \\
- <_R - & \quad \vdash (- <_R -) \in VALUE \leftrightarrow VALUE \\
& \quad \wedge (- <_R -) \\
& \quad \quad = \{s1, s2 : SEQ_VAL \\
& \quad \quad \quad | (Seq \sim) s1\ prefix_Z (Seq \sim) s2 \\
& \quad \quad \quad \wedge (Seq \sim) s1 \neq (Seq \sim) s2\} \\
- \in_R - & \quad \vdash (- \in_R -) \in VALUE \leftrightarrow VALUE \\
& \quad \wedge (- \in_R -) \\
& \quad \quad = \{e : VALUE; s : SET_VAL \\
& \quad \quad \quad | e \in (Set \sim) s\} \\
- \notin_R - & \quad \vdash (- \notin_R -) \in VALUE \leftrightarrow VALUE \\
& \quad \wedge (- \notin_R -) \\
& \quad \quad = \{e : VALUE; s : SET_VAL \\
& \quad \quad \quad | e \notin (Set \sim) s\} \\
- \subseteq_R - & \quad \vdash (- \subseteq_R -) \in VALUE \leftrightarrow VALUE \\
& \quad \wedge (- \subseteq_R -) \\
& \quad \quad = \{s1, s2 : SET_VAL \\
& \quad \quad \quad | (Set \sim) s1 \subseteq (Set \sim) s2\} \\
MkSingleton & \quad \vdash MkSingleton \in VALUE \rightarrow VALUE \\
& \quad \wedge (\forall v : VALUE \bullet MkSingleton\ v = Seq\ \langle v \rangle) \\
MkPair & \quad \vdash MkPair \in VALUE \times VALUE \rightarrow VALUE \\
& \quad \wedge (\forall v1, v2 : VALUE \\
& \quad \quad \bullet MkPair\ (v1, v2) = Pair\ (v1, v2))
\end{aligned}$$

- SeqDif_R -	$\vdash (- \text{SeqDif}_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s1, s2 : \text{VALUE}$ $\quad \{s1, s2\} \subseteq \text{SEQ_VAL}$ $\quad \wedge (\text{Seq } \sim) s2 \text{ prefix}_Z (\text{Seq } \sim) s1$ $\quad \bullet s1 \text{ SeqDif}_R s2$ $\quad = \text{Seq } ((\text{Seq } \sim) s1 -_Z (\text{Seq } \sim) s2))$
- \cap_R -	$\vdash (- \cap_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s1, s2 : \text{VALUE}$ $\quad \{s1, s2\} \subseteq \text{SEQ_VAL}$ $\quad \bullet s1 \cap_R s2 = \text{Seq } ((\text{Seq } \sim) s1 \cap (\text{Seq } \sim) s2))$
- \vdash_R -	$\vdash (- \vdash_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s, st : \text{VALUE}$ $\quad s \in \text{SEQ_VAL} \wedge st \in \text{SET_VAL}$ $\quad \bullet s \vdash_R st = \text{Seq } ((\text{Seq } \sim) s \vdash (\text{Set } \sim) st))$
- \cup_R -	$\vdash (- \cup_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s1, s2 : \text{VALUE}$ $\quad \{s1, s2\} \subseteq \text{SET_VAL}$ $\quad \bullet s1 \cup_R s2 = \text{Set } ((\text{Set } \sim) s1 \cup (\text{Set } \sim) s2))$
- \cap_R -	$\vdash (- \cap_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s1, s2 : \text{VALUE}$ $\quad \{s1, s2\} \subseteq \text{SET_VAL}$ $\quad \bullet s1 \cap_R s2 = \text{Set } ((\text{Set } \sim) s1 \cap (\text{Set } \sim) s2))$
- \setminus_R -	$\vdash (- \setminus_R -) \in \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE}$ $\wedge (\forall s1, s2 : \text{VALUE}$ $\quad \{s1, s2\} \subseteq \text{SET_VAL}$ $\quad \bullet s1 \setminus_R s2 = \text{Set } ((\text{Set } \sim) s1 \setminus (\text{Set } \sim) s2))$
True_R	$\vdash \text{True}_R \in \text{ALPHABET} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a : \text{ALPHABET}$ $\quad \bullet \text{True}_R a = (a, \{b : \text{BINDING} \mid \text{dom } b = a\}))$
False_R	$\vdash \text{False}_R \in \text{ALPHABET} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a : \text{ALPHABET} \bullet \text{False}_R a = (a, \emptyset))$
- \equiv_R -	$\vdash (- \equiv_R -)$ $\in \text{REL_PREDICATE} \times \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall u1, u2 : \text{REL_PREDICATE}$ $\quad \bullet (u1.1 = u2.1 \wedge u1.2 = u2.2$ $\quad \Rightarrow u1 \equiv_R u2 = \text{True}_R u1.1)$ $\quad \wedge (u1.1 \neq u2.1 \vee u1.2 \neq u2.2$

$=_R$	$\begin{aligned} & \Rightarrow u1 \equiv_R u2 = \text{False}_R \ u1.1)) \\ \vdash & =_R \in WF_Equals_R \rightarrow REL_PREDICATE \\ & \wedge (\forall a_n_e : WF_Equals_R \\ & \bullet =_R a_n_e \\ & = (a_n_e.1, \\ & \quad \{b : BINDING \\ & \quad \mid dom \ b = a_n_e.1 \\ & \quad \wedge b \ a_n_e.2 = Eval \ (b, \ a_n_e.3)\})) \end{aligned}$
$=_{+R}$	$\begin{aligned} \vdash & =_{+R} \in WF_ALPHABET_EXPRESSION \rightarrow REL_PREDICATE \\ & \wedge (\forall a_e_e : WF_ALPHABET_EXPRESSION \\ & \bullet =_{+R} a_e_e \\ & = (a_e_e.1, \\ & \quad \{b : BINDING \\ & \quad \mid dom \ b = a_e_e.1 \\ & \quad \wedge Eval \ (b, \ a_e_e.2) \\ & \quad = Eval \ (b, \ a_e_e.3)\})) \end{aligned}$
\neg_R	$\begin{aligned} \vdash & \neg_R \in REL_PREDICATE \rightarrow REL_PREDICATE \\ & \wedge (\forall u : REL_PREDICATE \\ & \bullet \neg_R u = (u.1, (\text{True}_R \ u.1).2 \setminus u.2)) \end{aligned}$
$- \oplus_R -$	$\begin{aligned} \vdash & (- \oplus_R -) \in REL_PREDICATE \times ALPHABET \rightarrow REL_PREDICATE \\ & \wedge (\forall u : REL_PREDICATE; a : ALPHABET \\ & \bullet u \oplus_R a \\ & = (u.1 \cup a, \\ & \quad \{b : BINDING \\ & \quad \mid u.1 \triangleleft b \in u.2 \wedge dom \ b = u.1 \cup a\})) \end{aligned}$
$- \wedge_R -$	$\begin{aligned} \vdash & (- \wedge_R -) \\ & \in REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE \\ & \wedge (\forall u1, u2 : REL_PREDICATE \\ & \bullet u1 \wedge_R u2 \\ & = (u1.1 \cup u2.1, \\ & \quad (u1 \oplus_R u2.1).2 \cap (u2 \oplus_R u1.1).2)) \end{aligned}$
$- \vee_R -$	$\begin{aligned} \vdash & (- \vee_R -) \\ & \in REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE \\ & \wedge (\forall u1, u2 : REL_PREDICATE \\ & \bullet u1 \vee_R u2 \\ & = (u1.1 \cup u2.1, \\ & \quad (u1 \oplus_R u2.1).2 \cup (u2 \oplus_R u1.1).2)) \end{aligned}$
$- \Rightarrow_R -$	$\vdash (- \Rightarrow_R -)$

	$\in REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall u1, u2 : REL_PREDICATE$ $\bullet u1 \Rightarrow_R u2 = \neg_R u1 \vee_R u2)$
$- \Leftrightarrow_R -$	$\vdash (- \Leftrightarrow_R -)$ $\in REL_PREDICATE \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall u1, u2 : REL_PREDICATE$ $\bullet u1 \Leftrightarrow_R u2 = (u1 \Rightarrow_R u2) \wedge_R u2 \Rightarrow_R u1)$
$- \triangleleft_R - \triangleright_R -$	$\vdash (- \triangleleft_R - \triangleright_R -) \in WF_Cond_R \rightarrow REL_PREDICATE$ $\wedge (\forall u1_b_u2 : WF_Cond_R$ $\bullet u1_b_u2.1 \triangleleft_R u1_b_u2.2 \triangleright_R u1_b_u2.3$ $= (u1_b_u2.2 \wedge_R u1_b_u2.1)$ $\vee_R \neg_R u1_b_u2.2 \wedge_R u1_b_u2.3)$
$- ;_R -$	$\vdash (- ;_R -) \in WF_Semi_R \rightarrow REL_PREDICATE$ $\wedge (\forall u1_u2 : WF_Semi_R$ $\bullet u1_u2.1 ;_R u1_u2.2$ $= (in_a u1_u2.1.1 \cup out_a u1_u2.2.1,$ $\{b1 : u1_u2.1.2; b2 : u1_u2.2.2$ $ \forall n : dom b2$ $ n \in undashed$ $\bullet b2 n = b1 (dash n)$ $\bullet undashed \triangleleft b1 \cup dashed \triangleleft b2\}))$
Π_R	$\vdash \Pi_R \in WF_Skip_R \rightarrow REL_PREDICATE$ $\wedge (\forall a : WF_Skip_R$ $\bullet \Pi_R a$ $= (a,$ $\{b : BINDING$ $ dom b = a$ $\wedge (\forall n : a$ $ n \in undashed$ $\bullet b n = b (dash n))\}))$
$Assign_R$	$\vdash Assign_R \in WF_Assign_R \rightarrow REL_PREDICATE$ $\wedge (\forall aa : WF_Assign_R$ $\bullet \# aa.2 = 1$ $\wedge (\exists n : NAME$ $ n = head aa.2$ $\bullet Assign_R aa$ $= =_R (aa.1, dash n, head aa.3)$

	$\wedge_R \Pi_R (aa.1 \setminus \{n, \text{dash } n\}))$ $\vee \# aa.2 > 1$ $\wedge (\exists n : NAME$ $ n = \text{head } aa.2$ $\bullet \text{Assign}_R aa$ $= =_R (aa.1, \text{dash } n, \text{head } aa.3)$ $\wedge_R \text{Assign}_R$ $(aa.1 \setminus \{n, \text{dash } n\},$ $\text{tail } aa.2,$ $\text{tail } aa.3)))$
$- \neg_R -$	$\vdash (- \neg_R -) \in REL_PREDICATE \times ALPHABET \rightarrow REL_PREDICATE$ $\wedge (\forall u : REL_PREDICATE; a : ALPHABET$ $\bullet u \neg_R a = (u.1 \setminus a, \{b : u.2 \bullet a \triangleleft b\}))$
\exists_{-R}	$\vdash \exists_{-R} \in ALPHABET \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall a : ALPHABET; u : REL_PREDICATE$ $\bullet \exists_{-R} (a, u) = u \neg_R a)$
\forall_{-R}	$\vdash \forall_{-R} \in ALPHABET \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall a : ALPHABET; u : REL_PREDICATE$ $\bullet \forall_{-R} (a, u) = \neg_R (\exists_{-R} (a, \neg_R u)))$
\exists_R	$\vdash \exists_R \in ALPHABET \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall a : ALPHABET; u : REL_PREDICATE$ $\bullet \exists_R (a, u) = \exists_{-R} (a, u) \oplus_R a)$
\forall_R	$\vdash \forall_R \in ALPHABET \times REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall a : ALPHABET; u : REL_PREDICATE$ $\bullet \forall_R (a, u) = \neg_R (\exists_R (a, \neg_R u)))$
$/_R$	$\vdash /_R \in WF_Subst_R \rightarrow REL_PREDICATE$ $\wedge (\forall u_e_n : WF_Subst_R$ $\bullet /_R u_e_n$ $= (u_e_n.1.1,$ $\{b : BINDING$ $ \text{dom } b = u_e_n.1.1$ $\wedge b \oplus \{u_e_n.3 \mapsto \text{Eval } (b, u_e_n.2)\}$ $\in u_e_n.1.2\}))$
$- \text{intchoice}_R -$	$\vdash (- \text{intchoice}_R -)$ $\in WF_REL_PREDICATE_PAIR \rightarrow REL_PREDICATE$ $\wedge (\forall \text{pair_}u : WF_REL_PREDICATE_PAIR$ $\bullet \text{pair_}u.1 \text{intchoice}_R \text{pair_}u.2$

$- \text{conj}_R -$	$= \text{pair-}u.1 \vee_R \text{pair-}u.2$ $\vdash (- \text{conj}_R -) \in \text{WF_REL_PREDICATE_PAIR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$ $\bullet \text{pair-}u.1 \text{ conj}_R \text{pair-}u.2$ $= \text{pair-}u.1 \wedge_R \text{pair-}u.2)$
var_R	$\vdash \text{var}_R \in \text{WF_Var}_{R_EndR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a_n : \text{WF_Var}_{R_EndR}$ $\bullet \text{var}_R a_n = \exists_{-R} (\{a_n.2\}, \Pi_R a_n.1))$
end_R	$\vdash \text{end}_R \in \text{WF_Var}_{R_EndR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a_n : \text{WF_Var}_{R_EndR}$ $\bullet \text{end}_R a_n = \exists_{-R} (\{\text{dash } a_n.2\}, \Pi_R a_n.1))$
$- +_R -$	$\vdash (- +_R -) \in \text{WF_Extend_rest}_R \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall u_a : \text{WF_Extend_rest}_R$ $\bullet u_a.1 +_R u_a.2$ $= u_a.1 \wedge_R \Pi_R (u_a.2 \cup \{n : u_a.2 \bullet \text{dash } n\}))$
$\langle_R - \rangle_R$	$\vdash (\langle_R - \rangle_R) \in \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall u : \text{REL_PREDICATE}$ $\bullet \langle_R u \rangle_R = \forall_{-R} (u.1, u))$
$- \sqsubseteq_R -$	$\vdash (- \sqsubseteq_R -) \in \text{WF_REL_PREDICATE_PAIR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$ $\bullet \text{pair-}u.1 \sqsubseteq_R \text{pair-}u.2$ $= \langle_R \text{pair-}u.2 \Rightarrow_R \text{pair-}u.1 \rangle_R)$
monotonic	$\vdash \text{monotonic} \in \mathbb{P} (\text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE})$ $\wedge \text{monotonic}$ $= \{f : \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE}$ $ \forall u1, u2 : \text{REL_PREDICATE}$ $\bullet (u1 \sqsubseteq_R u2) \Rightarrow_R f u1 \sqsubseteq_R f u2 = \text{True}_R \emptyset\}$
Bot_R	$\vdash \text{Bot}_R \in \text{ALPHABET} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a : \text{ALPHABET} \bullet \text{Bot}_R a = \text{True}_R a)$
Top_R	$\vdash \text{Top}_R \in \text{ALPHABET} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a : \text{ALPHABET} \bullet \text{Top}_R a = \text{False}_R a)$
\cap_R	$\vdash \cap_R \in \text{WF_Glb}_{R_LubR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a_us : \text{WF_Glb}_{R_LubR}; u : \text{REL_PREDICATE}$ $\bullet u \sqsubseteq_R \cap_R a_us = \text{True}_R \emptyset$ $\Leftrightarrow u.1 = a_us.1$ $\wedge (\forall u1 : a_us.2 \bullet u \sqsubseteq_R u1 = \text{True}_R \emptyset))$
\cup_R	$\vdash \cup_R \in \text{WF_Glb}_{R_LubR} \rightarrow \text{REL_PREDICATE}$ $\wedge (\forall a_us : \text{WF_Glb}_{R_LubR}; u : \text{REL_PREDICATE}$

$$\begin{array}{ll}
\mu_R & \begin{array}{l}
\bullet \bigcup_R a_us \sqsubseteq_R u = True_R \emptyset \\
\Leftrightarrow u.1 = a_us.1 \\
\wedge (\forall u1 : a_us.2 \bullet u1 \sqsubseteq_R u = True_R \emptyset)) \\
\vdash \mu_R \in REL_FUNCTION \rightarrow REL_PREDICATE \\
\wedge (\forall f : REL_FUNCTION \\
\bullet \exists a : ALPHABET \\
| \forall u_dom : dom\ f \bullet a = u_dom.1 \\
\bullet \mu_R f \\
= \bigcap_R \\
(a, \\
\{u : REL_PREDICATE \\
| a = u.1 \wedge f\ u \sqsubseteq_R u = True_R \emptyset\}))
\end{array} \\
\nu_R & \begin{array}{l}
\vdash \nu_R \in REL_FUNCTION \rightarrow REL_PREDICATE \\
\wedge (\forall f : REL_FUNCTION \\
\bullet \exists a : ALPHABET \\
| \forall u_dom : dom\ f \bullet a = u_dom.1 \\
\bullet \nu_R f \\
= \bigcup_R \\
(a, \\
\{u : REL_PREDICATE \\
| a = u.1 \wedge u \sqsubseteq_R f\ u = True_R \emptyset\}))
\end{array} \\
varT_R & \begin{array}{l}
\vdash varT_R \in WF_VarT_{R_EndT_R} \rightarrow REL_PREDICATE \\
\wedge (\forall a_n_t : WF_VarT_{R_EndT_R} \\
\bullet varT_R\ a_n_t \\
= \bigcap_R \\
(a_n_t.1 \setminus \{a_n_t.2\}, \\
\{v : VALUE \\
| v \in (Set\ \sim)\ a_n_t.3 \\
\bullet var_R(a_n_t.1, a_n_t.2) \\
;_R Assign_R \\
(a_n_t.1, \\
\langle a_n_t.2 \rangle, \\
\langle Val\ v \rangle\}))
\end{array} \\
endT_R & \begin{array}{l}
\vdash endT_R \in WF_VarT_{R_EndT_R} \rightarrow REL_PREDICATE \\
\wedge (\forall a_n_t : WF_VarT_{R_EndT_R} \\
\bullet endT_R\ a_n_t \\
= \bigcap_R \\
(a_n_t.1 \setminus \{dash\ a_n_t.2\},
\end{array}
\end{array}$$

$$\begin{aligned}
& \{v : \text{VALUE} \\
& \quad | v \in (\text{Set } \sim) \ a_n_t.3 \\
& \quad \bullet \text{Assign}_R \\
& \quad \quad (a_n_t.1, \langle a_n_t.2 \rangle, \langle \text{Val } v \rangle) \\
& \quad \quad ;_R \text{end}_R (a_n_t.1, a_n_t.2)\}
\end{aligned}$$

REL_Semi_R-idem_thm

$$\begin{aligned}
& \vdash \forall a1, a2 : \text{ALPHABET}; n1, n2 : \text{NAME} \\
& \quad | (a1, n1, \text{Val } (\text{Bool false})) \in \text{WF_Equals}_R \\
& \quad \wedge n1 \in \text{undashed} \\
& \quad \wedge n2 \in \text{undashed} \\
& \quad \wedge (a2, \\
& \quad \quad \text{Rel } ((- \leq_R -), \text{Var } n2, \text{Var } (\text{dash } n2)), \\
& \quad \quad \text{Val } (\text{Bool true})) \\
& \quad \in \text{WF_ALPHABET_EXPRESSION} \\
& \quad \wedge a2 \subseteq a1 \\
& \quad \wedge (=_R (a1, n1, \text{Val } (\text{Bool false})) \\
& \quad \quad \wedge_R =_{+R} \\
& \quad \quad (a2, \\
& \quad \quad \quad \text{Rel} \\
& \quad \quad \quad ((- \leq_R -), \\
& \quad \quad \quad \text{Var } n2, \\
& \quad \quad \quad \text{Var } (\text{dash } n2)), \\
& \quad \quad \quad \text{Val } (\text{Bool true})), \\
& \quad \quad =_R (a1, n1, \text{Val } (\text{Bool false})) \\
& \quad \quad \wedge_R =_{+R} \\
& \quad \quad (a2, \\
& \quad \quad \quad \text{Rel} \\
& \quad \quad \quad ((- \leq_R -), \\
& \quad \quad \quad \text{Var } n2, \\
& \quad \quad \quad \text{Var } (\text{dash } n2)), \\
& \quad \quad \quad \text{Val } (\text{Bool true}))) \\
& \quad \in \text{WF_Semi}_R \\
& \bullet (=_R (a1, n1, \text{Val } (\text{Bool false})) \\
& \quad \wedge_R =_{+R} \\
& \quad (a2, \\
& \quad \quad \text{Rel} \\
& \quad \quad ((- \leq_R -), \\
& \quad \quad \text{Var } n2,
\end{aligned}$$

$$\begin{aligned}
& \text{Var } (\text{dash } n2)), \\
& \text{Val } (\text{Bool true})) \\
& ;_R =_R (a1, n1, \text{Val } (\text{Bool false})) \\
& \wedge_R =_{+R} \\
& (a2, \\
& \text{Rel} \\
& ((- \leq_R -), \\
& \text{Var } n2, \\
& \text{Var } (\text{dash } n2)), \\
& \text{Val } (\text{Bool true})) \\
& = =_R (a1, n1, \text{Val } (\text{Bool false})) \\
& \wedge_R =_{+R} \\
& (a2, \\
& \text{Rel } ((- \leq_R -), \text{Var } n2, \text{Var } (\text{dash } n2)), \\
& \text{Val } (\text{Bool true}))
\end{aligned}$$

REL- $\triangleleft_R \triangleright_R \vee_R$ -reach_thm

$$\begin{aligned}
& \vdash \forall u1, u2, b1, b2 : \text{REL_PREDICATE} \\
& \quad | (u1, b2, u2) \in \text{WF_Cond}_R \\
& \quad \wedge (u1, b1 \vee_R b2, u2) \in \text{WF_Cond}_R \\
& \bullet u1 \triangleleft_R b1 \triangleright_R u1 \triangleleft_R b2 \triangleright_R u2 \\
& \quad = u1 \triangleleft_R b1 \vee_R b2 \triangleright_R u2
\end{aligned}$$

REL- $\triangleleft_R \triangleright_R \wedge_R$ -interchange_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1, b2 : \text{REL_PREDICATE} \\
& \quad | (u1 \wedge_R u2, b1, u3 \wedge_R u4) \in \text{WF_Cond}_R \\
& \quad \wedge (u1, b1, u3) \in \text{WF_Cond}_R \\
& \quad \wedge (u2, b2, u4) \in \text{WF_Cond}_R \\
& \bullet (u1 \wedge_R u2) \triangleleft_R b1 \triangleright_R u3 \wedge_R u4 \\
& \quad = (u1 \triangleleft_R b1 \triangleright_R u3) \wedge_R u2 \triangleleft_R b2 \triangleright_R u4
\end{aligned}$$

REL- $\triangleleft_R \triangleright_R \vee_R$ -interchange_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1, b2 : \text{REL_PREDICATE} \\
& \quad | (u1 \vee_R u2, b1, u3 \vee_R u4) \in \text{WF_Cond}_R \\
& \quad \wedge (u1, b1, u3) \in \text{WF_Cond}_R \\
& \quad \wedge (u2, b2, u4) \in \text{WF_Cond}_R \\
& \bullet (u1 \vee_R u2) \triangleleft_R b1 \triangleright_R u3 \vee_R u4 \\
& \quad = (u1 \triangleleft_R b1 \triangleright_R u3) \vee_R u2 \triangleleft_R b2 \triangleright_R u4
\end{aligned}$$

REL- $\triangleleft_R \triangleright_R \Rightarrow_R$ -interchange_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1, b2 : \text{REL_PREDICATE} \\
& \quad | (u1 \Rightarrow_R u2, b1, u3 \Rightarrow_R u4) \in \text{WF_Cond}_R
\end{aligned}$$

$$\begin{aligned}
& \wedge (u1, b1, u3) \in WF_Cond_R \\
& \wedge (u2, b2, u4) \in WF_Cond_R \\
& \bullet (u1 \Rightarrow_R u2) \triangleleft_R b1 \triangleright_R u3 \Rightarrow_R u4 \\
& = (u1 \triangleleft_R b1 \triangleright_R u3) \Rightarrow_R u2 \triangleleft_R b2 \triangleright_R u4
\end{aligned}$$

REL- \triangleleft_R - \triangleright_R - \Leftrightarrow_R -interchange_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1, b2 : REL_PREDICATE \\
& \quad | (u1 \Leftrightarrow_R u2, b1, u3 \Leftrightarrow_R u4) \in WF_Cond_R \\
& \quad \wedge (u1, b1, u3) \in WF_Cond_R \\
& \quad \wedge (u2, b2, u4) \in WF_Cond_R \\
& \bullet (u1 \Leftrightarrow_R u2) \triangleleft_R b1 \triangleright_R u3 \Leftrightarrow_R u4 \\
& = (u1 \triangleleft_R b1 \triangleright_R u3) \Leftrightarrow_R u2 \triangleleft_R b2 \triangleright_R u4
\end{aligned}$$

REL- \triangleleft_R - \triangleright_R -Semi_R-interchange_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1, b2 : REL_PREDICATE \\
& \quad | (u1, u2) \in WF_Semi_R \\
& \quad \wedge (u3, u4) \in WF_Semi_R \\
& \quad \wedge (u1 ;_R u2, b1, u3 ;_R u4) \in WF_Cond_R \\
& \quad \wedge (u1, b1, u3) \in WF_Cond_R \\
& \quad \wedge (u2, b2, u4) \in WF_Cond_R \\
& \bullet (u1 ;_R u2) \triangleleft_R b1 \triangleright_R u3 ;_R u4 \\
& = (u1 \triangleleft_R b1 \triangleright_R u3) ;_R u2 \triangleleft_R b2 \triangleright_R u4
\end{aligned}$$

REL- Π_R -Semi_R-unit_thm

$$\begin{aligned}
& \vdash \forall u1 : REL_PREDICATE; a : ALPHABET \\
& \quad | a \in WF_Skip_R \wedge u1.1 = a \\
& \bullet u1 ;_R \Pi_R a = u1 \wedge u1 = \Pi_R a ;_R u1
\end{aligned}$$

REL- Π_R -Semi_R-unit_thm2

$$\begin{aligned}
& \vdash \forall u1 : REL_PREDICATE; a : ALPHABET \\
& \quad | a \in WF_Skip_R \wedge (\Pi_R a, u1) \in WF_Semi_R \\
& \bullet \Pi_R a ;_R u1 = u1
\end{aligned}$$

REL-Semi_R- \triangleleft_R - \triangleright_R -left_dist_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b : REL_PREDICATE \\
& \quad | (u1, u3) \in WF_Semi_R \wedge (u1, b, u2) \in WF_Cond_R \\
& \bullet (u1 \triangleleft_R b \triangleright_R u2) ;_R u3 \\
& = (u1 ;_R u3) \triangleleft_R b \triangleright_R u2 ;_R u3
\end{aligned}$$

REL- \triangleleft_R - \triangleright_R - \cap_R -dist_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b : REL_PREDICATE \\
& \quad | (u1, b, u2) \in WF_Cond_R \wedge (u1, b, u3) \in WF_Cond_R \\
& \bullet u1 \triangleleft_R b \triangleright_R u2 \text{ intchoice}_R u3 \\
& = (u1 \triangleleft_R b \triangleright_R u2) \text{ intchoice}_R u1 \triangleleft_R b \triangleright_R u3
\end{aligned}$$

REL_Assign_R-order_idem_thm

$$\begin{aligned}
&\vdash \forall a : \text{ALPHABET}; \\
&\quad n1, n2, n3 : \text{NAME}; \\
&\quad e1, e2, e3 : \text{EXPRESSION} \\
&\quad | (a, \langle n1, n2, n3 \rangle, \langle e1, e2, e3 \rangle) \in \text{WF_Assign}_R \\
&\quad \bullet \text{Assign}_R (a, \langle n1, n2, n3 \rangle, \langle e1, e2, e3 \rangle) \\
&\quad \quad = \text{Assign}_R (a, \langle n2, n3, n1 \rangle, \langle e2, e3, e1 \rangle)
\end{aligned}$$
REL_Assign_eval_subst_thm

$$\begin{aligned}
&\vdash \forall a : \text{ALPHABET}; \\
&\quad n : \text{NAME}; \\
&\quad e : \text{EXPRESSION}; \\
&\quad f : \text{VALUE} \leftrightarrow \text{VALUE} \\
&\quad | (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_R \\
&\quad \bullet \text{Assign}_R (a, \langle n \rangle, \langle e \rangle) \\
&\quad \quad ;_R \text{Assign}_R (a, \langle n \rangle, \langle \text{Fun}_1 (f, \text{Var } n) \rangle) \\
&\quad \quad = \text{Assign}_R (a, \langle n \rangle, \langle \text{Fun}_1 (f, e) \rangle)
\end{aligned}$$
REL_Assign_R-<_R->_R-subst_thm

$$\begin{aligned}
&\vdash \forall u1, u2, b : \text{REL_PREDICATE}; \\
&\quad a : \text{ALPHABET}; \\
&\quad n : \text{NAME}; \\
&\quad e : \text{EXPRESSION} \\
&\quad | (u1, b, u2) \in \text{WF_Cond}_R \\
&\quad \quad \wedge (\text{Assign}_R (a, \langle n \rangle, \langle e \rangle), u1) \in \text{WF_Semi}_R \\
&\quad \quad \wedge (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_R \\
&\quad \quad \wedge (b, e, n) \in \text{WF_Subst}_R \\
&\quad \bullet \text{Assign}_R (a, \langle n \rangle, \langle e \rangle) ;_R u1 \triangleleft_R b \triangleright_R u2 \\
&\quad \quad = (\text{Assign}_R (a, \langle n \rangle, \langle e \rangle) ;_R u1) \triangleleft_R \\
&\quad \quad \quad /_R (b, e, n) \triangleright_R \\
&\quad \quad \quad \text{Assign}_R (a, \langle n \rangle, \langle e \rangle) ;_R u2
\end{aligned}$$
REL_<_R-Semi_R-left_dist_thm

$$\begin{aligned}
&\vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
&\quad | u1.1 = u2.1 \wedge (u1, u3) \in \text{WF_Semi}_R \\
&\quad \bullet (u1 \text{ intchoice}_R u2) ;_R u3 \\
&\quad \quad = (u1 ;_R u3) \text{ intchoice}_R u2 ;_R u3
\end{aligned}$$
REL_<_R-Semi_R-right_dist_thm

$$\begin{aligned}
&\vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
&\quad | u2.1 = u3.1 \wedge (u1, u3) \in \text{WF_Semi}_R \\
&\quad \bullet u1 ;_R u2 \text{ intchoice}_R u3
\end{aligned}$$

$$= (u1 ;_R u2) \text{ intchoice}_R u1 ;_R u3$$

REL- \cap_R - \triangleleft_R - \triangleright_R -dist_thm

$$\vdash \forall u1, u2, u3, b : \text{REL_PREDICATE}$$

$$| u1.1 = u2.1 \wedge (u2, b, u3) \in \text{WF_Cond}_R$$

- $u1 \text{ intchoice}_R u2 \triangleleft_R b \triangleright_R u3$

$$= (u1 \text{ intchoice}_R u2) \triangleleft_R b \triangleright_R u1 \text{ intchoice}_R u3$$

REL-fixed_point_thm

$$\vdash \forall F : \text{REL_FUNCTION}$$

$$| F \in \text{monotonic}$$

- $F (\mu_R F) = \mu_R F$

REL-Var_R-com_thm

$$\vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}$$

$$| (a, n1) \in \text{WF_Var}_{R_EndR}$$

$$\wedge (a, n2) \in \text{WF_Var}_{R_EndR}$$

- $\text{var}_R (a, n1) ;_R \text{var}_R (a, n2)$

$$= \text{var}_R (a, n2) ;_R \text{var}_R (a, n1)$$

REL-End_R-com_thm

$$\vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}$$

$$| (a, n1) \in \text{WF_Var}_{R_EndR}$$

$$\wedge (a, n2) \in \text{WF_Var}_{R_EndR}$$

- $\text{end}_R (a, n1) ;_R \text{end}_R (a, n2)$

$$= \text{end}_R (a, n2) ;_R \text{end}_R (a, n1)$$

REL-Var_R-End_R-com_thm

$$\vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}$$

$$| (a, n1) \in \text{WF_Var}_{R_EndR}$$

$$\wedge (a, n2) \in \text{WF_Var}_{R_EndR}$$

$$\wedge n1 \neq n2$$

- $\text{var}_R (a, n1) ;_R \text{end}_R (a, n2)$

$$= \text{end}_R (a, n2) ;_R \text{var}_R (a, n1)$$

REL-Var_R-initial_value_thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$$

$$| (a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{WF_Assign}_R$$

$$\wedge (a, n1) \in \text{WF_Var}_{R_EndR}$$

- $\text{var}_R (a, n)$

$$= \bigcap_R$$

$$(a,$$

$$\{v : \text{VALUE}$$

- $\text{var}_R (a, n)$

$$;_R \text{Assign}_R (a, \langle n \rangle, \langle \text{Val } v \rangle))\}})$$

REL_Var_R-<_R->_R-dist.thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; u1, u2, b : \text{REL_PREDICATE}$$

$$\mid (a, n) \in \text{WF_Var}_{R_EndR}$$

$$\wedge (u1, b, u2) \in \text{WF_Cond}_R$$

$$\wedge (\text{var}_R (a, n), u1) \in \text{WF_Semi}_R$$

$$\wedge (\text{var}_R (a, n), u2) \in \text{WF_Semi}_R$$

$$\wedge n \notin b.1$$

- $\text{var}_R (a, n) ;_R u1 <_R b >_R u2$
 $= (\text{var}_R (a, n) ;_R u1) <_R$
 $b >_R$
 $\text{var}_R (a, n) ;_R u2$

REL_End_R-<_R->_R-dist.thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; u1, u2, b : \text{REL_PREDICATE}$$

$$\mid (a, n) \in \text{WF_Var}_{R_EndR}$$

$$\wedge (u1, b, u2) \in \text{WF_Cond}_R$$

$$\wedge (\text{end}_R (a, n), u1) \in \text{WF_Semi}_R$$

$$\wedge (\text{end}_R (a, n), u2) \in \text{WF_Semi}_R$$

$$\wedge n \notin b.1$$

- $\text{end}_R (a, n) ;_R u1 <_R b >_R u2$
 $= (\text{end}_R (a, n) ;_R u1) <_R$
 $b >_R$
 $\text{end}_R (a, n) ;_R u2$

REL_Var_R-End_R-idem.thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$$

$$\mid (a, n) \in \text{WF_Var}_{R_EndR}$$

- $\text{var}_R (a, n) ;_R \text{end}_R (a, n) = \Pi_R a$

REL_End_R-Var_R-Assign_R-idem.thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION}$$

$$\mid (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_R \wedge n \notin \text{FV } e$$

- $\text{end}_R (a, n)$
 $;_R \text{var}_R (a, n) ;_R \text{Assign}_R (a, \langle n \rangle, \langle e \rangle)$
 $= \text{Assign}_R (a, \langle n \rangle, \langle e \rangle)$

REL_Assign_R-End_R-idem.thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION}$$

$$\mid (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_R$$

- $\text{Assign}_R (a, \langle n \rangle, \langle e \rangle) ;_R \text{end}_R (a, n)$
 $= \text{end}_R (a, n)$

REL_Extend_R-Var_R-dist_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; u1, u2 : \text{REL_PREDICATE} \\
& \quad | (a, n) \in \text{WF_Var}_{R_EndR} \\
& \quad \wedge (u1 +_R \{n\}, u2) \in \text{WF_Semi}_R \\
& \quad \wedge \{n, \text{dash } n\} \cap u1.1 = \emptyset \\
& \quad \bullet \text{var}_R(a, n) \\
& \quad \quad ;_R (u1 +_R \{n\}) ;_R u2 ;_R \text{end}_R(a, n) \\
& \quad = u1 ;_R \text{var}_R(a, n) ;_R u2 ;_R \text{end}_R(a, n)
\end{aligned}$$

REL_End_R-Extend_R-dist_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; u1, u2 : \text{REL_PREDICATE} \\
& \quad | (a, n) \in \text{WF_Var}_{R_EndR} \\
& \quad \wedge (u1, u2 +_R \{n\}) \in \text{WF_Semi}_R \\
& \quad \wedge \{n, \text{dash } n\} \cap u2.1 = \emptyset \\
& \quad \bullet \text{var}_R(a, n) \\
& \quad \quad ;_R u1 ;_R (u2 +_R \{n\}) ;_R \text{end}_R(a, n) \\
& \quad = \text{var}_R(a, n) ;_R u1 ;_R \text{end}_R(a, n) ;_R u2
\end{aligned}$$

REL- \bigcup_R - \bigcap_R -dist_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; \\
& \quad us : \mathbb{P} \text{REL_PREDICATE}; \\
& \quad u : \text{REL_PREDICATE} \\
& \quad | (a, us) \in \text{UNIFORM_REL_PREDICATE_SET} \wedge u.1 = a \\
& \quad \bullet \bigcup_R(a, us) \text{intchoice}_R u \\
& \quad = \bigcup_R(a, \{u1 : us \bullet u1 \text{intchoice}_R u\})
\end{aligned}$$

REL- \bigcap_R - \bigcup_R -dist_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; \\
& \quad us : \mathbb{P} \text{REL_PREDICATE}; \\
& \quad u : \text{REL_PREDICATE} \\
& \quad | (a, us) \in \text{UNIFORM_REL_PREDICATE_SET} \wedge u.1 = a \\
& \quad \bullet \bigcap_R(a, us) \text{conj}_R u \\
& \quad = \bigcap_R(a, \{u1 : us \bullet u1 \text{conj}_R u\})
\end{aligned}$$

REL_Semi_R-left-univ-disj_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; \\
& \quad us : \mathbb{P} \text{REL_PREDICATE}; \\
& \quad u : \text{REL_PREDICATE} \\
& \quad | (a, us) \in \text{UNIFORM_REL_PREDICATE_SET} \\
& \quad \wedge (\bigcap_R(a, us), u) \in \text{WF_Semi}_R \\
& \quad \bullet \bigcap_R(a, us) ;_R u = \bigcap_R(a, \{u1 : us \bullet u1 ;_R u\})
\end{aligned}$$

REL_Semi_R-right-univ-disj_thm

$\vdash \forall a : \text{ALPHABET};$
 $\quad us : \mathbb{P} \text{ REL_PREDICATE};$
 $\quad u : \text{REL_PREDICATE}$
 $\quad | (a, us) \in \text{UNIFORM_REL_PREDICATE_SET}$
 $\quad \wedge (u, \bigcap_R (a, us)) \in \text{WF_Semi}_R$
 $\quad \bullet u ;_R \bigcap_R (a, us) = \bigcap_R (a, \{u1 : us \bullet u ;_R u1\})$
REL- \forall_R -idem.thm
 $\vdash \forall a : \text{ALPHABET}; d : \text{REL_PREDICATE}$
 $\quad \bullet \forall_R (a, \forall_R (a, u)) = \forall_R (a, u)$
REL- \exists_R -idem.thm
 $\vdash \forall n : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad \bullet \exists_R (a, \exists_R (a, u)) = \exists_R (a, u)$
REL- \neg_R - \forall_R -deMorgans.thm
 $\vdash \forall n : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad \bullet \neg_R (\forall_R (a, u)) = \exists_R (a, \neg_R u)$
REL- \neg_R - \exists_R -deMorgans.thm
 $\vdash \forall n : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad \bullet \neg_R (\exists_R (a, u)) = \forall_R (a, \neg_R u)$
REL- \forall_R -comm.thm
 $\vdash \forall a1, a2 : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad | a1 \cap a2 = \emptyset$
 $\quad \bullet \forall_R (a1, \forall_R (a2, u)) = \forall_R (a2, \forall_R (a1, u))$
REL- \exists_R -comm.thm
 $\vdash \forall a1, a2 : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad | a1 \cap a2 = \emptyset$
 $\quad \bullet \exists_R (a1, \exists_R (a2, u)) = \exists_R (a2, \exists_R (a1, u))$
REL- \neg_R - \triangleleft_R - \triangleright_R .thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \bullet \neg_R (u1 \triangleleft_R b \triangleright_R u2) = \neg_R u1 \triangleleft_R b \triangleright_R \neg_R u2$
REL- \triangleleft_R - \triangleright_R - \wedge_R - \neg_R - \triangleleft_R - \triangleright_R .thm
 $\vdash \forall u1, u2, u3, u4, b : \text{REL_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \wedge (u3, b, u4) \in \text{WF_Cond}_R$
 $\quad \wedge (u1 \wedge_R \neg_R u3, b, u1 \wedge_R \neg_R u4) \in \text{WF_Cond}_R$
 $\quad \bullet (u1 \triangleleft_R b \triangleright_R u2) \wedge_R \neg_R (u3 \triangleleft_R b \triangleright_R u4)$
 $\quad = (u1 \wedge_R \neg_R u3) \triangleleft_R b \triangleright_R u1 \wedge_R \neg_R u4$
REL- \Rightarrow_R - \triangleleft_R - \triangleright_R -dist.thm

$\vdash \forall u1, u2, u3, b : REL_PREDICATE$
 $\quad | (u2, b, u3) \in WF_Cond_R$
 $\quad \bullet u1 \Rightarrow_R u2 \triangleleft_R b \triangleright_R u3$
 $\quad = (u1 \Rightarrow_R u2) \triangleleft_R b \triangleright_R u1 \Rightarrow_R u3$
REL- $\triangleleft_R \triangleright_R \Rightarrow_R$ -dist_thm

$\vdash \forall u1, u2, u3, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet (u1 \triangleleft_R b \triangleright_R u2) \Rightarrow_R u3$
 $\quad = (u1 \Rightarrow_R u3) \triangleleft_R b \triangleright_R u2 \Rightarrow_R u3$
REL- $\triangleleft_R \triangleright_R \wedge_R$ -dist_thm

$\vdash \forall u1, u2, u3, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet (u1 \triangleleft_R b \triangleright_R u2) \wedge_R u3$
 $\quad = (u1 \wedge_R u3) \triangleleft_R b \triangleright_R u2 \wedge_R u3$
REL- $\triangleleft_R \triangleright_R \vee_R$ -dist_thm

$\vdash \forall u1, u2, u3, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet (u1 \triangleleft_R b \triangleright_R u2) \vee_R u3$
 $\quad = (u1 \vee_R u3) \triangleleft_R b \triangleright_R u2 \vee_R u3$
REL- $\triangleleft_R \triangleright_R$ -know_cond_thm

$\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet b \wedge_R u1 \triangleleft_R b \triangleright_R u2 = b \wedge_R u1$
REL- $\triangleleft_R \triangleright_R$ -know_cond_thm1

$\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet \neg_R b \wedge_R u1 \triangleleft_R b \triangleright_R u2 = \neg_R b \wedge_R u2$
REL- $\triangleleft_R \triangleright_R$ -ass_if_cond_thm

$\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R b \triangleright_R u2 = (b \wedge_R u1) \triangleleft_R b \triangleright_R u2$
REL- $\triangleleft_R \triangleright_R$ -ass_else_cond_thm

$\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R b \triangleright_R u2 = u1 \triangleleft_R b \triangleright_R \neg_R b \wedge_R u2$
REL-Equals_R-Semi_R-left-one-point_thm

$\vdash \forall a : ALPHABET;$
 $\quad old : NAME;$

$e : \text{EXPRESSION};$
 $u : \text{REL_PREDICATE}$
 $| \text{ old } \in \text{undashed}$
 $\wedge (a, \text{dash old}, e) \in \text{WF_Equals}_R$
 $\wedge (u, e, \text{old}) \in \text{WF_Subst}_R$
 $\wedge (=_R (a, \text{dash old}, e), u) \in \text{WF_Semi}_R$
 $\bullet =_R (a, \text{dash old}, e) ;_R u = /_R (u, e, \text{old})$
REL_Equals_R-Semi_R-right-one-point-thm
 $\vdash \forall a : \text{ALPHABET};$
 $\text{old} : \text{NAME};$
 $e : \text{EXPRESSION};$
 $u : \text{REL_PREDICATE}$
 $| \text{ old } \in \text{undashed}$
 $\wedge (a, \text{old}, e) \in \text{WF_Equals}_R$
 $\wedge (u, e, \text{dash old}) \in \text{WF_Subst}_R$
 $\wedge (u, =_R (a, \text{old}, e)) \in \text{WF_Semi}_R$
 $\bullet u ;_R =_R (a, \text{old}, e) = /_R (u, e, \text{dash old})$

8.5 Definitions

NAME $\vdash \text{NAME} = \mathbb{U}$
ALPHABET $\vdash \text{ALPHABET} = \mathbb{P} \text{NAME}$
VALUE $\vdash \text{VALUE} = \mathbb{U}$
INT_VAL $\vdash \text{INT_VAL} = \{n : \mathbb{Z} \bullet \text{Int } n\}$
BOOL_VAL $\vdash \text{BOOL_VAL} = \{\text{Bool true}, \text{Bool false}\}$
CHANNEL_VAL $\vdash \text{CHANNEL_VAL} = \{n : \text{NAME} \bullet \text{Channel } n\}$
SEQ_VAL $\vdash \text{SEQ_VAL} = \{s : \text{seq VALUE} \bullet \text{Seq } s\}$
SET_VAL $\vdash \text{SET_VAL} = \{s : \mathbb{P} \text{VALUE} \bullet \text{Set } s\}$
PAIR_VAL $\vdash \text{PAIR_VAL} = \{v1, v2 : \text{VALUE} \bullet \text{Pair } (v1, v2)\}$
EVENT_VAL $\vdash \text{EVENT_VAL}$
 $= \{c : \text{CHANNEL_VAL}; v : \text{VALUE}$
 $\bullet \text{Pair } (c, v)\}$
SEQ_EVENT_VAL
 $\vdash \text{SEQ_EVENT_VAL} = \{s : \text{seq EVENT_VAL} \bullet \text{Seq } s\}$
SET_EVENT_VAL
 $\vdash \text{SET_EVENT_VAL} = \{s : \mathbb{P} \text{EVENT_VAL} \bullet \text{Set } s\}$
SET_SEQ_EVENT_VAL
 $\vdash \text{SET_SEQ_EVENT_VAL}$

$$\begin{aligned}
&= \{s : \mathbb{F} \text{ SEQ_EVENT_VAL} \\
&\quad \bullet \text{ Set } s\} \\
\mathbf{PAIR_SEQ_EVENT_VAL} &\vdash \text{PAIR_SEQ_EVENT_VAL} \\
&= \{s1, s2 : \text{SEQ_EVENT_VAL} \\
&\quad \bullet \text{ Pair } (s1, s2)\} \\
\mathbf{UNARY_F} &\vdash \text{UNARY_F} = \text{VALUE} \leftrightarrow \text{VALUE} \\
\mathbf{BINARY_F} &\vdash \text{BINARY_F} = \text{VALUE} \times \text{VALUE} \leftrightarrow \text{VALUE} \\
\mathbf{RELATION} &\vdash \text{RELATION} = \text{VALUE} \leftrightarrow \text{VALUE} \\
\mathbf{EXPRESSION} &\vdash \text{EXPRESSION} = \mathbb{U} \\
\mathbf{BINDING} &\vdash \text{BINDING} = \text{NAME} \leftrightarrow \text{VALUE} \\
\mathbf{BINDINGS} &\vdash \text{BINDINGS} = \mathbb{P} \text{ BINDING} \\
\mathbf{WF_BINDING_EXPRESSION}_R &\vdash \text{WF_BINDING_EXPRESSION}_R \\
&= \{b : \text{BINDING}; e : \text{EXPRESSION} \\
&\quad | \text{FV } e \subseteq \text{dom } b\} \\
\mathbf{REL_PREDICATE} &\vdash \text{REL_PREDICATE} \\
&= \{a : \text{ALPHABET}; bs : \text{BINDINGS} \\
&\quad | \forall b : bs \bullet \text{dom } b = a\} \\
\mathbf{WF_Equals}_R &\vdash \text{WF_Equals}_R \\
&= \{a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
&\quad | n \in a \wedge \text{FV } e \subseteq a\} \\
\mathbf{WF_ALPHABET_EXPRESSION} &\vdash \text{WF_ALPHABET_EXPRESSION} \\
&= \{a : \text{ALPHABET}; e1, e2 : \text{EXPRESSION} \\
&\quad | \text{FV } e1 \cup \text{FV } e2 \subseteq a\} \\
\mathbf{WF_Cond}_R &\vdash \text{WF_Cond}_R \\
&= \{u1, b, u2 : \text{REL_PREDICATE} \\
&\quad | b.1 \subseteq u1.1 \wedge u1.1 = u2.1\} \\
\mathbf{WF_Semi}_R &\vdash \text{WF_Semi}_R \\
&= \{u1, u2 : \text{REL_PREDICATE} \\
&\quad | (u1.1, u2.1) \in \text{composable}\} \\
\mathbf{WF_Skip}_R &\vdash \text{WF_Skip}_R = \{a : \text{ALPHABET} \mid a \in \text{homogeneous}\} \\
\mathbf{WF_Assign}_R &\vdash \text{WF_Assign}_R \\
&= \{a : \text{ALPHABET}; \\
&\quad ns : \text{seq NAME}; \\
&\quad exps : \text{seq EXPRESSION}
\end{aligned}$$

$$\begin{aligned}
& | (\forall n : \text{ran } ns \bullet n \in a \wedge n \in \text{undashed}) \\
& \wedge (\forall e : \text{ran } \text{exps} \bullet FV \ e \subseteq a \wedge FV \ e \subseteq \text{undashed}) \\
& \wedge (\# \ ns = \# \ \text{exps} \\
& \wedge \# \ \text{exps} \neq 0) \\
& \wedge a \in \text{homogeneous}\} \\
\mathbf{WF_Subst}_R & \vdash WF_Subst_R \\
& = \{u : REL_PREDICATE; e : EXPRESSION; n : NAME \\
& \quad | FV \ e \subseteq u.1 \wedge n \in u.1\} \\
\mathbf{WF_Var}_{R_EndR} & \vdash WF_Var_{R_EndR} \\
& = \{a : ALPHABET; n : NAME \\
& \quad | a \in \text{homogeneous} \\
& \quad \wedge n \in \text{undashed} \\
& \quad \wedge \{n, \text{dash } n\} \subseteq a\} \\
\mathbf{WF_VarT}_{R_EndTR} & \vdash WF_VarT_{R_EndTR} \\
& = \{a : ALPHABET; n : NAME; T : SET_VAL \\
& \quad | (a, n) \in WF_Var_{R_EndR}\} \\
\mathbf{WF_Extend_rest}_R & \vdash WF_Extend_rest_R \\
& = \{u : REL_PREDICATE; a : ALPHABET \\
& \quad | a \subseteq \text{undashed} \\
& \quad \wedge (a \cup \{n : a \bullet \text{dash } n\}) \cap u.1 = \emptyset\} \\
\mathbf{WF_REL_PREDICATE_PAIR} & \vdash WF_REL_PREDICATE_PAIR \\
& = \{u1, u2 : REL_PREDICATE \\
& \quad | u1.1 = u2.1\} \\
\mathbf{WF_Glb}_{R_LubR} & \vdash WF_Glb_{R_LubR} \\
& = \{a : ALPHABET; us : \mathbb{P} \ REL_PREDICATE \\
& \quad | \forall u_us : us \bullet u_us.1 = a\} \\
\mathbf{REL_FUNCTION} & \vdash REL_FUNCTION \\
& = \{f : REL_PREDICATE \leftrightarrow REL_PREDICATE \\
& \quad | \exists a : ALPHABET \\
& \quad \bullet \forall u_dom : dom \ f; u_ran : ran \ f \\
& \quad \bullet a = u_dom.1 \wedge u_dom.1 = u_ran.1\}
\end{aligned}$$

8.6 Theorems

ALPHABET_∪_thm

$$\vdash \forall a1, a2 : ALPHABET \bullet a1 \cup a2 \in ALPHABET$$

ALPHABET_∩_thm

$$\vdash \forall a1, a2 : ALPHABET \bullet a1 \cap a2 \in ALPHABET$$

ALPHABET_Sub_thm

$$\vdash \forall a1, a2 : ALPHABET \bullet a1 \setminus a2 \in ALPHABET$$

ALPHABET_⊆_thm

$$\vdash \forall a1, a2 : ALPHABET \\ \bullet a1 \subseteq a2 \Rightarrow a1 \cup a2 = a2 \cup a1 \wedge a2 \cup a1 = a2$$

ALPHABET_name_thm

$$\vdash \forall n : NAME \bullet \{n\} \in ALPHABET$$

ALPHABET_dash_thm

$$\vdash \forall n : NAME \bullet \{\text{dash } n\} \in ALPHABET$$

ALPHABET_dash_thm1

$$\vdash \forall n : NAME \bullet \text{dash } n \in NAME$$

ALPHABET_dash_∈_dashed_thm

$$\vdash \forall n : NAME \bullet \text{dash } n \in \text{dashed}$$

ALPHABET_dash_∉_undashed_thm

$$\vdash \forall n : NAME \bullet \text{dash } n \notin \text{undashed}$$

ALPHABET_dashed_∨_undashed_thm

$$\vdash \forall n : NAME \bullet \neg n \in \text{undashed} \Rightarrow n \in \text{dashed}$$

ALPHABET_dashed_∨_undashed_thm1

$$\vdash \forall n : NAME \bullet n \in \text{undashed} \Rightarrow \neg n \in \text{dashed}$$

ALPHABET_dashed_∨_undashed_thm2

$$\vdash \forall n : NAME \bullet \neg n \in \text{dashed} \Rightarrow n \in \text{undashed}$$

ALPHABET_dashed_∨_undashed_thm3

$$\vdash \forall n : NAME \bullet n \in \text{dashed} \Rightarrow \neg n \in \text{undashed}$$

ALPHABET_undashed_∩_dashed_∅_thm

$$\vdash \text{undashed} \cap \text{dashed} = \emptyset$$

ALPHABET_in_a_∪_out_a_thm

$$\vdash \forall a : ALPHABET \bullet \text{in}_a a \cup \text{out}_a a = a$$

ALPHABET_in_a_∩_out_a_thm

$$\vdash \forall a : ALPHABET \bullet \text{in}_a a \cap \text{out}_a a = \emptyset$$

ALPHABET_out_a_dist_in_a_∪_out_a_eq_out_a_thm

$$\vdash \forall a1, a2 : ALPHABET \\ \bullet \text{out}_a (\text{in}_a a1 \cup \text{out}_a a2) = \text{out}_a a2$$

ALPHABET_in_a_dist_in_a_∪_out_a_eq_in_a_thm

$\vdash \forall a1, a2 : \text{ALPHABET}$

- $\text{in}_a (\text{in}_a a1 \cup \text{out}_a a2) = \text{in}_a a1$

ALPHABET_in_∪_out_left_dist_thm

$\vdash \forall a1, a2, a3 : \text{ALPHABET}$

- $\text{in}_a (a1 \cup a2) \cup \text{out}_a a3$
 $= \text{in}_a a1 \cup \text{out}_a a3 \cup (\text{in}_a a2 \cup \text{out}_a a3)$

ALPHABET_hom_⇒_comp_thm

$\vdash \forall a : \text{ALPHABET}$

- $a \in \text{homogeneous} \Rightarrow (a, a) \in \text{composable}$

ALPHABET_comp_⇒_hom_thm

$\vdash \forall a : \text{ALPHABET}$

- $(a, a) \in \text{composable} \Rightarrow a \in \text{homogeneous}$

ALPHABET_comp_⇔_hom_thm

$\vdash \forall a : \text{ALPHABET}$

- $(a, a) \in \text{composable} \Leftrightarrow a \in \text{homogeneous}$

ALPHABET_homogeneous_∪_thm

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$

- $a \in \text{homogeneous} \wedge n \in \text{undashed}$
 $\bullet \{n, \text{dash } n\} \cup a \in \text{homogeneous}$

ALPHABET_homogeneous_set_dif_thm

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$

- $a \in \text{homogeneous} \wedge n \in \text{undashed}$
 $\bullet a \setminus \{n, \text{dash } n\} \in \text{homogeneous}$

ALPHABET_homogeneous_dash_thm

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$

- $a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge n \in a$
 $\bullet \text{dash } n \in a$

ALPHABET_comp_⇒_contr_comp_thm

$\vdash \forall a1, a2 : \text{ALPHABET}; n : \text{NAME}$

- $n \in a1 \wedge n \in \text{undashed} \wedge (a1, a2) \in \text{composable}$
 $\bullet (a1 \setminus \{n\}, a2) \in \text{composable}$

ALPHABET_comp_⇒_contr_comp_thm1

$\vdash \forall a1, a2 : \text{ALPHABET}; a : \text{ALPHABET}$

- $a \cap \text{dashed} = \emptyset \wedge (a1, a2) \in \text{composable}$
 $\bullet (a1 \setminus a, a2) \in \text{composable}$

ALPHABET_comp_⇒_contr_comp_thm2

$\vdash \forall a1, a2 : \text{ALPHABET}; a : \text{ALPHABET}$

$| a \cap \text{undashed} = \emptyset \wedge (a1, a2) \in \text{composable}$
 $\bullet (a1, a2 \setminus a) \in \text{composable}$
ALPHABET_a1_a2_a3_composable_thm
 $\vdash \forall a1, a2, a3 : \text{ALPHABET}$
 $| (a1, a2) \in \text{composable} \wedge (a2, a3) \in \text{composable}$
 $\bullet (\text{in}_a a1 \cup \text{out}_a a2, a3) \in \text{composable}$
ALPHABET_a1_a2_a3_composable_thm1
 $\vdash \forall a1, a2, a3 : \text{ALPHABET}$
 $| (a1, a2) \in \text{composable} \wedge (a2, a3) \in \text{composable}$
 $\bullet (a1, \text{in}_a a2 \cup \text{out}_a a3) \in \text{composable}$
ALPHABET_∪_composable_thm
 $\vdash \forall a1, a2, a3 : \text{ALPHABET}$
 $| (a1, a3) \in \text{composable} \wedge (a2, a3) \in \text{composable}$
 $\bullet (a1 \cup a2, a3) \in \text{composable}$
ALPHABET_∪_composable_thm1
 $\vdash \forall a1, a2, a3 : \text{ALPHABET}$
 $| (a1, a2) \in \text{composable} \wedge (a1, a3) \in \text{composable}$
 $\bullet (a1, a2 \cup a3) \in \text{composable}$
ALPHABET_∈_set_diff_thm
 $\vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}$
 $| n1 \neq n2$
 $\wedge a \in \text{homogeneous}$
 $\wedge \{n1, n2\} \subseteq \text{undashed}$
 $\wedge \{n2, \text{dash } n2\} \subseteq a$
 $\bullet n2 \in a \setminus \{n1, \text{dash } n1\}$
ALPHABET_∈_set_diff_thm1
 $\vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}$
 $| n1 \neq n2$
 $\wedge a \in \text{homogeneous}$
 $\wedge \{n1, n2\} \subseteq \text{undashed}$
 $\wedge \{n2, \text{dash } n2\} \subseteq a$
 $\bullet \text{dash } n2 \in a \setminus \{n1, \text{dash } n1\}$
ALPHABET_in_thm
 $\vdash \forall a : \text{ALPHABET} \bullet \text{in}_a a \in \text{ALPHABET}$
ALPHABET_out_thm
 $\vdash \forall a : \text{ALPHABET} \bullet \text{out}_a a \in \text{ALPHABET}$
ALPHABET_out_undashed_∪_dashed_thm
 $\vdash \forall a1, a2 : \text{ALPHABET}$

$$\begin{array}{l}
| a1 \subseteq undashed \wedge a2 \subseteq dashed \\
\bullet out_a (a1 \cup a2) = a2 \\
ALPHABET_in_undashed_cup_dashed_thm \\
\vdash \forall a1, a2 : ALPHABET \\
| a1 \subseteq undashed \wedge a2 \subseteq dashed \\
\bullet in_a (a1 \cup a2) = a1 \\
ALPHABET_dash_(|_)_thm \\
\vdash \forall a : ALPHABET \bullet dash (| a |) \in ALPHABET \\
ALPHABET_subseteq_cup_(|_)_thm \\
\vdash \forall n : NAME; a : ALPHABET \\
| n \in a \wedge a \subseteq undashed \\
\bullet \{n, dash\ n\} \subseteq a \cup dash (| a |) \\
ALPHABET_dash_(|_)_subseteq_dashed_thm \\
\vdash \forall a : ALPHABET | a \subseteq undashed \bullet dash (| a |) \subseteq dashed \\
ALPHABET_(|_)_homogeneous_thm \\
\vdash \forall a : ALPHABET \\
| a \subseteq undashed \\
\bullet a \cup dash (| a |) \in homogeneous \\
ALPHABET_in_a_vee_out_a_iff_in_a_thm \\
\vdash \forall a : ALPHABET; n : NAME \\
\bullet n \in in_a\ a \vee n \in out_a\ a \iff n \in a \\
ALPHABET_out_Equals_dash_(|_)_thm \\
\vdash \forall a1, a2 : ALPHABET \\
| a2 \subseteq undashed \wedge out_a\ a1 = dash (| a2 |) \\
\bullet (a1, a2) \in composable \\
BINDINGS_thm \quad \vdash \forall bs : BINDINGS; b : bs \bullet b \in NAME \leftrightarrow VALUE \\
REL_neg_true_eq_false_thm \\
\vdash \neg Bool\ true = Bool\ false \\
REL_neg_false_eq_true_thm \\
\vdash \neg Bool\ false = Bool\ true \\
BINDINGS_neg_Bool_false_eq_Bool_true_thm \\
\vdash \forall b : BINDING \\
\bullet \neg \\
Eval (b, Val (Bool\ false)) \\
= Eval (b, Val (Bool\ true)) \\
BINDINGS_neg_Bool_true_eq_Bool_false_thm \\
\vdash \forall b : BINDING \\
\bullet \neg
\end{array}$$

$$\begin{aligned}
& \text{Eval } (b, \text{Val } (\text{Bool true})) \\
& = \text{Eval } (b, \text{Val } (\text{Bool false})) \\
\mathbf{BINDINGS_}\triangleleft_\cup_\triangleleft_\mathbf{app_thm} \\
& \vdash \forall fs, gs : \mathbf{BINDINGS}; f : fs; g : gs; n : \text{dom } f \\
& \quad | n \in \text{undashed} \\
& \quad \bullet (\text{undashed } \triangleleft f \cup \text{dashed } \triangleleft g) n = f n \\
\mathbf{BINDINGS_}\triangleleft_\cup_\triangleleft_\mathbf{dash_app_thm} \\
& \vdash \forall fs, gs : \mathbf{BINDINGS}; f : fs; g : gs; n : \mathbf{NAME} \\
& \quad | n \in \text{undashed} \wedge \text{dash } n \in \text{dom } g \\
& \quad \bullet (\text{undashed } \triangleleft f \cup \text{dashed } \triangleleft g) (\text{dash } n) = g (\text{dash } n) \\
\mathbf{BINDINGS_}\triangleleft_\mathbf{idem_thm} \\
& \vdash \forall a : \mathbf{ALPHABET}; b : \mathbf{BINDING}; n : \text{dom } b \\
& \quad | n \in a \\
& \quad \bullet (a \triangleleft b) n = b n \\
\mathbf{BINDINGS_}\triangleleft_\mathbf{idem_thm} \\
& \vdash \forall n : \mathbf{NAME}; a1, a2 : \mathbf{ALPHABET}; b : \mathbf{BINDING} \\
& \quad | n \in \text{dom } b \wedge n \in a1 \wedge a2 \cap a1 = \{\} \\
& \quad \bullet (a2 \triangleleft b) n = b n \\
\mathbf{REL_FV_Val_}\subseteq_\mathbf{a_thm} \\
& \vdash \forall a : \mathbf{ALPHABET}; v : \mathbf{VALUE} \bullet \mathbf{FV } (\text{Val } v) \subseteq a \\
\mathbf{REL_Eval_Val_thm} \\
& \vdash \forall b : \mathbf{BINDING}; v : \mathbf{VALUE} \bullet \text{Eval } (b, \text{Val } v) = v \\
\mathbf{REL_FV_Var_thm} \\
& \vdash \forall n : \mathbf{NAME} \bullet \mathbf{FV } (\text{Var } n) = \{n\} \\
\mathbf{REL_PREDICATE_}\in_\mathbf{WF_BINDING_EXPRESSION_R_thm} \\
& \vdash \forall b : \mathbf{BINDING}; n : \mathbf{NAME} \\
& \quad | n \in \text{dom } b \\
& \quad \bullet (b, \text{Var } n) \in \mathbf{WF_BINDING_EXPRESSION_R} \\
\mathbf{REL_Eval_Var_thm} \\
& \vdash \forall b : \mathbf{BINDING}; n : \mathbf{NAME} \\
& \quad | n \in \text{dom } b \\
& \quad \bullet \text{Eval } (b, \text{Var } n) = b n \\
\mathbf{REL_Var_}\in_\mathbf{EXPRESSION_thm} \\
& \vdash \forall n : \mathbf{NAME} \bullet \text{Var } n \in \mathbf{EXPRESSION} \\
\mathbf{REL_}\leq_\mathbf{R_}\in_\mathbf{EXPRESSION_thm} \\
& \vdash \forall n1, n2 : \mathbf{NAME} \\
& \quad \bullet \text{Rel } ((_ \leq_\mathbf{R} _), \text{Var } n1, \text{Var } n2) \in \mathbf{EXPRESSION} \\
\mathbf{REL_PREDICATE_thm}
\end{aligned}$$

$\vdash \forall u : \text{REL_PREDICATE} \bullet u.1 \in \text{ALPHABET} \wedge u.2 \in \text{BINDINGS}$
WF_REL_PREDICATE_PAIR_thm
 $\vdash \forall \text{pair_}u : \text{WF_REL_PREDICATE_PAIR}$
 $\bullet \text{pair_}u.1 \in \text{REL_PREDICATE}$
 $\wedge \text{pair_}u.2 \in \text{REL_PREDICATE}$
 $\wedge \text{pair_}u.1.1 = \text{pair_}u.2.1$
 $\wedge (\text{pair_}u.2, \text{pair_}u.1) \in \text{WF_REL_PREDICATE_PAIR}$
REL_∩_R-com_thm1
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\mid u1.1 = u2.1$
 $\bullet (u1, u2) \in \text{WF_REL_PREDICATE_PAIR}$
WF_Glb_R-Lub_R_thm
 $\vdash \forall a_us : \text{WF_Glb}_R\text{-Lub}_R$
 $\bullet a_us.1 \in \text{ALPHABET} \wedge a_us.2 \in \mathbb{P} \text{REL_PREDICATE}$
REL_PREDICATE_∧_R_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\bullet (u1.1 \cup u2.1, (u1 \oplus_R u2.1).2 \cap (u2 \oplus_R u1.1).2)$
 $\in \text{REL_PREDICATE}$
REL_PREDICATE_∧_R_thm1
 $\vdash \forall u1, u2 : \text{REL_PREDICATE} \bullet u1 \wedge_R u2 \in \text{REL_PREDICATE}$
REL_PREDICATE_∨_R_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\bullet (u1.1 \cup u2.1, (u1 \oplus_R u2.1).2 \cup (u2 \oplus_R u1.1).2)$
 $\in \text{REL_PREDICATE}$
REL_PREDICATE_∨_R_thm1
 $\vdash \forall u1, u2 : \text{REL_PREDICATE} \bullet u1 \vee_R u2 \in \text{REL_PREDICATE}$
REL_PREDICATE_Extend_unrest_thm
 $\vdash \forall u : \text{REL_PREDICATE}; a : \text{ALPHABET}$
 $\bullet u \oplus_R a \in \text{REL_PREDICATE}$
REL_PREDICATE_∪_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\mid u1.1 = u2.1$
 $\bullet (u1.1, u1.2 \cup u2.2) \in \text{REL_PREDICATE}$
REL_PREDICATE_∩_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\mid u1.1 = u2.1$
 $\bullet (u1.1, u1.2 \cap u2.2) \in \text{REL_PREDICATE}$
REL_PREDICATE_True_R_thm

$$\begin{aligned}
& \vdash \forall u : \text{REL_PREDICATE} \\
& \quad \bullet (u.1, \{b : \text{BINDING} \mid \text{dom } b = u.1\}) \\
& \quad \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_True}_R\textbf{-thm1} \\
& \vdash \forall a : \text{ALPHABET} \bullet \text{True}_R a \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_False}_R\textbf{-thm} \\
& \vdash \forall u : \text{REL_PREDICATE} \bullet (u.1, \{\}) \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_False}_R\textbf{-thm1} \\
& \vdash \forall a : \text{ALPHABET} \bullet \text{False}_R a \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_Equals}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
& \quad \mid (a, n, e) \in \text{WF_Equals}_R \\
& \quad \bullet =_R (a, n, e) \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_Equals_exps}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET}; e1, e2 : \text{EXPRESSION} \\
& \quad \mid (a, e1, e2) \in \text{WF_ALPHABET_EXPRESSION} \\
& \quad \bullet =_{+R} (a, e1, e2) \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_WF_Equals}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
& \quad \mid (a, n, e) \in \text{WF_Equals}_R \\
& \quad \bullet n \in a \wedge \text{FV } e \subseteq a \\
\textbf{REL_PREDICATE_}\in\textbf{-WF_Equals}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
& \quad \mid n \in a \wedge \text{FV } e \subseteq a \\
& \quad \bullet (a, n, e) \in \text{WF_Equals}_R \\
\textbf{REL_PREDICATE_WF_Equals}_R\textbf{-WF_ALPHABET_EXTENSION_thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
& \quad \bullet (a, n, e) \in \text{WF_Equals}_R \\
& \quad \Leftrightarrow (a, \text{Var } n, e) \in \text{WF_ALPHABET_EXPRESSION} \\
\textbf{REL_PREDICATE_Extend_rest_thm} \\
& \vdash \forall u : \text{REL_PREDICATE}; a : \text{ALPHABET} \\
& \quad \mid (u, a) \in \text{WF_Extend_rest}_R \\
& \quad \bullet u +_R a \in \text{REL_PREDICATE} \\
\textbf{REL_PREDICATE_}\in\textbf{-WF_ALPHABET_EXPRESSION_thm} \\
& \vdash \forall a : \text{ALPHABET}; e1, e2 : \text{EXPRESSION} \\
& \quad \mid \text{FV } e1 \cup \text{FV } e2 \subseteq a \\
& \quad \bullet (a, e1, e2) \in \text{WF_ALPHABET_EXPRESSION} \\
\textbf{REL_PREDICATE_}\in\textbf{-WF_ALPHABET_EXPRESSION_thm1}
\end{aligned}$$

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\quad | \{n, \text{dash } n\} \subseteq a$
 $\quad \bullet (a, \text{Var } n, \text{Var } (\text{dash } n)) \in \text{WF_ALPHABET_EXPRESSION}$
REL_PREDICATE_ \neg _R-thm
 $\vdash \forall u : \text{REL_PREDICATE}$
 $\quad \bullet (u.1, \{b : \text{BINDING} \mid \text{dom } b = u.1\} \setminus u.2)$
 $\quad \in \text{REL_PREDICATE}$
REL_PREDICATE_ \neg _R-thm1
 $\vdash \forall u : \text{REL_PREDICATE} \bullet \neg_R u \in \text{REL_PREDICATE}$
REL_PREDICATE_ \Rightarrow _R-thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE} \bullet u1 \Rightarrow_R u2 \in \text{REL_PREDICATE}$
REL_PREDICATE_ \Leftrightarrow _R-thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE} \bullet u1 \Leftrightarrow_R u2 \in \text{REL_PREDICATE}$
REL_PREDICATE_WF_Cond_R-thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \bullet b.1 \subseteq u1.1 \wedge u1.1 = u2.1$
REL_PREDICATE_ \in -WF_Cond_R-thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\quad | b.1 \subseteq u1.1 \wedge u1.1 = u2.1$
 $\quad \bullet (u1, b, u2) \in \text{WF_Cond}_R$
REL_PREDICATE_ \triangleleft _R \triangleright _R-thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \bullet u1 \triangleleft_R b \triangleright_R u2 \in \text{REL_PREDICATE}$
REL_PREDICATE_ \neg _R-alphabet-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet (\neg_R u).1 = u.1$
REL_PREDICATE_WF_Cond_R-clauses-thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \bullet (u1, b, \neg_R u2) \in \text{WF_Cond}_R$
 $\quad \wedge (u1, \neg_R b, u2) \in \text{WF_Cond}_R$
 $\quad \wedge (u1, \neg_R b, \neg_R u2) \in \text{WF_Cond}_R$
 $\quad \wedge (\neg_R u1, b, u2) \in \text{WF_Cond}_R$
 $\quad \wedge (\neg_R u1, b, \neg_R u2) \in \text{WF_Cond}_R$
 $\quad \wedge (\neg_R u1, \neg_R b, u2) \in \text{WF_Cond}_R$
 $\quad \wedge (\neg_R u1, \neg_R b, \neg_R u2) \in \text{WF_Cond}_R$
REL_PREDICATE_WF_Cond_R-sym-thm

$$\begin{aligned}
& \vdash \forall u1, u2, b : REL_PREDICATE \\
& \quad | (u1, b, u2) \in WF_Cond_R \\
& \quad \bullet (u2, b, u1) \in WF_Cond_R \\
REL_PREDICATE_Semi_R_thm \\
& \vdash \forall u1, u2 : REL_PREDICATE \\
& \quad | (u1, u2) \in WF_Semi_R \\
& \quad \bullet u1 ;_R u2 \in REL_PREDICATE \\
REL_PREDICATE_WF_Semi_R_thm \\
& \vdash \forall u1, u2 : REL_PREDICATE \\
& \quad | (u1, u2) \in WF_Semi_R \\
& \quad \bullet (u1.1, u2.1) \in composable \\
REL_PREDICATE_in_WF_Semi_R_thm \\
& \vdash \forall u1, u2 : REL_PREDICATE \\
& \quad | (u1.1, u2.1) \in composable \\
& \quad \bullet (u1, u2) \in WF_Semi_R \\
REL_PREDICATE_Pi_R_thm \\
& \vdash \forall a : ALPHABET \\
& \quad | a \in WF_Skip_R \\
& \quad \bullet \Pi_R a \in REL_PREDICATE \\
REL_PREDICATE_WF_Skip_R_thm \\
& \vdash \forall a : ALPHABET \mid a \in WF_Skip_R \bullet a \in homogeneous \\
REL_PREDICATE_in_WF_Skip_R_thm \\
& \vdash \forall a : ALPHABET \mid a \in homogeneous \bullet a \in WF_Skip_R \\
REL_PREDICATE_WF_Assign_R_thm \\
& \vdash \forall a : ALPHABET; ns : seq NAME; exps : seq EXPRESSION \\
& \quad | (a, ns, exps) \in WF_Assign_R \\
& \quad \bullet (\forall n : ran ns \bullet n \in a \wedge n \in undashed) \\
& \quad \wedge (\forall e : ran exps \bullet FV e \subseteq a \wedge FV e \subseteq undashed) \\
& \quad \wedge (\# ns = \# exps \\
& \quad \wedge \# exps \neq 0) \\
& \quad \wedge a \in homogeneous \\
REL_PREDICATE_Contract_R_thm \\
& \vdash \forall u : REL_PREDICATE; a : ALPHABET \\
& \quad \bullet u \text{---}_R a \in REL_PREDICATE \\
REL_PREDICATE_exists_Sub_R_thm \\
& \vdash \forall a : ALPHABET; u : REL_PREDICATE \\
& \quad \bullet \exists_R (a, u) \in REL_PREDICATE \\
REL_PREDICATE_exists_R_thm
\end{aligned}$$

$\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet \exists_R (a, u) \in \text{REL_PREDICATE}$
REL_PREDICATE_True_R-alphabet-thm
 $\vdash \forall a : \text{ALPHABET} \bullet (\text{True}_R a).1 = a$
REL_PREDICATE_False_R-alphabet-thm
 $\vdash \forall a : \text{ALPHABET} \bullet (\text{False}_R a).1 = a$
REL_PREDICATE_∀-Sub_R-thm
 $\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet \forall_{-R} (a, u) \in \text{REL_PREDICATE}$
REL_PREDICATE_∀_R-thm
 $\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet \forall_R (a, u) \in \text{REL_PREDICATE}$
REL_PREDICATE_Subst_R-thm
 $\vdash \forall u : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid (u, e, n) \in \text{WF_Subst}_R$
 $\bullet /_R (u, e, n) \in \text{REL_PREDICATE}$
REL_PREDICATE_WF_Subst_R-thm
 $\vdash \forall u : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid (u, e, n) \in \text{WF_Subst}_R$
 $\bullet \text{FV } e \subseteq u.1 \wedge n \in u.1$
REL_PREDICATE_∈-WF_Subst_R-thm
 $\vdash \forall u : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid \text{FV } e \subseteq u.1 \wedge n \in u.1$
 $\bullet (u, e, n) \in \text{WF_Subst}_R$
REL_PREDICATE_⟨_R⟩_R-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet \langle_R u \rangle_R \in \text{REL_PREDICATE}$
REL_PREDICATE_⊆_R-thm
 $\vdash \forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$
 $\bullet \text{pair-}u.1 \subseteq_R \text{pair-}u.2 \in \text{REL_PREDICATE}$
REL_PREDICATE_Var_R-thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\mid (a, n) \in \text{WF_Var}_{R_EndR}$
 $\bullet \text{var}_R (a, n) \in \text{REL_PREDICATE}$
REL_PREDICATE_WF_Var_R-End_R-thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\mid (a, n) \in \text{WF_Var}_{R_EndR}$
 $\bullet a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \{n, \text{dash } n\} \subseteq a$
REL_PREDICATE_WF_Var_R-End_R-WF_Skip_R-thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
& \quad | (a, n) \in \text{WF_Var}_{R_EndR} \\
& \quad \bullet a \in \text{WF_Skip}_R \\
& \textbf{REL_PREDICATE_End}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
& \quad | (a, n) \in \text{WF_Var}_{R_EndR} \\
& \quad \bullet \text{end}_R(a, n) \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_}\cap_R\textbf{-thm} \\
& \vdash \forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR} \\
& \quad \bullet \text{pair-}u.1 \text{ intchoice}_R \text{ pair-}u.2 \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_Bot}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET} \bullet \text{Bot}_R a \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_Top}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET} \bullet \text{Top}_R a \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_}\cap_R\textbf{-thm} \\
& \vdash \forall a_us : \text{WF_Glb}_{R_LubR} \bullet \cap_R a_us \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_}\cup_R\textbf{-thm} \\
& \vdash \forall a_us : \text{WF_Glb}_{R_LubR} \bullet \cup_R a_us \in \text{REL_PREDICATE} \\
& \textbf{REL_PREDICATE_Extend_unrest_alphabet-thm} \\
& \vdash \forall u : \text{REL_PREDICATE}; a : \text{ALPHABET} \\
& \quad \bullet (u \oplus_R a).1 = u.1 \cup a \\
& \textbf{REL_PREDICATE_}\wedge_R\textbf{-alphabet-thm} \\
& \vdash \forall u1, u2 : \text{REL_PREDICATE} \\
& \quad | u1.1 = u2.1 \\
& \quad \bullet (u1 \wedge_R u2).1 = u1.1 \\
& \textbf{REL_PREDICATE_}\wedge_R\textbf{-alphabet-thm1} \\
& \vdash \forall u1, u2 : \text{REL_PREDICATE} \\
& \quad \bullet (u1 \wedge_R u2).1 = u1.1 \cup u2.1 \\
& \textbf{REL_PREDICATE_}\vee_R\textbf{-alphabet-thm} \\
& \vdash \forall u1, u2 : \text{REL_PREDICATE} \\
& \quad | u1.1 = u2.1 \\
& \quad \bullet (u1 \vee_R u2).1 = u1.1 \\
& \textbf{REL_PREDICATE_}\vee_R\textbf{-alphabet-thm1} \\
& \vdash \forall u1, u2 : \text{REL_PREDICATE} \\
& \quad \bullet (u1 \vee_R u2).1 = u1.1 \cup u2.1 \\
& \textbf{REL_PREDICATE_Equals}_R\textbf{-alphabet-thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION} \\
& \quad | (a, n, e) \in \text{WF_Equals}_R
\end{aligned}$$

$\bullet (=_R (a, n, e)).1 = a$
REL_PREDICATE_Equals_exps_R-alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; e1, e2 : \text{EXPRESSION}$
 $\mid (a, e1, e2) \in \text{WF_ALPHABET_EXPRESSION}$
 $\bullet (=_{+R} (a, e1, e2)).1 = a$
REL_PREDICATE₋ \Rightarrow_R -alphabet_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\bullet (u1 \Rightarrow_R u2).1 = u1.1 \cup u2.1$
REL_PREDICATE₋ \Leftrightarrow_R -alphabet_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\bullet (u1 \Leftrightarrow_R u2).1 = u1.1 \cup u2.1$
REL_PREDICATE_ Π_R -alphabet_thm
 $\vdash \forall a : \text{ALPHABET} \mid a \in \text{WF_Skip}_R \bullet (\Pi_R a).1 = a$
REL_PREDICATE_ $\triangleleft_R \triangleright_R$ -alphabet_thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}$
 $\mid (u1, b, u2) \in \text{WF_Cond}_R$
 $\bullet (u1 \triangleleft_R b \triangleright_R u2).1 = u1.1$
REL_PREDICATE_Semi_R-alphabet_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\mid (u1, u2) \in \text{WF_Semi}_R$
 $\bullet (u1 ;_R u2).1 = \text{in-}a \ u1.1 \cup \text{out-}a \ u2.1$
REL_PREDICATE_Var_R-alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\mid (a, n) \in \text{WF_Var}_{R_EndR}$
 $\bullet (\text{var}_R (a, n)).1 = a \setminus \{n\}$
REL_PREDICATE_End_R-alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\mid (a, n) \in \text{WF_Var}_{R_EndR}$
 $\bullet (\text{end}_R (a, n)).1 = a \setminus \{\text{dash } n\}$
REL_PREDICATE_Contract_R-alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet (u -_R a).1 = u.1 \setminus a$
REL_PREDICATE_ \exists -Sub_R-alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet (\exists_{-R} (a, u)).1 = u.1 \setminus a$
REL_PREDICATE_ \exists_R -alphabet_thm
 $\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\bullet (\exists_R (a, u)).1 = u.1 \cup a$

REL_PREDICATE- \forall -Sub_R-alphabet.thm

$\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$

$\bullet (\forall_{-R} (a, u)).1 = u.1 \setminus a$

REL_PREDICATE- \forall _R-alphabet.thm

$\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$

$\bullet (\forall_R (a, u)).1 = u.1 \cup a$

REL_PREDICATE-Subst_R-alphabet.thm

$\vdash \forall u : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$

$\mid (u, e, n) \in \text{WF_Subst}_R$

$\bullet (\text{/_R} (u, e, n)).1 = u.1$

REL_PREDICATE- $\langle _R \rangle$ -alphabet.thm

$\vdash \forall u : \text{REL_PREDICATE} \bullet \langle _R u \rangle_R.1 = \emptyset$

REL_PREDICATE- \cap _R-alphabet.thm

$\vdash \forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$

$\bullet (\text{pair-}u.1 \text{ intchoice}_R \text{ pair-}u.2).1 = \text{pair-}u.1.1$

REL_PREDICATE- \sqsubseteq _R-alphabet.thm

$\vdash \forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$

$\bullet (\text{pair-}u.1 \sqsubseteq_R \text{ pair-}u.2).1 = \emptyset$

REL_PREDICATE-Bot_R-alphabet.thm

$\vdash \forall a : \text{ALPHABET} \bullet (\text{Bot}_R a).1 = a$

REL_PREDICATE-Top_R-alphabet.thm

$\vdash \forall a : \text{ALPHABET} \bullet (\text{Top}_R a).1 = a$

REL- \neg _R-True_R.thm

$\vdash \forall a : \text{ALPHABET} \bullet \neg_R (\text{True}_R a) = \text{False}_R a$

REL- \neg _R-False_R.thm

$\vdash \forall a : \text{ALPHABET} \bullet \neg_R (\text{False}_R a) = \text{True}_R a$

REL-True_R-False_R.thm

$\vdash \forall a : \text{ALPHABET} \bullet \text{True}_R a \neq \text{False}_R a$

REL-Equals_R-Equals_R-exps.thm

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; e : \text{EXPRESSION}$

$\mid (a, n, e) \in \text{WF_Equals}_R$

$\bullet =_R (a, n, e) = =_{+R} (a, \text{Var } n, e)$

REL-Extend-unrest-idem.thm

$\vdash \forall u : \text{REL_PREDICATE} \bullet u \oplus_R u.1 = u$

REL-Extend-unrest-idem.thm1

$\vdash \forall u : \text{REL_PREDICATE}; a : \text{ALPHABET}$

$\bullet u \oplus_R u.1 \cup a = u \oplus_R a$

REL-Extend-unrest-idem.thm2

$\vdash \forall u : REL_PREDICATE; a : ALPHABET$
 $\quad | a \subseteq u.1$
 $\quad \bullet u \oplus_R a = u$
REL_Extend_unrest_idem_thm3
 $\vdash \forall u : REL_PREDICATE \bullet u \oplus_R \{\} = u$
REL_\wedge_R_Extend_unrest_contraction_thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a1, a2 : ALPHABET$
 $\quad | a1 = u1.1 \vee a1 = u2.1$
 $\quad \bullet (u1 \wedge_R u2) \oplus_R a1 \cup a2 = (u1 \wedge_R u2) \oplus_R a2$
REL_\vee_R_Extend_unrest_contraction_thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a1, a2 : ALPHABET$
 $\quad | a1 = u1.1 \vee a1 = u2.1$
 $\quad \bullet (u1 \vee_R u2) \oplus_R a1 \cup a2 = (u1 \vee_R u2) \oplus_R a2$
REL_\wedge_R_idem_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \wedge_R u = u$
REL_\vee_R_idem_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \vee_R u = u$
REL_\wedge_R_com_thm
 $\vdash \forall u1, u2 : REL_PREDICATE \bullet u1 \wedge_R u2 = u2 \wedge_R u1$
REL_\vee_R_com_thm
 $\vdash \forall u1, u2 : REL_PREDICATE \bullet u1 \vee_R u2 = u2 \vee_R u1$
REL_\wedge_R_subst_thm
 $\vdash \forall u1, u2, u3 : REL_PREDICATE$
 $\quad | u1 = u2$
 $\quad \bullet u1 \wedge_R u3 = u2 \wedge_R u3$
REL_\vee_R_subst_thm
 $\vdash \forall u1, u2, u3 : REL_PREDICATE$
 $\quad | u1 = u2$
 $\quad \bullet u1 \vee_R u3 = u2 \vee_R u3$
BINDINGS_\triangleleft_idemp_thm
 $\vdash \forall a1, a2 : ALPHABET; b : BINDING$
 $\quad \bullet a1 \triangleleft (a1 \cup a2) \triangleleft b = a1 \triangleleft b$
 $\quad \wedge a1 \triangleleft (a2 \cup a1) \triangleleft b = a1 \triangleleft b$
BINDINGS_\triangleleft_idemp_thm1
 $\vdash \forall b : BINDING \bullet dom\ b \triangleleft b = b$
REL_Extend_unrest_\cap_distr_thm
 $\vdash \forall u1, u2, u3 : REL_PREDICATE$
 $\quad \bullet ((u1.1 \cup u2.1, (u1 \oplus_R u2.1).2) \cap (u2 \oplus_R u1.1).2)$

$$\begin{aligned}
& \oplus_R u3.1).2 \\
& = (u1 \oplus_R u2.1 \cup u3.1).2 \cap (u2 \oplus_R u1.1 \cup u3.1).2 \\
\mathbf{REL_Extend_unrest_}\cup\mathbf{_distr_thm} \\
& \vdash \forall u1, u2, u3 : \mathbf{REL_PREDICATE} \\
& \quad \bullet ((u1.1 \cup u2.1, (u1 \oplus_R u2.1).2 \cup (u2 \oplus_R u1.1).2) \\
& \quad \quad \oplus_R u3.1).2 \\
& \quad = (u1 \oplus_R u2.1 \cup u3.1).2 \cup (u2 \oplus_R u1.1 \cup u3.1).2 \\
\mathbf{REL_Extend_unrest_comm_thm} \\
& \vdash \forall u : \mathbf{REL_PREDICATE}; a1, a2 : \mathbf{ALPHABET} \\
& \quad \bullet u \oplus_R a1 \cup a2 = u \oplus_R a2 \cup a1 \\
\mathbf{REL_Contract}_R\mathbf{_comb_thm} \\
& \vdash \forall a1, a2 : \mathbf{ALPHABET}; u : \mathbf{REL_PREDICATE} \\
& \quad \bullet (u -_R a1) -_R a2 = u -_R a1 \cup a2 \\
\mathbf{REL_Contract}_R\mathbf{_}\cup\mathbf{_idem_thm} \\
& \vdash \forall a1, a2 : \mathbf{ALPHABET}; u : \mathbf{REL_PREDICATE} \\
& \quad \bullet u -_R a1 \cup a2 = u -_R a2 \cup a1 \\
\mathbf{REL_}\wedge_R\mathbf{_assoc_thm} \\
& \vdash \forall u1, u2, u3 : \mathbf{REL_PREDICATE} \\
& \quad \bullet (u1 \wedge_R u2) \wedge_R u3 = u1 \wedge_R u2 \wedge_R u3 \\
\mathbf{REL_}\wedge_R\mathbf{_True}_R\mathbf{_thm} \\
& \vdash \forall u1, u2 : \mathbf{REL_PREDICATE}; a : \mathbf{ALPHABET} \\
& \quad | a = u1.1 \wedge u1.1 = u2.1 \\
& \quad \bullet u1 = \mathbf{True}_R a \wedge u2 = \mathbf{True}_R a \\
& \quad \Leftrightarrow u1 \wedge_R u2 = \mathbf{True}_R a \\
\mathbf{REL_}\vee_R\mathbf{_assoc_thm} \\
& \vdash \forall u1, u2, u3 : \mathbf{REL_PREDICATE} \\
& \quad \bullet (u1 \vee_R u2) \vee_R u3 = u1 \vee_R u2 \vee_R u3 \\
\mathbf{REL_}\wedge_R\mathbf{_}\vee_R\mathbf{_abs_thm} \\
& \vdash \forall u1, u2 : \mathbf{REL_PREDICATE} \\
& \quad | u1.1 = u2.1 \\
& \quad \bullet u1 \wedge_R u1 \vee_R u2 = u1 \\
\mathbf{REL_}\wedge_R\mathbf{_}\vee_R\mathbf{_abs_thm1} \\
& \vdash \forall u1, u2 : \mathbf{REL_PREDICATE} \\
& \quad | u2.1 \subseteq u1.1 \\
& \quad \bullet u1 \wedge_R u1 \vee_R u2 = u1 \\
\mathbf{REL_}\vee_R\mathbf{_}\wedge_R\mathbf{_abs_thm} \\
& \vdash \forall u1, u2 : \mathbf{REL_PREDICATE} \\
& \quad | u1.1 = u2.1
\end{aligned}$$

$\bullet u1 \vee_R u1 \wedge_R u2 = u1$
REL- \vee_R - \wedge_R -abs_thm1
 $\vdash \forall u1, u2 : REL_PREDICATE$
 $\quad | u2.1 \subseteq u1.1$
 $\bullet u1 \vee_R u1 \wedge_R u2 = u1$
REL- \wedge_R - \vee_R -dist_thm
 $\vdash \forall u1, u2, u3 : REL_PREDICATE$
 $\bullet u1 \wedge_R u2 \vee_R u3 = (u1 \wedge_R u2) \vee_R u1 \wedge_R u3$
REL- \vee_R - \wedge_R -dist_thm
 $\vdash \forall u1, u2, u3 : REL_PREDICATE$
 $\bullet u1 \vee_R u2 \wedge_R u3 = (u1 \vee_R u2) \wedge_R u1 \vee_R u3$
REL-True_R- \wedge_R -id_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \wedge_R True_R u.1 = u$
REL-True_R- \wedge_R -id_thm1
 $\vdash \forall a : ALPHABET; u : REL_PREDICATE$
 $\quad | a \subseteq u.1$
 $\bullet u \wedge_R True_R a = u$
REL-True_R- \vee_R -zero_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \vee_R True_R u.1 = True_R u.1$
REL-True_R- \vee_R -zero_thm1
 $\vdash \forall a : ALPHABET; u : REL_PREDICATE$
 $\quad | u.1 \subseteq a$
 $\bullet u \vee_R True_R a = True_R a$
REL-True_R- \vee_R -zero_thm2
 $\vdash \forall a : ALPHABET; u : REL_PREDICATE$
 $\bullet u \vee_R True_R a = True_R (a \cup u.1)$
REL-False_R- \wedge_R -zero_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \wedge_R False_R u.1 = False_R u.1$
REL-False_R- \wedge_R -zero_thm1
 $\vdash \forall a : ALPHABET; u : REL_PREDICATE$
 $\quad | u.1 \subseteq a$
 $\bullet u \wedge_R False_R a = False_R a$
REL-False_R- \wedge_R -zero_thm2
 $\vdash \forall a : ALPHABET; u : REL_PREDICATE$
 $\bullet u \wedge_R False_R a = False_R (a \cup u.1)$
REL-False_R- \vee_R -id_thm
 $\vdash \forall u : REL_PREDICATE \bullet u \vee_R False_R u.1 = u$
REL-False_R- \vee_R -id_thm1

$\vdash \forall a : \text{ALPHABET}; u : \text{REL_PREDICATE}$
 $\quad | a \subseteq u.1$
 $\quad \bullet u \vee_R \text{False}_R a = u$
REL- \Rightarrow_R -vacuous_thm
 $\vdash \forall u : \text{REL_PREDICATE}; a : \text{ALPHABET}$
 $\quad | u.1 \subseteq a$
 $\quad \bullet \text{False}_R a \Rightarrow_R u = \text{True}_R a$
REL- $u \wedge_R \neg_R u$ -Eq-False_R-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet u \wedge_R \neg_R u = \text{False}_R u.1$
REL- $\wedge_R \vee_R \neg_R$ -abs-thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\quad \bullet u1 \wedge_R \neg_R u1 \vee_R u2 = u1 \wedge_R u2$
REL- $u \vee_R \neg_R u$ -Eq-True_R-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet u \vee_R \neg_R u = \text{True}_R u.1$
REL- \neg_R -double_negation-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet \neg_R (\neg_R u) = u$
REL- $\neg_R \wedge_R$ -deMorgan-thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\quad \bullet \neg_R (u1 \wedge_R u2) = \neg_R u1 \vee_R \neg_R u2$
REL- $\neg_R \vee_R$ -deMorgan-thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}$
 $\quad \bullet \neg_R (u1 \vee_R u2) = \neg_R u1 \wedge_R \neg_R u2$
REL- $u \Rightarrow_R u \Leftrightarrow$ -True_R-thm
 $\vdash \forall u : \text{REL_PREDICATE} \bullet u \Rightarrow_R u = \text{True}_R u.1$
REL- $\Rightarrow_R \Rightarrow_R$ -thm
 $\vdash \forall u1, u2, u3 : \text{REL_PREDICATE}$
 $\quad \bullet u1 \Rightarrow_R u2 \Rightarrow_R u3 = (u1 \wedge_R u2) \Rightarrow_R u3$
REL- \Rightarrow_R -trans_thm
 $\vdash \forall u1, u2, u3 : \text{REL_PREDICATE}; a : \text{ALPHABET}$
 $\quad | a = u1.1 \wedge u1.1 = u2.1 \wedge u2.1 = u3.1$
 $\quad \bullet ((u1 \Rightarrow_R u2) \wedge_R u2 \Rightarrow_R u3) \Rightarrow_R u1 \Rightarrow_R u3$
 $\quad = \text{True}_R a$
REL-modus-ponens_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}; a : \text{ALPHABET}$
 $\quad | a = u1.1 \wedge u1.1 = u2.1$
 $\quad \bullet u1 \Rightarrow_R u2 = \text{True}_R a \wedge u1 = \text{True}_R a$
 $\quad \Rightarrow u2 = \text{True}_R a$
REL- $u \Leftrightarrow_R u \Leftrightarrow$ -True_R-thm

$\vdash \forall u : REL_PREDICATE \bullet u \Leftrightarrow_R u = True_R u.1$
REL- \Leftrightarrow_R -Eq-thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $\quad | a = u1.1 \wedge u1.1 = u2.1$
 $\quad \bullet (u1 \wedge_R u2) \vee_R \neg_R u1 \wedge_R \neg_R u2 = True_R a$
 $\quad \Leftrightarrow u1 = u2$
REL- \Leftrightarrow_R -Eq-thm1
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $\quad | a = u1.1 \wedge u1.1 = u2.1$
 $\quad \bullet u1 \Leftrightarrow_R u2 = True_R a \Leftrightarrow u1 = u2$
REL- $\triangleleft_R \triangleright_R$ -subst-thm
 $\vdash \forall u1, u2, cond1, cond2 : REL_PREDICATE$
 $\quad | cond1 = cond2$
 $\quad \bullet u1 \triangleleft_R cond1 \triangleright_R u2 = u1 \triangleleft_R cond2 \triangleright_R u2$
REL- $\triangleleft_R \triangleright_R$ -idem-thm
 $\vdash \forall u, b : REL_PREDICATE$
 $\quad | (u, b, u) \in WF_Cond_R$
 $\quad \bullet u \triangleleft_R b \triangleright_R u = u$
REL- $\triangleleft_R \triangleright_R$ -symm-thm
 $\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R b \triangleright_R u2 = u2 \triangleleft_R \neg_R b \triangleright_R u1$
REL- $\triangleleft_R \triangleright_R$ -assoc-thm
 $\vdash \forall u1, u2, u3, b1, b2 : REL_PREDICATE$
 $\quad | (u1 \triangleleft_R b1 \triangleright_R u2, b2, u3) \in WF_Cond_R$
 $\quad \wedge (u1, b1 \wedge_R b2, u2 \triangleleft_R b2 \triangleright_R u3) \in WF_Cond_R$
 $\quad \wedge (u1, b1, u2) \in WF_Cond_R$
 $\quad \bullet (u1 \triangleleft_R b1 \triangleright_R u2) \triangleleft_R b2 \triangleright_R u3$
 $\quad = u1 \triangleleft_R b1 \wedge_R b2 \triangleright_R u2 \triangleleft_R b2 \triangleright_R u3$
REL- $\triangleleft_R \triangleright_R$ -dist-thm
 $\vdash \forall u1, u2, u3, b1, b2 : REL_PREDICATE$
 $\quad | (u2, b2, u3) \in WF_Cond_R$
 $\quad \wedge (u1, b1, u2) \in WF_Cond_R$
 $\quad \wedge (u1, b1, u3) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R b1 \triangleright_R u2 \triangleleft_R b2 \triangleright_R u3$
 $\quad = (u1 \triangleleft_R b1 \triangleright_R u2) \triangleleft_R b2 \triangleright_R u1 \triangleleft_R b1 \triangleright_R u3$
REL- $\triangleleft_R \triangleright_R$ -unit-thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$

$$\begin{aligned}
& | (u1, \text{True}_R a, u2) \in \text{WF_Cond}_R \\
& \bullet u1 \triangleleft_R \text{True}_R a \triangleright_R u2 = u2 \triangleleft_R \text{False}_R a \triangleright_R u1
\end{aligned}$$

REL- $\triangleleft_R \triangleright_R$ -unreach_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3, b1 : \text{REL_PREDICATE} \\
& | (u2, b1, u3) \in \text{WF_Cond}_R \\
& \quad \wedge (u1, b1, u3) \in \text{WF_Cond}_R \\
& \bullet u1 \triangleleft_R b1 \triangleright_R u2 \triangleleft_R b1 \triangleright_R u3 = u1 \triangleleft_R b1 \triangleright_R u3
\end{aligned}$$

REL- Π_R -step_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
& | n \in \text{undashed} \\
& \quad \wedge a \in \text{WF_Skip}_R \\
& \quad \wedge (a, n, \text{Var } n) \in \text{WF_Equals}_R \\
& \bullet =_R (a, \text{dash } n, \text{Var } n) \wedge_R \Pi_R (a \setminus \{n, \text{dash } n\}) \\
& \quad = \Pi_R a
\end{aligned}$$

REL_Assign_R-unchanged_not_mentioned_thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}; e : \text{EXPRESSION} \\
& | (a, \langle n1, n2 \rangle, \langle e, \text{Var } n2 \rangle) \in \text{WF_Assign}_R \\
& \quad \wedge n1 \neq n2 \\
& \bullet \text{Assign}_R (a, \langle n1 \rangle, \langle e \rangle) \\
& \quad = \text{Assign}_R (a, \langle n1, n2 \rangle, \langle e, \text{Var } n2 \rangle)
\end{aligned}$$

REL_Semi_R-assoc_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
& | (u1, u2) \in \text{WF_Semi}_R \wedge (u2, u3) \in \text{WF_Semi}_R \\
& \bullet u1 ;_R u2 ;_R u3 = (u1 ;_R u2) ;_R u3
\end{aligned}$$

UTP_Semi_R- \vee_R -left-dist_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
& | u1.1 = u2.1 \wedge u2.1 = u3.1 \wedge u3.1 \in \text{homogeneous} \\
& \bullet (u1 \vee_R u2) ;_R u3 = (u1 ;_R u3) \vee_R u2 ;_R u3
\end{aligned}$$

REL_Semi_R- \vee_R -left-dist_thm1

$$\begin{aligned}
& \vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
& | u1.1 = u2.1 \wedge (u1, u3) \in \text{WF_Semi}_R \\
& \bullet (u1 \vee_R u2) ;_R u3 = (u1 ;_R u3) \vee_R u2 ;_R u3
\end{aligned}$$

REL_Semi_R- \vee_R -right-dist_thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3 : \text{REL_PREDICATE} \\
& | u1.1 = u2.1 \wedge u2.1 = u3.1 \wedge u3.1 \in \text{homogeneous} \\
& \bullet u1 ;_R u2 \vee_R u3 = (u1 ;_R u2) \vee_R u1 ;_R u3
\end{aligned}$$

REL_Semi_R- \vee_R -right-dist_thm1

$$\vdash \forall u1, u2, u3 : \text{REL_PREDICATE}$$

$| u2.1 = u3.1 \wedge (u1, u2) \in WF_Semi_R$
 $\bullet u1 ;_R u2 \vee_R u3 = (u1 ;_R u2) \vee_R u1 ;_R u3$
REL_Contract_R-Semi_R-expansion_thm
 $\vdash \forall a : ALPHABET; u1, u2 : REL_PREDICATE$
 $| a \cap dashed = \emptyset \wedge (u1, u2) \in WF_Semi_R$
 $\bullet (u1 -_R a) ;_R u2 = (u1 ;_R u2) -_R a$
REL_Semi_R-Contract_R-expansion_thm
 $\vdash \forall a : ALPHABET; u1, u2 : REL_PREDICATE$
 $| a \cap undashed = \emptyset \wedge (u1, u2) \in WF_Semi_R$
 $\bullet u1 ;_R u2 -_R a = (u1 ;_R u2) -_R a$
REL_∃_Sub_R-Semi_R-expansion_thm
 $\vdash \forall a : ALPHABET; u1, u2 : REL_PREDICATE$
 $| a \cap dashed = \emptyset \wedge (u1, u2) \in WF_Semi_R$
 $\bullet \exists_{-R} (a, u1) ;_R u2 = \exists_{-R} (a, u1 ;_R u2)$
REL_Semi_R-∃_Sub_R-expansion_thm
 $\vdash \forall a : ALPHABET; u1, u2 : REL_PREDICATE$
 $| a \cap undashed = \emptyset \wedge (u1, u2) \in WF_Semi_R$
 $\bullet u1 ;_R \exists_{-R} (a, u2) = \exists_{-R} (a, u1 ;_R u2)$
REL_∃_Sub_R-comb_thm
 $\vdash \forall a1, a2 : ALPHABET; u : REL_PREDICATE$
 $\bullet \exists_{-R} (a1, \exists_{-R} (a2, u)) = \exists_{-R} (a1 \cup a2, u)$
REL_∃_Sub_R-∨_R-dist_thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $| a = u1.1 \wedge u1.1 = u2.1$
 $\bullet \exists_{-R} (a, u1) \vee_R \exists_{-R} (a, u2)$
 $= \exists_{-R} (a, u1 \vee_R u2)$
REL_∨_Sub_R-∧_R-dist_thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $| a = u1.1 \wedge u1.1 = u2.1$
 $\bullet \forall_{-R} (a, u1) \wedge_R \forall_{-R} (a, u2)$
 $= \forall_{-R} (a, u1 \wedge_R u2)$
REL_∨_Sub_R-True_R-thm
 $\vdash \forall a : ALPHABET \bullet \forall_{-R} (a, True_R a) = True_R \emptyset$
REL_∨_Sub_R-u-Eq-True_R-thm
 $\vdash \forall u : REL_PREDICATE$
 $\bullet \forall_{-R} (u.1, u) = True_R \emptyset \Leftrightarrow u = True_R u.1$
REL_⟨_R⟩_R-True_R-thm
 $\vdash \forall a : ALPHABET \bullet \langle_R True_R a \rangle_R = True_R \emptyset$

REL- $\langle_R \rangle_R$ -Eq-True_R-thm

$\vdash \forall u : \text{REL_PREDICATE}$

$\bullet \langle_R u \rangle_R = \text{True}_R \emptyset \Leftrightarrow u = \text{True}_R u.1$

REL- $\langle_R \Leftrightarrow_R \rangle_R$ -thm

$\vdash \forall u1, u2 : \text{REL_PREDICATE}$

$| u1.1 = u2.1$

$\bullet \langle_R u1 \Leftrightarrow_R u2 \rangle_R = \text{True}_R \emptyset \Leftrightarrow u1 = u2$

REL- $\langle_R \rangle_R \wedge_R$ -dist-thm

$\vdash \forall u1, u2 : \text{REL_PREDICATE}$

$| u1.1 = u2.1$

$\bullet \langle_R u1 \rangle_R \wedge_R \langle_R u2 \rangle_R = \langle_R u1 \wedge_R u2 \rangle_R$

REL- $\langle_R \rangle_R \Rightarrow_R$ -dist-thm

$\vdash \forall u1, u2 : \text{REL_PREDICATE}$

$| u1.1 = u2.1$

$\bullet \langle_R u1 \Rightarrow_R u2 \rangle_R \Rightarrow_R \langle_R u1 \rangle_R \Rightarrow_R \langle_R u2 \rangle_R$
 $= \text{True}_R \emptyset$

REL-Subst_R-idem-thm

$\vdash \forall u : \text{REL_PREDICATE}; n1, n2 : \text{NAME}$

$| (u, \text{Var } n2, n1) \in \text{WF_Subst}_R$

$\wedge (u, \text{Var } n1, n2) \in \text{WF_Subst}_R$

$\wedge n2 \in \text{UnrestVar } u$

$\bullet /_R (/_R (u, \text{Var } n2, n1), \text{Var } n1, n2) = u$

REL-True_R-Subst-idem-thm

$\vdash \forall a : \text{ALPHABET}; e : \text{EXPRESSION}; n : \text{NAME}$

$| (\text{True}_R a, e, n) \in \text{WF_Subst}_R$

$\bullet /_R (\text{True}_R a, e, n) = \text{True}_R a$

REL- \cap_R -com-thm

$\vdash \forall \text{pair-}u : \text{WF_REL_PREDICATE_PAIR}$

$\bullet \text{pair-}u.1 \text{ intchoice}_R \text{pair-}u.2$

$= \text{pair-}u.2 \text{ intchoice}_R \text{pair-}u.1$

REL- \cap_R -assoc-thm

$\vdash \forall u1, u2, u3 : \text{REL_PREDICATE}$

$| (u1, u2) \in \text{WF_REL_PREDICATE_PAIR}$

$\wedge (u2, u3) \in \text{WF_REL_PREDICATE_PAIR}$

$\bullet u1 \text{ intchoice}_R u2 \text{ intchoice}_R u3$

$= (u1 \text{ intchoice}_R u2) \text{ intchoice}_R u3$

REL- \cap_R -idem-thm

$\vdash \forall u : \text{REL_PREDICATE} \bullet u \text{ intchoice}_R u = u$

REL- \cap_R -dist-thm

$$\begin{aligned}
& \vdash \forall u1, u2, u3 : REL_PREDICATE \\
& \quad | (u1, u2) \in WF_REL_PREDICATE_PAIR \\
& \quad \quad \wedge (u2, u3) \in WF_REL_PREDICATE_PAIR \\
& \quad \bullet u1 \text{ intchoice}_R u2 \text{ intchoice}_R u3 \\
& \quad \quad = (u1 \text{ intchoice}_R u2) \\
& \quad \quad \quad \text{intchoice}_R u1 \text{ intchoice}_R u3
\end{aligned}$$

REL-var_R-Semi_R-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n : NAME; u : REL_PREDICATE \\
& \quad | (a, n) \in WF_Var_{R_EndR} \wedge a = u.1 \\
& \quad \bullet var_R(a, n) ;_R u = \exists_{-R}(\{n\}, u)
\end{aligned}$$

REL-Semi_R-end_R-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n : NAME; u : REL_PREDICATE \\
& \quad | (a, n) \in WF_Var_{R_EndR} \wedge a = u.1 \\
& \quad \bullet u ;_R end_R(a, n) = \exists_{-R}(\{dash\ n\}, u)
\end{aligned}$$

REL-var_R-composable-idem-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n : NAME; u : REL_PREDICATE \\
& \quad | (a, n) \in WF_Var_{R_EndR} \wedge (\Pi_R a, u) \in WF_Semi_R \\
& \quad \bullet (var_R(a, n), u) \in WF_Semi_R
\end{aligned}$$

REL-end_R-composable-idem-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n : NAME; u : REL_PREDICATE \\
& \quad | (a, n) \in WF_Var_{R_EndR} \wedge (u, \Pi_R a) \in WF_Semi_R \\
& \quad \bullet (u, end_R(a, n)) \in WF_Semi_R
\end{aligned}$$

REL-var_R-end_R-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n : NAME; u : REL_PREDICATE \\
& \quad | (a, n) \in WF_Var_{R_EndR} \wedge a = u.1 \\
& \quad \bullet var_R(a, n) ;_R u ;_R end_R(a, n) \\
& \quad \quad = \exists_{-R}(\{n, dash\ n\}, u)
\end{aligned}$$

REL-Bot_R-element-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; u : REL_PREDICATE \\
& \quad | a = u.1 \\
& \quad \bullet Bot_R a \sqsubseteq_R u = True_R \emptyset
\end{aligned}$$

REL-Top_R-element-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; u : REL_PREDICATE \\
& \quad | a = u.1 \\
& \quad \bullet u \sqsubseteq_R Top_R a = True_R \emptyset
\end{aligned}$$

REL- \sqsubseteq_R -reflex-thm

$$\vdash \forall u : REL_PREDICATE \bullet u \sqsubseteq_R u = True_R \emptyset$$

REL- \sqsubseteq_R -anti-symmetry_thm

$$\begin{aligned} &\vdash \forall u1, u2 : REL_PREDICATE \\ &\quad | u1.1 = u2.1 \\ &\quad \bullet u1 = u2 \Leftrightarrow (u1 \sqsubseteq_R u2) \wedge_R u2 \sqsubseteq_R u1 = True_R \emptyset \end{aligned}$$

REL- \sqsubseteq_R -transitivity_thm

$$\begin{aligned} &\vdash \forall u1, u2, u3 : REL_PREDICATE \\ &\quad | u1.1 = u2.1 \wedge u2.1 = u3.1 \\ &\quad \bullet ((u1 \sqsubseteq_R u2) \wedge_R u2 \sqsubseteq_R u3) \Rightarrow_R u1 \sqsubseteq_R u3 \\ &\quad = True_R \emptyset \end{aligned}$$

REL_PREDICATE- \cap_R -alphabet_thm

$$\vdash \forall a_us : WF_Glb_{R_LubR} \bullet (\cap_R a_us).1 = a_us.1$$

REL_PREDICATE- \cap_R - \emptyset -alphabet_thm

$$\vdash \forall a : ALPHABET \bullet (\cap_R (a, \{\})).1 = a$$

REL_PREDICATE- \cup_R -alphabet_thm

$$\vdash \forall a_us : WF_Glb_{R_LubR} \bullet (\cup_R a_us).1 = a_us.1$$

REL_PREDICATE- \cup_R - \emptyset -alphabet_thm

$$\vdash \forall a : ALPHABET \bullet (\cup_R (a, \{\})).1 = a$$

REL_lower_bound_thm

$$\begin{aligned} &\vdash \forall a : ALPHABET; \\ &\quad us : \mathbb{P} REL_PREDICATE; \\ &\quad u : REL_PREDICATE \\ &\quad | (a, us) \in WF_Glb_{R_LubR} \wedge u \in us \\ &\quad \bullet \cap_R (a, us) \sqsubseteq_R u = True_R \emptyset \end{aligned}$$

REL_greatest_lower_bound_thm

$$\begin{aligned} &\vdash \forall a : ALPHABET; \\ &\quad us : \mathbb{P} REL_PREDICATE; \\ &\quad u : REL_PREDICATE \\ &\quad | u.1 = a \\ &\quad \wedge (a, us) \in WF_Glb_{R_LubR} \\ &\quad \wedge (\forall u1 : us \bullet u \sqsubseteq_R u1 = True_R \emptyset) \\ &\quad \bullet u \sqsubseteq_R \cap_R (a, us) = True_R \emptyset \end{aligned}$$

REL- \cap_R - \emptyset -thm

$$\vdash \forall a : ALPHABET \bullet \cap_R (a, \{\}) = Top_R a$$

REL- \cup_R - \emptyset -thm

$$\vdash \forall a : ALPHABET \bullet \cup_R (a, \{\}) = Bot_R a$$

REL_weakest_fixed_point_thm

$$\begin{aligned} &\vdash \forall F : REL_FUNCTION; Y : REL_PREDICATE \\ &\quad | Y \in dom F \wedge F Y \sqsubseteq_R Y = True_R \emptyset \end{aligned}$$

$\bullet \mu_R F \sqsubseteq_R Y = \text{True}_R \emptyset$
VALUE_EVENT_VAL_thm
 $\vdash \text{EVENT_VAL} \subseteq \text{VALUE}$
VALUE_SEQ_EVENT_VAL_thm
 $\vdash \forall v : \text{SEQ_EVENT_VAL} \bullet v \in \text{SEQ_VAL}$
REL_⊕_∈_BINDING_thm
 $\vdash \forall b : \text{BINDING}; n : \text{NAME}; v : \text{VALUE}$
 $\bullet b \oplus \{n \mapsto v\} \in \text{BINDING}$
REL_◁_∈_BINDING_thm
 $\vdash \forall b : \text{BINDING}; a : \text{ALPHABET} \bullet a \triangleleft b \in \text{BINDING}$
REL_⊕_dom_thm
 $\vdash \forall b : \text{BINDING}; n1, n2 : \text{NAME}; v : \text{VALUE}$
 $\mid n1 \in \text{dom } b$
 $\bullet n1 \in \text{dom } (b \oplus \{n2 \mapsto v\})$
REL_◁_dom_thm
 $\vdash \forall b : \text{BINDING}; a : \text{ALPHABET}; n : \text{NAME}$
 $\mid n \in \text{dom } b \wedge n \in a$
 $\bullet n \in \text{dom } (a \triangleleft b)$
REL_FV_Val_thm
 $\vdash \forall v : \text{VALUE} \bullet \text{FV } (\text{Val } v) = \{\}$
REL_FV_Rel_thm
 $\vdash \forall r : \text{RELATION}; e1, e2 : \text{EXPRESSION}$
 $\bullet \text{FV } (\text{Rel } (r, e1, e2)) = \text{FV } e1 \cup \text{FV } e2$
REL_PREDICATE_WF_ALPHABET_EXPRESSION_thm
 $\vdash \forall a : \text{ALPHABET}; e1, e2 : \text{EXPRESSION}$
 $\mid (a, e1, e2) \in \text{WF_ALPHABET_EXPRESSION}$
 $\bullet \text{FV } e1 \cup \text{FV } e2 \subseteq a$
REL_Eval_Rel_thm
 $\vdash \forall b : \text{BINDING}; r : \text{RELATION}; e1, e2 : \text{EXPRESSION}$
 $\mid (b, e1) \in \text{WF_BINDING_EXPRESSION}_R$
 $\wedge (b, e2) \in \text{WF_BINDING_EXPRESSION}_R$
 $\bullet \text{Eval } (b, \text{Rel } (r, e1, e2))$
 $= \text{Bool } ((\text{Eval } (b, e1), \text{Eval } (b, e2)) \in r)$
REL_Eval_Fun2_thm
 $\vdash \forall b : \text{BINDING}; f : \text{BINARY_F}; e1, e2 : \text{EXPRESSION}$
 $\mid (b, e1) \in \text{WF_BINDING_EXPRESSION}_R$
 $\wedge (b, e2) \in \text{WF_BINDING_EXPRESSION}_R$
 $\bullet \text{Eval } (b, \text{Fun}_2 (f, e1, e2))$

$$= f (Eval (b, e1), Eval (b, e2))$$

REL_Fun₂_∈_VALUE_thm

$$\begin{aligned} &\vdash \forall f : \text{BINARY_F}; b : \text{BINDING}; n1, n2 : \text{NAME} \\ &\quad | n1 \in \text{dom } b \wedge n2 \in \text{dom } b \\ &\quad \bullet f (b \ n1, b \ n2) \in \text{VALUE} \end{aligned}$$

REL_⊕_Eval_thm

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; n1, n2 : \text{NAME}; v : \text{VALUE} \\ &\quad | n1 \in \text{dom } b \wedge n1 \neq n2 \\ &\quad \bullet Eval (b, Var \ n1) = Eval (b \oplus \{n2 \mapsto v\}, Var \ n1) \end{aligned}$$

REL_◁_Eval_thm

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; n : \text{NAME}; a : \text{ALPHABET} \\ &\quad | n \in \text{dom } b \wedge n \in a \\ &\quad \bullet Eval (a \triangleleft b, Var \ n) = b \ n \end{aligned}$$

REL_⊕_app_thm

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; n1, n2 : \text{NAME} \\ &\quad | n2 \in \text{dom } b \wedge n2 \neq n1 \\ &\quad \bullet b \ n2 = (b \oplus \{n1 \mapsto v\}) \ n2 \end{aligned}$$

REL_⊕_app_thm1

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; n1, n2 : \text{NAME}; v : \text{VALUE} \\ &\quad | n2 \in \text{dom } b \wedge n2 \neq n1 \\ &\quad \bullet b \ n2 = (b \oplus \{n1 \mapsto v\}) \ n2 \end{aligned}$$

REL_⊕_⊕_com_thm

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; n1, n2 : \text{NAME}; v1, v2 : \text{VALUE} \\ &\quad | n1 \neq n2 \\ &\quad \bullet b \oplus \{n1 \mapsto v1\} \oplus \{n2 \mapsto v2\} \\ &\quad \quad = b \oplus \{n2 \mapsto v2\} \oplus \{n1 \mapsto v1\} \end{aligned}$$

REL_◁_⊕_assoc_thm

$$\begin{aligned} &\vdash \forall a : \text{ALPHABET}; b : \text{BINDING}; n : \text{NAME}; v : \text{VALUE} \\ &\quad | n \in a \\ &\quad \bullet a \triangleleft (b \oplus \{n \mapsto v\}) = a \triangleleft b \oplus \{n \mapsto v\} \end{aligned}$$

REL_FV_Fun₂_thm

$$\begin{aligned} &\vdash \forall f : \text{BINARY_F}; e1, e2 : \text{EXPRESSION} \\ &\quad \bullet FV (Fun_2 (f, e1, e2)) = FV \ e1 \cup FV \ e2 \end{aligned}$$

REL_Fun₂_∈_EXPRESSION_thm

$$\begin{aligned} &\vdash \forall n1, n2 : \text{NAME}; f : \text{BINARY_F} \\ &\quad \bullet Fun_2 (f, Var \ n1, Var \ n2) \in \text{EXPRESSION} \end{aligned}$$

REL_Fun₂_∈_EXPRESSION_thm1

$$\vdash \forall e1, e2 : \text{EXPRESSION}; f : \text{BINARY_F}$$

$\bullet \text{Fun}_2 (f, e1, e2) \in \text{EXPRESSION}$
REL_SeqDif_R ∈ EXPRESSION_thm
 $\vdash \forall n1, n2 : \text{NAME}$
 $\bullet \text{Fun}_2 ((_ \text{SeqDif}_R _), \text{Var } n1, \text{Var } n2) \in \text{EXPRESSION}$
ALPHABET_n ≠ dash_n_thm
 $\vdash \forall n : \text{NAME} \mid n \in \text{undashed} \bullet n \neq \text{dash } n$
ALPHABET_n1 ≠ dash_n2_thm
 $\vdash \forall n1, n2 : \text{NAME}$
 $\mid n1 \neq n2 \wedge n1 \in \text{undashed} \wedge n2 \in \text{undashed}$
 $\bullet n1 \neq \text{dash } n2 \wedge \text{dash } n2 \neq n1$
ALPHABET_dash_n1 ≠ dash_n2_thm
 $\vdash \forall n1, n2 : \text{NAME}$
 $\mid n1 \neq n2 \wedge n1 \in \text{undashed} \wedge n2 \in \text{undashed}$
 $\bullet \text{dash } n1 \neq \text{dash } n2 \wedge \text{dash } n2 \neq \text{dash } n1$
REL_BOOL_true ∈ VALUE_thm
 $\vdash \text{Bool true} \in \text{VALUE}$
REL_BOOL_false ∈ VALUE_thm
 $\vdash \text{Bool false} \in \text{VALUE}$
REL_Val ∈ EXPRESSION_thm
 $\vdash \forall v : \text{VALUE} \bullet \text{Val } v \in \text{EXPRESSION}$
BINDINGS_◁_R_thm
 $\vdash \forall b : \text{BINDING}; a : \text{ALPHABET} \bullet a \triangleleft b \in \text{BINDING}$
REL_PREDICATE_WF_Subst_R ∧_R dist_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid (u1, e, n) \in \text{WF_Subst}_R \wedge (u2, e, n) \in \text{WF_Subst}_R$
 $\bullet (u1 \wedge_R u2, e, n) \in \text{WF_Subst}_R$
REL_PREDICATE_WF_Subst_R ∨_R dist_thm
 $\vdash \forall u1, u2 : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid (u1, e, n) \in \text{WF_Subst}_R \wedge (u2, e, n) \in \text{WF_Subst}_R$
 $\bullet (u1 \vee_R u2, e, n) \in \text{WF_Subst}_R$
REL_PREDICATE_WF_Subst_R ◁_R ▷_R dist_thm
 $\vdash \forall u1, u2, b : \text{REL_PREDICATE}; e : \text{EXPRESSION}; n : \text{NAME}$
 $\mid (u1, e, n) \in \text{WF_Subst}_R$
 $\wedge (u2, e, n) \in \text{WF_Subst}_R$
 $\wedge (u1, b, u2) \in \text{WF_Cond}_R$
 $\bullet (u1 \triangleleft_R b \triangleright_R u2, e, n) \in \text{WF_Subst}_R$
REL_PREDICATE_WF_Subst_R ¬_R dist_thm

$\vdash \forall u : REL_PREDICATE; e : EXPRESSION; n : NAME$
 $\quad | (u, e, n) \in WF_Subst_R$
 $\quad \bullet (\neg_R u, e, n) \in WF_Subst_R$
REL- \triangleleft_R -True $_R$ - \triangleright_R -app-thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $\quad | (u1, True_R a, u2) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R True_R a \triangleright_R u2 = u1$
REL- \triangleleft_R -False $_R$ - \triangleright_R -app-thm
 $\vdash \forall u1, u2 : REL_PREDICATE; a : ALPHABET$
 $\quad | (u1, False_R a, u2) \in WF_Cond_R$
 $\quad \bullet u1 \triangleleft_R False_R a \triangleright_R u2 = u2$
REL-Subst $_R$ - \wedge_R -dist-thm
 $\vdash \forall u1, u2 : REL_PREDICATE; v : VALUE; n : NAME$
 $\quad | (u1, Val v, n) \in WF_Subst_R$
 $\quad \quad \wedge (u2, Val v, n) \in WF_Subst_R$
 $\quad \bullet /_R (u1 \wedge_R u2, Val v, n)$
 $\quad \quad = /_R (u1, Val v, n) \wedge_R /_R (u2, Val v, n)$
REL-Subst $_R$ - \wedge_R -dist-thm1
 $\vdash \forall u1, u2 : REL_PREDICATE;$
 $\quad f : BINARY_F;$
 $\quad n1, n2, n3 : NAME$
 $\quad | (u1, Fun_2 (f, Var n1, Var n2), n3) \in WF_Subst_R$
 $\quad \quad \wedge (u2, Fun_2 (f, Var n1, Var n2), n3)$
 $\quad \quad \in WF_Subst_R$
 $\quad \bullet /_R (u1 \wedge_R u2, Fun_2 (f, Var n1, Var n2), n3)$
 $\quad \quad = /_R (u1, Fun_2 (f, Var n1, Var n2), n3)$
 $\quad \quad \wedge_R /_R (u2, Fun_2 (f, Var n1, Var n2), n3)$
REL-Subst $_R$ - \neg_R -dist-thm
 $\vdash \forall u : REL_PREDICATE; v : VALUE; n : NAME$
 $\quad | (u, Val v, n) \in WF_Subst_R$
 $\quad \bullet /_R (\neg_R u, Val v, n) = \neg_R (/_R (u, Val v, n))$
REL- \vee_R - \wedge_R -repl-thm
 $\vdash \forall u1, u2 : REL_PREDICATE$
 $\quad \bullet u1 \vee_R u2 = \neg_R (\neg_R u1 \wedge_R \neg_R u2)$
REL- \wedge_R - \vee_R -repl-thm
 $\vdash \forall u1, u2 : REL_PREDICATE$
 $\quad \bullet u1 \wedge_R u2 = \neg_R (\neg_R u1 \vee_R \neg_R u2)$
REL-Subst $_R$ - \vee_R -dist-thm

$$\begin{aligned}
& \vdash \forall u1, u2 : REL_PREDICATE; v : VALUE; n : NAME \\
& \quad | (u1, Val\ v, n) \in WF_Subst_R \\
& \quad \wedge (u2, Val\ v, n) \in WF_Subst_R \\
& \quad \bullet \ /_R (u1 \vee_R u2, Val\ v, n) \\
& \quad = \ /_R (u1, Val\ v, n) \vee_R \ /_R (u2, Val\ v, n)
\end{aligned}$$

REL_Subst_R- \Rightarrow_R -dist-thm

$$\begin{aligned}
& \vdash \forall u1, u2 : REL_PREDICATE; v : VALUE; n : NAME \\
& \quad | (u1, Val\ v, n) \in WF_Subst_R \\
& \quad \wedge (u2, Val\ v, n) \in WF_Subst_R \\
& \quad \bullet \ /_R (u1 \Rightarrow_R u2, Val\ v, n) \\
& \quad = \ /_R (u1, Val\ v, n) \Rightarrow_R \ /_R (u2, Val\ v, n)
\end{aligned}$$

REL_Subst_R- \triangleleft_R - \triangleright_R -dist-thm

$$\begin{aligned}
& \vdash \forall u1, u2, b : REL_PREDICATE; v : VALUE; n : NAME \\
& \quad | (b, Val\ v, n) \in WF_Subst_R \\
& \quad \wedge (u1, b, u2) \in WF_Cond_R \\
& \quad \bullet \ /_R (u1 \triangleleft_R b \triangleright_R u2, Val\ v, n) \\
& \quad = \ /_R (u1, Val\ v, n) \triangleleft_R \\
& \quad \quad \ /_R (b, Val\ v, n) \triangleright_R \\
& \quad \quad \ /_R (u2, Val\ v, n)
\end{aligned}$$

REL_Subst_R-id-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET; n1, n2 : NAME; oldv, newv : VALUE \\
& \quad | (a, n1, Val\ oldv) \in WF_Equals_R \\
& \quad \wedge n2 \in a \\
& \quad \wedge n1 \neq n2 \\
& \quad \bullet \ /_R (=_R (a, n1, Val\ oldv), Val\ newv, n2) \\
& \quad = \ =_R (a, n1, Val\ oldv)
\end{aligned}$$

REL_Subst_R-id-thm2

$$\begin{aligned}
& \vdash \forall a : ALPHABET; \\
& \quad n1, n2, n3 : NAME; \\
& \quad r : RELATION; \\
& \quad v1, v2 : VALUE \\
& \quad | (a, Rel\ (r, Var\ n1, Var\ n2), Val\ v1) \\
& \quad \quad \in WF_ALPHABET_EXPRESSION \\
& \quad \wedge n1 \neq n3 \\
& \quad \wedge n2 \neq n3 \\
& \quad \wedge n3 \in a \\
& \quad \bullet \ /_R \\
& \quad \quad (=_{+R} (a, Rel\ (r, Var\ n1, Var\ n2), Val\ v1),
\end{aligned}$$

$$\begin{aligned}
& \text{Val } v2, \\
& n3) \\
& = =_{+R} (a, \text{Rel } (r, \text{Var } n1, \text{Var } n2), \text{Val } v1) \\
\mathbf{REL_Subst}_R\text{-com_thm1} \\
& \vdash \forall r : \text{REL_PREDICATE}; n1, n2 : \text{NAME}; v1, v2 : \text{VALUE} \\
& \quad | (r, \text{Val } v1, n1) \in \text{WF_Subst}_R \wedge n2 \in r.1 \wedge n2 \neq n1 \\
& \quad \bullet /_R (/_R (r, \text{Val } v1, n1), \text{Val } v2, n2) \\
& \quad = /_R (/_R (r, \text{Val } v2, n2), \text{Val } v1, n1) \\
\mathbf{REL_Subst}_R\text{-com_thm} \\
& \vdash \forall r : \text{REL_PREDICATE}; \\
& \quad n1, n2, n3, n4 : \text{NAME}; \\
& \quad f : \text{BINARY_F}; \\
& \quad v : \text{VALUE} \\
& \quad | (r, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \in \text{WF_Subst}_R \\
& \quad \wedge n4 \in r.1 \\
& \quad \wedge n4 \neq n1 \\
& \quad \wedge n4 \neq n2 \\
& \quad \wedge n4 \neq n3 \\
& \quad \bullet /_R \\
& \quad \quad (/_R (r, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3), \\
& \quad \quad \text{Val } v, \\
& \quad \quad n4) \\
& \quad = /_R \\
& \quad \quad (/_R (r, \text{Val } v, n4), \\
& \quad \quad \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), \\
& \quad \quad n3) \\
\mathbf{REL_Subst}_R\text{-app_thm} \\
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME}; \text{oldv}, \text{newv} : \text{VALUE} \\
& \quad | (a, n, \text{Val } \text{oldv}) \in \text{WF_Equals}_R \\
& \quad \bullet /_R (=_{\text{R}} (a, n, \text{Val } \text{oldv}), \text{Val } \text{newv}, n) \\
& \quad = =_{+R} (a, \text{Val } \text{newv}, \text{Val } \text{oldv}) \\
\mathbf{REL_PREDICATE_decompose_thm} \\
& \vdash \forall u : \text{REL_PREDICATE} \bullet u = (u.1, u.2) \\
\mathbf{REL_Subst}_R\text{-id_thm1} \\
& \vdash \forall r : \text{REL_PREDICATE}; n : \text{NAME}; v : \text{VALUE}; T : \mathbb{P} \text{ VALUE} \\
& \quad | (r, \text{Val } v, n) \in \text{WF_Subst}_R \\
& \quad \wedge \text{unrestTypedVar } (r, n, T) \\
& \quad \wedge (\forall b : \text{BINDING} \mid n \in \text{dom } b \bullet b \ n \in T)
\end{aligned}$$

$\bullet \ /_R (r, \text{Val } v, n) = r$
REL_Equals_R- \wedge_R -alphabet_ext_thm
 $\vdash \forall a : \text{ALPHABET};$
 $n : \text{NAME};$
 $v : \text{VALUE};$
 $r : \text{REL-PREDICATE}$
 $| (a, n, \text{Val } v) \in \text{WF_Equals}_R \wedge a \subseteq r.1$
 $\bullet =_R (a, n, \text{Val } v) \wedge_R r$
 $=_R (r.1, n, \text{Val } v) \wedge_R r$
REL_Equals_R- \vee_R -alphabet_ext_thm
 $\vdash \forall a : \text{ALPHABET};$
 $n : \text{NAME};$
 $v : \text{VALUE};$
 $r : \text{REL-PREDICATE}$
 $| (a, n, \text{Val } v) \in \text{WF_Equals}_R \wedge a \subseteq r.1$
 $\bullet =_R (a, n, \text{Val } v) \vee_R r$
 $=_R (r.1, n, \text{Val } v) \vee_R r$
REL_Equals_R-exps_thm
 $\vdash \forall a : \text{ALPHABET}; e : \text{EXPRESSION}$
 $| (a, e, e) \in \text{WF_ALPHABET_EXPRESSION}$
 $\bullet =_{+R} (a, e, e) = \text{True}_R a$
REL_Equals_R-exps_thm1
 $\vdash \forall a : \text{ALPHABET}; v1, v2 : \text{VALUE}$
 $| (a, \text{Val } v1, \text{Val } v2) \in \text{WF_ALPHABET_EXPRESSION}$
 $\wedge v1 \neq v2$
 $\bullet =_{+R} (a, \text{Val } v1, \text{Val } v2) = \text{False}_R a$
REL_Subst_R-app_thm1
 $\vdash \forall a : \text{ALPHABET};$
 $r : \text{RELATION};$
 $n1, n2 : \text{NAME};$
 $v1, v2 : \text{VALUE}$
 $| (a, \text{Rel } (r, \text{Var } n1, \text{Var } n2), \text{Val } v1)$
 $\in \text{WF_ALPHABET_EXPRESSION}$
 $\wedge n1 \neq n2$
 $\bullet \ /_R$
 $(=_{+R} (a, \text{Rel } (r, \text{Var } n1, \text{Var } n2), \text{Val } v1),$
 $\text{Val } v2,$
 $n1)$

$$\begin{aligned}
&= =_{+R} (a, \text{Rel } (r, \text{Val } v2, \text{Var } n2), \text{Val } v1) \\
\mathbf{REL_Subst}_R\text{-app_thm2} \\
&\vdash \forall a : \text{ALPHABET}; \\
&\quad r : \text{RELATION}; \\
&\quad f : \text{BINARY_F}; \\
&\quad n1, n2, n3 : \text{NAME}; \\
&\quad v1, v2 : \text{VALUE} \\
&\mid n1 \in a \wedge n2 \in a \wedge n3 \in a \\
&\bullet /_R \\
&\quad (=_{+R} (a, \text{Rel } (r, \text{Val } v1, \text{Var } n1), \text{Val } v2), \\
&\quad \text{Fun}_2 (f, \text{Var } n2, \text{Var } n3), \\
&\quad n1) \\
&= =_{+R} \\
&\quad (a, \\
&\quad \text{Rel } (r, \text{Val } v1, \text{Fun}_2 (f, \text{Var } n2, \text{Var } n3)), \\
&\quad \text{Val } v2) \\
\mathbf{REL_Bool_equivalence_thm} \\
&\vdash \forall b1, b2 : \mathbb{B} \bullet \text{Bool } b1 = \text{Bool } b2 \Leftrightarrow b1 \Leftrightarrow b2 \\
\mathbf{REL_}\neg_R\text{-Equals}_R\text{-true_thm} \\
&\vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
&\mid (a, n, \text{Val } (\text{Bool true})) \in \text{WF_Equals}_R \\
&\quad \wedge (\forall b : \text{BINDING} \mid n \in \text{dom } b \bullet b \ n \in \text{BOOL_VAL}) \\
&\bullet \neg_R (=_R (a, n, \text{Val } (\text{Bool true}))) \\
&\quad =_R (a, n, \text{Val } (\text{Bool false})) \\
\mathbf{REL_Subst}_R\text{-}\neg_R\text{-dist_thm1} \\
&\vdash \forall u : \text{REL_PREDICATE}; f : \text{BINARY_F}; n1, n2, n3 : \text{NAME} \\
&\mid (u, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \in \text{WF_Subst}_R \\
&\bullet /_R (\neg_R u, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \\
&\quad = \neg_R (/_R (u, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3)) \\
\mathbf{REL_Subst}_R\text{-}\vee_R\text{-dist_thm1} \\
&\vdash \forall u1, u2 : \text{REL_PREDICATE}; \\
&\quad f : \text{BINARY_F}; \\
&\quad n1, n2, n3 : \text{NAME} \\
&\mid (u1, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \in \text{WF_Subst}_R \\
&\quad \wedge (u2, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \\
&\quad \in \text{WF_Subst}_R \\
&\bullet /_R (u1 \vee_R u2, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \\
&\quad = /_R (u1, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3)
\end{aligned}$$

$\vee_R /_R (u2, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3)$

REL_Subst_R-id.thm3
 $\vdash \forall a : \text{ALPHABET};$
 $\quad n1, n2, n3, n4 : \text{NAME};$
 $\quad \text{oldv} : \text{VALUE};$
 $\quad f : \text{BINARY_F}$
 $\mid (a, n1, \text{Val oldv}) \in \text{WF_Equals}_R$
 $\quad \wedge n2 \in a$
 $\quad \wedge n3 \in a$
 $\quad \wedge n4 \in a$
 $\quad \wedge n1 \neq n2$
 $\bullet /_R$
 $\quad (=_R (a, n1, \text{Val oldv}),$
 $\quad \quad \text{Fun}_2 (f, \text{Var } n3, \text{Var } n4),$
 $\quad \quad n2)$
 $\quad =_R (a, n1, \text{Val oldv})$

ALPHABET_seqd_double_size.thm
 $\vdash \forall n1, n2 : \text{NAME} \bullet \# \langle n1, n2 \rangle = 2$

ALPHABET_seqd_singleton_size.thm
 $\vdash \forall n1 : \text{NAME} \bullet \# \langle n1 \rangle = 1$

ALPHABET_seqd_double_head.thm
 $\vdash \forall n1, n2 : \text{NAME} \bullet \text{head } \langle n1, n2 \rangle = n1$

ALPHABET_seqd_double_tail.thm
 $\vdash \forall n1, n2 : \text{NAME} \bullet \text{tail } \langle n1, n2 \rangle = \langle n2 \rangle$

EXPRESSION_seqd_double_size.thm
 $\vdash \forall e1, e2 : \text{EXPRESSION} \bullet \# \langle e1, e2 \rangle = 2$

EXPRESSION_seqd_singleton_size.thm
 $\vdash \forall e1 : \text{EXPRESSION} \bullet \# \langle e1 \rangle = 1$

EXPRESSION_seqd_double_head.thm
 $\vdash \forall e1, e2 : \text{EXPRESSION} \bullet \text{head } \langle e1, e2 \rangle = e1$

EXPRESSION_seqd_double_tail.thm
 $\vdash \forall e1, e2 : \text{EXPRESSION} \bullet \text{tail } \langle e1, e2 \rangle = \langle e2 \rangle$

REL_<_R>_R-idem.thm1
 $\vdash \forall P, Q, b : \text{REL_PREDICATE}$
 $\mid (P, b, Q) \in \text{WF_Cond}_R$
 $\bullet P <_R b >_R P <_R b >_R Q = P <_R b >_R Q$

REL_⇒_R-∧_R-distribution.thm
 $\vdash \forall a, b, c, d, e, f : \text{REL_PREDICATE}$

$$\begin{array}{l}
| a.1 = c.1 \\
\wedge c.1 \subseteq b.1 \\
\wedge b.1 = d.1 \\
\wedge d.1 = e.1 \\
\wedge e.1 = f.1 \\
\bullet ((a \wedge_R b) \Rightarrow_R c \wedge_R d) \wedge_R (a \wedge_R e) \Rightarrow_R c \wedge_R f \\
= (a \wedge_R b \vee_R e) \Rightarrow_R c \wedge_R (b \Rightarrow_R d) \wedge_R e \Rightarrow_R f \\
\mathbf{REL_PREDICATE_WF_Skip_thm2} \\
\vdash \forall a : WF_Skip_R \bullet a \in ALPHABET \wedge a \in homogeneous \\
\mathbf{REL_UnrestTypedVar_True_thm} \\
\vdash \forall a : ALPHABET; n : NAME; T : \mathbb{P} \text{ VALUE} \\
| n \in a \\
\bullet unrestTypedVar (True_R a, n, T) \Leftrightarrow true \\
\mathbf{REL_UnrestTypedVar_}\wedge_R\text{-thm} \\
\vdash \forall u1, u2 : REL_PREDICATE; n : NAME; T : \mathbb{P} \text{ VALUE} \\
\bullet unrestTypedVar (u1, n, T) \\
\wedge unrestTypedVar (u2, n, T) \\
\Rightarrow unrestTypedVar (u1 \wedge_R u2, n, T) \\
\mathbf{REL_}\neg_R\text{-}\vee_R\text{-}\wedge_R\text{-abs_thm} \\
\vdash \forall P, Q : REL_PREDICATE \\
\bullet \neg_R P \vee_R P \wedge_R Q = \neg_R P \vee_R Q \\
\mathbf{REL_}\Rightarrow_R\text{-abs_thm} \\
\vdash \forall P, Q, R : REL_PREDICATE \\
\bullet (P \wedge_R Q) \Rightarrow_R P \wedge_R R = (P \wedge_R Q) \Rightarrow_R R \\
\mathbf{REL_}\wedge_R\text{-}\triangleleft_R\text{-}\triangleright_R\text{-dist_thm} \\
\vdash \forall u, u1, u2, b : REL_PREDICATE \\
| (u1, b, u2) \in WF_Cond_R \\
\bullet u \wedge_R u1 \triangleleft_R b \triangleright_R u2 \\
= (u \wedge_R u1) \triangleleft_R b \triangleright_R u \wedge_R u2 \\
\mathbf{REL_UnrestTypedVar_Equals_Var_thm} \\
\vdash \forall n1, n2, n3 : NAME; T : \mathbb{P} \text{ VALUE}; a : ALPHABET \\
| (a, n2, Var n3) \in WF_Equals_R \\
\wedge (\forall b : BINDING \mid n1 \in dom b \bullet b n1 \in T) \\
\wedge n1 \neq n2 \\
\wedge n1 \neq n3 \\
\wedge n1 \in a \\
\bullet unrestTypedVar (=_R (a, n2, Var n3), n1, T) \\
\mathbf{REL_}\oplus\text{-Eval_thm1}
\end{array}$$

$$\begin{aligned}
& \vdash \forall b : \text{BINDING}; n : \text{NAME}; v1, v2 : \text{VALUE} \\
& \bullet \text{Eval } (b, \text{Val } v1) = \text{Eval } (b \oplus \{n \mapsto v2\}, \text{Val } v1) \\
\mathbf{REL_UnrestTypedVar_Equals_R_Val_thm} \\
& \vdash \forall n1, n2 : \text{NAME}; \\
& \quad T : \mathbb{P} \text{ VALUE}; \\
& \quad a : \text{ALPHABET}; \\
& \quad val : \text{VALUE} \\
& \quad | (a, n2, \text{Val } val) \in \text{WF_Equals}_R \\
& \quad \wedge (\forall b : \text{BINDING} \mid n1 \in \text{dom } b \bullet b \ n1 \in T) \\
& \quad \wedge n1 \neq n2 \\
& \quad \wedge n1 \in a \\
& \bullet \text{unrestTypedVar } (=_R (a, n2, \text{Val } val), n1, T) \\
\mathbf{REL_Subst_R_id_thm4} \\
& \vdash \forall r : \text{REL_PREDICATE}; \\
& \quad n1, n2, n3 : \text{NAME}; \\
& \quad f : \text{BINARY_F}; \\
& \quad T : \mathbb{P} \text{ VALUE} \\
& \quad | (r, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) \in \text{WF_Subst}_R \\
& \quad \wedge \text{unrestTypedVar } (r, n3, T) \\
& \quad \wedge (\forall b : \text{BINDING} \mid n3 \in \text{dom } b \bullet b \ n3 \in T) \\
& \bullet /_R (r, \text{Fun}_2 (f, \text{Var } n1, \text{Var } n2), n3) = r \\
\mathbf{REL_Subst_R_app_thm3} \\
& \vdash \forall a : \text{ALPHABET}; n1, n2 : \text{NAME}; v : \text{VALUE} \\
& \quad | (a, n1, \text{Var } n2) \in \text{WF_Equals}_R \wedge n1 \neq n2 \\
& \bullet /_R (=_R (a, n1, \text{Var } n2), \text{Val } v, n2) \\
& \quad =_R (a, n1, \text{Val } v) \\
\mathbf{ALPHABET_dash_n_neq_n_thm} \\
& \vdash \forall n : \text{NAME} \mid n \in \text{undashed} \bullet \text{dash } n \neq n \\
\mathbf{REL_Subst_R_app_thm4} \\
& \vdash \forall a : \text{ALPHABET}; \\
& \quad f : \text{BINARY_F}; \\
& \quad n1, n2, n3 : \text{NAME}; \\
& \quad v : \text{VALUE} \\
& \quad | n1 \in a \wedge n2 \in a \wedge n3 \in a \\
& \bullet /_R \\
& \quad (=_R (a, n1, \text{Val } v), \\
& \quad \text{Fun}_2 (f, \text{Var } n2, \text{Var } n3), \\
& \quad n1)
\end{aligned}$$

$$=_{+R} (a, \text{Fun}_2 (f, \text{Var } n2, \text{Var } n3), \text{Val } v)$$

REL_inv_Seq_thm

$$\vdash \forall s : \text{seq VALUE} \bullet (\text{Seq } \sim) (\text{Seq } s) = s$$

REL_SeqDif_R_thm

$$\begin{aligned} &\vdash \forall s1, s2 : \text{VALUE} \\ &\quad | \{s1, s2\} \subseteq \text{SEQ_VAL} \\ &\quad \wedge (\text{Seq } \sim) s2 \text{ prefix}_Z (\text{Seq } \sim) s1 \\ &\quad \bullet s1 \text{ SeqDif}_R s2 \in \text{SEQ_VAL} \end{aligned}$$

REL_Prefix_Z-clauses_thm

$$\vdash \forall s : \text{seq VALUE} \bullet \langle \rangle \text{ prefix}_Z s$$

REL_Prefix_Z-SeqDif_R_thm

$$\begin{aligned} &\vdash \forall s1, s2 : \text{VALUE} \\ &\quad | \{s1, s2\} \subseteq \text{SEQ_VAL} \\ &\quad \wedge (\text{Seq } \sim) s2 \text{ prefix}_Z (\text{Seq } \sim) s1 \\ &\quad \bullet (\text{Seq } \sim) (\text{Seq } \langle \rangle) \\ &\quad \text{prefix}_Z (\text{Seq } \sim) (s1 \text{ SeqDif}_R s2) \end{aligned}$$

REL_<-Eval_thm1

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; a : \text{ALPHABET}; v : \text{VALUE} \\ &\quad \bullet \text{Eval } (a < b, \text{Val } v) = v \end{aligned}$$

REL_<-Eval_thm2

$$\begin{aligned} &\vdash \forall b : \text{BINDING}; \\ &\quad a : \text{ALPHABET}; \\ &\quad r : \text{RELATION}; \\ &\quad n1, n2 : \text{NAME} \\ &\quad | n1 \in \text{dom } b \wedge n1 \in a \wedge n2 \in \text{dom } b \wedge n2 \in a \\ &\quad \bullet \text{Eval } (a < b, \text{Rel } (r, \text{Var } n1, \text{Var } n2)) \\ &\quad = \text{Bool } ((\text{Eval } (b, \text{Var } n1), \text{Eval } (b, \text{Var } n2))) \in r \end{aligned}$$

REL_Prefix_Z-id_thm

$$\vdash \forall s : \text{seq VALUE} \bullet s \text{ prefix}_Z s$$

REL_Prefix_Z-id_thm1

$$\begin{aligned} &\vdash \forall s : \text{VALUE} \\ &\quad | s \in \text{SEQ_VAL} \\ &\quad \bullet (\text{Seq } \sim) s \text{ prefix}_Z (\text{Seq } \sim) s \end{aligned}$$

REL_Seq_Z-id_thm1

$$\vdash \forall s1, s2 : \text{seq VALUE} \bullet \text{Seq } s1 = \text{Seq } s2 \Leftrightarrow s1 = s2$$

REL_Seq_Z-inv-id_thm

$$\begin{aligned} &\vdash \forall v1, v2 : \text{VALUE} \\ &\quad | v1 \in \text{ran Seq} \wedge v2 \in \text{ran Seq} \end{aligned}$$

$$\bullet (Seq \sim) v1 = (Seq \sim) v2 \Leftrightarrow v1 = v2$$

REL_Equals_R- \wedge_R -alphabet_ext_thm1

$$\begin{aligned} &\vdash \forall a : ALPHABET; n1, n2 : NAME; r : REL_PREDICATE \\ &\quad | (a, n1, Var\ n2) \in WF_Equals_R \wedge a \subseteq r.1 \\ &\bullet =_R (a, n1, Var\ n2) \wedge_R r \\ &\quad = =_R (r.1, n1, Var\ n2) \wedge_R r \end{aligned}$$

REL- \triangleleft_R - \triangleright_R -idem_thm2

$$\begin{aligned} &\vdash \forall u1, u2 : REL_PREDICATE \\ &\quad | u1.1 = u2.1 \\ &\bullet (u1 \wedge_R u2) \triangleleft_R u1 \triangleright_R u1 \vee_R u2 = u2 \end{aligned}$$

ALPHABET_homogeneous_set_def_thm2

$$\begin{aligned} &\vdash \forall a1, a2 : ALPHABET \\ &\quad | a1 \in homogeneous \wedge a2 \in homogeneous \\ &\bullet a1 \setminus a2 \in homogeneous \end{aligned}$$

ALPHABET_homogeneous- \cup -thm1

$$\begin{aligned} &\vdash \forall a1, a2 : ALPHABET \\ &\quad | a1 \in homogeneous \wedge a2 \in homogeneous \\ &\bullet a1 \cup a2 \in homogeneous \end{aligned}$$

REL_Equals_exps- \wedge_R -alphabet_ext_thm

$$\begin{aligned} &\vdash \forall a : ALPHABET; \\ &\quad n1, n2 : NAME; \\ &\quad rel : RELATION; \\ &\quad v : VALUE; \\ &\quad r : REL_PREDICATE \\ &\quad | (a, Rel\ (rel, Var\ n1, Var\ n2), Val\ v) \\ &\quad \quad \in WF_ALPHABET_EXPRESSION \\ &\quad \quad \wedge a \subseteq r.1 \\ &\bullet =_{+R} (a, Rel\ (rel, Var\ n1, Var\ n2), Val\ v) \wedge_R r \\ &\quad = =_{+R} (r.1, Rel\ (rel, Var\ n1, Var\ n2), Val\ v) \\ &\quad \quad \wedge_R r \end{aligned}$$

REL_Equals_exps- \vee_R -alphabet_ext_thm

$$\begin{aligned} &\vdash \forall a : ALPHABET; \\ &\quad n1, n2 : NAME; \\ &\quad rel : RELATION; \\ &\quad v : VALUE; \\ &\quad r : REL_PREDICATE \\ &\quad | (a, Rel\ (rel, Var\ n1, Var\ n2), Val\ v) \\ &\quad \quad \in WF_ALPHABET_EXPRESSION \end{aligned}$$

$$\begin{aligned}
& \wedge a \subseteq r.1 \\
& \bullet =_{+R} (a, \text{Rel } (rel, \text{Var } n1, \text{Var } n2), \text{Val } v) \vee_R r \\
& =_{+R} (r.1, \text{Rel } (rel, \text{Var } n1, \text{Var } n2), \text{Val } v) \\
& \vee_R r
\end{aligned}$$

REL- \oplus -idem_thm

$$\begin{aligned}
& \vdash \forall b : \text{BINDING}; n : \text{NAME}; v : \text{VALUE} \\
& \quad | n \in \text{dom } b \wedge b \ n = v \\
& \bullet b \oplus \{n \mapsto v\} = b
\end{aligned}$$

REL-Subst_R- \wedge_R -idem_thm

$$\begin{aligned}
& \vdash \forall r : \text{REL-PREDICATE}; n : \text{NAME}; v : \text{VALUE} \\
& \quad | n \in r.1 \\
& \bullet /_R (r, \text{Val } v, n) \wedge_R =_R (r.1, n, \text{Val } v) \\
& = r \wedge_R =_R (r.1, n, \text{Val } v)
\end{aligned}$$

REL- $\wedge_R \Rightarrow_R$ -simpl_thm

$$\begin{aligned}
& \vdash \forall u1, u2 : \text{REL-PREDICATE} \\
& \bullet u1 \wedge_R u1 \Rightarrow_R u2 = u1 \wedge_R u2
\end{aligned}$$

REL- $\wedge_R \Rightarrow_R$ -simpl_thm1

$$\begin{aligned}
& \vdash \forall u1, u2 : \text{REL-PREDICATE} \\
& \quad | u1.1 \subseteq u2.1 \\
& \bullet u2 \wedge_R u1 \Rightarrow_R u2 = u2
\end{aligned}$$

REL- $\vee_R \wedge_R \Rightarrow_R$ -simpl_thm

$$\begin{aligned}
& \vdash \forall u1, u2 : \text{REL-PREDICATE} \\
& \quad | u1.1 \subseteq u2.1 \\
& \bullet (u1 \vee_R u2) \wedge_R u1 \Rightarrow_R u2 = u2
\end{aligned}$$

REL- $\langle_R \Rightarrow_R \rangle_R$ -thm

$$\begin{aligned}
& \vdash \forall u1, u2 : \text{REL-PREDICATE} \\
& \quad | u1.1 = u2.1 \\
& \bullet \langle_R u1 \Rightarrow_R u2 \rangle_R = \text{True}_R \ \emptyset \\
& \Leftrightarrow u1 \Rightarrow_R u2 = \text{True}_R \ u1.1
\end{aligned}$$

BINDINGS-empty_dom_thm

$$\vdash \{b : \text{BINDING} \mid \text{dom } b = \{\}\} = \{\{\}\}$$

REL- $\triangleleft_R \triangleright_R$ -cases_thm

$$\begin{aligned}
& \vdash \forall u1, u2, b : \text{REL-PREDICATE} \\
& \quad | (u1, b, u2) \in \text{WF-Cond}_R \\
& \bullet u1 \triangleleft_R b \triangleright_R u2 \\
& = (b \wedge_R u1) \triangleleft_R b \triangleright_R \neg_R b \wedge_R u2
\end{aligned}$$

REL-UnrestTypedVar-False_R-thm

$$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; T : \mathbb{P} \ \text{VALUE}$$

$$\begin{array}{l}
| n \in a \\
\bullet \text{unrestTypedVar } (\text{False}_R \ a, \ n, \ T) \Leftrightarrow \text{true} \\
\mathbf{VALUE_SEQ_EVENT_VAL_thm1} \\
\vdash \text{SEQ_EVENT_VAL} \subseteq \text{VALUE} \\
\mathbf{VALUE_SEQ_VAL_thm} \\
\vdash \forall s : \text{SEQ_VAL} \bullet (\text{Seq } \sim) \ s \in \text{seq VALUE} \\
\mathbf{REL_inv_Seq_thm1} \\
\vdash \forall s : \text{SEQ_VAL} \bullet \text{Seq } ((\text{Seq } \sim) \ s) = s \\
\mathbf{VALUE_SEQ_VAL_thm1} \\
\vdash \forall s : \text{seq VALUE} \bullet \text{Seq } s \in \text{SEQ_VAL} \\
\mathbf{VALUE_Prefix_thm} \\
\vdash (- \leq_R -) \\
= \{s1, s2 : \text{seq VALUE} \\
| s1 \text{ prefix}_Z s2 \\
\bullet (\text{Seq } s1, \text{Seq } s2)\} \\
\mathbf{ALPHABET_}\subseteq_\mathbf{thm1} \\
\vdash \forall a1, a2 : \text{ALPHABET} \mid a1 \subseteq a2 \wedge \neg x2 \in a2 \bullet \neg x2 \in a1 \\
\mathbf{ALPHABET_}\neq_\mathbf{thm} \\
\vdash \forall a1, a2 : \text{ALPHABET} \\
\bullet a1 \neq a2 \Rightarrow a1 \cap a2 = \emptyset \wedge a1 \cap a2 = \emptyset \Rightarrow a1 \neq a2 \\
\mathbf{BINDINGS_}\cup_\mathbf{thm} \\
\vdash \forall b1, b2 : \text{BINDING} \\
| \text{dom } b1 \triangleleft b2 = \text{dom } b2 \triangleleft b1 \\
\bullet b1 \cup b2 \in \text{BINDING} \\
\mathbf{BINDINGS_}\cup_\mathbf{thm1} \\
\vdash \forall b1, b2 : \text{BINDING} \\
| \text{dom } b1 \cap \text{dom } b2 = \{\} \\
\bullet b1 \cup b2 \in \text{BINDING} \\
\mathbf{BINDINGS_dom_}\cup_\mathbf{thm} \\
\vdash \forall b1, b2 : \text{BINDING}; \ n : \text{NAME} \\
\bullet n \in \text{dom } (b1 \cup b2) \Leftrightarrow n \in \text{dom } b1 \vee n \in \text{dom } b2 \\
\mathbf{BINDINGS_dom_}\cup_\mathbf{thm1} \\
\vdash \forall b1, b2 : \text{BINDING}; \ n : \text{NAME} \\
\bullet \text{dom } (b1 \cup b2) = \text{dom } b1 \cup \text{dom } b2 \\
\mathbf{BINDINGS_dom_}\cup_\mathbf{thm2} \\
\vdash \forall b1, b2 : \text{BINDING} \bullet \text{dom } (b1 \cup b2) = \text{dom } b1 \cup \text{dom } b2 \\
\mathbf{BINDINGS_}\triangleleft_\cup_\mathbf{com_thm} \\
\vdash \forall f, g : \text{BINDING}
\end{array}$$

$\bullet \text{undashed} \triangleleft f \cup \text{dashed} \triangleleft g$
 $= \text{dashed} \triangleleft g \cup \text{undashed} \triangleleft f$

BINDINGS_ $\triangleleft \cup \triangleleft$ _app_thm2

$\vdash \forall f : \text{BINDING}; g : \text{BINDING}; n : \text{dom } f$
 $\quad | n \in \text{undashed}$
 $\bullet (\text{undashed} \triangleleft f \cup \text{dashed} \triangleleft g) n = f n$

BINDINGS_ $\triangleleft \cup \triangleleft$ _app_thm3

$\vdash \forall f : \text{BINDING}; g : \text{BINDING}; n : \text{dom } f$
 $\quad | n \in \text{dashed}$
 $\bullet (\text{dashed} \triangleleft f \cup \text{undashed} \triangleleft g) n = f n$

REL_ \triangleleft _dom_thm1

$\vdash \forall b : \text{BINDING}; a : \text{ALPHABET}; n : \text{NAME}$
 $\quad | n \in \text{dom } (a \triangleleft b) \wedge n \in a$
 $\bullet n \in \text{dom } b$

REL_ \triangleleft _dom_thm2

$\vdash \forall b : \text{BINDING}; a : \text{ALPHABET} \bullet \text{dom } (a \triangleleft b) = a \cap \text{dom } b$

REL_ \triangleleft _dom_thm3

$\vdash \forall b : \text{BINDING}; a : \text{ALPHABET}; n : \text{NAME}$
 $\quad | n \in \text{dom } b \wedge \neg n \in a$
 $\bullet \neg$
 $n \in \text{dom } (a \triangleleft b)$

REL_ \triangleleft _idem_thm

$\vdash \forall b : \text{BINDING}; a : \text{ALPHABET} \bullet a \triangleleft a \triangleleft b = a \triangleleft b$

REL_ \leq_R _trans_thm

$\vdash \forall a, b, c : \text{SEQ_VAL} \bullet a \leq_R b \wedge b \leq_R c \Rightarrow a \leq_R c$

BINDINGS_ \triangleleft _idem_thm3

$\vdash \forall a : \text{ALPHABET}; b : \text{BINDING} \bullet a \triangleleft a \triangleleft b = a \triangleleft b$

BINDINGS_dom_ \triangleleft _thm

$\vdash \forall a : \text{ALPHABET}; b : \text{BINDING}; n : \text{NAME}$
 $\quad | n \in a \wedge n \in \text{dom } (a \triangleleft b)$
 $\bullet n \in \text{dom } b$

REL_ $\Rightarrow_R \wedge_R$ _distr_thm

$\vdash \forall a, b, c : \text{REL_PREDICATE}$
 $\bullet (a \Rightarrow_R b) \wedge_R a \Rightarrow_R c = a \Rightarrow_R b \wedge_R c$

ALPHABET_homogeneous_dash_thm1

$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
 $\quad | a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \text{dash } n \in a$
 $\bullet n \in a$

REL_Eval_Fun₁_thm

- $$\vdash \forall b : \text{BINDING}; f : \text{UNARY_F}; e1 : \text{EXPRESSION}$$
- $$| (b, e1) \in \text{WF_BINDING_EXPRESSION}_R$$
- $\text{Eval } (b, \text{Fun}_1(f, e1)) = f(\text{Eval } (b, e1))$

REL_FV_Fun₁_thm

- $$\vdash \forall f : \text{UNARY_F}; e1 : \text{EXPRESSION}$$
- $\text{FV } (\text{Fun}_1(f, e1)) = \text{FV } e1$

REL_PREDICATE_VarT_R_thm

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; T : \text{SET_VAL}$$
- $$| (a, n, T) \in \text{WF_VarT}_{R_EndTR}$$
- $\text{varT}_R(a, n, T) \in \text{REL_PREDICATE}$

REL_PREDICATE_EndT_R_thm

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; T : \text{SET_VAL}$$
- $$| (a, n, T) \in \text{WF_VarT}_{R_EndTR}$$
- $\text{endT}_R(a, n, T) \in \text{REL_PREDICATE}$

ALPHABET_comp \Rightarrow _contr_comp_thm3

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$$
- $$| n \in a \wedge n \in \text{undashed} \wedge (a, a) \in \text{composable}$$
- $(a, a \cup (a \setminus \{n, \text{dash } n\})) \in \text{composable}$

REL_PREDICATE_Var_R-Semi_R-Assign_R_thm

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}; T : \text{SET_VAL}$$
- $$| a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \{n, \text{dash } n\} \subseteq a$$
- $\text{var}_R(a, n) ;_R \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{REL_PREDICATE}$

REL_PREDICATE_Var_R-Semi_R-Assign_R_thm2

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}; T : \text{SET_VAL}$$
- $$| a \in \text{homogeneous}$$
- $$\wedge n \in \text{undashed}$$
- $$\wedge \{n, \text{dash } n\} \subseteq a$$
- $$\wedge v \in (\text{Set } \sim) T$$
- $\text{var}_R(a, n) ;_R \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{REL_PREDICATE}$

REL_PREDICATE_Var_R-Semi_R-Assign_R_thm3

- $$\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; T : \text{SET_VAL}$$
- $$| a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \{n, \text{dash } n\} \subseteq a$$
- $\{v : \text{VALUE}$

$$| v \in (\text{Set } \sim) T$$
 - $\text{var}_R(a, n) ;_R \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle)\}$

$$\subseteq \text{REL_PREDICATE}$$

ALPHABET_dash_n_∩_undashed_thm
 $\vdash \forall n : \text{NAME} \bullet \{\text{dash } n\} \cap \text{undashed} = \emptyset$

REL_PREDICATE_∈_WF_Assign_R_thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}$
 $\mid n \in a \wedge n \in \text{undashed} \wedge a \in \text{homogeneous}$
 $\bullet (a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{WF_Assign}_R$

REL_PREDICATE_Assign_R_thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}$
 $\mid (a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{WF_Assign}_R$
 $\wedge n \in \text{undashed}$
 $\wedge \{n, \text{dash } n\} \subseteq a$
 $\wedge a \in \text{homogeneous}$
 $\bullet \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{REL_PREDICATE}$

REL_PREDICATE_Assign_R_alphabet_thm2
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}$
 $\mid (a, \langle n \rangle, \langle \text{Val } v \rangle) \in \text{WF_Assign}_R$
 $\wedge n \in \text{undashed}$
 $\wedge \{n, \text{dash } n\} \subseteq a$
 $\wedge a \in \text{homogeneous}$
 $\bullet (\text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle)).1 = a$

REL_PREDICATE_Assign_R_Semi_R_End_R_thm
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}; T : \text{SET_VAL}$
 $\mid a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \{n, \text{dash } n\} \subseteq a$
 $\bullet \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle) ;_R \text{end}_R(a, n)$
 $\in \text{REL_PREDICATE}$

REL_PREDICATE_Assign_R_Semi_R_End_R_thm2
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; v : \text{VALUE}; T : \text{SET_VAL}$
 $\mid a \in \text{homogeneous}$
 $\wedge n \in \text{undashed}$
 $\wedge \{n, \text{dash } n\} \subseteq a$
 $\wedge v \in (\text{Set } \sim) T$
 $\bullet \text{Assign}_R(a, \langle n \rangle, \langle \text{Val } v \rangle) ;_R \text{end}_R(a, n)$
 $\in \text{REL_PREDICATE}$

REL_PREDICATE_Assign_R_Semi_R_End_R_thm3
 $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}; T : \text{SET_VAL}$
 $\mid a \in \text{homogeneous} \wedge n \in \text{undashed} \wedge \{n, \text{dash } n\} \subseteq a$
 $\bullet \{v : \text{VALUE}$

$$\begin{aligned}
& | v \in (Set \sim) T \\
& \bullet Assign_R (a, \langle n \rangle, \langle Val v \rangle) ;_R end_R (a, n) \} \\
& \subseteq REL_PREDICATE \\
\mathbf{REL_PREDICATE_VarT_R_alphabet_thm} \\
& \vdash \forall a : ALPHABET; n : NAME; T : SET_VAL \\
& | (a, n, T) \in WF_VarT_{R_EndTR} \\
& \bullet (varT_R (a, n, T)).1 = a \setminus \{n\} \\
\mathbf{REL_PREDICATE_EndT_R_alphabet_thm} \\
& \vdash \forall a : ALPHABET; n : NAME; T : SET_VAL \\
& | (a, n, T) \in WF_VarT_{R_EndTR} \\
& \bullet (endT_R (a, n, T)).1 = a \setminus \{dash\ n\} \\
\mathbf{REL_}\oplus_ \oplus_ \mathbf{thm} & \vdash \forall b : BINDING; n : NAME; v1, v2 : VALUE \\
& \bullet b \oplus \{n \mapsto v1\} \oplus \{n \mapsto v2\} = b \oplus \{n \mapsto v2\} \\
\mathbf{REL_UnrestTypedVar_}\vee_ \mathbf{thm} \\
& \vdash \forall u1, u2 : REL_PREDICATE; n : NAME; T : \mathbb{P}\ VALUE \\
& \bullet unrestTypedVar (u1, n, T) \\
& \quad \wedge unrestTypedVar (u2, n, T) \\
& \quad \Rightarrow unrestTypedVar (u1 \vee_R u2, n, T) \\
\mathbf{REL_UnrestVar_Contract_R_dist_thm} \\
& \vdash \forall a : ALPHABET; P : REL_PREDICATE \\
& \bullet UnrestVar P \setminus a \subseteq UnrestVar (P -_R a) \\
\mathbf{REL_CONDITION_Contract_R_thm} \\
& \vdash \forall a : ALPHABET; P : REL_CONDITION \\
& \bullet P -_R a \in REL_CONDITION \\
\mathbf{REL_}\triangleleft_R_ \triangleright_R_ \mathbf{cases_thm1} \\
& \vdash \forall u1, u2, b : REL_PREDICATE \\
& | (u1, b, u2) \in WF_Cond_R \\
& \bullet u1 \triangleleft_R b \triangleright_R u2 = u1 \triangleleft_R b \triangleright_R \neg_R b \wedge_R u2 \\
\mathbf{REL_}\triangleleft_R_ \triangleright_R_ \mathbf{cases_thm2} \\
& \vdash \forall u1, u2, b : REL_PREDICATE \\
& | (u1, b, u2) \in WF_Cond_R \\
& \bullet u1 \triangleleft_R b \triangleright_R u2 = (b \wedge_R u1) \triangleleft_R b \triangleright_R u2 \\
\mathbf{ALPHABET_n1_}\neq_ dash_ n2_ \mathbf{thm1} \\
& \vdash \forall n1, n2 : NAME \\
& | n1 \neq n2 \wedge n1 \in undashed \wedge n2 \in undashed \\
& \bullet n2 \neq dash\ n1 \wedge dash\ n1 \neq n2
\end{aligned}$$

9 THE Z THEORY utp-okay

9.1 Parents

utp-rel

9.2 Global Variables

$okay$ $NAME$
 $ALPHABET_OKAY$
 $\mathbb{P} NAME$
 $unrestOKAY$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{B}$
 $unrestOKAY'$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{B}$
 $OKAY$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
 $OKAY'$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
 $(- \sigma_f)$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
 $(- \sigma_t)$ $\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$

9.3 Fixity

fun 0 *rightassoc*
 $(- \sigma_f) (- \sigma_t)$

9.4 Axioms

$okay$ $\vdash okay \in NAME \wedge okay \in undashed$
 $Constraint$ 1 $\vdash (\forall b : BINDING \mid okay \in dom\ b \bullet b\ okay \in BOOL_VAL)$
 $\wedge (\forall b : BINDING$
 $\mid dash\ okay \in dom\ b$
 $\bullet b\ (dash\ okay) \in BOOL_VAL)$
 $unrestOKAY$
 $unrestOKAY'$ $\vdash \{unrestOKAY, unrestOKAY'\} \subseteq REL_PREDICATE \rightarrow$
 $\wedge (\forall r : REL_PREDICATE$
 $\bullet (unrestOKAY\ r$
 $\Leftrightarrow unrestTypedVar\ (r, okay, BOOL_VAL))$
 $\wedge (unrestOKAY'\ r$
 $\Leftrightarrow unrestTypedVar\ (r, dash\ okay, BOOL_VAL)))$

OKAY	$\vdash OKAY \in REL_PREDICATE$ $\wedge OKAY = =_R (ALPHABET_OKAY, okay, Val (Bool true))$
OKAY'	$\vdash OKAY' \in REL_PREDICATE$ $\wedge OKAY'$ $= =_R (ALPHABET_OKAY, dash\ okay, Val (Bool true))$
- σ_f	$\vdash (-\sigma_f) \in REL_PREDICATE \leftrightarrow REL_PREDICATE$ $\wedge (\forall d : REL_PREDICATE$ $\quad dash\ okay \in d.1$ $\quad \bullet d\ \sigma_f = /_R (d, Val (Bool false), dash\ okay))$
- σ_t	$\vdash (-\sigma_t) \in REL_PREDICATE \rightarrow REL_PREDICATE$ $\wedge (\forall d : REL_PREDICATE$ $\quad dash\ okay \in d.1$ $\quad \bullet d\ \sigma_t = /_R (d, Val (Bool true), dash\ okay))$

9.5 Definitions

ALPHABET_OKAY

$\vdash ALPHABET_OKAY = \{okay, dash\ okay\}$

9.6 Theorems

OKAY_ \subseteq _NAME_thm

$\vdash \{okay, dash\ okay\} \subseteq NAME$

OKAY_values_thm

$\vdash \forall b : BINDING$
 $\quad | okay \in dom\ b$
 $\quad \bullet b\ okay = Bool\ true \vee b\ okay = Bool\ false$

OKAY_DASH_alues_thm

$\vdash \forall b : BINDING$
 $\quad | dash\ okay \in dom\ b$
 $\quad \bullet b\ (dash\ okay) = Bool\ true$
 $\quad \vee b\ (dash\ okay) = Bool\ false$

OKAY_clauses_thm

$\vdash \forall b : BINDING$
 $\quad | \{okay, dash\ okay\} \subseteq dom\ b$
 $\quad \bullet (\neg b\ okay = Bool\ true \Rightarrow b\ okay = Bool\ false)$
 $\quad \wedge (\neg b\ okay = Bool\ false \Rightarrow b\ okay = Bool\ true)$
 $\quad \wedge (\neg$

$$\begin{aligned}
& b \text{ (dash okay)} = \text{Bool true} \\
& \Rightarrow b \text{ (dash okay)} = \text{Bool false} \\
& \wedge (\neg \\
& \quad b \text{ (dash okay)} = \text{Bool false} \\
& \quad \Rightarrow b \text{ (dash okay)} = \text{Bool true}) \\
& \wedge (b \text{ okay} = \text{Bool true} \Rightarrow \neg b \text{ okay} = \text{Bool false}) \\
& \wedge (b \text{ okay} = \text{Bool false} \Rightarrow \neg b \text{ okay} = \text{Bool true}) \\
& \wedge (b \text{ (dash okay)} = \text{Bool true} \\
& \quad \Rightarrow \neg \\
& \quad \quad b \text{ (dash okay)} = \text{Bool false}) \\
& \wedge (b \text{ (dash okay)} = \text{Bool false} \\
& \quad \Rightarrow \neg \\
& \quad \quad b \text{ (dash okay)} = \text{Bool true})
\end{aligned}$$

OKAY_Equals_true.thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
& \quad | \text{ALPHABET_OKAY} \subseteq a \wedge n \in \text{ALPHABET_OKAY} \\
& \quad \bullet =_R (a, n, \text{Val} (\text{Bool true})) \\
& \quad = \neg_R (=_R (a, n, \text{Val} (\text{Bool false})))
\end{aligned}$$

OKAY_Equals_false.thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; n : \text{NAME} \\
& \quad | \text{ALPHABET_OKAY} \subseteq a \wedge n \in \text{ALPHABET_OKAY} \\
& \quad \bullet =_R (a, n, \text{Val} (\text{Bool false})) \\
& \quad = \neg_R (=_R (a, n, \text{Val} (\text{Bool true})))
\end{aligned}$$

OKAY_okay_WF_Equals_R.thm

$$\vdash (\{\text{okay}\}, \text{okay}, \text{Val} (\text{Bool false})) \in \text{WF_Equals}_R$$

OKAY_ALPHABET_OKAY.thm

$$\vdash \text{ALPHABET_OKAY} \in \text{ALPHABET}$$

OKAY_okay_WF_Equals_R.thm1

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET} \\
& \quad | \{\text{okay}, \text{dash okay}\} \subseteq a \\
& \quad \bullet (a, \text{dash okay}, \text{Var okay}) \in \text{WF_Equals}_R
\end{aligned}$$

OKAY_okay_WF_Equals_R.thm2

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET}; v : \text{VALUE} \\
& \quad | \{\text{okay}, \text{dash okay}\} \subseteq a \\
& \quad \bullet (a, \text{okay}, \text{Val } v) \in \text{WF_Equals}_R \\
& \quad \wedge (a, \text{dash okay}, \text{Val } v) \in \text{WF_Equals}_R
\end{aligned}$$

OKAY_OKAY_alphabet.thm

$$\vdash \text{OKAY.1} = \{\text{okay}, \text{dash okay}\}$$

OKAY_OKAY_DASH_alphabet_thm

$\vdash \text{OKAY}'.1 = \{\text{okay}, \text{dash okay}\}$

OKAY_okay_dash_converge- \wedge_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \wedge_R r2) \sigma_t = (r1 \sigma_t) \wedge_R r2 \sigma_t$

OKAY_okay_dash_converge- \vee_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \vee_R r2) \sigma_t = (r1 \sigma_t) \vee_R r2 \sigma_t$

OKAY_okay_dash_converge- \Rightarrow_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \Rightarrow_R r2) \sigma_t = (r1 \sigma_t) \Rightarrow_R r2 \sigma_t$

OKAY_okay_dash_converge- \neg_R -distr_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet \neg_R r \sigma_t = \neg_R (r \sigma_t)$

OKAY_okay_dash_diverge- \wedge_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \wedge_R r2) \sigma_f = (r1 \sigma_f) \wedge_R r2 \sigma_f$

OKAY_okay_dash_diverge- \vee_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \vee_R r2) \sigma_f = (r1 \sigma_f) \vee_R r2 \sigma_f$

OKAY_okay_dash_diverge- \Rightarrow_R -distr_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r1.1 \wedge \text{dash okay} \in r2.1$

$\bullet (r1 \Rightarrow_R r2) \sigma_f = (r1 \sigma_f) \Rightarrow_R r2 \sigma_f$

OKAY_okay_dash_diverge- \neg_R -distr_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet \neg_R r \sigma_f = \neg_R (r \sigma_f)$

OKAY_OKAY_def- \in -WF_Equals $_R$ -thm

$\vdash (\text{ALPHABET_OKAY}, \text{okay}, \text{Val}(\text{Bool true})) \in \text{WF_Equals}_R$

OKAY_not_OKAY_def- \in -WF_Equals $_R$ -thm

$\vdash (\text{ALPHABET_OKAY}, \text{okay}, \text{Val}(\text{Bool false})) \in \text{WF_Equals}_R$

OKAY_OKAY_∈_REL_PREDICATE_thm

$\vdash \text{OKAY} \in \text{REL_PREDICATE}$

OKAY_okay_dash_converge_is_REL_PREDICATE_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet r \sigma_t \in \text{REL_PREDICATE}$

OKAY_okay_dash_diverge_is_REL_PREDICATE_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet r \sigma_f \in \text{REL_PREDICATE}$

OKAY_okay_dash_converge_alphabet_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet (r \sigma_t).1 = r.1$

OKAY_okay_dash_diverge_alphabet_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet (r \sigma_f).1 = r.1$

OKAY_okay_dash_converge_subst_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet r \wedge_R =_R (r.1, \text{dash okay}, \text{Val} (\text{Bool true}))$
 $= (r \sigma_t)$

$\wedge_R =_R (r.1, \text{dash okay}, \text{Val} (\text{Bool true}))$

OKAY_okay_dash_diverge_subst_thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet r \wedge_R =_R (r.1, \text{dash okay}, \text{Val} (\text{Bool false}))$
 $= (r \sigma_f)$

$\wedge_R =_R (r.1, \text{dash okay}, \text{Val} (\text{Bool false}))$

OKAY_okay_dash_diverge_subst_thm1

$\vdash \forall r : \text{REL_PREDICATE}$

$\mid \text{dash okay} \in r.1$

$\bullet r \wedge_R \neg_R (=_R (r.1, \text{dash okay}, \text{Val} (\text{Bool true})))$
 $= (r \sigma_f)$

$\wedge_R \neg_R$

$(=_R (r.1, \text{dash okay}, \text{Val} (\text{Bool true})))$

OKAY_okay_dist_over_◁_R▷_R-dist_thm

$$\begin{aligned}
& \vdash \forall u1, u2, b : REL_PREDICATE \\
& \quad | (u1, b, u2) \in WF_Cond_R \\
& \quad \bullet OKAY \wedge_R u1 \triangleleft_R b \triangleright_R u2 \\
& \quad = (OKAY \wedge_R u1) \triangleleft_R b \triangleright_R OKAY \wedge_R u2
\end{aligned}$$

10 THE Z THEORY utp-des

10.1 Parents

utp-okay

10.2 Global Variables

ALPHABET_DES	$\mathbb{P} (\mathbb{P} \text{ NAME})$
DES_PREDICATE	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_DES_PREDICATE_PAIR	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_DESIGN_PAIR	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
WF_Assign_D	$\mathbb{P} (\mathbb{P} \text{ NAME} \times (\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION}))$
WF_Skip_D	$\mathbb{P} (\mathbb{P} \text{ NAME})$
DES_FUNCTION	\mathbb{P} $((\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
(- ⊢_D -)	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
DESIGN	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
Π_D	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
Assign_D	$\mathbb{P} \text{ NAME} \times (\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION})$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
Top_D	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
Bot_D	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
H1	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
H1_healthy	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
H2_healthy	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
J	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
H2	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

H2_J_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

H3 $\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

H3_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

H4_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

10.3 Fixity

fun 0 rightassoc

$(- \vdash_D -)$

10.4 Axioms

$- \vdash_D -$ $\vdash (- \vdash_D -) \in WF_DES_PREDICATE_PAIR \rightarrow REL_PREDICATE$
 $\wedge (\forall d : WF_DES_PREDICATE_PAIR$
 $\bullet d.1 \vdash_D d.2$
 $= (=_R (d.1.1, okay, Val (Bool true))) \wedge_R d.1)$
 $\Rightarrow_R =_R (d.1.1, dash\ okay, Val (Bool true))$
 $\wedge_R d.2)$

Π_D $\vdash \Pi_D \in WF_Skip_D \rightarrow REL_PREDICATE$
 $\wedge (\forall a : WF_Skip_D \bullet \Pi_D a = True_R a \vdash_D \Pi_R a)$

$Assign_D$ $\vdash Assign_D \in WF_Assign_D \rightarrow REL_PREDICATE$
 $\wedge (\forall aa : WF_Assign_D$
 $\bullet Assign_D aa = True_R aa.1 \vdash_D Assign_R aa)$

Top_D $\vdash Top_D \in ALPHABET_DES \rightarrow DES_PREDICATE$
 $\wedge (\forall a : ALPHABET_DES$
 $\bullet Top_D a = =_R (a, okay, Val (Bool false)))$

Bot_D $\vdash Bot_D \in ALPHABET_DES \rightarrow DES_PREDICATE$
 $\wedge (\forall a : ALPHABET_DES \bullet Bot_D a = True_R a)$

$H1$ $\vdash H1 \in REL_PREDICATE \rightarrow REL_PREDICATE$
 $\wedge (\forall d : REL_PREDICATE$
 $\bullet H1\ d$
 $= =_R (\{okay\}, okay, Val (Bool true)) \Rightarrow_R d)$

$H1_healthy$ $\vdash H1_healthy \in \mathbb{P} REL_PREDICATE$
 $\wedge H1_healthy = \{d : REL_PREDICATE \mid H1\ d = d\}$

$H2_healthy$ $\vdash H2_healthy \in \mathbb{P} REL_PREDICATE$
 $\wedge H2_healthy$
 $= \{d : REL_PREDICATE$
 $\mid dash\ okay \in d.1$

$$\begin{aligned}
& \wedge \langle_R \\
& \quad /_R (d, \text{Val}(\text{Bool false}), \text{dash okay}) \\
& \quad \Rightarrow_R /_R \\
& \quad \quad (d, \\
& \quad \quad \quad \text{Val}(\text{Bool true}), \\
& \quad \quad \quad \text{dash okay}) \rangle_R \\
& \quad = \text{True}_R \emptyset \} \\
\mathbf{J} \quad & \vdash J \in \text{ALPHABET} \leftrightarrow \text{DES_PREDICATE} \\
& \quad \wedge (\forall a' : \text{ALPHABET} \\
& \quad \quad | a' \subseteq \text{dashed} \wedge \text{dash okay} \in a' \\
& \quad \quad \bullet \exists a : \text{ALPHABET} \\
& \quad \quad \quad | a \subseteq \text{undashed} \wedge a' = \text{dash} \langle a \rangle \\
& \quad \quad \bullet J a' \\
& \quad \quad = (=_R (a \cup a', \text{okay}, \text{Val}(\text{Bool true}))) \\
& \quad \quad \Rightarrow_R =_R \\
& \quad \quad \quad (a \cup a', \\
& \quad \quad \quad \text{dash okay}, \\
& \quad \quad \quad \text{Val}(\text{Bool true})) \\
& \quad \quad \wedge_R \Pi_R (a \cup a' \setminus \text{ALPHABET_OKAY})) \\
\mathbf{H2} \quad & \vdash H2 \in \text{REL_PREDICATE} \leftrightarrow \text{REL_PREDICATE} \\
& \quad \wedge (\forall d : \text{REL_PREDICATE} \\
& \quad \quad | \text{dash okay} \in d.1 \\
& \quad \quad \bullet H2 d = d ;_R J (\text{out}_a d.1)) \\
\mathbf{H2_J_healthy} \quad & \vdash H2_J_healthy \in \mathbb{P} \text{REL_PREDICATE} \\
& \quad \wedge H2_J_healthy \\
& \quad = \{d : \text{REL_PREDICATE} \\
& \quad \quad | \text{dash okay} \in d.1 \wedge H2 d = d\} \\
\mathbf{H3} \quad & \vdash H3 \in \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE} \\
& \quad \wedge (\forall d : \text{REL_PREDICATE}; a : \text{ALPHABET} \\
& \quad \quad | a \in \text{WF_Skip}_R \wedge (d, \Pi_R a) \in \text{WF_Semi}_R \\
& \quad \quad \bullet H3 d = d ;_R \Pi_R a) \\
\mathbf{H3_healthy} \quad & \vdash H3_healthy \in \mathbb{P} \text{REL_PREDICATE} \\
& \quad \wedge H3_healthy = \{d : \text{REL_PREDICATE} \mid H3 d = d\} \\
\mathbf{H4_healthy} \quad & \vdash H4_healthy \in \mathbb{P} \text{REL_PREDICATE} \\
& \quad \wedge H4_healthy \\
& \quad = \{d : \text{REL_PREDICATE} \\
& \quad \quad | \forall a : \text{ALPHABET} \\
& \quad \quad \quad | (d, \text{True}_R a) \in \text{WF_Semi}_R
\end{aligned}$$

- $\text{True}_R a ;_R d = \text{True}_R a\}$

DES_refinement_thm

- $\vdash \forall d1, d2 : \text{WF_DES_PREDICATE_PAIR}$
- $| (d1, d2) \in \text{WF_DESIGN_PAIR}$
- $d1.1 \vdash_D d1.2 \sqsubseteq_R d2.1 \vdash_D d2.2$
- $= \langle_R (d1.1 \wedge_R d2.2) \Rightarrow_R d1.2 \rangle_R$
- $\wedge_R \langle_R d1.1 \Rightarrow_R d2.1 \rangle_R$

DES_Assign_D-eval_subst_thm

- $\vdash \forall a : \text{ALPHABET_DES};$
- $n : \text{NAME};$
- $e : \text{EXPRESSION};$
- $f : \text{VALUE} \leftrightarrow \text{VALUE}$
- $| (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_D$
- $\text{Assign}_D (a, \langle n \rangle, \langle e \rangle)$
- $;_R \text{Assign}_D (a, \langle n \rangle, \langle \text{Fun}_1 (f, \text{Var } n) \rangle)$
- $= \text{Assign}_D (a, \langle n \rangle, \langle \text{Fun}_1 (f, e) \rangle)$

DES_Assign_D- \triangleleft_R - \triangleright_R -subst_thm

- $\vdash \forall d1, d2, b : \text{REL_PREDICATE};$
- $a : \text{ALPHABET_DES};$
- $n : \text{NAME};$
- $e : \text{EXPRESSION}$
- $| (d1, b, d2) \in \text{WF_Cond}_D$
- $\wedge (\text{Assign}_D (a, \langle n \rangle, \langle e \rangle), d1) \in \text{WF_Semi}_D$
- $\wedge (a, \langle n \rangle, \langle e \rangle) \in \text{WF_Assign}_D$
- $\wedge (b, e, n) \in \text{WF_Subst}_D$
- $\text{Assign}_D (a, \langle n \rangle, \langle e \rangle) ;_R d1 \triangleleft_R b \triangleright_R d2$
- $= (\text{Assign}_D (a, \langle n \rangle, \langle e \rangle) ;_R d1) \triangleleft_R$
- $/_R (b, e, n) \triangleright_R$
- $\text{Assign}_D (a, \langle n \rangle, \langle e \rangle) ;_R d2$

DES_ Π_D -left_unit_thm

- $\vdash \forall a : \text{ALPHABET_DES}; d : \text{WF_DES_PREDICATE_PAIR}$
- $| a \in \text{WF_Skip}_D \wedge (\Pi_D a, d1) \in \text{WF_Semi}_D$
- $\Pi_D a ;_R d.1 \vdash_D d.2 = d.1 \vdash_D d.2$

DES- \triangleleft_R - \triangleright_R -designs_thm

- $\vdash \forall d1, d2 : \text{WF_DES_PREDICATE_PAIR}; b : \text{REL_PREDICATE}$
- $| (d1, b, d2) \in \text{WF_Cond}_D$
- $(d1.1 \vdash_D d1.2) \triangleleft_R b \triangleright_R d2.1 \vdash_D d2.2$
- $= (d1.1 \triangleleft_R b \triangleright_R d2.1) \vdash_D d1.2 \triangleleft_R b \triangleright_R d2.2$

DES_Semi_R-designs_thm

$$\begin{aligned}
&\vdash \forall d1, d2 : WF_DES_PREDICATE_PAIR \\
&\quad | (d1.1, d2.1) \in WF_Semi_D \\
&\quad \bullet (d1.1 \vdash_D d1.2) ;_R d2.1 \vdash_D d2.2 \\
&\quad = (\neg_R (\neg_R d1.1 ;_R True_R d2.1.1) \\
&\quad \quad \wedge_R \neg_R (d1.2 ;_R \neg_R d2.1)) \\
&\quad \vdash_D d1.2 ;_R d2.2
\end{aligned}$$
DES_weakest_fixed_point_designs_thm

$$\begin{aligned}
&\vdash \forall F, G : DES_FUNCTION; \\
&\quad fd : DES_PREDICATE \rightarrow DES_PREDICATE; \\
&\quad pair_d : WF_DES_PREDICATE_PAIR; \\
&\quad d, Q : DES_PREDICATE; \\
&\quad fp, fq, P : DES_PREDICATE \rightarrow DES_PREDICATE \\
&\quad \bullet fd (pair_d.1 \vdash_D pair_d.2) \\
&\quad \quad = F pair_d \vdash_D G pair_d \\
&\quad \quad \wedge fp d = F (d, pair_d.2) \\
&\quad \quad \wedge P d = \nu_R fp \\
&\quad \quad \wedge fq d = P d \Rightarrow_R G (P d, d) \\
&\quad \quad \wedge Q = \mu_D fq \\
&\quad \quad \wedge \mu_R fd = P Q \vdash_D Q
\end{aligned}$$
DES_algebraic_H1_thm

$$\begin{aligned}
&\vdash \forall d : REL_PREDICATE; a : ALPHABET \\
&\quad | a \in WF_Skip_R \wedge (\Pi_D a, d) \in WF_Semi_R \\
&\quad \bullet d \in H1_healthy \\
&\quad \Leftrightarrow True_R a ;_R d = True_R a \wedge \Pi_D a ;_R d = d
\end{aligned}$$
DES_Semi_R-designs_thm1

$$\begin{aligned}
&\vdash \forall d1, d2 : WF_DES_PREDICATE_PAIR \\
&\quad | (d1.1, d2.1) \in WF_Semi_D \wedge d1.1.1 \cap dashed = \emptyset \\
&\quad \bullet (d1.1 \vdash_D d1.2) ;_R d2.1 \vdash_D d2.2 \\
&\quad = (d1.1 \wedge_R \neg_R (d1.2 ;_R \neg_R d2.1)) \\
&\quad \vdash_D d1.2 ;_R d2.2
\end{aligned}$$
DES_ \neg_R -design_thm

$$\begin{aligned}
&\vdash \forall d : WF_DES_PREDICATE_PAIR \\
&\quad \bullet \neg_R (d.1 \vdash_D d.2) \\
&\quad = =_R (d.1.1, okay, Val (Bool true)) \\
&\quad \quad \wedge_R d.1 \\
&\quad \quad \wedge_R =_R (d.1.1, okay, Val (Bool true)) \\
&\quad \quad \Rightarrow_R \neg_R d.2
\end{aligned}$$

DES_design_contradiction_thm

$$\vdash \forall d : DES_PREDICATE \bullet d \vdash_D \neg_R d = d \vdash_D False_R d.1$$

DES_Π_D-H1-Π_R-thm

$$\vdash \forall a : WF_Skip_D \bullet \Pi_D a = H1 (\Pi_R a)$$

DES_post-H1-thm

$$\vdash \forall d : WF_DES_PREDICATE_PAIR \\ \bullet d.1 \vdash_D d.2 = d.1 \vdash_D H1 d.2$$

DES_post-¬_R-H1-thm

$$\vdash \forall d : WF_DES_PREDICATE_PAIR \\ \bullet d.1 \vdash_D \neg_R d.2 = d.1 \vdash_D \neg_R (H1 d.2)$$

DES_H2-J-thm

$$\vdash H2_healthy = H2_J_healthy$$

DES_H2-H1-com-thm

$$\vdash \forall d : REL_PREDICATE \\ | dash_okay \in d.1 \\ \bullet H1 (H2 d) = H2 (H1 d)$$

DES-J-Semi_R-idem-thm

$$\vdash \forall a' : ALPHABET \\ | a' \subseteq dashed \wedge dash_okay \in a' \\ \bullet J a' ;_R J a' = J a'$$

DES_Designs-H1-H2-thm

$$\vdash \forall d : REL_PREDICATE \\ \bullet d \in DESIGN \Leftrightarrow d \in H1_healthy \cap H2_healthy$$

10.5 Definitions

ALPHABET_DES

$$\vdash ALPHABET_DES = \{a : ALPHABET \mid ALPHABET_C$$

DES_PREDICATE

$$\vdash DES_PREDICATE \\ = \{u : REL_PREDICATE \\ | u.1 \in ALPHABET_DES\}$$

WF_DES_PREDICATE_PAIR

$$\vdash WF_DES_PREDICATE_PAIR \\ = \{d1, d2 : DES_PREDICATE \\ | d1.1 = d2.1\}$$

WF_DESIGN_PAIR

$$\vdash WF_DESIGN_PAIR \\ = \{d1, d2 : WF_DES_PREDICATE_PAIR \\ | d1.1.1 = d2.1.1\}$$

$$\begin{aligned}
\mathbf{WF_Assign}_D &\vdash \mathbf{WF_Assign}_D \\
&= \{aa : \mathbf{WF_Assign}_R \\
&\quad | aa.1 \in \mathbf{ALPHABET_DES}\} \\
\mathbf{WF_Skip}_D &\vdash \mathbf{WF_Skip}_D = \{a : \mathbf{ALPHABET_DES} \mid a \in \mathbf{WF_Skip}_R\} \\
\mathbf{DES_FUNCTION} &\vdash \mathbf{DES_FUNCTION} \\
&= \{f : \mathbf{WF_DES_PREDICATE_PAIR} \leftrightarrow \mathbf{DES_PREDICATE} \\
&\quad | \exists a : \mathbf{ALPHABET} \\
&\quad \bullet \forall pair : \mathbf{dom} f; u_ran : \mathbf{ran} f \\
&\quad \bullet a = pair.1.1 \wedge pair.1.1 = u_ran.1\} \\
\mathbf{DESIGN} &\vdash \mathbf{DESIGN} = \mathbf{ran} (- \vdash_D -)
\end{aligned}$$

10.6 Theorems

ALPHABET_DES.thm

$$\vdash \forall a : \mathbf{ALPHABET_DES} \bullet a \in \mathbf{ALPHABET}$$

DES_PREDICATE.thm

$$\vdash \forall d : \mathbf{DES_PREDICATE}$$

- $d \in \mathbf{REL_PREDICATE}$
- $\wedge \{okay, \text{dash okay}\} \subseteq d.1$
- $\wedge d.1 \in \mathbf{ALPHABET_DES}$

WF_DES_PREDICATE_PAIR.thm

$$\vdash \forall pair_d : \mathbf{WF_DES_PREDICATE_PAIR}$$

- $pair_d.1 \in \mathbf{DES_PREDICATE}$
- $\wedge pair_d.2 \in \mathbf{DES_PREDICATE}$
- $\wedge pair_d.1.1 = pair_d.2.1$
- $\wedge (pair_d.2, pair_d.1) \in \mathbf{WF_DES_PREDICATE_PAIR}$

DES_okay_dash_okay_⊆_ALPHABET_DES.thm

$$\vdash \forall a : \mathbf{ALPHABET_DES} \bullet \{okay, \text{dash okay}\} \subseteq a$$

DES_PREDICATE_⊢_D.thm

$$\vdash \forall d : \mathbf{WF_DES_PREDICATE_PAIR}$$

- $d.1 \vdash_D d.2 \in \mathbf{DES_PREDICATE}$

DES_H1.thm $\vdash \forall d : \mathbf{REL_PREDICATE} \bullet H1\ d \in \mathbf{REL_PREDICATE}$

DES_J.thm $\vdash \forall a' : \mathbf{ALPHABET}$

- $| a' \subseteq \text{dashed} \wedge \text{dash okay} \in a'$
- $J\ a' \in \mathbf{DES_PREDICATE}$

DES_⊢_D_alphabet.thm

$$\vdash \forall d : \mathbf{WF_DES_PREDICATE_PAIR} \bullet (d.1 \vdash_D d.2).1 = d.1.1$$

DES_H1_alphabet.thm

$\vdash \forall d : \text{REL_PREDICATE} \bullet (H1\ d).1 = d.1 \cup \{\text{okay}\}$
DES_J_alphabet_thm
 $\vdash \forall a' : \text{ALPHABET}$
 $\mid a' \subseteq \text{dashed} \wedge \text{dash okay} \in a'$
 $\bullet \exists a : \text{ALPHABET}$
 $\bullet a \subseteq \text{undashed}$
 $\wedge a' = \text{dash} \parallel a \parallel$
 $\wedge (J\ a').1 = a \cup a'$
DES_Semi_R_J_comp_thm
 $\vdash \forall d : \text{DES_PREDICATE}$
 $\bullet (d.1, (J\ (\text{out_a}\ d.1)).1) \in \text{composable}$
DES_Semi_R_J_comp_thm1
 $\vdash \forall d : \text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1$
 $\bullet (d.1, (J\ (\text{out_a}\ d.1)).1) \in \text{composable}$
DES_Semi_R_J_alphabet_thm
 $\vdash \forall d : \text{DES_PREDICATE} \bullet (d ;_R J\ (\text{out_a}\ d.1)).1 = d.1$
DES_Semi_R_J_alphabet_thm1
 $\vdash \forall d : \text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1$
 $\bullet (d ;_R J\ (\text{out_a}\ d.1)).1 = d.1$
DES_H2_thm $\vdash \forall d : \text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1$
 $\bullet H2\ d \in \text{REL_PREDICATE}$
DES_H2_alphabet_thm
 $\vdash \forall d : \text{REL_PREDICATE}$
 $\mid \text{dash okay} \in d.1$
 $\bullet (H2\ d).1 = d.1$
DES_J_Semi_R_J_in_comp_thm
 $\vdash \forall a' : \text{ALPHABET}$
 $\mid a' \subseteq \text{dashed} \wedge \text{dash okay} \in a'$
 $\bullet ((J\ a').1, (J\ a').1) \in \text{composable}$
DESIGNS_True_R_Semi_R_left_zero_thm
 $\vdash \forall d : \text{WF_DES_PREDICATE_PAIR}; a : \text{ALPHABET_DES}$
 $\mid (\text{True}_R\ a, d.1 \vdash_D d.2) \in \text{WF_Semi}_R$
 $\bullet \text{True}_R\ a ;_R d.1 \vdash_D d.2$
 $= \text{True}_R\ (a \cup \text{out_a}\ d.1.1)$
DES_H1_idem_thm

$\vdash \forall d : \text{REL_PREDICATE} \bullet H1 \ (H1 \ d) = H1 \ d$
DES_H2_idem_thm
 $\vdash \forall d : \text{REL_PREDICATE}$
 $\quad | \text{dash okay} \in d.1$
 $\quad \bullet H2 \ (H2 \ d) = H2 \ d$
ALPHABET_DES_thm1
 $\vdash \forall a : \text{ALPHABET_DES} \bullet \text{ALPHABET_OKAY} \subseteq a$
DES_PREDICATE_∧_R_thm
 $\vdash \forall d1, d2 : \text{DES_PREDICATE} \bullet d1 \wedge_R d2 \in \text{DES_PREDICATE}$
DES_PREDICATE_∨_R_thm
 $\vdash \forall d1, d2 : \text{DES_PREDICATE} \bullet d1 \vee_R d2 \in \text{DES_PREDICATE}$
DES_PREDICATE_⇒_R_thm
 $\vdash \forall d1, d2 : \text{DES_PREDICATE} \bullet d1 \Rightarrow_R d2 \in \text{DES_PREDICATE}$
DES_PREDICATE_¬_R_thm
 $\vdash \forall d : \text{DES_PREDICATE} \bullet \neg_R d \in \text{DES_PREDICATE}$
DES_OKAY_∈_DES_PREDICATE_thm
 $\vdash \text{OKAY} \in \text{DES_PREDICATE}$
DES_OKAY_DASH_∈_DES_PREDICATE_thm
 $\vdash \text{OKAY}' \in \text{DES_PREDICATE}$
DES_PREDICATE_okay_dash_converge_thm
 $\vdash \forall d : \text{DES_PREDICATE} \bullet d \ \sigma_t \in \text{DES_PREDICATE}$
DES_PREDICATE_okay_dash_diverge_thm
 $\vdash \forall d : \text{DES_PREDICATE} \bullet d \ \sigma_f \in \text{DES_PREDICATE}$
DES_design_thm
 $\vdash \forall P, Q : \text{DES_PREDICATE}$
 $\quad | (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$
 $\quad \bullet P \vdash_D Q = (\text{OKAY} \wedge_R P) \Rightarrow_R \text{OKAY}' \wedge_R Q$
DES_OKAY_converge_thm
 $\vdash \text{OKAY} \ \sigma_t = \text{OKAY}$
DES_OKAY_diverge_thm
 $\vdash \text{OKAY} \ \sigma_f = \text{OKAY}$
DES_OKAY_DASH_converge_thm
 $\vdash \text{OKAY}' \ \sigma_t = \text{True}_R \ \text{ALPHABET_OKAY}$
DES_OKAY_DASH_diverge_thm
 $\vdash \text{OKAY}' \ \sigma_f = \text{False}_R \ \text{ALPHABET_OKAY}$
DES_okay_dash_converge_∧_R_distr_thm
 $\vdash \forall d1, d2 : \text{DES_PREDICATE}$
 $\quad \bullet (d1 \wedge_R d2) \ \sigma_t = (d1 \ \sigma_t) \wedge_R d2 \ \sigma_t$

DES_okay_dash_diverge- \wedge_R -distr_thm
 $\vdash \forall d1, d2 : DES_PREDICATE$
 $\bullet (d1 \wedge_R d2) \sigma_f = (d1 \sigma_f) \wedge_R d2 \sigma_f$

DES_okay_dash_converge- \vee_R -distr_thm
 $\vdash \forall d1, d2 : DES_PREDICATE$
 $\bullet (d1 \vee_R d2) \sigma_t = (d1 \sigma_t) \vee_R d2 \sigma_t$

DES_okay_dash_diverge- \vee_R -distr_thm
 $\vdash \forall d1, d2 : DES_PREDICATE$
 $\bullet (d1 \vee_R d2) \sigma_f = (d1 \sigma_f) \vee_R d2 \sigma_f$

DES_okay_dash_converge- \neg_R -distr_thm
 $\vdash \forall d : DES_PREDICATE \bullet \neg_R d \sigma_t = \neg_R (d \sigma_t)$

DES_okay_dash_diverge- \neg_R -distr_thm
 $\vdash \forall d : DES_PREDICATE \bullet \neg_R d \sigma_f = \neg_R (d \sigma_f)$

DES_okay_dash_converge- \Rightarrow_R -distr_thm
 $\vdash \forall d1, d2 : DES_PREDICATE$
 $\bullet (d1 \Rightarrow_R d2) \sigma_t = (d1 \sigma_t) \Rightarrow_R d2 \sigma_t$

DES_okay_dash_diverge- \Rightarrow_R -distr_thm
 $\vdash \forall d1, d2 : DES_PREDICATE$
 $\bullet (d1 \Rightarrow_R d2) \sigma_f = (d1 \sigma_f) \Rightarrow_R d2 \sigma_f$

DES_okay_dash_converge_id_thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid \text{unrestTypedVar } (d, \text{dash okay}, \text{BOOL_VAL})$
 $\bullet d \sigma_t = d$

DES_okay_dash_diverge_id_thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid \text{unrestTypedVar } (d, \text{dash okay}, \text{BOOL_VAL})$
 $\bullet d \sigma_f = d$

DES_design_converge_thm
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\wedge \text{unrestTypedVar } (P, \text{dash okay}, \text{BOOL_VAL})$
 $\wedge \text{unrestTypedVar } (Q, \text{dash okay}, \text{BOOL_VAL})$
 $\bullet (P \vdash_D Q) \sigma_t = (\text{OKAY} \wedge_R P) \Rightarrow_R Q$

DES_design_diverge_thm
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\wedge \text{unrestTypedVar } (P, \text{dash okay}, \text{BOOL_VAL})$
 $\wedge \text{unrestTypedVar } (Q, \text{dash okay}, \text{BOOL_VAL})$

$\bullet (P \vdash_D Q) \sigma_f = \neg_R (OKAY \wedge_R P)$
DES_conj_R-designs_thm
 $\vdash \forall d1, d2 : WF_DES_PREDICATE_PAIR$
 $\mid (d1, d2) \in WF_DESIGN_PAIR$
 $\bullet (d1.1 \vdash_D d1.2) \text{ conj}_R d2.1 \vdash_D d2.2$
 $= (d1.1 \vee_R d2.1)$
 $\vdash_D (d1.1 \Rightarrow_R d1.2) \wedge_R d2.1 \Rightarrow_R d2.2$
DES_design_alphabet_thm
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\bullet (P \vdash_D Q).1 = P.1$
DES_post_condition_OKAY_intro_thm
 $\vdash \forall P, Q : REL_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\bullet P \vdash_D Q = P \vdash_D OKAY \wedge_R Q$
DES_intchoice_R-designs_thm
 $\vdash \forall d1, d2 : WF_DES_PREDICATE_PAIR$
 $\mid (d1, d2) \in WF_DESIGN_PAIR$
 $\bullet (d1.1 \vdash_D d1.2) \text{ intchoice}_R d2.1 \vdash_D d2.2$
 $= (d1.1 \wedge_R d2.1) \vdash_D d1.2 \vee_R d2.2$
DES_false_pre_thm
 $\vdash \forall a : ALPHABET; u : DES_PREDICATE$
 $\mid (False_R a, u) \in WF_DES_PREDICATE_PAIR$
 $\bullet False_R a \vdash_D u = True_R a$
DES_H1_healthy_thm
 $\vdash \forall d : DES_PREDICATE \mid d \in H1_healthy \bullet d = H1 \ d$
DES_H2_healthy_thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid d \in H2_healthy$
 $\bullet \langle_R (d \sigma_f) \Rightarrow_R d \sigma_t \rangle_R = True_R \emptyset$
 $\wedge \text{dash } okay \in d.1$
DES_H2_healthy_okay_dash_options_ \vee_R _thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid d \in H2_healthy$
 $\bullet (d \sigma_t) \vee_R d \sigma_f = d \sigma_t$
DES_H2_healthy_okay_dash_options_ \wedge_R _thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid d \in H2_healthy$

$\bullet (d \sigma_t) \wedge_R d \sigma_f = d \sigma_f$
DES_pre_post_rewrite_thm
 $\vdash \forall d : DES_PREDICATE$
 $\mid d \in H1_healthy \wedge d \in H2_healthy$
 $\bullet d = \neg_R (d \sigma_f) \vdash_D d \sigma_t$
DES_H1_H2_is_healthy_thm
 $\vdash \forall d : DES_PREDICATE$
 $\bullet H1 (H2 d) \in H1_healthy \wedge H1 (H2 d) \in H2_healthy$
DES_PREDICATE_Design_thm
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\bullet P \vdash_D Q \in DES_PREDICATE$
DES_H1_healthy_thm1
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\bullet P \vdash_D Q \in H1_healthy$
DES_okay_∈_WF_Equals_R_thm
 $\vdash \forall a : ALPHABET_DES$
 $\bullet (a, okay, Val (Bool true)) \in WF_Equals_R$
DES_dash_okay_∈_WF_Equals_R_thm
 $\vdash \forall a : ALPHABET_DES$
 $\bullet (a, dash\ okay, Val (Bool true)) \in WF_Equals_R$
DES_pre_condition_OKAY_intro_thm
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid P.1 = Q.1$
 $\bullet P \vdash_D Q = (OKAY \wedge_R P) \vdash_D Q$
DES_okay_dash_converge_∧_R_distr_thm1
 $\vdash \forall d1, d2 : REL_PREDICATE$
 $\mid dash\ okay \in d1.1 \wedge dash\ okay \in d2.1$
 $\bullet (d1 \wedge_R d2) \sigma_t = (d1 \sigma_t) \wedge_R d2 \sigma_t$
DES_okay_dash_converge_⇒_R_distr_thm1
 $\vdash \forall d1, d2 : REL_PREDICATE$
 $\mid dash\ okay \in d1.1 \wedge dash\ okay \in d2.1$
 $\bullet (d1 \Rightarrow_R d2) \sigma_t = (d1 \sigma_t) \Rightarrow_R d2 \sigma_t$
DES_design_diverge_thm1
 $\vdash \forall P, Q : DES_PREDICATE$
 $\mid (P, Q) \in WF_DES_PREDICATE_PAIR$
 $\bullet (P \vdash_D Q) \sigma_f = \neg_R (OKAY \wedge_R P \sigma_f)$

11 THE Z THEORY utp-wtr

11.1 Parents

utp-rel

11.2 Global Variables

<i>wait</i>	<i>NAME</i>	
<i>ALPHABET_WAIT</i>	$\mathbb{P} \text{ NAME}$	
<i>WAIT</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$	
<i>WAIT'</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$	
<i>tr</i>	<i>NAME</i>	
<i>ALPHABET_TR</i>	$\mathbb{P} \text{ NAME}$	
<i>ref</i>	<i>NAME</i>	
<i>ALPHABET_REF</i>	$\mathbb{P} \text{ NAME}$	
<i>unrestWAIT</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
<i>unrestWAIT'</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
<i>unrestTR</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
<i>unrestTR'</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
<i>unrestREF</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
<i>unrestREF'</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$	
$(- \omega_f)$	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$	
$(- \omega_t)$	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$	

11.3 Fixity

fun 0 rightassoc
 $(- \omega_f) (- \omega_t)$

11.4 Axioms

wait $\vdash \text{wait} \in \text{NAME} \wedge \text{wait} \in \text{undashed}$

Constraint 1 $\vdash (\forall b : \text{BINDING} \mid \text{wait} \in \text{dom } b \bullet b \text{ wait} \in \text{BOOL_VAL})$
 $\wedge (\forall b : \text{BINDING}$
 $\mid \text{dash wait} \in \text{dom } b$
 $\bullet b (\text{dash wait}) \in \text{BOOL_VAL})$

WAIT $\vdash \text{WAIT} \in \text{REL_PREDICATE}$
 $\wedge \text{WAIT} = =_R (\{\text{wait}\}, \text{wait}, \text{Val } (\text{Bool true}))$

WAIT' $\vdash \text{WAIT}' \in \text{REL_PREDICATE}$
 $\wedge \text{WAIT}'$
 $= =_R (\{\text{dash wait}\}, \text{dash wait}, \text{Val } (\text{Bool true}))$

tr $\vdash \text{tr} \in \text{NAME} \wedge \text{tr} \in \text{undashed}$

Constraint 2 $\vdash (\forall b : \text{BINDING}$
 $\mid \text{tr} \in \text{dom } b$
 $\bullet b \text{ tr} \in \text{SEQ_EVENT_VAL})$
 $\wedge (\forall b : \text{BINDING}$
 $\mid \text{dash tr} \in \text{dom } b$
 $\bullet b (\text{dash tr}) \in \text{SEQ_EVENT_VAL})$

ref $\vdash \text{ref} \in \text{NAME} \wedge \text{ref} \in \text{undashed}$

Constraint 3 $\vdash (\forall b : \text{BINDING} \mid \text{ref} \in \text{dom } b \bullet b \text{ ref} \in \text{SET_EVENT_VAL})$
 $\wedge (\forall b : \text{BINDING}$
 $\mid \text{dash ref} \in \text{dom } b$
 $\bullet b (\text{dash ref}) \in \text{SET_EVENT_VAL})$

unrestWAIT

unrestWAIT' $\vdash \{\text{unrestWAIT}, \text{unrestWAIT}'\} \subseteq \text{REL_PREDICATE} \rightarrow \mathbb{B}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $\bullet (\text{unrestWAIT } r$
 $\Leftrightarrow \text{unrestTypedVar } (r, \text{wait}, \text{BOOL_VAL}))$
 $\wedge (\text{unrestWAIT}' r$
 $\Leftrightarrow \text{unrestTypedVar } (r, \text{dash wait}, \text{BOOL_VAL})))$

unrestTR

unrestTR' $\vdash \{\text{unrestTR}, \text{unrestTR}'\} \subseteq \text{REL_PREDICATE} \rightarrow \mathbb{B}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $\bullet (\text{unrestTR } r$
 $\Leftrightarrow \text{unrestTypedVar } (r, \text{tr}, \text{SEQ_EVENT_VAL}))$
 $\wedge (\text{unrestTR}' r$
 $\Leftrightarrow \text{unrestTypedVar } (r, \text{dash tr}, \text{SEQ_EVENT_VAL})))$

$$(r, \text{dash } tr, \text{SEQ_EVENT_VAL}))$$

unrestREF
unrestREF' $\vdash \{unrestREF, unrestREF'\} \subseteq \text{REL_PREDICATE} \rightarrow \mathbb{B}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $\bullet (unrestREF \ r$
 $\Leftrightarrow unrestTypedVar \ (r, \text{ref}, \text{SET_EVENT_VAL}))$
 $\wedge (unrestREF' \ r$
 $\Leftrightarrow unrestTypedVar$
 $(r, \text{dash ref}, \text{SET_EVENT_VAL})))$

Constraint 4 $\vdash wait \neq tr \wedge wait \neq ref$
Constraint 5 $\vdash tr \neq wait \wedge tr \neq ref$
Constraint 6 $\vdash ref \neq tr \wedge ref \neq wait$
- ω_f $\vdash (- \omega_f) \in \text{REL_PREDICATE} \leftrightarrow \text{REL_PREDICATE}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $| wait \in r.1$
 $\bullet r \omega_f = /_R (r, Val (Bool \text{false}), wait))$

- ω_t $\vdash (- \omega_t) \in \text{REL_PREDICATE} \leftrightarrow \text{REL_PREDICATE}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $| wait \in r.1$
 $\bullet r \omega_t = /_R (r, Val (Bool \text{true}), wait))$

11.5 Definitions

ALPHABET_WAIT

$\vdash \text{ALPHABET_WAIT} = \{wait, \text{dash } wait\}$

ALPHABET_TR

$\vdash \text{ALPHABET_TR} = \{tr, \text{dash } tr\}$

ALPHABET_REF

$\vdash \text{ALPHABET_REF} = \{ref, \text{dash } ref\}$

11.6 Theorems

WTR_ \subseteq -NAME_thm

$\vdash \{tr, \text{dash } tr, wait, \text{dash } wait, ref, \text{dash } ref\} \subseteq \text{NAME}$

WTR_wait_values_thm

$\vdash \forall b : \text{BINDING}$

$| wait \in \text{dom } b$

$\bullet b \text{ wait} = \text{Bool true} \vee b \text{ wait} = \text{Bool false}$

WTR_wait_dash_values_thm

$\vdash \forall b : \text{BINDING}$

- | $\text{dash wait} \in \text{dom } b$
- $b(\text{dash wait}) = \text{Bool true}$
- $\vee b(\text{dash wait}) = \text{Bool false}$

WTR_wait_clauses_thm

- $\vdash \forall b : \text{BINDING}$
- | $\{\text{wait}, \text{dash wait}\} \subseteq \text{dom } b$
- $(\neg b \text{ wait} = \text{Bool true} \Rightarrow b \text{ wait} = \text{Bool false})$
- $\wedge (\neg b \text{ wait} = \text{Bool false} \Rightarrow b \text{ wait} = \text{Bool true})$
- $\wedge (\neg$
- $b(\text{dash wait}) = \text{Bool true}$
- $\Rightarrow b(\text{dash wait}) = \text{Bool false})$
- $\wedge (\neg$
- $b(\text{dash wait}) = \text{Bool false}$
- $\Rightarrow b(\text{dash wait}) = \text{Bool true})$
- $\wedge (b \text{ wait} = \text{Bool true} \Rightarrow \neg b \text{ wait} = \text{Bool false})$
- $\wedge (b \text{ wait} = \text{Bool false} \Rightarrow \neg b \text{ wait} = \text{Bool true})$
- $\wedge (b(\text{dash wait}) = \text{Bool true}$
- $\Rightarrow \neg$
- $b(\text{dash wait}) = \text{Bool false})$
- $\wedge (b(\text{dash wait}) = \text{Bool false}$
- $\Rightarrow \neg$
- $b(\text{dash wait}) = \text{Bool true})$

WTR_wait_Equals_true_thm

- $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
- | $\text{ALPHABET_WAIT} \subseteq a \wedge n \in \text{ALPHABET_WAIT}$
- $=_R(a, n, \text{Val}(\text{Bool true}))$
- $= \neg_R(=_R(a, n, \text{Val}(\text{Bool false})))$

WTR_wait_Equals_false_thm

- $\vdash \forall a : \text{ALPHABET}; n : \text{NAME}$
- | $\text{ALPHABET_WAIT} \subseteq a \wedge n \in \text{ALPHABET_WAIT}$
- $=_R(a, n, \text{Val}(\text{Bool false}))$
- $= \neg_R(=_R(a, n, \text{Val}(\text{Bool true})))$

WTR_tr_dash_tr_WF_ALPHABET_EXPRESSION_thm

- $\vdash (\{tr, \text{dash tr}\}, \text{Var } tr, \text{Var}(\text{dash tr}))$
- $\in \text{WF_ALPHABET_EXPRESSION}$

WTR_tr_≤_R-dash-tr-∈-WF_ALPHABET_EXPRESSION_thm

- $\vdash \forall a : \text{ALPHABET}$
- | $\text{ALPHABET_TR} \subseteq a$

$$\begin{aligned}
& \bullet (a, \\
& \quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)), \\
& \quad \text{Val } (\text{Bool true})) \\
& \in \text{WF_ALPHABET_EXPRESSION} \\
\mathbf{WTR_tr_dash_tr_}\leq_R\mathbf{-thm} \\
& \vdash =_{+R} \\
& \quad (\text{ALPHABET_TR}, \\
& \quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)), \\
& \quad \text{Val } (\text{Bool true})) \\
& \in \text{REL_PREDICATE} \\
\mathbf{WTR_tr_dash_tr_}\leq_R\mathbf{-thm1} \\
& \vdash \forall a : \text{ALPHABET} \\
& \quad | \text{ALPHABET_TR} \subseteq a \\
& \bullet =_{+R} \\
& \quad (a, \\
& \quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)), \\
& \quad \text{Val } (\text{Bool true})) \\
& \in \text{REL_PREDICATE} \\
\mathbf{WTR_ALPHABET_TR.thm} \\
& \vdash \text{ALPHABET_TR} \in \text{ALPHABET} \\
\mathbf{WTR_ALPHABET_WAIT.thm} \\
& \vdash \text{ALPHABET_WAIT} \in \text{ALPHABET} \\
\mathbf{WTR_ALPHABET_REF.thm} \\
& \vdash \text{ALPHABET_REF} \in \text{ALPHABET} \\
\mathbf{WTR_wait_WF_Equals_R.thm} \\
& \vdash (\{wait\}, wait, \text{Val } (\text{Bool true})) \in \text{WF_Equals}_R \\
& \quad \wedge (\{wait\}, wait, \text{Val } (\text{Bool false})) \in \text{WF_Equals}_R \\
\mathbf{WTR_wait_WF_Equals_R.thm1} \\
& \vdash \forall a : \text{ALPHABET} \\
& \quad | \{wait, \text{dash } wait\} \subseteq a \\
& \quad \bullet (a, \text{dash } wait, \text{Var } wait) \in \text{WF_Equals}_R \\
\mathbf{WTR_tr_WF_Equals_R.thm1} \\
& \vdash \forall a : \text{ALPHABET} \\
& \quad | \{tr, \text{dash } tr\} \subseteq a \\
& \quad \bullet (a, \text{dash } tr, \text{Var } tr) \in \text{WF_Equals}_R \\
\mathbf{WTR_ref_WF_Equals_R.thm1} \\
& \vdash \forall a : \text{ALPHABET} \\
& \quad | \{ref, \text{dash } ref\} \subseteq a
\end{aligned}$$

$\bullet (a, \text{dash ref}, \text{Var ref}) \in WF_Equals_R$
WTR_tr_prefix_dash_tr_thm
 $\vdash \forall b : BINDING$
 $\quad | \text{tr} \in \text{dom } b$
 $\quad \wedge \text{dash tr} \in \text{dom } b$
 $\quad \wedge b \text{ tr} \leq_R b (\text{dash tr})$
 $\bullet b \text{ tr} \leq_R b (\text{dash tr})$
 $\quad \Leftrightarrow Seq \langle \rangle \leq_R b (\text{dash tr}) SeqDif_R b \text{ tr}$
WTR_WAIT_wait_false_thm
 $\vdash WAIT \omega_f = False_R \{wait\}$
WTR_WAIT_wait_true_thm
 $\vdash WAIT \omega_t = True_R \{wait\}$
WTR_◁_R-▷_R-thm
 $\vdash \forall r1, r2, b : REL_PREDICATE$
 $\quad | (r1, b, r2) \in WF_Cond_R \wedge wait \in b.1$
 $\bullet wait \in (r1 \triangleleft_R b \triangleright_R r2).1$
WTR_wait_true_∧_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | wait \in r1.1 \wedge wait \in r2.1$
 $\bullet (r1 \wedge_R r2) \omega_t = (r1 \omega_t) \wedge_R r2 \omega_t$
WTR_wait_false_∧_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | wait \in r1.1 \wedge wait \in r2.1$
 $\bullet (r1 \wedge_R r2) \omega_f = (r1 \omega_f) \wedge_R r2 \omega_f$
WTR_wait_true_∨_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | wait \in r1.1 \wedge wait \in r2.1$
 $\bullet (r1 \vee_R r2) \omega_t = (r1 \omega_t) \vee_R r2 \omega_t$
WTR_wait_false_∨_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | wait \in r1.1 \wedge wait \in r2.1$
 $\bullet (r1 \vee_R r2) \omega_f = (r1 \omega_f) \vee_R r2 \omega_f$
WTR_wait_true_⇒_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | wait \in r1.1 \wedge wait \in r2.1$
 $\bullet (r1 \Rightarrow_R r2) \omega_t = (r1 \omega_t) \Rightarrow_R r2 \omega_t$
WTR_wait_false_⇒_R-distr_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$

$$\begin{array}{l}
| \text{wait} \in r1.1 \wedge \text{wait} \in r2.1 \\
\bullet (r1 \Rightarrow_R r2) \omega_f = (r1 \omega_f) \Rightarrow_R r2 \omega_f \\
\mathbf{WTR_wait_true_}\neg_R\mathbf{-distr_thm} \\
\vdash \forall r : \mathbf{REL_PREDICATE} \\
| \text{wait} \in r.1 \\
\bullet \neg_R r \omega_t = \neg_R (r \omega_t) \\
\mathbf{WTR_wait_false_}\neg_R\mathbf{-distr_thm} \\
\vdash \forall r : \mathbf{REL_PREDICATE} \\
| \text{wait} \in r.1 \\
\bullet \neg_R r \omega_f = \neg_R (r \omega_f) \\
\mathbf{WTR_wait_true_}\triangleleft_R\triangleright_R\mathbf{-distr_thm} \\
\vdash \forall r1, r2, b : \mathbf{REL_PREDICATE} \\
| (r1, b, r2) \in \mathbf{WF_Cond}_R \wedge \text{wait} \in b.1 \\
\bullet (r1 \triangleleft_R b \triangleright_R r2) \omega_t \\
= (r1 \omega_t) \triangleleft_R b \omega_t \triangleright_R r2 \omega_t \\
\mathbf{WTR_wait_false_}\triangleleft_R\triangleright_R\mathbf{-distr_thm} \\
\vdash \forall r1, r2, b : \mathbf{REL_PREDICATE} \\
| (r1, b, r2) \in \mathbf{WF_Cond}_R \wedge \text{wait} \in b.1 \\
\bullet (r1 \triangleleft_R b \triangleright_R r2) \omega_f \\
= (r1 \omega_f) \triangleleft_R b \omega_f \triangleright_R r2 \omega_f \\
\mathbf{WTR_tr_}\in\mathbf{-ran_Seq_thm} \\
\vdash \forall b : \mathbf{BINDING} \mid \text{tr} \in \text{dom } b \bullet b \text{ tr} \in \text{ran } \text{Seq} \\
\mathbf{WTR_dash_tr_}\in\mathbf{-ran_Seq_thm} \\
\vdash \forall b : \mathbf{BINDING} \\
| \text{dash tr} \in \text{dom } b \\
\bullet b (\text{dash tr}) \in \text{ran } \text{Seq} \\
\mathbf{WTR_dash_tr_equals_tr_thm} \\
\vdash \forall b : \mathbf{BINDING} \\
| \text{tr} \in \text{dom } b \\
\wedge \text{dash tr} \in \text{dom } b \\
\wedge b \text{ tr} \leq_R b (\text{dash tr}) \\
\bullet b (\text{dash tr}) = b \text{ tr} \\
\Leftrightarrow b (\text{dash tr}) \text{SeqDif}_R b \text{ tr} = \text{Seq } \langle \rangle \\
\mathbf{WTR_wait_false_is_REL_PREDICATE_thm} \\
\vdash \forall r : \mathbf{REL_PREDICATE} \\
| \text{wait} \in r.1 \\
\bullet r \omega_f \in \mathbf{REL_PREDICATE} \\
\mathbf{WTR_wait_true_is_REL_PREDICATE_thm}
\end{array}$$

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& \quad | \text{wait} \in r.1 \\
& \quad \bullet r \omega_t \in REL_PREDICATE \\
WTR_wait_true_alphabet_thm \\
& \vdash \forall r : REL_PREDICATE \mid \text{wait} \in r.1 \bullet (r \omega_t).1 = r.1 \\
WTR_wait_false_alphabet_thm \\
& \vdash \forall r : REL_PREDICATE \mid \text{wait} \in r.1 \bullet (r \omega_f).1 = r.1 \\
WTR_wait_true_subst_thm \\
& \vdash \forall r : REL_PREDICATE \\
& \quad | \text{wait} \in r.1 \\
& \quad \bullet r \wedge_R =_R (r.1, \text{wait}, \text{Val} (\text{Bool true})) \\
& \quad \quad = (r \omega_t) \wedge_R =_R (r.1, \text{wait}, \text{Val} (\text{Bool true})) \\
WTR_wait_false_subst_thm \\
& \vdash \forall r : REL_PREDICATE \\
& \quad | \text{wait} \in r.1 \\
& \quad \bullet r \wedge_R =_R (r.1, \text{wait}, \text{Val} (\text{Bool false})) \\
& \quad \quad = (r \omega_f) \wedge_R =_R (r.1, \text{wait}, \text{Val} (\text{Bool false})) \\
WTR_wait_false_subst_thm1 \\
& \vdash \forall r : REL_PREDICATE \\
& \quad | \text{wait} \in r.1 \\
& \quad \bullet r \wedge_R \neg_R (=_R (r.1, \text{wait}, \text{Val} (\text{Bool true}))) \\
& \quad \quad = (r \omega_f) \\
& \quad \quad \wedge_R \neg_R (=_R (r.1, \text{wait}, \text{Val} (\text{Bool true})))
\end{aligned}$$

12 THE Z THEORY *utp-rea*

12.1 Parents

$$utp-wtr \quad utp-okay$$

12.2 Global Variables

ALPHABET_OWTR

$$\mathbb{P} \text{ NAME}$$

ALPHABET_REA

$$\mathbb{P} (\mathbb{P} \text{ NAME})$$

REA_PREDICATE

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

WF_Skip_{REA}

$$\mathbb{P} (\mathbb{P} \text{ NAME})$$

Π_{REA}

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R1

$$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R1_healthy

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R2

$$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R2_healthy

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R3

$$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R3_healthy

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R

$$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

R_healthy

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

REA_PROCESS

$$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$$

12.3 Axioms

Constraint 1 $\vdash okay \neq wait \wedge okay \neq tr \wedge wait \neq ref$

Π_{REA} $\vdash \Pi_{REA} \in WF_Skip_{REA} \rightarrow REA_PREDICATE$

$$\wedge (\forall a : WF_Skip_{REA}$$

$$\bullet \Pi_{REA} a$$

$$= (=_R (a, okay, Val (Bool false))$$

$$\wedge_R =_{+R}$$

$$(ALPHABET_OWTR,$$

$$Rel$$

$$((- \leq_R -),$$

$$Var \ tr,$$

$$Var \ (dash \ tr)),$$

$$\begin{array}{l}
\text{Val (Bool true))} \\
\vee_R =_R (a, \text{dash okay}, \text{Val (Bool true)}) \\
\wedge_R =_R (a, \text{dash tr}, \text{Var tr}) \\
\wedge_R =_R (a, \text{dash wait}, \text{Var wait}) \\
\wedge_R =_R (a, \text{dash ref}, \text{Var ref}) \\
\mathbf{R1} \quad \vdash R1 \in \text{REL_PREDICATE} \rightarrow \text{REL_PREDICATE} \\
\quad \wedge (\forall r : \text{REL_PREDICATE} \\
\quad \bullet R1 \ r \\
\quad = r \\
\quad \wedge_R =_{+R} \\
\quad \quad (\text{ALPHABET_OWTR}, \\
\quad \quad \text{Rel} \\
\quad \quad ((- \leq_R -), \\
\quad \quad \text{Var tr}, \\
\quad \quad \text{Var (dash tr)}), \\
\quad \quad \text{Val (Bool true)}) \\
\mathbf{R1_healthy} \quad \vdash R1_healthy \in \mathbb{P} \text{REL_PREDICATE} \\
\quad \wedge R1_healthy = \{r : \text{REL_PREDICATE} \mid r = R1 \ r\} \\
\mathbf{R2} \quad \vdash R2 \in \text{REL_PREDICATE} \leftrightarrow \text{REL_PREDICATE} \\
\quad \wedge (\forall r : \text{REL_PREDICATE} \\
\quad \mid \text{ALPHABET_OWTR} \subseteq r.1 \\
\quad \bullet R2 \ r \\
\quad = /_R \\
\quad \quad (/_R (r, \text{Val (Seq } \langle \rangle), \text{tr}), \\
\quad \quad \text{Fun}_2 \\
\quad \quad ((- \text{SeqDif}_R -), \\
\quad \quad \text{Var (dash tr)}, \\
\quad \quad \text{Var tr}), \\
\quad \quad \text{dash tr})) \\
\mathbf{R2_healthy} \quad \vdash R2_healthy \in \mathbb{P} \text{REL_PREDICATE} \\
\quad \wedge R2_healthy = \{r : \text{REL_PREDICATE} \mid r = R2 \ r\} \\
\mathbf{R3} \quad \vdash R3 \in \text{REA_PREDICATE} \leftrightarrow \text{REA_PREDICATE} \\
\quad \wedge (\forall r : \text{REA_PREDICATE} \\
\quad \mid r.1 \in \text{WF_Skip}_{\text{REA}} \\
\quad \bullet R3 \ r \\
\quad = \Pi_{\text{REA}} r.1 \triangleleft_R \\
\quad =_R (\{wait\}, wait, \text{Val (Bool true)}) \triangleright_R \\
\quad r)
\end{array}$$

$$\begin{array}{ll}
\mathbf{R3_healthy} & \vdash R3_healthy \in \mathbb{P} \text{ REA_PREDICATE} \\
& \wedge R3_healthy \\
& = \{r : \text{REA_PREDICATE} \\
& \quad | r.1 \in WF_Skip_{REA} \wedge r = R3 \ r\} \\
\mathbf{R} & \vdash R \in \text{REA_PREDICATE} \leftrightarrow \text{REA_PREDICATE} \\
& \wedge (\forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in WF_Skip_{REA} \\
& \quad \bullet R \ r = R1 \ (R2 \ (R3 \ r))) \\
\mathbf{R_healthy} & \vdash R_healthy \in \mathbb{P} \text{ REA_PREDICATE} \\
& \wedge R_healthy \\
& = \{r : \text{REA_PREDICATE} \\
& \quad | r.1 \in WF_Skip_{REA} \wedge r = R \ r\} \\
\mathbf{REA_H1_wait_dash_okay_thm} & \vdash \forall r : \text{REA_PREDICATE}; b1, b2 : \text{BOOL_VAL} \\
& \bullet /_R \\
& \quad (/_R (H1 \ r, \text{Val } b1, \text{dash } okay), \\
& \quad \text{Val } b2, \\
& \quad \text{dash } wait) \\
& = H1 \\
& \quad (/_R \\
& \quad \quad (/_R (r, \text{Val } b1, \text{dash } okay), \\
& \quad \quad \text{Val } b2, \\
& \quad \quad \text{dash } wait)) \\
\mathbf{REA_}\Pi_{REA}\text{-}\Pi_R\text{-thm} & \vdash \forall a : \text{ALPHABET_REA} \\
& \quad | a \in WF_Skip_{REA} \\
& \bullet \Pi_{REA} \ a \\
& = (=_R (a, okay, \text{Val } (\text{Bool } false)) \\
& \quad \wedge_R =_{+R} \\
& \quad \quad (\text{ALPHABET_OWTR}, \\
& \quad \quad \text{Rel} \\
& \quad \quad ((- \leq_R -), \\
& \quad \quad \text{Var } tr, \\
& \quad \quad \text{Var } (\text{dash } tr)), \\
& \quad \quad \text{Val } (\text{Bool } true))) \\
& \quad \vee_R \Pi_R \ a \\
\mathbf{REA_}\Pi_{REA}\text{-}\triangleleft_R\text{-}\triangleright_R\text{-thm} & \vdash \forall a : \text{ALPHABET_REA}
\end{array}$$

$$\begin{aligned}
& | a \in WF_Skip_{REA} \\
& \bullet \Pi_{REA} a \\
& = \Pi_R a \triangleleft_R \\
& \quad =_R (a, okay, Val (Bool true)) \triangleright_R \\
& \quad =_{+R} \\
& \quad \quad (ALPHABET_OWTR, \\
& \quad \quad \quad Rel ((- \leq_R -), Var tr, Var (dash tr)), \\
& \quad \quad \quad Val (Bool true))
\end{aligned}$$

REA_okay_Π_{REA}-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet =_R (a, okay, Val (Bool true)) \wedge_R \Pi_{REA} a \\
& \quad = =_R (a, okay, Val (Bool true)) \wedge_R \Pi_R a
\end{aligned}$$

REA_okay_Π_{REA}-unit-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA; r : REA_PREDICATE \\
& \quad | a \in WF_Skip_{REA} \wedge (\Pi_{REA} a, r) \in WF_Semi_R \\
& \quad \bullet (=_R (a, okay, Val (Bool true)) \wedge_R \Pi_{REA} a) \\
& \quad \quad ;_R r \\
& \quad = =_R (a, okay, Val (Bool true)) \wedge_R r
\end{aligned}$$

REA_Π_{REA}-H2_healthy-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet \Pi_{REA} a \in H2_healthy
\end{aligned}$$

REA_Π_{REA}-R1_healthy-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet \Pi_{REA} a \in R1_healthy
\end{aligned}$$

REA_Π_{REA}-R1-H1-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet \Pi_{REA} a = H1 (R1 (\Pi_{REA} a))
\end{aligned}$$

REA_R1-H2_com-thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& \quad | dash okay \in r.1 \\
& \quad \bullet H2 (R1 r) = R1 (H2 r)
\end{aligned}$$

REA_Π_{REA}-R1-H1-Π_R-thm

$$\begin{aligned}
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA}
\end{aligned}$$

$\bullet \Pi_{REA} a = R1 (H1 (\Pi_R a))$
REA_closure- $\triangleleft_R \triangleright_R$ -R1_thm
 $\vdash \forall r1, r2 : REL_PREDICATE; b : REL_PREDICATE$
 $\quad | (r1, b, r2) \in WF_Cond_R \wedge \{r1, r2\} \subseteq R1_healthy$
 $\bullet R1 (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2$
REA_closure-Semi $_R$ -R1_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | (r1, r2) \in WF_Semi_R \wedge \{r1, r2\} \subseteq R1_healthy$
 $\bullet R1 (r1 ;_R r2) = r1 ;_R r2$
REA_R1- $\wedge_R \neg_R$ -R1_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\bullet R1 r1 \wedge_R \neg_R (R1 r2) = R1 (r1 \wedge_R \neg_R r2)$
REA_R1_wait_thm
 $\vdash \forall r : REL_PREDICATE; b : BOOL_VAL$
 $\quad | wait \in r.1$
 $\bullet /_R (R1 r, Val b, wait)$
 $\quad = R1 (/_R (r, Val b, wait))$
REA_R1_dash_okay_thm
 $\vdash \forall r : REL_PREDICATE; b : BOOL_VAL$
 $\quad | dash_okay \in r.1$
 $\bullet /_R (R1 r, Val b, dash_okay)$
 $\quad = R1 (/_R (r, Val b, dash_okay))$
REA_R2_idem_thm
 $\vdash \forall r : REL_PREDICATE$
 $\quad | ALPHABET_OWTR \subseteq r.1$
 $\bullet R2 (R2 r) = R2 r$
REA_R2_H1_com_thm
 $\vdash \forall r : REA_PREDICATE$
 $\quad | ALPHABET_OWTR \subseteq r.1$
 $\bullet H1 (R2 r) = R2 (H1 r)$
REA_R2_H2_com_thm
 $\vdash \forall r : REL_PREDICATE$
 $\quad | \{dash_okay\} \cup ALPHABET_OWTR \subseteq r.1$
 $\bullet H2 (R2 r) = R2 (H2 r)$
REA_closure- \wedge_R -R2_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | ALPHABET_OWTR \subseteq r1.1 \cup r2.1$
 $\quad \wedge \{r1, r2\} \subseteq R2_healthy$

$\bullet R2 (r1 \wedge_R r2) = r1 \wedge_R r2$
REA_closure_ \vee_R -R2_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\mid ALPHABET_OWTR \subseteq r1.1 \cup r2.1$
 $\mid \{r1, r2\} \subseteq R2_healthy$
 $\bullet R2 (r1 \vee_R r2) = r1 \vee_R r2$
REA_closure_ $\triangleleft_R \triangleright_R$ -R2_thm
 $\vdash \forall r1, r2, b : REL_PREDICATE$
 $\mid ALPHABET_OWTR \subseteq r1.1$
 $\mid \wedge (r1, b, r2) \in WF_Cond_R$
 $\mid \wedge \{r1, r2, b\} \subseteq R2_healthy$
 $\bullet R2 (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2$
REA_closure_Semi_R-R2_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\mid tr \in r1.1$
 $\mid \wedge dash\ tr \in r2.1$
 $\mid \wedge (r1, r2) \in WF_Semi_R$
 $\mid \wedge \{r1, r2\} \subseteq R2_healthy$
 $\bullet R2 (r1 ;_R r2) = r1 ;_R r2$
REA_R3_H2_com_thm
 $\vdash \forall r : REL_PREDICATE$
 $\mid dash\ okay \in r.1 \wedge r.1 \in WF_Skip_{REA}$
 $\bullet H2 (R3\ r) = R3 (H2\ r)$
REA_R3_R1_H1_quasi_com_thm
 $\vdash \forall r : REA_PREDICATE$
 $\mid r.1 \in WF_Skip_{REA}$
 $\bullet R3 (R1 (H1\ r)) = R1 (H1 (R3\ r))$
REA_R3_ \neg _okay_wait_thm
 $\vdash \forall r : REA_PREDICATE$
 $\mid r \in R3_healthy$
 $\bullet =_R (r.1, okay, Val (Bool\ false))$
 $\quad \wedge_R =_R (r.1, wait, Val (Bool\ true)) \wedge_R R3\ r$
 $\quad =_R (r.1, okay, Val (Bool\ false))$
 $\quad \wedge_R =_R (r.1, wait, Val (Bool\ true))$
 $\quad \wedge_R =_{+R}$
 $\quad (r.1,$
 $\quad \quad Rel$
 $\quad \quad ((- \leq_R -),$

$Var\ tr,$
 $Var\ (dash\ tr)),$
 $Val\ (Bool\ true))$

REA_closure- \wedge_R -R3_thm

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | \{r1, r2\} \subseteq R3_healthy$
 $\quad \bullet R3\ (r1 \wedge_R r2) = r1 \wedge_R r2$

REA_closure- \vee_R -R3_thm

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | \{r1, r2\} \subseteq R3_healthy$
 $\quad \bullet R3\ (r1 \vee_R r2) = r1 \vee_R r2$

REA_closure- $\triangleleft_R \triangleright_R$ -R3_thm

$\vdash \forall r1, r2 : REA_PREDICATE; b : REL_PREDICATE$
 $\quad | (r1, b, r2) \in WF_Cond_R \wedge \{r1, r2\} \subseteq R3_healthy$
 $\quad \bullet R3\ (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2$

REA_closure-Semi $_R$ -R3_thm

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | (r1, r2) \in WF_Semi_R$
 $\quad \wedge \{r1, r2\} \subseteq R3_healthy$
 $\quad \wedge r2 \in R1_healthy$
 $\quad \bullet R3\ (r1 ;_R r2) = r1 ;_R r2$

REA_R3_wait_true_thm

$\vdash \forall r : REA_PREDICATE$
 $\quad | r.1 \in WF_Skip_{REA}$
 $\quad \bullet /_R\ (R3\ r, Val\ (Bool\ true), wait)$
 $\quad \quad = /_R\ (\Pi_{REA}\ r.1, Val\ (Bool\ true), wait)$

REA_R3_wait_false_thm

$\vdash \forall r : REA_PREDICATE$
 $\quad | r.1 \in WF_Skip_{REA}$
 $\quad \bullet /_R\ (R3\ r, Val\ (Bool\ false), wait)$
 $\quad \quad = /_R\ (r, Val\ (Bool\ false), wait)$

REA_R3_dash_okay_thm

$\vdash \forall r : REA_PREDICATE; b : BOOL_VAL$
 $\quad | r.1 \in WF_Skip_{REA}$
 $\quad \bullet /_R\ (R3\ r, Val\ b, dash\ okay)$
 $\quad \quad = /_R\ (\Pi_{REA}\ r.1, Val\ b, dash\ okay) \triangleleft_R$
 $\quad \quad \quad =_R\ (r.1, wait, Val\ (Bool\ true)) \triangleright_R$
 $\quad \quad \quad /_R\ (r, Val\ b, dash\ okay)$

REA_tr_≤_Rdash_tr_Semi_Rzero_thm

$\vdash \forall p : \text{REA_PROCESS}$
 $\mid p.1 \in \text{homogeneous}$
 $\bullet =_{+R}$
 $(p.1,$
 $\quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\quad \text{Val } (\text{Bool } true))$
 $;_R p$
 $= =_{+R}$
 $(p.1,$
 $\quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\quad \text{Val } (\text{Bool } true))$

REA_¬okay_tr_≤_Rdash_tr_Semi_Rzero_thm

$\vdash \forall p : \text{REA_PROCESS}$
 $\mid p.1 \in \text{homogeneous}$
 $\bullet (=_R (p.1, \text{okay}, \text{Val } (\text{Bool } false)))$
 $\wedge_R =_{+R}$
 $(p.1,$
 $\quad \text{Rel}$
 $\quad \quad ((- \leq_R -),$
 $\quad \quad \text{Var } tr,$
 $\quad \quad \text{Var } (\text{dash } tr)),$
 $\quad \text{Val } (\text{Bool } true)))$
 $;_R p$
 $= =_R (p.1, \text{okay}, \text{Val } (\text{Bool } false))$
 $\wedge_R =_{+R}$
 $(p.1,$
 $\quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\quad \text{Val } (\text{Bool } true))$

REA_Π_{REA}-Semi_Rrea_thm

$\vdash \forall p : \text{REA_PROCESS}$
 $\mid p.1 \in \text{WF_Skip}_{\text{REA}}$
 $\bullet \Pi_{\text{REA}} p.1 ;_R p$
 $= p \triangleleft_R$
 $=_R (p.1, \text{okay}, \text{Val } (\text{Bool } true)) \triangleright_R$
 $=_{+R}$
 $(p.1,$
 $\quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$

$Val (Bool \ true))$

REA_closure_* \wedge_R ***R.thm**

$$\vdash \forall r1, r2 : REA_PREDICATE$$

$$| \{r1, r2\} \subseteq R_healthy$$

$$\bullet R (r1 \wedge_R r2) = r1 \wedge_R r2$$

REA_closure_* \vee_R ***R.thm**

$$\vdash \forall r1, r2 : REA_PREDICATE$$

$$| \{r1, r2\} \subseteq R_healthy$$

$$\bullet R (r1 \vee_R r2) = r1 \vee_R r2$$

REA_closure_* $\triangleleft_R \triangleright_R$ ***R.thm**

$$\vdash \forall r1, r2 : REA_PREDICATE; b : REL_PREDICATE$$

$$| (r1, b, r2) \in WF_Cond_R \wedge \{r1, r2, b\} \subseteq R_healthy$$

$$\bullet R (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2$$

REA_closure_Semi* R ***R.thm**

$$\vdash \forall r1, r2 : REA_PREDICATE$$

$$| (r1, r2) \in WF_Semi_R \wedge \{r1, r2\} \subseteq R_healthy$$

$$\bullet R (r1 ;_R r2) = r1 ;_R r2$$

REA_R_dash_okay.thm

$$\vdash \forall r : REA_PREDICATE; b : BOOL_VAL$$

$$| r.1 \in WF_Skip_{REA}$$

$$\bullet /_R (R \ r, \ Val \ b, \ dash \ okay)$$

$$= /_R (\Pi_{REA} \ r.1, \ Val \ b, \ dash \ okay) \triangleleft_R$$

$$=_R (r.1, \ wait, \ Val \ (Bool \ true)) \triangleright_R$$

$$R1 \ (R2 \ (/_R \ (r, \ Val \ b, \ dash \ okay)))$$

12.4 Definitions

ALPHABET_OWTR

$$\vdash ALPHABET_OWTR$$

$$= ALPHABET_OKAY \cup ALPHABET_WAIT \cup ALPHABET_TR$$

$$\cup ALPHABET_REF$$

ALPHABET_REA $\vdash ALPHABET_REA = \{a : ALPHABET \mid ALPHABET_OWTR \ a\}$

REA_PREDICATE

$$\vdash REA_PREDICATE$$

$$= \{d : REL_PREDICATE$$

$$| d.1 \in ALPHABET_REA\}$$

***WF_Skip* $_{REA}$** $\vdash WF_Skip_{REA} = \{a : ALPHABET_REA \mid a \in homogeneous\}$

REA_PROCESS $\vdash REA_PROCESS = \{r : REA_PREDICATE \mid r \in R_healthy\}$

12.5 Theorems

ALPHABET_REA_thm

$\vdash \forall a : \text{ALPHABET_REA} \bullet a \in \text{ALPHABET}$

ALPHABET_REA_thm1

$\vdash \forall a : \text{ALPHABET_REA}$

- $\text{ALPHABET_TR} \subseteq a$
- $\wedge \text{ALPHABET_OKAY} \subseteq a$
- $\wedge \text{ALPHABET_WAIT} \subseteq a$
- $\wedge \text{ALPHABET_REF} \subseteq a$

ALPHABET_REA_thm2

$\vdash \forall a : \text{ALPHABET_REA} \bullet \text{ALPHABET_OWTR} \subseteq a$

REA_PREDICATE_thm

$\vdash \forall r : \text{REA_PREDICATE}$

- $r \in \text{REL_PREDICATE}$
- $\wedge \{ \text{okay},$
- $\text{dash okay},$
- $\text{tr},$
- $\text{dash tr},$
- $\text{wait},$
- $\text{dash wait},$
- $\text{ref},$
- $\text{dash ref} \}$
- $\subseteq r.1$

$\wedge r.1 \in \text{ALPHABET_REA}$

REA_PROCESS_thm

$\vdash \forall r : \text{REA_PROCESS} \bullet r \in \text{REA_PREDICATE}$

REA_tr_≤_R_dash_tr_∈_WF_ALPHABET_EXPRESSION_thm

$\vdash \forall a : \text{ALPHABET_REA}$

- $(a,$
- $\text{Rel } ((- \leq_R -), \text{Var } \text{tr}, \text{Var } (\text{dash tr})),$
- $\text{Val } (\text{Bool true}))$
- $\in \text{WF_ALPHABET_EXPRESSION}$

REA_ALPHABET_OWTR_thm

$\vdash \text{ALPHABET_OWTR} \in \text{ALPHABET_REA}$

REA_PREDICATE_ALPHABET_OWTR_thm

$\vdash \forall r : \text{REA_PREDICATE} \bullet \text{ALPHABET_OWTR} \subseteq r.1$

REA_tr_dash_tr_≤_R_thm

$$\begin{aligned}
& \vdash =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
& \quad \in REL_PREDICATE \\
REA_tr_dash_tr_ \leq_R_thm1 \\
& \vdash \forall a : ALPHABET \\
& \quad | ALPHABET_OWTR \subseteq a \\
& \quad \bullet =_{+R} \\
& \quad \quad (a, \\
& \quad \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad \quad Val\ (Bool\ true)) \\
& \quad \in REL_PREDICATE \\
REA_tr_dash_tr_ \leq_R \in WF_ALPHABET_EXPRESSION_thm \\
& \vdash \forall a : ALPHABET \\
& \quad | ALPHABET_OWTR \subseteq a \\
& \quad \bullet (a, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
& \quad \in WF_ALPHABET_EXPRESSION \\
REA_R1_thm $\vdash \forall r : REL_PREDICATE \bullet R1\ r \in REL_PREDICATE$ \\
REA_R1_alphabet_thm1 \\
REA_R1_alphabet_thm \\
& \vdash \forall r : REL_PREDICATE \bullet (R1\ r).1 = r.1 \cup ALPHABET_OWTR \\
REA_R1_thm1 $\vdash \forall r : REA_PREDICATE \bullet R1\ r \in REA_PREDICATE$ \\
REA_R2_thm $\vdash \forall r : REL_PREDICATE$ \\
& \quad | ALPHABET_OWTR \subseteq r.1 \\
& \quad \bullet R2\ r \in REL_PREDICATE \\
REA_R2_alphabet_thm \\
& \vdash \forall r : REL_PREDICATE \\
& \quad | ALPHABET_OWTR \subseteq r.1 \\
& \quad \bullet (R2\ r).1 = r.1 \\
REA_R2_thm1 $\vdash \forall r : REA_PREDICATE \bullet R2\ r \in REA_PREDICATE$ \\
REA_ \Pi_{REA}_thm \\
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet \Pi_{REA}\ a \in REA_PREDICATE \\
REA_ \Pi_{REA}_alphabet_thm
\end{aligned}$$

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad | a \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet (\Pi_{\text{REA}} a).1 = a \\
\textbf{REA_R3_thm} & \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet R3 \ r \in \text{REA_PREDICATE} \\
\textbf{REA_R3_alphabet_thm} & \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet (R3 \ r).1 = r.1 \\
\textbf{REA_R_thm} & \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet R \ r \in \text{REA_PREDICATE} \\
\textbf{REA_R_alphabet_thm} & \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet (R \ r).1 = r.1 \\
\textbf{REA_reactive_vars_}\subseteq\textbf{_NAME_thm} & \\
& \vdash \{okay, \\
& \quad \text{dash okay}, \\
& \quad tr, \\
& \quad \text{dash tr}, \\
& \quad wait, \\
& \quad \text{dash wait}, \\
& \quad ref, \\
& \quad \text{dash ref}\} \\
& \subseteq \text{NAME} \\
\textbf{REA_reactive_vars_}\subseteq\textbf{_ALPHABET_REA_thm} & \\
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad \bullet \{okay, \\
& \quad \text{dash okay}, \\
& \quad tr, \\
& \quad \text{dash tr}, \\
& \quad wait, \\
& \quad \text{dash wait}, \\
& \quad ref, \\
& \quad \text{dash ref}\} \\
& \subseteq a
\end{aligned}$$

REA_R1_idem_thm

$\vdash \forall r : REL_PREDICATE \bullet R1 (R1\ r) = R1\ r$

REA_R1_def_∈_REA_PREDICATE_thm

$\vdash =_{+R}$
 $(ALPHABET_OWTR,$
 $Rel\ ((- \leq_R -), Var\ tr, Var\ (dash\ tr)),$
 $Val\ (Bool\ true))$
 $\in REA_PREDICATE$

REA_R2_def_∈_WF_Subst_R_thm1

$\vdash \forall r : REA_PREDICATE$
 $\bullet (r, Val\ (Seq\ \langle \rangle), tr) \in WF_Subst_R$

REA_R2_def_alphabet_thm1

$\vdash \forall r : REA_PREDICATE$
 $\bullet (/_R\ (r, Val\ (Seq\ \langle \rangle), tr)).1 = r.1$

REA_R2_def_∈_WF_Subst_R_thm

$\vdash \forall r : REA_PREDICATE$
 $\bullet (/_R\ (r, Val\ (Seq\ \langle \rangle), tr),$
 $Fun_2\ ((- SeqDif_R -), Var\ (dash\ tr), Var\ tr),$
 $dash\ tr)$
 $\in WF_Subst_R$

REA_R2_def_alphabet_thm

$\vdash \forall r : REA_PREDICATE$
 $\bullet (/_R$
 $(/_R\ (r, Val\ (Seq\ \langle \rangle), tr),$
 Fun_2
 $((- SeqDif_R -),$
 $Var\ (dash\ tr),$
 $Var\ tr),$
 $dash\ tr)).1$
 $= r.1$

REA_R2_def_∈_REL_PREDICATE_thm1

$\vdash \forall r : REA_PREDICATE$
 $\bullet /_R\ (r, Val\ (Seq\ \langle \rangle), tr) \in REL_PREDICATE$

REA_R2_def_∈_REL_PREDICATE_thm

$\vdash \forall r : REA_PREDICATE$
 $\bullet /_R$
 $(/_R\ (r, Val\ (Seq\ \langle \rangle), tr),$
 Fun_2

$$\begin{aligned}
& ((- \text{SeqDif}_R -), \\
& \quad \text{Var } (\text{dash } tr), \\
& \quad \text{Var } tr), \\
& \text{dash } tr) \\
& \in \text{REL_PREDICATE} \\
\mathbf{REA_R2_def_}\in\mathbf{_REA_PREDICATE_thm1} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{ } /_R (r, \text{Val } (\text{Seq } \langle \rangle), tr) \in \text{REA_PREDICATE} \\
\mathbf{REA_R2_def_}\in\mathbf{_REA_PREDICATE_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{ } /_R \\
& \quad \quad (/_R (r, \text{Val } (\text{Seq } \langle \rangle), tr), \\
& \quad \quad \text{Fun}_2 \\
& \quad \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \quad \text{Var } (\text{dash } tr), \\
& \quad \quad \quad \text{Var } tr), \\
& \quad \quad \text{dash } tr) \\
& \quad \in \text{REA_PREDICATE} \\
\mathbf{REA_R1_def_alphabet_thm} \\
& \vdash (=_{+R} \\
& \quad (\text{ALPHABET_OWTR}, \\
& \quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)), \\
& \quad \text{Val } (\text{Bool true})).1 \\
& \quad = \text{ALPHABET_OWTR} \\
\mathbf{REA_PREDICATE_}\wedge\mathbf{_R_thm} \\
& \vdash \forall r1, r2 : \text{REA_PREDICATE} \bullet r1 \wedge_R r2 \in \text{REA_PREDICATE} \\
\mathbf{REA_PREDICATE_}\wedge\mathbf{_R_thm1} \\
& \vdash \forall r1 : \text{REL_PREDICATE}; r2 : \text{REA_PREDICATE} \\
& \quad \bullet r1 \wedge_R r2 \in \text{REA_PREDICATE} \\
\mathbf{REA_PREDICATE_}\wedge\mathbf{_R_thm2} \\
& \vdash \forall r1 : \text{REA_PREDICATE}; r2 : \text{REL_PREDICATE} \\
& \quad \bullet r1 \wedge_R r2 \in \text{REA_PREDICATE} \\
\mathbf{REA_PREDICATE_}\vee\mathbf{_R_thm} \\
& \vdash \forall r1, r2 : \text{REA_PREDICATE} \bullet r1 \vee_R r2 \in \text{REA_PREDICATE} \\
\mathbf{REA_PREDICATE_}\vee\mathbf{_R_thm1} \\
& \vdash \forall r1 : \text{REL_PREDICATE}; r2 : \text{REA_PREDICATE} \\
& \quad \bullet r1 \vee_R r2 \in \text{REA_PREDICATE} \\
\mathbf{REA_PREDICATE_}\vee\mathbf{_R_thm2}
\end{aligned}$$

$$\begin{array}{l}
\vdash \forall r1 : REA_PREDICATE; r2 : REL_PREDICATE \\
\quad \bullet r1 \vee_R r2 \in REA_PREDICATE \\
REA_PREDICATE_{\Rightarrow_R}thm \\
\vdash \forall r1, r2 : REA_PREDICATE \bullet r1 \Rightarrow_R r2 \in REA_PREDICATE \\
REA_PREDICATE_{\Rightarrow_R}thm1 \\
\vdash \forall r1 : REL_PREDICATE; r2 : REA_PREDICATE \\
\quad \bullet r1 \Rightarrow_R r2 \in REA_PREDICATE \\
REA_PREDICATE_{\Rightarrow_R}thm2 \\
\vdash \forall r1 : REA_PREDICATE; r2 : REL_PREDICATE \\
\quad \bullet r1 \Rightarrow_R r2 \in REA_PREDICATE \\
REA_PREDICATE_{\neg_R}thm \\
\vdash \forall r : REA_PREDICATE \bullet \neg_R r \in REA_PREDICATE \\
REA_R2_{\wedge_R}dist_thm \\
\vdash \forall r1, r2 : REA_PREDICATE \\
\quad \bullet R2 (r1 \wedge_R r2) = R2 r1 \wedge_R R2 r2 \\
REA_tr_prefix_dash_tr_thm \\
\vdash =_{+R} \\
\quad (ALPHABET_OWTR, \\
\quad \quad Rel \\
\quad \quad ((- \leq_R -), \\
\quad \quad \quad Val (Seq \langle \rangle), \\
\quad \quad \quad Fun_2 \\
\quad \quad \quad ((- SeqDif_R -), \\
\quad \quad \quad \quad Var (dash tr), \\
\quad \quad \quad \quad Var tr)), \\
\quad \quad Val (Bool true)) \\
\quad \wedge_R =_{+R} \\
\quad (ALPHABET_OWTR, \\
\quad \quad Rel ((- \leq_R -), Var tr, Var (dash tr)), \\
\quad \quad Val (Bool true)) \\
\quad = =_{+R} \\
\quad (ALPHABET_OWTR, \\
\quad \quad Rel ((- \leq_R -), Var tr, Var (dash tr)), \\
\quad \quad Val (Bool true)) \\
REA_R2_R1_def_unit_thm \\
\vdash R1 \\
\quad (R2 \\
\quad \quad (=_{+R}
\end{array}$$

$$\begin{aligned}
& (ALPHABET_OWTR, \\
& \quad Rel \\
& \quad ((- \leq_R -), \\
& \quad \quad Var \ tr, \\
& \quad \quad Var \ (dash \ tr)), \\
& \quad Val \ (Bool \ true))) \\
& = R1 \ (True_R \ ALPHABET_OWTR) \\
\mathbf{REA_R1_extends_over_}\wedge_R\mathbf{-thm} \\
& \vdash \forall \ r1, \ r2 : REL_PREDICATE \\
& \quad \bullet \ R1 \ r1 \wedge_R \ r2 = R1 \ (r1 \wedge_R \ r2) \\
\mathbf{REA_R1_}\wedge_R\mathbf{-dist_thm} \\
& \vdash \forall \ r1, \ r2 : REL_PREDICATE \\
& \quad \bullet \ R1 \ r1 \wedge_R \ R1 \ r2 = R1 \ (r1 \wedge_R \ r2) \\
\mathbf{REA_R1_}\vee_R\mathbf{-dist_thm} \\
& \vdash \forall \ r1, \ r2 : REL_PREDICATE \\
& \quad \bullet \ R1 \ r1 \vee_R \ R1 \ r2 = R1 \ (r1 \vee_R \ r2) \\
\mathbf{REA_R1_R2_com_thm} \\
& \vdash \forall \ r : REA_PREDICATE \bullet \ R1 \ (R2 \ r) = R1 \ (R2 \ (R1 \ r)) \\
\mathbf{REA_R3_idem_thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \quad | \ r.1 \in WF_Skip_{REA} \\
& \quad \bullet \ R3 \ (R3 \ r) = R3 \ r \\
\mathbf{REA_R1_alphabet_WF_Skip}_{REA}\mathbf{-thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \quad | \ r.1 \in WF_Skip_{REA} \\
& \quad \bullet \ (R1 \ r).1 \in WF_Skip_{REA} \\
\mathbf{REA_R2_alphabet_WF_Skip}_{REA}\mathbf{-thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \quad | \ r.1 \in WF_Skip_{REA} \\
& \quad \bullet \ (R2 \ r).1 \in WF_Skip_{REA} \\
\mathbf{REA_R3_def_}\in\mathbf{-WF_Cond}_R\mathbf{-thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \quad | \ r.1 \in WF_Skip_{REA} \\
& \quad \bullet \ (\Pi_{REA} \ r.1, \\
& \quad \quad =_R \ (\{wait\}, \ wait, \ Val \ (Bool \ true)), \\
& \quad \quad r) \\
& \quad \in WF_Cond_R \\
\mathbf{REA_R3_def_}\in\mathbf{-WF_Cond}_R\mathbf{-thm1}
\end{aligned}$$

$$\begin{aligned}
& \vdash \forall r : REA_PREDICATE \\
& \quad | r.1 \in WF_Skip_{REA} \\
& \quad \bullet (\Pi_{REA} r.1, =_R (r.1, wait, Val (Bool true)), r) \\
& \quad \in WF_Cond_R \\
REA_dash_tr_Equals_tr_in_WF_Equals_R_thm \\
& \vdash \forall a : ALPHABET_REA \\
& \quad \bullet (a, dash\ tr, Var\ tr) \in WF_Equals_R \\
REA_dash_tr_equals_tr_thm1 \\
& \vdash \forall a : ALPHABET_REA \\
& \quad \bullet =_R (a, dash\ tr, Var\ tr) \\
& \quad \quad \wedge_R =_{+R} \\
& \quad \quad (ALPHABET_OWTR, \\
& \quad \quad \quad Rel ((- \leq_R -), Var\ tr, Var (dash\ tr)), \\
& \quad \quad \quad Val (Bool true)) \\
& \quad =_R (a, dash\ tr, Var\ tr) \\
REA_Pi_REA_R1_healthy_thm \\
& \vdash \forall a : ALPHABET_REA \\
& \quad | a \in WF_Skip_{REA} \\
& \quad \bullet \Pi_{REA} a \in R1_healthy \\
REA_R3_R1_com_thm \\
& \vdash \forall r : REA_PREDICATE \\
& \quad | r.1 \in WF_Skip_{REA} \\
& \quad \bullet R1 (R3\ r) = R3 (R1\ r) \\
REA_R3_rewrite_thm \\
& \vdash \forall r : REA_PREDICATE \\
& \quad | r.1 \in WF_Skip_{REA} \\
& \quad \bullet R3\ r \\
& \quad = \Pi_{REA} r.1 \triangleleft_R \\
& \quad =_R (r.1, wait, Val (Bool true)) \triangleright_R \\
& \quad r \\
REA_R3_def_cond_in_REA_PREDICATE_thm1 \\
& \vdash \forall a : ALPHABET_REA; v : VALUE \\
& \quad \bullet =_R (a, wait, Val\ v) \in REA_PREDICATE \\
REA_R2_V_R_dist_thm \\
& \vdash \forall r1, r2 : REA_PREDICATE \\
& \quad \bullet R2 (r1 \vee_R r2) = R2\ r1 \vee_R R2\ r2 \\
REA_R2_neg_R_dist_thm \\
& \vdash \forall r : REA_PREDICATE \bullet R2 (\neg_R r) = \neg_R (R2\ r)
\end{aligned}$$

REA_R2_<_R->_R-dist_thm

$\vdash \forall r1, r2, b : REA_PREDICATE$
 $\quad | (r1, b, r2) \in WF_Cond_R$
 $\quad \bullet R2 (r1 <_R b >_R r2) = R2 r1 <_R R2 b >_R R2 r2$

REA_Equals_R-<_R->_R-REA-PREDICATE_thm

$\vdash \forall a : ALPHABET_REA; v : VALUE; n : NAME$
 $\quad | n \in a$
 $\quad \bullet =_R (a, n, Val v) \in REA_PREDICATE$

REA_Equals_R-<_R->_R-REA-PREDICATE_thm1

$\vdash \forall a : ALPHABET_REA; n1, n2 : NAME$
 $\quad | n1 \in a \wedge n2 \in a$
 $\quad \bullet =_R (a, n1, Var n2) \in REA_PREDICATE$

REA_R2_Equals_R-unit_thm

$\vdash \forall v : VALUE; a : ALPHABET_REA; n : NAME$
 $\quad | n \neq tr \wedge n \neq dash\ tr \wedge n \in a$
 $\quad \bullet R2 (=_R (a, n, Val v)) = =_R (a, n, Val v)$

REA_R1_<_R->_R-dist_thm

$\vdash \forall u1, u2, b : REL_PREDICATE$
 $\quad | (u1, b, u2) \in WF_Cond_R$
 $\quad \bullet R1 (u1 <_R b >_R u2) = R1 u1 <_R b >_R R1 u2$

REA_R2_Equals_R-unit_thm1

$\vdash \forall a : ALPHABET_REA; n1, n2 : NAME$
 $\quad | n1 \neq tr$
 $\quad \wedge n1 \neq dash\ tr$
 $\quad \wedge n2 \neq tr$
 $\quad \wedge n2 \neq dash\ tr$
 $\quad \wedge n1 \in a$
 $\quad \wedge n2 \in a$
 $\quad \bullet R2 (=_R (a, n1, Var n2)) = =_R (a, n1, Var n2)$

REA_PREDICATE_dash_tr_Equals_tr_thm

$\vdash \forall a : ALPHABET_REA$
 $\quad \bullet =_R (a, dash\ tr, Var tr) \in REA_PREDICATE$

REA_dash_tr_Equals_tr_alphabet_thm

$\vdash \forall a : ALPHABET_REA \bullet (=_R (a, dash\ tr, Var tr)).1 = a$

REA_dash_tr_equals_tr_thm

$\vdash \forall a : ALPHABET_REA$
 $\quad \bullet =_{+R}$
 $\quad (a,$

$$\begin{aligned}
& \text{Fun}_2 \\
& \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \text{Var } (\text{dash } tr), \\
& \quad \quad \text{Var } tr), \\
& \quad \text{Val } (\text{Seq } \langle \rangle)) \\
& \wedge_R =_{+R} \\
& \quad (\text{ALPHABET_OWTR}, \\
& \quad \quad \text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)), \\
& \quad \quad \text{Val } (\text{Bool } \text{true})) \\
& =_R (a, \text{dash } tr, \text{Var } tr) \\
\mathbf{REA_R2_dash_tr_Equals_tr_unit_thm} \\
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad \bullet R1 (R2 (=R (a, \text{dash } tr, \text{Var } tr))) \\
& \quad =_R (a, \text{dash } tr, \text{Var } tr) \\
\mathbf{REA_R1_dash_tr_Equals_tr_unit_thm} \\
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad \bullet R1 (=R (a, \text{dash } tr, \text{Var } tr)) \\
& \quad =_R (a, \text{dash } tr, \text{Var } tr) \\
\mathbf{REA_R2_dash_wait_unit_thm} \\
& \vdash \forall v : \text{VALUE}; a : \text{ALPHABET_REA} \\
& \quad \bullet R2 (=R (a, \text{dash } wait, \text{Val } v)) \\
& \quad =_R (a, \text{dash } wait, \text{Val } v) \\
\mathbf{REA_wait_false_id_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | \text{unrestTypedVar } (r, \text{wait}, \text{BOOL_VAL}) \\
& \quad \bullet r \omega_f = r \\
\mathbf{REA_wait_true_id_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | \text{unrestTypedVar } (r, \text{wait}, \text{BOOL_VAL}) \\
& \quad \bullet r \omega_t = r \\
\mathbf{REA_Equals_wait_false_thm} \\
& \vdash =_R (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } (\text{Bool } \text{false})) \\
& \quad =_R (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } (\text{Bool } \text{false})) \omega_f \\
\mathbf{REA_Equals_wait_false_thm1} \\
& \vdash =_R (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } (\text{Bool } \text{true})) \\
& \quad =_R (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } (\text{Bool } \text{true})) \omega_f \\
\mathbf{REA_Equals_wait_true_thm} \\
& \vdash =_R (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } (\text{Bool } \text{false}))
\end{aligned}$$

$=_R (ALPHABET_OWTR, okay, Val (Bool false)) \omega_t$
REA_Equals_wait_true_thm1
 $\vdash =_R (ALPHABET_OWTR, okay, Val (Bool true))$
 $=_R (ALPHABET_OWTR, okay, Val (Bool true)) \omega_t$
REA_R2_Π_{REA}-idem_thm
 $\vdash \forall a : ALPHABET_REA$
 $\quad | a \in WF_Skip_{REA}$
 $\quad \bullet R1 (R2 (\Pi_{REA} a)) = R1 (\Pi_{REA} a)$
REA_R3_R2_com_thm
 $\vdash \forall r : REA_PREDICATE$
 $\quad | r.1 \in WF_Skip_{REA}$
 $\quad \bullet R1 (R2 (R3 r)) = R1 (R3 (R2 r))$
REA_R_idem_thm
 $\vdash \forall r : REA_PREDICATE$
 $\quad | r.1 \in WF_Skip_{REA}$
 $\quad \bullet R (R r) = R r$
REA_PREDICATE_wait_∈_ALPHABET_thm
 $\vdash \forall r : REA_PREDICATE \bullet wait \in r.1$
REA_PREDICATE_okay_dash_∈_ALPHABET_thm
 $\vdash \forall r : REA_PREDICATE \bullet dash\ okay \in r.1$
REA_PREDICATE_wait_true_thm
 $\vdash \forall r : REA_PREDICATE \bullet r \omega_t \in REA_PREDICATE$
REA_PREDICATE_wait_false_thm
 $\vdash \forall r : REA_PREDICATE \bullet r \omega_f \in REA_PREDICATE$
REA_PREDICATE_okay_dash_converge_thm
 $\vdash \forall r : REA_PREDICATE \bullet r \sigma_t \in REA_PREDICATE$
REA_PREDICATE_okay_dash_diverge_thm
 $\vdash \forall r : REA_PREDICATE \bullet r \sigma_f \in REA_PREDICATE$
REA_PREDICATE_wait_true_alphabet_thm
 $\vdash \forall r : REA_PREDICATE \bullet (r \omega_t).1 = r.1$
REA_PREDICATE_wait_false_alphabet_thm
 $\vdash \forall r : REA_PREDICATE \bullet (r \omega_f).1 = r.1$
REA_PREDICATE_okay_dash_converge_alphabet_thm
 $\vdash \forall r : REA_PREDICATE \bullet (r \sigma_t).1 = r.1$
REA_PREDICATE_okay_dash_diverge_alphabet_thm
 $\vdash \forall r : REA_PREDICATE \bullet (r \sigma_f).1 = r.1$
REA_R1_def_∈_REL_PREDICATE_thm
 $\vdash =_{+R}$

$$\begin{aligned}
& (ALPHABET_OWTR, \\
& \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad Val\ (Bool\ true)) \\
& \in REL_PREDICATE \\
REA_R1_def_wait_false_id_thm \\
& \vdash =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true))\ \omega_f \\
& = =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
REA_R1_def_wait_true_id_thm \\
& \vdash =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true))\ \omega_t \\
& = =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
REA_R1_def_okay_dash_diverge_id_thm \\
& \vdash =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true))\ \sigma_f \\
& = =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
REA_R1_def_okay_dash_converge_id_thm \\
& \vdash =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true))\ \sigma_t \\
& = =_{+R} \\
& \quad (ALPHABET_OWTR,
\end{aligned}$$

$$\begin{aligned}
& Rel \ ((- \leq_R -), Var \ tr, Var \ (dash \ tr)), \\
& Val \ (Bool \ true)) \\
\mathbf{REA_R2_def_wait_false_com_thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \bullet \ /_R \\
& \quad (/_R \ (r, Val \ (Seq \ \langle \rangle), tr), \\
& \quad \quad Fun_2 \\
& \quad \quad ((- \ SeqDif_R \ -), \\
& \quad \quad \quad Var \ (dash \ tr), \\
& \quad \quad \quad Var \ tr), \\
& \quad \quad dash \ tr) \ \omega_f \\
& = \ /_R \\
& \quad (/_R \ (r \ \omega_f, Val \ (Seq \ \langle \rangle), tr), \\
& \quad \quad Fun_2 \\
& \quad \quad ((- \ SeqDif_R \ -), \\
& \quad \quad \quad Var \ (dash \ tr), \\
& \quad \quad \quad Var \ tr), \\
& \quad \quad dash \ tr) \\
\mathbf{REA_R2_def_wait_true_com_thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \bullet \ /_R \\
& \quad (/_R \ (r, Val \ (Seq \ \langle \rangle), tr), \\
& \quad \quad Fun_2 \\
& \quad \quad ((- \ SeqDif_R \ -), \\
& \quad \quad \quad Var \ (dash \ tr), \\
& \quad \quad \quad Var \ tr), \\
& \quad \quad dash \ tr) \ \omega_t \\
& = \ /_R \\
& \quad (/_R \ (r \ \omega_t, Val \ (Seq \ \langle \rangle), tr), \\
& \quad \quad Fun_2 \\
& \quad \quad ((- \ SeqDif_R \ -), \\
& \quad \quad \quad Var \ (dash \ tr), \\
& \quad \quad \quad Var \ tr), \\
& \quad \quad dash \ tr) \\
\mathbf{REA_R2_def_okay_dash_diverge_com_thm} \\
& \vdash \forall \ r : REA_PREDICATE \\
& \bullet \ /_R \\
& \quad (/_R \ (r, Val \ (Seq \ \langle \rangle), tr),
\end{aligned}$$

$$\begin{aligned}
& \text{Fun}_2 \\
& \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \text{Var} (\text{dash } tr), \\
& \quad \quad \text{Var } tr), \\
& \quad \text{dash } tr) \sigma_f \\
= & \text{ /}_R \\
& \quad (\text{ /}_R (r \sigma_f, \text{Val} (\text{Seq } \langle \rangle), tr), \\
& \quad \text{Fun}_2 \\
& \quad \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \quad \text{Var} (\text{dash } tr), \\
& \quad \quad \quad \text{Var } tr), \\
& \quad \quad \text{dash } tr) \\
\mathbf{REA_R2_def_okay_dash_converge_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{ /}_R \\
& \quad \quad (\text{ /}_R (r, \text{Val} (\text{Seq } \langle \rangle), tr), \\
& \quad \quad \text{Fun}_2 \\
& \quad \quad \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \quad \quad \text{Var} (\text{dash } tr), \\
& \quad \quad \quad \quad \text{Var } tr), \\
& \quad \quad \quad \text{dash } tr) \sigma_t \\
= & \text{ /}_R \\
& \quad (\text{ /}_R (r \sigma_t, \text{Val} (\text{Seq } \langle \rangle), tr), \\
& \quad \text{Fun}_2 \\
& \quad \quad ((- \text{SeqDif}_R -), \\
& \quad \quad \quad \text{Var} (\text{dash } tr), \\
& \quad \quad \quad \text{Var } tr), \\
& \quad \quad \text{dash } tr) \\
\mathbf{REA_R1_wait_false_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet R1 \ r \ \omega_f = R1 \ (r \ \omega_f) \\
\mathbf{REA_R1_wait_true_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet R1 \ r \ \omega_t = R1 \ (r \ \omega_t) \\
\mathbf{REA_R1_okay_dash_diverge_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet R1 \ r \ \sigma_f = R1 \ (r \ \sigma_f) \\
\mathbf{REA_R1_okay_dash_converge_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet R1 \ r \ \sigma_t = R1 \ (r \ \sigma_t) \\
\mathbf{REA_R2_wait_false_com_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet R2 \ r \ \omega_f = R2 \ (r \ \omega_f)
\end{aligned}$$

REA_R2_wait_true_com_thm

$\vdash \forall r : \text{REA_PREDICATE} \bullet R2 \ r \ \omega_t = R2 \ (r \ \omega_t)$

REA_R2_okay_dash_diverge_com_thm

$\vdash \forall r : \text{REA_PREDICATE} \bullet R2 \ r \ \sigma_f = R2 \ (r \ \sigma_f)$

REA_R2_okay_dash_converge_com_thm

$\vdash \forall r : \text{REA_PREDICATE} \bullet R2 \ r \ \sigma_t = R2 \ (r \ \sigma_t)$

REA_R1_def_∈_WF_ALPHABET_EXPRESSION_thm

$\vdash (\text{ALPHABET_OWTR},$
 $\text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\text{Val } (\text{Bool } \text{true}))$
 $\in \text{WF_ALPHABET_EXPRESSION}$

REA_R2_OKAY_def_ext_unit_thm

$\vdash \forall v : \text{VALUE}$
 $\bullet R2 \ (=_{\text{R}} (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } v))$
 $\quad =_{\text{R}} (\text{ALPHABET_OWTR}, \text{okay}, \text{Val } v)$

REA_R2_OKAY_def_ext_unit_thm1

$\vdash \forall v : \text{VALUE}; a : \text{ALPHABET_REA}$
 $\bullet R2 \ (=_{\text{R}} (a, \text{okay}, \text{Val } v)) =_{\text{R}} (a, \text{okay}, \text{Val } v)$

REA_closure_∧_R_R1_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$
 $\quad | \{r1, r2\} \subseteq R1_healthy$
 $\bullet R1 \ (r1 \wedge_R r2) = r1 \wedge_R r2$

REA_closure_∨_R_R1_thm

$\vdash \forall r1, r2 : \text{REL_PREDICATE}$
 $\quad | \{r1, r2\} \subseteq R1_healthy$
 $\bullet R1 \ (r1 \vee_R r2) = r1 \vee_R r2$

REA_PREDICATE_True_R_thm

$\vdash \forall a : \text{ALPHABET_REA} \bullet \text{True}_R \ a \in \text{REA_PREDICATE}$

REA_PREDICATE_Equals_R_thm

$\vdash \forall a : \text{ALPHABET_REA}; n : \text{NAME}; e : \text{EXPRESSION}$
 $\quad | (a, n, e) \in \text{WF_Equals}_R$
 $\bullet =_R (a, n, e) \in \text{REA_PREDICATE}$

REA_wait_∈_WF_Equals_R_thm

$\vdash \forall a : \text{ALPHABET_REA}$
 $\bullet (a, \text{wait}, \text{Val } (\text{Bool } \text{true})) \in \text{WF_Equals}_R$

REA_not_wait_∈_WF_Equals_R_thm

$\vdash \forall a : \text{ALPHABET_REA}$
 $\bullet (a, \text{wait}, \text{Val } (\text{Bool } \text{false})) \in \text{WF_Equals}_R$

REA_wait_dash_∈_WF_Equals_R_thm

$\vdash \forall a : \text{ALPHABET_REA}$

$\bullet (a, \text{dash wait}, \text{Val}(\text{Bool true})) \in \text{WF_Equals}_R$

REA_not_wait_dash_∈_WF_Equals_R_thm

$\vdash \forall a : \text{ALPHABET_REA}$

$\bullet (a, \text{dash wait}, \text{Val}(\text{Bool false})) \in \text{WF_Equals}_R$

REA_R2_True_R_unit_thm

$\vdash \forall a : \text{ALPHABET_REA} \bullet R2(\text{True}_R a) = \text{True}_R a$

REA_R2_⇒_R_dist_thm

$\vdash \forall r1, r2 : \text{REA_PREDICATE}$

$\bullet R2(r1 \Rightarrow_R r2) = R2 r1 \Rightarrow_R R2 r2$

REA_ALPHABET_OWTR_homogeneous_thm

$\vdash \text{ALPHABET_OWTR} \in \text{homogeneous}$

REA_PREDICATE_False_R_thm

$\vdash \forall a : \text{ALPHABET_REA} \bullet \text{False}_R a \in \text{REA_PREDICATE}$

REA_PROCESS_thm1

$\vdash \forall r : \text{REA_PROCESS} \bullet r \in \text{REA_PREDICATE} \wedge r \in R_healthy$

REA_R_healthy_thm

$\vdash \forall r : \text{REA_PREDICATE}$

$\mid r \in R_healthy \wedge r.1 \in \text{WF_Skip}_{\text{REA}}$

$\bullet r = R r$

REA_PREDICATE_∈_WF_Skip_{REA}_thm

$\vdash \forall a : \text{ALPHABET_REA}$

$\mid a \in \text{homogeneous}$

$\bullet a \in \text{WF_Skip}_{\text{REA}}$

REA_R_healthy_is_R1_healthy_thm

$\vdash \forall r : \text{REA_PROCESS} \mid r \in R_healthy \bullet r \in R1_healthy$

REA_R1_healthy_thm

$\vdash \forall r : \text{REA_PROCESS} \bullet r = R1 r$

REA_R_healthy_R1_zero_thm

$\vdash \forall r : \text{REA_PROCESS}$

$\bullet r$

$\vee_R =_{+R}$

$(\text{ALPHABET_OWTR},$

$\text{Rel}((- \leq_R -), \text{Var } tr, \text{Var}(\text{dash } tr)),$

$\text{Val}(\text{Bool true}))$

$= =_{+R}$

$(r.1,$

$Rel ((- \leq_R -), Var\ tr, Var\ (dash\ tr)),$
 $Val\ (Bool\ true))$

REA_R_R1_idem_thm

$\vdash \forall r : REA_PREDICATE$
 $\quad | \ r.1 \in WF_Skip_{REA}$
 $\quad \bullet \ R\ (R1\ r) = R\ r$

REA_okay_dash_diverge_wait_false_com_thm

$\vdash \forall r : REA_PREDICATE \bullet (r\ \sigma_f)\ \omega_f = (r\ \omega_f)\ \sigma_f$

REA_okay_dash_converge_wait_false_com_thm

$\vdash \forall r : REA_PREDICATE \bullet (r\ \sigma_t)\ \omega_f = (r\ \omega_f)\ \sigma_t$

REA_WF_Skip_REA- \cup _dist_thm

$\vdash \forall a1, a2 : WF_Skip_{REA} \bullet a1 \cup a2 \in WF_Skip_{REA}$

REA_ Π_{REA} -alphabet- \vee_R -dist_thm1

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | \ r1.1 \in WF_Skip_{REA}$
 $\quad \quad \wedge \ r2.1 \in WF_Skip_{REA}$
 $\quad \quad \wedge \ r1.1 = r2.1$
 $\quad \bullet \ \Pi_{REA}\ (r1\ \vee_R\ r2).1 = \Pi_{REA}\ r1.1\ \vee_R\ \Pi_{REA}\ r2.1$

REA_R3- \vee_R -dist_thm1

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | \ r1.1 \in WF_Skip_{REA}$
 $\quad \quad \wedge \ r2.1 \in WF_Skip_{REA}$
 $\quad \quad \wedge \ r1.1 = r2.1$
 $\quad \bullet \ R3\ (r1\ \vee_R\ r2) = R3\ r1\ \vee_R\ R3\ r2$

REA_R_healthy- \vee_R -dist_thm1

$\vdash \forall r1, r2 : REA_PREDICATE$
 $\quad | \ r1 \in R_healthy \wedge r2 \in R_healthy \wedge r1.1 = r2.1$
 $\quad \bullet \ r1\ \vee_R\ r2 \in R_healthy$

REA_PROCESS- \vee_R -thm1

$\vdash \forall r1, r2 : REA_PROCESS$
 $\quad | \ r1.1 = r2.1$
 $\quad \bullet \ r1\ \vee_R\ r2 \in REA_PROCESS$

REA_PREDICATE-Semi $_R$ -thm

$\vdash \forall A1, A2 : REA_PREDICATE$
 $\quad | \ (A1, A2) \in WF_Semi_R$
 $\quad \bullet \ A1\ ;_R\ A2 \in REA_PREDICATE$

REA_R_healthy-is-R3_healthy_thm

$\vdash \forall r : REA_PROCESS \mid r \in R_healthy \bullet r \in R3_healthy$

REA_R3_healthy_thm

$\vdash \forall r : \text{REA_PROCESS} \bullet r = R3\ r$

REA_R1_∨_R-dist_thm1

$\vdash \forall r1, r2 : \text{REA_PREDICATE}$
 $\quad | \quad r1.1 \in \text{WF_Skip}_{\text{REA}}$
 $\quad \quad \wedge r2.1 \in \text{WF_Skip}_{\text{REA}}$
 $\quad \quad \wedge r2.1 = r1.1$
 $\quad \bullet \quad R\ r1\ \vee_R\ R\ r2 = R\ (r1\ \vee_R\ r2)$

REA_Semi_R-is_R_healthy_thm1

$\vdash \forall r1, r2 : \text{REA_PROCESS}$
 $\quad | \quad (r1, r2) \in \text{WF_Semi}_R \wedge r1.1 = r2.1$
 $\quad \bullet \quad r1\ ;_R\ r2 \in R_healthy$

REA_PROCESS_Semi_R-thm1

$\vdash \forall r1, r2 : \text{REA_PROCESS}$
 $\quad | \quad (r1, r2) \in \text{WF_Semi}_R \wedge r1.1 = r2.1$
 $\quad \bullet \quad r1\ ;_R\ r2 \in \text{REA_PROCESS}$

REA_R1_R2-is_healthy_thm

$\vdash \forall r : \text{REA_PROCESS} \mid r \in R_healthy \bullet R1\ (R2\ r) = r$

REA_closure_Semi_R-R-thm1

$\vdash \forall r1, r2 : \text{REA_PROCESS}$
 $\quad | \quad (r1, r2) \in \text{WF_Semi}_R$
 $\quad \quad \wedge r1.1 = r2.1$
 $\quad \quad \wedge r1 \in R_healthy$
 $\quad \quad \wedge r2 \in R_healthy$
 $\quad \bullet \quad R\ (r1\ ;_R\ r2) = r1\ ;_R\ r2$

REA_PREDICATE_thm1

$\vdash \forall r : \text{REA_PREDICATE}$
 $\quad \bullet \quad r \in \text{REL_PREDICATE}$
 $\quad \quad \wedge \text{ALPHABET_OWTR} \subseteq r.1$
 $\quad \quad \wedge r.1 \in \text{ALPHABET_REA}$

REA_dash_wait_∈_WF_Equals_R-thm

$\vdash \forall a : \text{ALPHABET_REA}$
 $\quad \bullet \quad (a, \text{dash wait}, \text{Val}(\text{Bool true})) \in \text{WF_Equals}_R$

REA_PREDICATE_Equals_R-dash_wait_thm

$\vdash \forall a : \text{ALPHABET_REA}$
 $\quad \bullet \quad =_R\ (a, \text{dash wait}, \text{Val}(\text{Bool true}))$
 $\quad \quad \in \text{REA_PREDICATE}$

REA_okay_∈_WF_Equals_R-thm

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad \bullet (a, \text{okay}, \text{Val}(\text{Bool true})) \in \text{WF_Equals}_R \\
\mathbf{REA_not_okay_}\in\mathbf{WF_Equals}_R\mathbf{_thm} \\
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad \bullet (a, \text{okay}, \text{Val}(\text{Bool false})) \in \text{WF_Equals}_R \\
\mathbf{REA_UnrestTypedVar_wait_false_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{unrestTypedVar}(r \ \omega_f, \text{wait}, \text{BOOL_VAL}) \\
\mathbf{REA_UnrestTypedVar_wait_true_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{unrestTypedVar}(r \ \omega_t, \text{wait}, \text{BOOL_VAL}) \\
\mathbf{REA_UnrestTypedVar_okay_dash_converge_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{unrestTypedVar}(r \ \sigma_t, \text{dash okay}, \text{BOOL_VAL}) \\
\mathbf{REA_UnrestTypedVar_okay_dash_diverge_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad \bullet \text{unrestTypedVar}(r \ \sigma_f, \text{dash okay}, \text{BOOL_VAL}) \\
\mathbf{REA_R3_}\wedge_R\mathbf{_dist_thm1} \\
& \vdash \forall r1, r2 : \text{REA_PREDICATE} \\
& \quad | r1.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \quad \wedge r2.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \quad \wedge r1.1 = r2.1 \\
& \quad \bullet R3(r1 \wedge_R r2) = R3 \ r1 \wedge_R R3 \ r2 \\
\mathbf{REA_R_healthy_}\wedge_R\mathbf{_dist_thm1} \\
& \vdash \forall r1, r2 : \text{REA_PREDICATE} \\
& \quad | r1 \in R_healthy \wedge r2 \in R_healthy \wedge r1.1 = r2.1 \\
& \quad \bullet r1 \wedge_R r2 \in R_healthy \\
\mathbf{REA_PROCESS_}\wedge_R\mathbf{_thm1} \\
& \vdash \forall r1, r2 : \text{REA_PROCESS} \\
& \quad | r1.1 = r2.1 \\
& \quad \bullet r1 \wedge_R r2 \in \text{REA_PROCESS} \\
\mathbf{REA_Equals_wait_false_thm2} \\
& \vdash =_R (\text{ALPHABET_OWTR}, \text{dash okay}, \text{Val}(\text{Bool true})) \\
& \quad = =_R \\
& \quad \quad (\text{ALPHABET_OWTR}, \\
& \quad \quad \text{dash okay}, \\
& \quad \quad \text{Val}(\text{Bool true})) \ \omega_f \\
\mathbf{REA_R1_wait_false_thm}
\end{aligned}$$

$\vdash \forall u : \text{REA_PROCESS} \bullet R1 \ u \ \omega_f = R1 \ (u \ \omega_f)$
REA_R1_wait_false_thm1
 $\vdash \forall u : \text{REA_PREDICATE} \bullet R1 \ u \ \omega_f = R1 \ (u \ \omega_f)$
REA_dash_okay_diverge_id_thm
 $\vdash \forall r : \text{REA_PREDICATE}$
 $\quad | \text{unrestTypedVar} \ (r, \text{dash okay}, \text{BOOL_VAL})$
 $\quad \bullet r \ \sigma_f = r$
REA_dash_okay_converge_id_thm
 $\vdash \forall r : \text{REA_PREDICATE}$
 $\quad | \text{unrestTypedVar} \ (r, \text{dash okay}, \text{BOOL_VAL})$
 $\quad \bullet r \ \sigma_t = r$
REA_okay_dash_diverge_idem_thm
 $\vdash \forall r : \text{REA_PREDICATE} \bullet (r \ \sigma_f) \ \sigma_f = r \ \sigma_f$
REA_okay_dash_converge_idem_thm
 $\vdash \forall r : \text{REA_PREDICATE} \bullet (r \ \sigma_t) \ \sigma_t = r \ \sigma_t$
REA_wait_false_idem_thm
 $\vdash \forall r : \text{REA_PREDICATE} \bullet (r \ \omega_f) \ \omega_f = r \ \omega_f$
REA_wait_true_idem_thm
 $\vdash \forall r : \text{REA_PREDICATE} \bullet (r \ \omega_t) \ \omega_t = r \ \omega_t$
R_healthy_thm
 $\vdash \forall r : \text{REA_PROCESS} \bullet r \in R_healthy$
REA_R_not_wait_thm
 $\vdash \forall r : \text{REA_PREDICATE}$
 $\quad | r.1 \in \text{WF_Skip}_{\text{REA}}$
 $\quad \bullet R \ r \ \omega_f = R1 \ (R2 \ r) \ \omega_f$
REA_R1_def_∈_R1_healthy_thm
 $\vdash R1$
 $\quad (=_{+R}$
 $\quad \quad (\text{ALPHABET_OWTR},$
 $\quad \quad \text{Rel} \ ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\quad \quad \text{Val } (\text{Bool } \text{true}))$
 $\quad =_{+R}$
 $\quad \quad (\text{ALPHABET_OWTR},$
 $\quad \quad \text{Rel} \ ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\quad \quad \text{Val } (\text{Bool } \text{true}))$
REA_PREDICATE_◁_R▷_R_thm
 $\vdash \forall u1, u2, b : \text{REA_PREDICATE}$
 $\quad | (u1, b, u2) \in \text{WF_Cond}_R$

$\bullet u1 \triangleleft_R b \triangleright_R u2 \in REA_PREDICATE$
REA_PREDICATE_thm2
 $\vdash \forall r : REA_PREDICATE \bullet r \in REL_PREDICATE$
REA_wait_∈_REA_PREDICATE_thm
 $\vdash \forall r : REA_PREDICATE \bullet wait \in r.1$
REA_dash_okay_∈_REA_PREDICATE_thm
 $\vdash \forall r : REA_PREDICATE \bullet dash\ okay \in r.1$
REA_¬_R_wait_false_com_thm
 $\vdash \forall r : REA_PREDICATE \bullet \neg_R r\ \omega_f = \neg_R (r\ \omega_f)$
REA_¬_R_wait_true_com_thm
 $\vdash \forall r : REA_PREDICATE \bullet \neg_R r\ \omega_t = \neg_R (r\ \omega_t)$
REA_¬_R_dash_okay_false_com_thm
 $\vdash \forall r : REA_PREDICATE \bullet \neg_R r\ \sigma_f = \neg_R (r\ \sigma_f)$
REA_¬_R_dash_okay_true_com_thm
 $\vdash \forall r : REA_PREDICATE \bullet \neg_R r\ \sigma_t = \neg_R (r\ \sigma_t)$

13 THE Z THEORY *utp-csp*

13.1 Parents

utp-rea *utp-des*

13.2 Global Variables

unrestALPHABET_CSP

$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$

unrestALPHABET_CSP'

$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{B}$

VAR_NAME $\mathbb{P} \text{ NAME}$

CSP1 $\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP1_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP2 $\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP2_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP_PROCESS $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

WF_CSP_PROCESS_PAIR

$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

WF_PREFIXING $\mathbb{P} (\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION})$

STOP $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

SKIP $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CHAOS $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

(- \boxtimes_{CSP} -) $(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
 $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
 $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

Φ $\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

(do_A -) $\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

(- \rightarrow_{CSP} -) $(\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION})$
 $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$
 $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

(do_C -) $\mathbb{P} \text{ NAME} \times \text{NAME} \times \text{EXPRESSION} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP3_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

CSP4_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

13.3 Fixity

fun 0 rightassoc

$$(do_A _) \quad (do_C _) \quad (- \boxtimes_{CSP} -) \quad (- \rightarrow_{CSP} -)$$

13.4 Axioms

unrestALPHABET_CSP

$$\begin{aligned} &\vdash unrestALPHABET_CSP \in REL_PREDICATE \rightarrow \mathbb{B} \\ &\quad \wedge (\forall r : REL_PREDICATE \\ &\quad \bullet unrestALPHABET_CSP \ r \\ &\quad \Leftrightarrow unrestOKAY \ r \\ &\quad \quad \wedge unrestWAIT \ r \\ &\quad \quad \wedge unrestTR \ r \\ &\quad \quad \wedge unrestREF \ r) \end{aligned}$$

unrestALPHABET_CSP'

$$\begin{aligned} &\vdash unrestALPHABET_CSP' \in REL_PREDICATE \rightarrow \mathbb{B} \\ &\quad \wedge (\forall r : REL_PREDICATE \\ &\quad \bullet unrestALPHABET_CSP' \ r \\ &\quad \Leftrightarrow unrestOKAY' \ r \\ &\quad \quad \wedge unrestWAIT' \ r \\ &\quad \quad \wedge unrestTR' \ r \\ &\quad \quad \wedge unrestREF' \ r) \end{aligned}$$

CSP1

$$\begin{aligned} &\vdash CSP1 \in REL_PREDICATE \rightarrow REL_PREDICATE \\ &\quad \wedge (\forall r : REL_PREDICATE \\ &\quad \bullet CSP1 \ r \\ &\quad = r \\ &\quad \quad \vee_R =_R \\ &\quad \quad \quad (ALPHABET_OWTR, \\ &\quad \quad \quad \quad okay, \\ &\quad \quad \quad \quad Val (Bool false)) \\ &\quad \quad \wedge_R =_{+R} \\ &\quad \quad \quad (ALPHABET_OWTR, \\ &\quad \quad \quad \quad Rel \\ &\quad \quad \quad \quad ((- \leq_R -), \\ &\quad \quad \quad \quad \quad Var \ tr, \\ &\quad \quad \quad \quad \quad Var (dash \ tr)), \\ &\quad \quad \quad \quad Val (Bool true))) \end{aligned}$$

CSP1_healthy $\vdash \text{CSP1_healthy} \in \mathbb{P} \text{REL_PREDICATE}$
 $\wedge \text{CSP1_healthy} = \{r : \text{REL_PREDICATE} \mid r = \text{CSP1 } r\}$
CSP2 $\vdash \text{CSP2} \in \text{REL_PREDICATE} \leftrightarrow \text{REL_PREDICATE}$
 $\wedge (\forall r : \text{REL_PREDICATE}$
 $\mid \{\text{dash okay}, \text{dash wait}, \text{dash tr}, \text{dash ref}\} \subseteq r.1$
 $\bullet \text{CSP2 } r = r ;_R J (\text{out_a } r.1))$
CSP2_healthy $\vdash \text{CSP2_healthy} \in \mathbb{P} \text{REL_PREDICATE}$
 $\wedge \text{CSP2_healthy}$
 $= \{r : \text{REL_PREDICATE}$
 $\mid \{\text{dash okay}, \text{dash wait}, \text{dash tr}, \text{dash ref}\}$
 $\subseteq r.1$
 $\wedge r = \text{CSP2 } r\}$
STOP $\vdash \text{STOP} \in \text{WF_Skip}_{\text{REA}} \rightarrow \text{CSP_PROCESS}$
 $\wedge (\forall a : \text{WF_Skip}_{\text{REA}}$
 $\bullet \text{STOP } a$
 $= R (\text{Assign}_R (a, \langle \text{wait} \rangle, \langle \text{Val } (\text{Bool true}) \rangle)))$
CSP_STOP_design_thm
 $\vdash \forall a : \text{ALPHABET_REA}$
 $\mid a \in \text{WF_Skip}_{\text{REA}}$
 $\bullet \text{STOP } a$
 $= R$
 $(\text{True}_R a$
 $\vdash_D =_R (a, \text{dash tr}, \text{Var tr})$
 $\wedge_R =_R$
 $(a, \text{dash wait}, \text{Val } (\text{Bool true})))$
SKIP $\vdash \text{SKIP} \in \text{WF_Skip}_{\text{REA}} \rightarrow \text{CSP_PROCESS}$
 $\wedge (\forall a : \text{WF_Skip}_{\text{REA}}$
 $\bullet \text{SKIP } a = R (\exists_R (\{\text{ref}\}, \Pi_{\text{REA}} a)))$
CSP_SKIP_design_thm
 $\vdash \forall a : \text{ALPHABET_REA}$
 $\mid a \in \text{WF_Skip}_{\text{REA}}$
 $\bullet \text{SKIP } a$
 $= R$
 $(\text{True}_R a$
 $\vdash_D =_R (a, \text{dash tr}, \text{Var tr})$
 $\wedge_R =_R$
 $(a, \text{dash wait}, \text{Val } (\text{Bool false})))$
CHAOS $\vdash \text{CHAOS} \in \text{WF_Skip}_{\text{REA}} \rightarrow \text{CSP_PROCESS}$

$$\begin{array}{l}
\wedge (\forall a : WF_Skip_{REA} \bullet CHAOS\ a = R\ (True_R\ a)) \\
\mathbf{CSP_CHAOS_design_thm} \\
\vdash \forall a : ALPHABET_REA \\
\quad | a \in WF_Skip_{REA} \\
\quad \bullet CHAOS\ a = R\ (False_R\ a \vdash_D\ True_R\ a) \\
- \ \boxtimes_{CSP} \ - \quad \vdash (- \ \boxtimes_{CSP} \ -) \in WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS \\
\quad \wedge (\forall pp : WF_CSP_PROCESS_PAIR \\
\quad \quad \bullet pp.1 \ \boxtimes_{CSP} \ pp.2 \\
\quad \quad = CSP2 \\
\quad \quad ((pp.1 \wedge_R pp.2) \triangleleft_R \\
\quad \quad \quad STOP\ pp.1.1 \triangleright_R \\
\quad \quad \quad pp.1 \vee_R pp.2)) \\
\mathbf{CSP_}\boxtimes_{CSP}\mathbf{_design_thm_2} \\
\vdash \forall p1, p2 : CSP_PROCESS \\
\quad | (p1, p2) \in WF_CSP_PROCESS_PAIR \\
\quad \bullet p1 \ \boxtimes_{CSP} \ p2 \\
\quad = R \\
\quad ((\neg_R ((p1\ \sigma_f)\ \omega_f) \wedge_R \neg_R ((p2\ \sigma_f)\ \omega_f)) \\
\quad \vdash_D (((p1\ \sigma_t)\ \omega_f) \wedge_R (p2\ \sigma_t)\ \omega_f) \triangleleft_R \\
\quad \quad =_{+R} (p1.1, Var\ (dash\ tr), Var\ tr) \\
\quad \quad \wedge_R =_R \\
\quad \quad (p1.1, \\
\quad \quad \quad dash\ wait, \\
\quad \quad \quad Val\ (Bool\ true)) \triangleright_R \\
\quad \quad ((p1\ \sigma_t)\ \omega_f) \vee_R (p2\ \sigma_t)\ \omega_f) \\
\Phi \quad \vdash \Phi \in CSP_PROCESS \rightarrow CSP_PROCESS \\
\quad \wedge (\forall p : CSP_PROCESS \\
\quad \quad \bullet \Phi\ p \\
\quad \quad = R \\
\quad \quad (p \\
\quad \quad \quad \wedge_R (=_{\mathbf{R}} \\
\quad \quad \quad \quad (ALPHABET_OWTR, \\
\quad \quad \quad \quad \quad dash\ tr, \\
\quad \quad \quad \quad \quad Var\ tr) \\
\quad \quad \quad \wedge_R =_R \\
\quad \quad \quad \quad (ALPHABET_OWTR, \\
\quad \quad \quad \quad \quad dash\ wait, \\
\quad \quad \quad \quad \quad Val\ (Bool\ true))))
\end{array}$$

$$\begin{aligned}
& \vee_R =_{+R} \\
& \quad (ALPHABET_OWTR, \\
& \quad \quad Rel \\
& \quad \quad ((- <_R -), \\
& \quad \quad \quad Var \ tr, \\
& \quad \quad \quad Var \ (dash \ tr)), \\
& \quad \quad Val \ (Bool \ true))) \\
do_A \ - \quad & \vdash (do_A \ -) \in WF_PREFIXING \rightarrow CSP_PROCESS \\
& \quad \wedge (\forall \ a_n_e : WF_PREFIXING \\
& \quad \bullet \ do_A \ a_n_e \\
& \quad \quad = \Phi \\
& \quad \quad ((=_R \ (a_n_e.1, \ dash \ tr, \ Var \ tr) \\
& \quad \quad \quad \wedge_R =_{+R} \\
& \quad \quad \quad (a_n_e.1, \\
& \quad \quad \quad \quad Rel \\
& \quad \quad \quad ((- \not\leq_R -), \\
& \quad \quad \quad \quad Fun_2 \\
& \quad \quad \quad \quad (MkPair, \\
& \quad \quad \quad \quad \quad Val \\
& \quad \quad \quad \quad \quad (Channel \\
& \quad \quad \quad \quad \quad \quad a_n_e.2), \\
& \quad \quad \quad \quad \quad \quad a_n_e.3), \\
& \quad \quad \quad \quad \quad \quad Var \ (dash \ ref)), \\
& \quad \quad \quad \quad \quad \quad Val \ (Bool \ true))) \triangleleft_R \\
& \quad \quad =_R \\
& \quad \quad (a_n_e.1, \\
& \quad \quad \quad dash \ wait, \\
& \quad \quad \quad Val \ (Bool \ true)) \triangleright_R \\
& \quad \quad =_R \\
& \quad \quad (a_n_e.1, \\
& \quad \quad \quad dash \ tr, \\
& \quad \quad \quad Fun_2 \\
& \quad \quad \quad ((- \frown_R -), \\
& \quad \quad \quad \quad Var \ tr, \\
& \quad \quad \quad \quad Fun_1 \\
& \quad \quad \quad \quad (MkSingleton, \\
& \quad \quad \quad \quad \quad Fun_2
\end{aligned}$$

$$\begin{aligned}
& \quad \quad \quad (MkPair, \\
& \quad \quad \quad Val \\
& \quad \quad \quad (Channel \\
& \quad \quad \quad a_n_e.2), \\
& \quad \quad \quad a_n_e.3)))))) \\
- \rightarrow_{CSP} - & \quad \vdash (- \rightarrow_{CSP} -) \\
& \quad \in WF_PREFIXING \times CSP_PROCESS \rightarrow CSP_PROCESS \\
& \quad \wedge (\forall a_n_e : WF_PREFIXING \\
& \quad \bullet a_n_e \rightarrow_{CSP} SKIP\ a_n_e.1 \\
& \quad \quad =_{CSP2} \\
& \quad \quad (=_R \\
& \quad \quad \quad (a_n_e.1, \\
& \quad \quad \quad \quad dash\ okay, \\
& \quad \quad \quad \quad Val\ (Bool\ true)) \\
& \quad \quad \quad \wedge_R\ do_A\ a_n_e) \\
& \quad \wedge (\forall p : CSP_PROCESS \\
& \quad \bullet a_n_e \rightarrow_{CSP} p \\
& \quad \quad = (a_n_e \rightarrow_{CSP} SKIP\ a_n_e.1) ;_R p)) \\
do_C - & \quad \vdash (do_C -) \in WF_PREFIXING \rightarrow CSP_PROCESS \\
& \quad \wedge (\forall a_n_e : WF_PREFIXING \\
& \quad \bullet do_C\ a_n_e \\
& \quad \quad = (=_R (a_n_e.1, dash\ tr, Var\ tr) \\
& \quad \quad \quad \wedge_R\ =_{+R} \\
& \quad \quad \quad \quad (a_n_e.1, \\
& \quad \quad \quad \quad \quad Rel \\
& \quad \quad \quad \quad \quad ((- \not\leq_R -), \\
& \quad \quad \quad \quad \quad \quad Fun_2 \\
& \quad \quad \quad \quad \quad \quad (MkPair, \\
& \quad \quad \quad \quad \quad \quad \quad Val\ (Channel\ a_n_e.2), \\
& \quad \quad \quad \quad \quad \quad \quad \quad a_n_e.3), \\
& \quad \quad \quad \quad \quad \quad \quad \quad Var\ (dash\ ref)), \\
& \quad \quad \quad \quad \quad \quad \quad \quad Val\ (Bool\ true))) \triangleleft_R \\
& \quad \quad \quad =_R \\
& \quad \quad \quad \quad (a_n_e.1, \\
& \quad \quad \quad \quad \quad dash\ wait, \\
& \quad \quad \quad \quad \quad \quad Val\ (Bool\ true)) \triangleright_R \\
& \quad \quad \quad =_R \\
& \quad \quad \quad \quad (a_n_e.1,
\end{aligned}$$

$$\begin{aligned}
& \text{dash } tr, \\
& \text{Fun}_2 \\
& ((- \frown_R -), \\
& \text{Var } tr, \\
& \text{Fun}_1 \\
& (\text{MkSingleton}, \\
& \text{Fun}_2 \\
& (\text{MkPair}, \\
& \text{Val} \\
& (\text{Channel} \\
& \quad a_n_e.2), \\
& \quad a_n_e.3)))))) \\
\mathbf{CSP_} \rightarrow \mathbf{CSP_design_thm} & \\
& \vdash \forall a_n_e : \mathbf{WF_PREFIXING} \\
& \bullet a_n_e \rightarrow_{\mathbf{CSP}} \mathbf{SKIP } a_n_e.1 \\
& \quad = R (\text{True}_R a_n_e.1 \vdash_D \text{do_C } a_n_e) \\
\mathbf{CSP3_healthy} & \vdash \mathbf{CSP3_healthy} \in \mathbb{P} \mathbf{CSP_PROCESS} \\
& \wedge \mathbf{CSP3_healthy} \\
& = \{r : \mathbf{CSP_PROCESS} \\
& \quad | =_R (r.1, \text{wait}, \text{Val } (\text{Bool } \text{false})) \\
& \quad \Rightarrow_R r \equiv_R \exists_R (\{\text{ref}\}, r) \\
& \quad = \text{True}_R r.1\} \\
\mathbf{CSP4_healthy} & \vdash \mathbf{CSP4_healthy} \in \mathbb{P} \mathbf{CSP_PROCESS} \\
& \wedge \mathbf{CSP4_healthy} \\
& = \{r : \mathbf{CSP_PROCESS} \\
& \quad | (r, \mathbf{SKIP } r.1) \in \mathbf{WF_Semi}_R \\
& \quad \wedge r ;_R \mathbf{SKIP } r.1 = r\} \\
\mathbf{CSP_CSP2_healthy_Design_thm} & \\
& \vdash \forall P, Q : \mathbf{REA_PREDICATE} \\
& \quad | (P \vdash_D Q).1 \in \text{homogeneous} \\
& \quad \wedge (P, Q) \in \mathbf{WF_DES_PREDICATE_PAIR} \\
& \bullet R (P \vdash_D Q) \in \mathbf{CSP2_healthy} \\
\mathbf{CSP_rea_des_CSP1_thm} & \\
& \vdash \forall d : \mathbf{WF_DES_PREDICATE_PAIR} \\
& \bullet R (d.1 \vdash_D d.2) = \mathbf{CSP1} (R (d.1 \vdash_D d.2)) \\
\mathbf{CSP_closure_}\wedge_R \mathbf{CSP1_thm} & \\
& \vdash \forall r1, r2 : \mathbf{REL_PREDICATE}
\end{aligned}$$

$$\begin{aligned}
& | \{r1, r2\} \subseteq CSP1_healthy \\
& \bullet CSP1 (r1 \wedge_R r2) = r1 \wedge_R r2 \\
CSP_closure_V_R_CSP1_thm \\
& \vdash \forall r1, r2 : REL_PREDICATE \\
& | \{r1, r2\} \subseteq CSP1_healthy \\
& \bullet CSP1 (r1 \vee_R r2) = r1 \vee_R r2 \\
CSP_closure_<_R_>_R_CSP1_thm \\
& \vdash \forall r1, r2, b : REL_PREDICATE \\
& | (r1, b, r2) \in WF_Cond_R \wedge \{r1, r2\} \subseteq CSP1_healthy \\
& \bullet CSP1 (r1 <_R b >_R r2) = r1 <_R b >_R r2 \\
CSP_closure_Semi_R_CSP1_thm \\
& \vdash \forall r1, r2 : REL_PREDICATE \\
& | (r1, r2) \in WF_Semi_R \wedge \{r1, r2\} \subseteq CSP1_healthy \\
& \bullet CSP1 (r1 ;_R r2) = r1 ;_R r2 \\
CSP_CSP1_wait_dash_okay_thm \\
& \vdash \forall r : REL_PREDICATE; b1, b2 : BOOL_VAL \\
& | \{wai, dash\ okay\} \subseteq r.1 \\
& \bullet /_R \\
& \quad (/_R (CSP1\ r, Val\ b1, wait), \\
& \quad \quad Val\ b2, \\
& \quad \quad dash\ okay) \\
& = CSP1 \\
& \quad (/_R \\
& \quad \quad (/_R (r, Val\ b1, wait), \\
& \quad \quad \quad Val\ b2, \\
& \quad \quad \quad dash\ okay)) \\
CSP_des_post_&_R_CSP1_thm \\
& \vdash \forall d, r1, r2 : REL_PREDICATE \\
& | (d, r1 \wedge_R r2) \in WF_DES_PREDICATE_PAIR \\
& \quad \wedge \{r1, r2\} \subseteq R1_healthy \\
& \bullet d \vdash_D r1 \wedge_R CSP1\ r2 = d \vdash_D r1 \wedge_R r2 \\
CSP_des_post_&_R_neg_R_CSP1_thm \\
& \vdash \forall d, r1, r2 : REL_PREDICATE \\
& | (d, r1 \wedge_R \neg_R r2) \in WF_DES_PREDICATE_PAIR \\
& \quad \wedge \{r1, r2\} \subseteq R1_healthy_def \\
& \bullet d \vdash_D r1 \wedge_R \neg_R (CSP1\ r2) = d \vdash_D r1 \wedge_R \neg_R r2 \\
CSP_des_post_<_R_>_R_CSP1_thm \\
& \vdash \forall d, r1, r2, rb : REL_PREDICATE
\end{aligned}$$

$$\begin{aligned}
& | (r1, rb, r2) \in WF_Cond_R \\
& \wedge (d, r1 \triangleleft_R CSP1 \ rb \triangleright_R r2) \\
& \in WF_DES_PREDICATE_PAIR \\
& \wedge \{r1, rb, r2\} \subseteq R1_healthy \\
& \bullet d \vdash_D r1 \triangleleft_R CSP1 \ rb \triangleright_R r2 \\
& = d \vdash_D r1 \triangleleft_R rb \triangleright_R r2
\end{aligned}$$

CSP_CSP2_R1_com_thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1 \\
& \bullet CSP2 \ (R1 \ r) = R1 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_R2_com_thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1 \\
& \wedge ALPHABET_OWTR \subseteq r.1 \\
& \bullet CSP2 \ (R2 \ r) = R2 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_R3_com_thm

$$\begin{aligned}
& \vdash \forall r : REA_PREDICATE \\
& | r.1 \in WF_Skip_{REA} \\
& \bullet CSP2 \ (R3 \ r) = R3 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_H1_com_thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1 \\
& \bullet CSP2 \ (H1 \ r) = H1 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_H2_com_thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1 \\
& \bullet CSP2 \ (H2 \ r) = H2 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_CSP1_com_thm

$$\begin{aligned}
& \vdash \forall r : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1 \\
& \bullet CSP2 \ (CSP1 \ r) = CSP1 \ (CSP2 \ r)
\end{aligned}$$

CSP_CSP2_\wedge_closure_thm

$$\begin{aligned}
& \vdash \forall r1, r2 : REL_PREDICATE \\
& | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \\
& \quad \subseteq r1.1 \cup r2.1 \\
& \wedge \{r1, r2\} \subseteq CSP2_healthy \\
& \bullet r1 \wedge_R r2 \sqsubseteq_R CSP2 \ (r1 \wedge_R r2) = True_R \ \emptyset
\end{aligned}$$

CSP_CSP2_\vee_closure_thm

$\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\}$
 $\quad \quad \subseteq r1.1 \cup r2.1$
 $\quad \quad \wedge \{r1, r2\} \subseteq CSP2_healthy$
 $\quad \bullet CSP2\ (r1 \vee_R r2) = r1 \vee_R r2$
CSP_closure- $\triangleleft_R \triangleright_R$ -CSP2_thm
 $\vdash \forall r1, r2, b : REL_PREDICATE$
 $\quad | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r1.1$
 $\quad \quad \wedge (r1, b, r2) \in WF_Cond_R$
 $\quad \quad \wedge \{r1, r2\} \subseteq CSP2_healthy$
 $\quad \bullet CSP2\ (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2$
CSP- Π_{REA} -Semi $_R$ -left-unit_thm
 $\vdash \forall a : ALPHABET; r : REL_PREDICATE$
 $\quad | a \in WF_Skip_{REA_def} \wedge (\Pi_{REA}\ a, r) \in WF_Semi_R$
 $\quad \bullet \Pi_{REA}\ a ;_R r = r$
CSP-CSP2_dash_okay_thm
 $\vdash \forall r : REL_PREDICATE$
 $\quad | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1$
 $\quad \bullet /_R\ (CSP2\ r, Val\ (Bool\ true), dash\ okay)$
 $\quad \quad = /_R\ (r, Val\ (Bool\ true), dash\ okay)$
 $\quad \quad \vee_R /_R\ (r, Val\ (Bool\ false), dash\ okay)$
CSP-CSP2_not_dash_okay_thm
 $\vdash \forall r : REL_PREDICATE$
 $\quad | \{dash\ okay, dash\ tr, dash\ wait, dash\ ref\} \subseteq r.1$
 $\quad \bullet /_R\ (CSP2\ r, Val\ (Bool\ false), dash\ okay)$
 $\quad \quad = /_R\ (r, Val\ (Bool\ false), dash\ okay)$
CSP-CSP3_healthy_thm
 $\vdash \forall r : CSP_PROCESS$
 $\quad \bullet r \in CSP3_healthy \Leftrightarrow \Pi_{REA}\ r.1 ;_R r = r$
CSP_SKIP-CSP3_thm
 $\vdash \forall a : WF_Skip_{REA} \bullet CSP3\ (SKIP\ a) = SKIP\ a$
CSP-CSP3_idem_thm
 $\vdash \forall r : CSP_PROCESS \bullet CSP3\ (CSP3\ r) = CSP3\ r$
CSP_closure- \vee_R -CSP3_thm
 $\vdash \forall r1, r2 : REL_PREDICATE$
 $\quad | \{r1, r2\} \subseteq CSP3_healthy$
 $\quad \bullet CSP3\ (r1 \vee_R r2) = r1 \vee_R r2$
CSP_closure- $\triangleleft_R \triangleright_R$ -CSP3_thm

$$\begin{aligned}
& \vdash \forall r1, r2, b : REL_PREDICATE \\
& \quad | (r1, b, r2) \in WF_Cond_R \\
& \quad \wedge \{r1, r2\} \subseteq CSP3_healthy \\
& \quad \wedge ref \notin b.1 \\
& \quad \bullet CSP3 (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2 \\
CSP_closure_Semi_R_CSP3_thm \\
& \vdash \forall r1, r2 : REL_PREDICATE \\
& \quad | (r1, r2) \in WF_Semi_R \wedge \{r1, r2\} \subseteq CSP3_healthy \\
& \quad \bullet CSP3 (r1 ;_R r2) = r1 ;_R r2 \\
CSP_SKIP_CSP4_thm \\
& \vdash \forall a : WF_Skip_{REA} \bullet CSP4 (SKIP a) = SKIP a \\
CSP_STOP_CSP4_thm \\
& \vdash \forall a : WF_Skip_{REA} \bullet CSP4 (STOP a) = STOP a \\
CSP_CHAOS_CSP4_thm \\
& \vdash \forall a : WF_Skip_{REA} \bullet CSP4 (CHAOS a) = CHAOS a \\
CSP_closure_V_R_CSP4_thm \\
& \vdash \forall r1, r2 : REL_PREDICATE \\
& \quad | \{r1, r2\} \subseteq CSP4_healthy \\
& \quad \bullet CSP4 (r1 \vee_R r2) = r1 \vee_R r2 \\
CSP_closure_triangleleft_R_triangletriangleright_R_CSP4_thm \\
& \vdash \forall r1, r2, b : REL_PREDICATE \\
& \quad | (r1, b, r2) \in WF_Cond_R \wedge \{r1, r2\} \subseteq CSP4_healthy \\
& \quad \bullet CSP4 (r1 \triangleleft_R b \triangleright_R r2) = r1 \triangleleft_R b \triangleright_R r2 \\
CSP_closure_Semi_R_CSP4_thm \\
& \vdash \forall r1, r2 : REL_PREDICATE \\
& \quad | (r1, r2) \in WF_Semi_R \wedge \{r1, r2\} \subseteq CSP4_healthy \\
& \quad \bullet CSP4 (r1 ;_R r2) = r1 ;_R r2 \\
CSP_designs_thm \\
& \vdash \forall p : CSP_PROCESS \\
& \quad \bullet p \\
& \quad = R \\
& \quad (/_R \\
& \quad \quad (/_R (p, Val (Bool false), dash okay), \\
& \quad \quad \quad Val (Bool false), \\
& \quad \quad \quad wait) \\
& \quad \vdash_D /_R \\
& \quad \quad (/_R (p, Val (Bool true), dash okay), \\
& \quad \quad \quad Val (Bool false),
\end{aligned}$$

$$\begin{aligned}
& \text{wait})) \\
\mathbf{CSP_STOP_dash_okay_true_thm} \\
& \vdash \forall a : WF_Skip_{REA} \\
& \bullet /_R \\
& \quad (/_R (STOP\ a, Val\ (Bool\ true), dash\ okay), \\
& \quad \quad Val\ (Bool\ false), \\
& \quad \quad wait)) \\
& = CSP1 \\
& \quad (=_R\ (a, dash\ tr, Var\ tr) \\
& \quad \quad \wedge_R =_R\ (a, dash\ wait, Val\ (Bool\ true))) \\
\mathbf{CSP_STOP_not_dash_okay_true_thm} \\
& \vdash /_R \\
& \quad (/_R (STOP\ a, Val\ (Bool\ false), dash\ okay), \\
& \quad \quad Val\ (Bool\ false), \\
& \quad \quad wait)) \\
& = CSP1\ (\neg_R\ (= _R\ (a, okay, Val\ (Bool\ false)))) \\
\mathbf{CSP_STOP_Semi_R_left_zero_thm} \\
& \vdash \forall p : CSP_PROCESS \\
& \quad | p.1 = ALPHABET_OWTR \\
& \quad \bullet STOP\ p.1\ ;_R\ p = STOP\ p.1 \\
\mathbf{CSP_CHAOS_Semi_R_left_zero_thm} \\
& \vdash \forall p : CSP_PROCESS \\
& \quad | p.1 = ALPHABET_OWTR \\
& \quad \bullet CHAOS\ p.1\ ;_R\ p = CHAOS\ p.1 \\
\mathbf{CSP_}\boxtimes_{CSP}\mathbf{-diverge_thm} \\
& \vdash \forall p1, p2 : CSP_PROCESS \\
& \quad | p1.1 = p2.1 \wedge p2.1 = ALPHABET_OWTR \\
& \quad \bullet /_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1\ \boxtimes_{CSP}\ p2, \\
& \quad \quad \quad \quad Val\ (Bool\ false), \\
& \quad \quad \quad \quad dash\ okay), \\
& \quad \quad \quad Val\ (Bool\ false), \\
& \quad \quad \quad wait)) \\
& = (/_R \\
& \quad (/_R\ (p1, Val\ (Bool\ false), dash\ okay), \\
& \quad \quad Val\ (Bool\ false), \\
& \quad \quad wait))
\end{aligned}$$

$$\begin{aligned}
& \forall_R \ /_R \\
& \quad (/_R (p2, \text{Val} (\text{Bool false}), \text{dash okay}), \\
& \quad \text{Val} (\text{Bool false}), \\
& \quad \text{wait})) \triangleleft_R \\
& =_R (p1.1, \text{okay}, \text{Val} (\text{Bool true})) \triangleright_R \\
& \ /_R \\
& \quad (/_R (p1, \text{Val} (\text{Bool false}), \text{dash okay}), \\
& \quad \text{Val} (\text{Bool false}), \\
& \quad \text{wait}) \\
& \wedge_R \ /_R \\
& \quad (/_R \\
& \quad \quad (p2, \\
& \quad \quad \text{Val} (\text{Bool false}), \\
& \quad \quad \text{dash okay}), \\
& \quad \text{Val} (\text{Bool false}), \\
& \quad \text{wait})
\end{aligned}$$

CSP_ \boxtimes_{CSP} ***precondition_thm***

$$\begin{aligned}
& \vdash \forall p1, p2, p3 : \text{CSP_PROCESS} \\
& \quad | p1.1 = p2.1 \wedge p2.1 = p3.1 \wedge p3.1 = \text{ALPHABET_OWTR}
\end{aligned}$$

$$\begin{aligned}
& \bullet \neg_R \\
& \quad (/_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1 \boxtimes_{CSP} p2, \\
& \quad \quad \quad \text{Val} (\text{Bool false}), \\
& \quad \quad \quad \text{dash okay}), \\
& \quad \quad \text{Val} (\text{Bool false}), \\
& \quad \quad \text{wait})) \\
& \vdash_D p3 \\
& = \neg_R \\
& \quad (/_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1, \\
& \quad \quad \quad \text{Val} (\text{Bool false}), \\
& \quad \quad \quad \text{dash okay}), \\
& \quad \quad \text{Val} (\text{Bool false}), \\
& \quad \quad \text{wait}) \\
& \quad \forall_R \ /_R \\
& \quad \quad (/_R
\end{aligned}$$

$$\begin{aligned}
& (p2, \\
& \quad Val (Bool false), \\
& \quad \quad dash okay), \\
& Val (Bool false), \\
& wait)) \\
& \vdash_D p3 \\
\mathbf{CSP_}\boxtimes\mathbf{CSP_converge_thm} \\
& \vdash \forall p1, p2 : CSP_PROCESS \\
& \quad | p1.1 = p2.1 \wedge p2.1 = ALPHABET_OWTR \\
& \quad \bullet \ /_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1 \boxtimes_{CSP} p2, \\
& \quad \quad \quad \quad Val (Bool false), \\
& \quad \quad \quad \quad \quad dash okay), \\
& \quad \quad \quad \quad Val (Bool true), \\
& \quad \quad \quad \quad \quad wait)) \\
& = ((p1 \wedge_R p2) \triangleleft_R \\
& \quad STOP\ p1.1 \triangleright_R \\
& \quad \ /_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1 \vee_R p2, \\
& \quad \quad \quad \quad Val (Bool true), \\
& \quad \quad \quad \quad \quad dash okay), \\
& \quad \quad \quad \quad Val (Bool false), \\
& \quad \quad \quad \quad \quad wait)) \\
& \vee_R (p1 \wedge_R p2) \triangleleft_R \\
& \quad STOP\ p1.1 \triangleright_R \\
& \quad \ /_R \\
& \quad \quad (/_R \\
& \quad \quad \quad (p1 \vee_R p2, \\
& \quad \quad \quad \quad Val (Bool false), \\
& \quad \quad \quad \quad \quad dash okay), \\
& \quad \quad \quad \quad Val (Bool false), \\
& \quad \quad \quad \quad \quad wait)) \\
\mathbf{CSP_}\boxtimes\mathbf{CSP_design_lemma_thm} \\
& \vdash \forall p1, p2 : CSP_PROCESS \\
& \quad | p1.1 = p2.1 \wedge p2.1 = ALPHABET_OWTR \\
& \quad \bullet \neg_R
\end{aligned}$$

$$\begin{aligned}
& (/_R (/_R (p1 \boxtimes_{CSP} p2, \\
& \quad Val (Bool false), \\
& \quad \quad dash okay), \\
& \quad Val (Bool false), \\
& \quad \quad wait)) \\
\vdash_D & /_R (/_R (p1 \boxtimes_{CSP} p2, \\
& \quad Val (Bool true), \\
& \quad \quad dash okay), \\
& \quad Val (Bool false), \\
& \quad \quad wait) \\
= & (\neg_R (/_R (/_R (p1, \\
& \quad \quad Val (Bool false), \\
& \quad \quad \quad dash okay), \\
& \quad \quad Val (Bool false), \\
& \quad \quad \quad wait)) \\
& \wedge_R \neg_R (/_R (/_R (p2, \\
& \quad \quad Val (Bool false), \\
& \quad \quad \quad dash okay), \\
& \quad \quad Val (Bool false), \\
& \quad \quad \quad wait))) \\
\vdash_D & /_R (/_R (p1, Val (Bool true), dash okay), \\
& \quad Val (Bool false), \\
& \quad \quad wait) \\
& \wedge_R /_R (/_R (p2, \\
& \quad \quad Val (Bool true),
\end{aligned}$$

$$\begin{array}{c}
dash\ okay), \\
Val\ (Bool\ false), \\
wait) \triangleleft_R \\
=_{+R} (p1.1, Var\ (dash\ tr), Var\ tr) \\
\wedge_R =_R \\
(p1.1, \\
dash\ wait, \\
Val\ (Bool\ true)) \triangleright_R \\
/_R \\
(/_R \\
(p1, \\
Val\ (Bool\ true), \\
dash\ okay), \\
Val\ (Bool\ false), \\
wait) \\
\vee_R \ /_R \\
(/_R \\
(p2, \\
Val\ (Bool\ true), \\
dash\ okay), \\
Val\ (Bool\ false), \\
wait)
\end{array}$$

13.5 Definitions

$$\begin{array}{l}
VAR_NAME \vdash VAR_NAME \\
= \{n : NAME \\
\mid n \notin ALPHABET_OWTR \wedge n \in undashed\} \\
CSP_PROCESS \vdash CSP_PROCESS \\
= \{p : REA_PROCESS \\
\mid ALPHABET_OWTR \subseteq p.1 \\
\wedge p \in CSP1_healthy \\
\wedge p \in CSP2_healthy\} \\
WF_CSP_PROCESS_PAIR \\
\vdash WF_CSP_PROCESS_PAIR \\
= \{p1, p2 : CSP_PROCESS \\
\mid p1.1 = p2.1\} \\
WF_PREFIXING \vdash WF_PREFIXING
\end{array}$$

$$= \{a : \text{ALPHABET_REA}; n : \text{VAR_NAME}; e : \text{EXPRESSION} \\ | a \in \text{WF_Skip}_{\text{REA}} \wedge \text{FV } e \subseteq a\}$$

13.6 Theorems

CSP_CSP1.thm $\vdash \forall r : \text{REL_PREDICATE} \bullet \text{CSP1 } r \in \text{REL_PREDICATE}$

CSP_CSP1.alphabet.thm

$\vdash \forall r : \text{REL_PREDICATE}$

$\bullet (\text{CSP1 } r).1 = r.1 \cup \text{ALPHABET_OWTR}$

CSP_CSP1.idem.thm

$\vdash \forall r : \text{REL_PREDICATE} \bullet \text{CSP1 } (\text{CSP1 } r) = \text{CSP1 } r$

CSP_REA_PREDICATE_∈_DES_PREDICATE.thm

$\vdash \forall r : \text{REA_PREDICATE} \bullet r \in \text{DES_PREDICATE}$

CSP_CSP2.H2.thm

$\vdash \forall r : \text{REL_PREDICATE}$

$| \{dash\ okay, dash\ wait, dash\ tr, dash\ ref\} \subseteq r.1$

$\bullet \text{CSP2 } r = H2\ r$

CSP_CSP2.idem.thm

$\vdash \forall r : \text{REL_PREDICATE}$

$| \{dash\ okay, dash\ wait, dash\ tr, dash\ ref\} \subseteq r.1$

$\bullet \text{CSP2 } (\text{CSP2 } r) = \text{CSP2 } r$

CSP_R_design_not_wait.thm

$\vdash \forall r : \text{REA_PREDICATE}$

$| r.1 \in \text{WF_Skip}_{\text{REA}}$

$\bullet R\ r\ \omega_f = R1\ (R2\ (r\ \omega_f))$

CSP_R_design_∈_REA_PREDICATE.thm

$\vdash \forall P, Q : \text{REA_PREDICATE}$

$| (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$

$\bullet P \vdash_D Q \in \text{REA_PREDICATE}$

CSP_design_wait_false_dist.thm

$\vdash \forall P, Q : \text{REA_PREDICATE}$

$| (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$

$\wedge P.1 \in \text{WF_Skip}_{\text{REA}}$

$\bullet (P \vdash_D Q)\ \omega_f = (P\ \omega_f) \vdash_D Q\ \omega_f$

CSP_design_wait_true_dist.thm

$\vdash \forall P, Q : \text{REA_PREDICATE}$

$| (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$

$\wedge P.1 \in \text{WF_Skip}_{\text{REA}}$

$\bullet (P \vdash_D Q) \omega_t = (P \omega_t) \vdash_D Q \omega_t$
CSP_design_wait_false_id_thm
 $\vdash \forall P, Q : \text{REA_PREDICATE}$
 $\mid (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$
 $\wedge P.1 \in \text{WF_Skip}_{\text{REA}}$
 $\wedge \text{unrestTypedVar } (P, \text{wait}, \text{BOOL_VAL})$
 $\wedge \text{unrestTypedVar } (Q, \text{wait}, \text{BOOL_VAL})$
 $\bullet (P \vdash_D Q) \omega_f = P \vdash_D Q$
CSP_design_wait_true_id_thm
 $\vdash \forall P, Q : \text{REA_PREDICATE}$
 $\mid (P, Q) \in \text{WF_DES_PREDICATE_PAIR}$
 $\wedge P.1 \in \text{WF_Skip}_{\text{REA}}$
 $\wedge \text{unrestTypedVar } (P, \text{wait}, \text{BOOL_VAL})$
 $\wedge \text{unrestTypedVar } (Q, \text{wait}, \text{BOOL_VAL})$
 $\bullet (P \vdash_D Q) \omega_t = P \vdash_D Q$
CSP_CSP1_using_OKAY_thm
 $\vdash \forall r : \text{REL_PREDICATE}$
 $\bullet \text{CSP1 } r$
 $= r$
 $\vee_R \neg_R \text{OKAY}$
 $\wedge_R =_{+R}$
 $(\text{ALPHABET_OWTR},$
 Rel
 $((- \leq_R -),$
 $\text{Var } tr,$
 $\text{Var } (\text{dash } tr)),$
 $\text{Val } (\text{Bool } \text{true}))$
CSP_CSP1_using_OKAY_∈_REA_PREDICATE_thm
 $\vdash \neg_R \text{OKAY}$
 $\wedge_R =_{+R}$
 $(\text{ALPHABET_OWTR},$
 $\text{Rel } ((- \leq_R -), \text{Var } tr, \text{Var } (\text{dash } tr)),$
 $\text{Val } (\text{Bool } \text{true}))$
 $\in \text{REA_PREDICATE}$
CSP_R2_OKAY_∧_R_r_dist_thm
 $\vdash \forall r : \text{REA_PREDICATE} \bullet R2 (\text{OKAY} \wedge_R r) = \text{OKAY} \wedge_R R2 r$
CSP_R_design_not_wait_converge_thm
 $\vdash \forall P, Q : \text{REA_PREDICATE}$

$$\begin{aligned}
& | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \wedge P.1 \in WF_Skip_{REA} \\
& \wedge unrestTypedVar (P, dash\ okay, BOOL_VAL) \\
& \wedge unrestTypedVar (Q, dash\ okay, BOOL_VAL) \\
& \wedge unrestTypedVar (P, wait, BOOL_VAL) \\
& \wedge unrestTypedVar (Q, wait, BOOL_VAL) \\
& \bullet (R (P \vdash_D Q) \omega_f) \sigma_t \\
& = CSP1 (R1 (R2 (P \Rightarrow_R Q)))
\end{aligned}$$

CSP_R_design_not_wait_diverge_thm

$$\begin{aligned}
& \vdash \forall P, Q : REA_PREDICATE \\
& | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \wedge P.1 \in WF_Skip_{REA} \\
& \wedge unrestTypedVar (P, dash\ okay, BOOL_VAL) \\
& \wedge unrestTypedVar (Q, dash\ okay, BOOL_VAL) \\
& \wedge unrestTypedVar (P, wait, BOOL_VAL) \\
& \wedge unrestTypedVar (Q, wait, BOOL_VAL) \\
& \bullet (R (P \vdash_D Q) \omega_f) \sigma_f \\
& = R1 (\neg_R (OKAY \wedge_R R2 P))
\end{aligned}$$

CSP_CSP1_idem_thm1

$$\vdash \forall r : REL_PREDICATE \bullet CSP1 (CSP1 r) = CSP1 r$$

CSP_CSP1_R1_com_thm

$$\vdash \forall r : REL_PREDICATE \bullet CSP1 (R1 r) = R1 (CSP1 r)$$

CSP_CSP1_REA_PREDICATE_thm

$$\vdash \forall r : REA_PREDICATE \bullet CSP1 r \in REA_PREDICATE$$

CSP_CSP1_R3_com_thm

$$\begin{aligned}
& \vdash \forall r : REA_PREDICATE \\
& | r.1 \in WF_Skip_{REA} \\
& \bullet CSP1 (R3 r) = R3 (CSP1 r)
\end{aligned}$$

CSP_PROCESS_∈_REA_PREDICATE_thm

$$\vdash \forall c : CSP_PROCESS \bullet c \in REA_PREDICATE$$

CSP_PROCESS_thm

$$\begin{aligned}
& \vdash \forall c : CSP_PROCESS \\
& \bullet c \in REA_PROCESS \\
& \wedge ALPHABET_OWTR \subseteq c.1 \\
& \wedge c \in CSP1_healthy \\
& \wedge c \in CSP2_healthy
\end{aligned}$$

CSP_R_design_alphabet_thm

$$\vdash \forall P, Q : REA_PREDICATE$$

$$\begin{aligned}
& | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \quad \wedge P.1 \in WF_Skip_{REA} \\
& \quad \bullet (R (P \vdash_D Q)).1 = P.1 \\
\mathbf{CSP_CSP1_CSP_PROCESS_thm} \\
& \vdash \forall c : CSP_PROCESS \bullet CSP1 \ c = c \\
\mathbf{CSP_OKAY_}\wedge_R\mathbf{CSP1_abs_thm} \\
& \vdash \forall r : REL_PREDICATE \\
& \quad | ALPHABET_OWTR \subseteq r.1 \\
& \quad \bullet OKAY \wedge_R CSP1 \ r = OKAY \wedge_R r \\
\mathbf{CSP_CSP1_okay_dash_diverge_com_thm} \\
& \vdash \forall r : REA_PREDICATE \bullet CSP1 \ (r \ \sigma_f) = CSP1 \ r \ \sigma_f \\
\mathbf{CSP_CSP1_okay_dash_converge_com_thm} \\
& \vdash \forall r : REA_PREDICATE \bullet CSP1 \ (r \ \sigma_t) = CSP1 \ r \ \sigma_t \\
\mathbf{CSP_CSP1_wait_false_com_thm} \\
& \vdash \forall r : REA_PREDICATE \bullet CSP1 \ (r \ \omega_f) = CSP1 \ r \ \omega_f \\
\mathbf{CSP_CSP1_wait_true_com_thm} \\
& \vdash \forall r : REA_PREDICATE \bullet CSP1 \ (r \ \omega_t) = CSP1 \ r \ \omega_t \\
\mathbf{CSP_CSP1_ext_alphabet_thm} \\
& \vdash \forall r : REA_PREDICATE \\
& \quad \bullet CSP1 \ r \\
& \quad = r \\
& \quad \vee_R =_R (r.1, okay, Val (Bool false)) \\
& \quad \wedge_R =_{+R} \\
& \quad \quad (ALPHABET_OWTR, \\
& \quad \quad Rel \\
& \quad \quad ((- \leq_R -), \\
& \quad \quad Var \ tr, \\
& \quad \quad Var \ (dash \ tr)), \\
& \quad \quad Val \ (Bool \ true)) \\
\mathbf{CSP_UnrestALPHABET_CSP_True_R_thm} \\
& \vdash \forall a : ALPHABET_REA \\
& \quad | ALPHABET_OWTR \subseteq a \\
& \quad \bullet unrestALPHABET_CSP \ (True_R \ a) \\
\mathbf{CSP_UnrestALPHABET_CSP_False_R_thm} \\
& \vdash \forall a : ALPHABET_REA \\
& \quad | ALPHABET_OWTR \subseteq a \\
& \quad \bullet unrestALPHABET_CSP \ (False_R \ a) \\
\mathbf{CSP_UnrestALPHABET_CSP_dash_True_R_thm}
\end{aligned}$$

$$\begin{aligned}
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad | \text{ALPHABET_OWTR} \subseteq a \\
& \quad \bullet \text{unrestALPHABET_CSP}' (\text{True}_R a) \\
\textbf{CSP_UnrestALPHABET_CSP_dash_False}_R\textbf{-thm} \\
& \vdash \forall a : \text{ALPHABET_REA} \\
& \quad | \text{ALPHABET_OWTR} \subseteq a \\
& \quad \bullet \text{unrestALPHABET_CSP}' (\text{False}_R a) \\
\textbf{CSP_CSP1_healthy_thm} \\
& \vdash \forall c : \text{CSP_PROCESS} \bullet c = \text{CSP1 } c \\
\textbf{CSP_CSP2_healthy_thm} \\
& \vdash \forall c : \text{CSP_PROCESS} \bullet c = \text{CSP2 } c \\
\textbf{CSP_CSP1_R1_H1_thm} \\
& \vdash \forall c : \text{CSP_PROCESS} \bullet \text{CSP1 } c = R1 (H1 \ c) \\
\textbf{CSP_CSP2_thm} \quad \vdash \forall c : \text{CSP_PROCESS} \bullet \text{CSP2 } c \in \text{CSP_PROCESS} \\
\textbf{CSP_REA_PREDICATE_H2_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet H2 \ r \in \text{REA_PREDICATE} \\
\textbf{CSP_REA_PREDICATE_H1_thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \bullet H1 \ r \in \text{REA_PREDICATE} \\
\textbf{CSP_REA_PREDICATE_H2_}\in\textbf{WF_Skip}_{\text{REA}}\textbf{-thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet (H2 \ r).1 \in \text{WF_Skip}_{\text{REA}} \\
\textbf{CSP_REA_PREDICATE_H1_}\in\textbf{WF_Skip}_{\text{REA}}\textbf{-thm} \\
& \vdash \forall r : \text{REA_PREDICATE} \\
& \quad | r.1 \in \text{WF_Skip}_{\text{REA}} \\
& \quad \bullet (H1 \ r).1 \in \text{WF_Skip}_{\text{REA}} \\
\textbf{CSP_PROCESS_reactive_design_thm} \\
& \vdash \forall c : \text{CSP_PROCESS} \\
& \quad | c.1 \in \text{homogeneous} \\
& \quad \bullet c = R (\neg_R ((c \ \omega_f) \ \sigma_f) \vdash_D (c \ \omega_f) \ \sigma_t) \\
\textbf{CSP_CSP1_}\vee_R\textbf{-dist_thm} \\
& \vdash \forall r1, r2 : \text{REA_PROCESS} \\
& \quad \bullet \text{CSP1 } (r1 \vee_R r2) = \text{CSP1 } r1 \vee_R \text{CSP1 } r2 \\
\textbf{CSP_CSP1_healthy_}\vee_R\textbf{-dist_thm} \\
& \vdash \forall r1, r2 : \text{REA_PROCESS} \\
& \quad | r1 \in \text{CSP1_healthy} \wedge r2 \in \text{CSP1_healthy} \\
& \quad \bullet r1 \vee_R r2 \in \text{CSP1_healthy} \\
\textbf{CSP_CSP2_}\vee_R\textbf{-dist_thm1}
\end{aligned}$$

$$\begin{array}{l}
\vdash \forall r1, r2 : REA_PROCESS \\
\quad | \quad r2.1 = r1.1 \\
\quad \bullet \quad CSP2 \ (r1 \vee_R r2) = CSP2 \ r1 \vee_R CSP2 \ r2 \\
\mathbf{CSP_CSP2_healthy_}\mathbb{V}_R\mathbf{-dist_thm1} \\
\vdash \forall r1, r2 : REA_PROCESS \\
\quad | \quad r1 \in CSP2_healthy \\
\quad \quad \wedge \ r2 \in CSP2_healthy \\
\quad \quad \wedge \ r1.1 = r2.1 \\
\quad \bullet \quad r1 \vee_R r2 \in CSP2_healthy \\
\mathbf{CSP_PROCESS_}\mathbb{V}_R\mathbf{-thm} \\
\vdash \forall A1, A2 : CSP_PROCESS \\
\quad | \quad A1.1 = A2.1 \\
\quad \bullet \quad A1 \vee_R A2 \in CSP_PROCESS \\
\mathbf{CSP_empty_}\mathbb{F}\mathbf{-VAR_NAME_thm} \\
\vdash \{\} \in \mathbb{F} \ VAR_NAME \\
\mathbf{CSP_REA_PREDICATE_Design_thm} \\
\vdash \forall P, Q : REA_PREDICATE \\
\quad | \quad (P, Q) \in WF_DES_PREDICATE_PAIR \\
\quad \bullet \quad P \vdash_D Q \in REA_PREDICATE \\
\mathbf{CSP_REA_PREDICATE_R_Design_thm} \\
\vdash \forall P, Q : REA_PREDICATE \\
\quad | \quad (P \vdash_D Q).1 \in homogeneous \\
\quad \quad \wedge \ (P, Q) \in WF_DES_PREDICATE_PAIR \\
\quad \bullet \quad R \ (P \vdash_D Q) \in REA_PREDICATE \\
\mathbf{CSP_R_healthy_Design_thm} \\
\vdash \forall P, Q : REA_PREDICATE \\
\quad | \quad (P \vdash_D Q).1 \in homogeneous \\
\quad \quad \wedge \ (P, Q) \in WF_DES_PREDICATE_PAIR \\
\quad \bullet \quad R \ (P \vdash_D Q) \in R_healthy \\
\mathbf{CSP_REA_PROCESS_Design_thm} \\
\vdash \forall P, Q : REA_PREDICATE \\
\quad | \quad (P \vdash_D Q).1 \in homogeneous \\
\quad \quad \wedge \ (P, Q) \in WF_DES_PREDICATE_PAIR \\
\quad \bullet \quad R \ (P \vdash_D Q) \in REA_PROCESS \\
\mathbf{CSP_REL_PREDICATE_Design_thm} \\
\vdash \forall P, Q : REA_PREDICATE \\
\quad | \quad (P, Q) \in WF_DES_PREDICATE_PAIR \\
\quad \bullet \quad P \vdash_D Q \in REL_PREDICATE
\end{array}$$

CSP_reactive_design_* \vee_R ***thm**

$$\begin{aligned}
& \vdash \forall p1, q1, p2, q2 : REA_PREDICATE \\
& \quad | (p1, q1) \in WF_DES_PREDICATE_PAIR \\
& \quad \wedge (p2, q2) \in WF_DES_PREDICATE_PAIR \\
& \quad \wedge p2.1 = p1.1 \\
& \quad \wedge q2.1 = q1.1 \\
& \quad \wedge p1.1 \in WF_Skip_{REA} \\
& \bullet R (p1 \vdash_D q1) \vee_R R (p2 \vdash_D q2) \\
& \quad = R ((p1 \wedge_R p2) \vdash_D q1 \vee_R q2)
\end{aligned}$$

CSP_CSP1_R1_H1_thm2

$$\vdash \forall r : REA_PREDICATE \bullet R1 (H1 \ r) = R1 (CSP1 \ r)$$

CSP_R2_CSP1_def_id_thm

$$\begin{aligned}
& \vdash R1 \\
& \quad (\neg_R OKAY \\
& \quad \quad \wedge_R =_{+R} \\
& \quad \quad (ALPHABET_OWTR, \\
& \quad \quad \quad Rel \\
& \quad \quad \quad ((- \leq_R -), \\
& \quad \quad \quad \quad Var \ tr, \\
& \quad \quad \quad \quad Var \ (dash \ tr)), \\
& \quad \quad \quad Val \ (Bool \ true))) \\
& \quad = R1 \\
& \quad (R2 \\
& \quad \quad (\neg_R OKAY \\
& \quad \quad \quad \wedge_R =_{+R} \\
& \quad \quad \quad (ALPHABET_OWTR, \\
& \quad \quad \quad \quad Rel \\
& \quad \quad \quad \quad ((- \leq_R -), \\
& \quad \quad \quad \quad \quad Var \ tr, \\
& \quad \quad \quad \quad \quad Var \ (dash \ tr)), \\
& \quad \quad \quad \quad Val \ (Bool \ true))))
\end{aligned}$$

CSP_CSP1_R2_com_thm

$$\begin{aligned}
& \vdash \forall r : REA_PREDICATE \\
& \bullet R1 (R2 (CSP1 \ r)) = R1 (CSP1 (R2 \ r))
\end{aligned}$$

CSP_CSP1_R1_H1_thm1

$$\begin{aligned}
& \vdash \forall r : REA_PREDICATE \\
& \quad | r \in R1_healthy \\
& \bullet R1 (H1 \ r) = CSP1 \ r
\end{aligned}$$

CSP_CSP1_healthy_Design_thm

$$\begin{aligned} &\vdash \forall P, Q : \text{REA_PREDICATE} \\ &\quad | (P \vdash_D Q).1 \in \text{homogeneous} \\ &\quad \wedge (P, Q) \in \text{WF_DES_PREDICATE_PAIR} \\ &\quad \bullet R(P \vdash_D Q) \in \text{CSP1_healthy} \end{aligned}$$
CSP_CSP_PROCESS_Design_thm

$$\begin{aligned} &\vdash \forall P, Q : \text{REA_PREDICATE} \\ &\quad | (P \vdash_D Q).1 \in \text{homogeneous} \\ &\quad \wedge (P, Q) \in \text{WF_DES_PREDICATE_PAIR} \\ &\quad \bullet R(P \vdash_D Q) \in \text{CSP_PROCESS} \end{aligned}$$
CSP_CSP_PROCESS_∈_homogeneous_thm

$$\vdash \forall A : \text{CSP_PROCESS} \bullet A.1 \in \text{homogeneous}$$
CSP_closure_Semi_R_CSP1_thm

$$\begin{aligned} &\vdash \forall r1, r2 : \text{REA_PREDICATE} \\ &\quad | r2.1 = r1.1 \\ &\quad \wedge (r1, r2) \in \text{WF_Semi}_R \\ &\quad \wedge \{r1, r2\} \subseteq \text{CSP1_healthy} \\ &\quad \bullet \text{CSP1}(r1 ;_R r2) = r1 ;_R r2 \end{aligned}$$
CSP_closure_Semi_R_CSP2_thm

$$\begin{aligned} &\vdash \forall r1, r2 : \text{REA_PREDICATE} \\ &\quad | \{\text{dash okay}, \text{dash tr}, \text{dash wait}, \text{dash ref}\} \\ &\quad \quad \subseteq \text{out_a } r2.1 \\ &\quad \wedge (r1, r2) \in \text{WF_Semi}_R \\ &\quad \wedge \{r1, r2\} \subseteq \text{CSP2_healthy} \\ &\quad \bullet \text{CSP2}(r1 ;_R r2) = r1 ;_R r2 \end{aligned}$$
CSP_REA_PREDICATE_do_C_thm

$$\begin{aligned} &\vdash \forall n : \text{VAR_NAME}; e : \text{EXPRESSION}; a : \text{WF_Skip}_{\text{REA}} \\ &\quad | (a, n, e) \in \text{WF_PREFIXING} \\ &\quad \bullet \text{do_C}(a, n, e) \in \text{REA_PREDICATE} \end{aligned}$$
CSP_REA_PREDICATE_do_C_alphabet_thm

$$\begin{aligned} &\vdash \forall n : \text{VAR_NAME}; e : \text{EXPRESSION}; a : \text{WF_Skip}_{\text{REA}} \\ &\quad | (a, n, e) \in \text{WF_PREFIXING} \\ &\quad \bullet (\text{do_C}(a, n, e)).1 = a \end{aligned}$$
CSP_PROCESS_Semi_R_thm

$$\begin{aligned} &\vdash \forall A1, A2 : \text{CSP_PROCESS} \\ &\quad | (A1, A2) \in \text{WF_Semi}_R \wedge A1.1 = A2.1 \\ &\quad \bullet A1 ;_R A2 \in \text{CSP_PROCESS} \end{aligned}$$
CSP_R_design_R1_thm

$$\begin{aligned}
& \vdash \forall P, Q : REA_PREDICATE \\
& \quad | Q \in R1_healthy \\
& \quad \bullet R (P \vdash_D R1 \ Q) = R (P \vdash_D \ Q) \\
\mathbf{CSP_design_wait_false_id_thm1} \\
& \vdash \forall P, Q : REA_PREDICATE \\
& \quad | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \quad \quad \wedge P.1 \in WF_Skip_{REA} \\
& \quad \bullet (P \vdash_D \ Q) \ \omega_f = (P \ \omega_f) \vdash_D \ Q \ \omega_f \\
\mathbf{CSP_design_okay_dash_converge_id_thm} \\
& \vdash \forall P, Q : REA_PREDICATE \\
& \quad | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \quad \quad \wedge P.1 \in WF_Skip_{REA} \\
& \quad \bullet (P \vdash_D \ Q) \ \sigma_t = (OKAY \wedge_R P \ \sigma_t) \Rightarrow_R Q \ \sigma_t \\
\mathbf{CSP_okay_dash_diverge_thm} \\
& \vdash \forall u : REA_PROCESS \bullet R1 \ u \ \sigma_f = R1 \ (u \ \sigma_f) \\
\mathbf{CSP_R_okay_dash_diverge_thm} \\
& \vdash \forall u : REA_PROCESS \bullet R \ u \ \sigma_f = R1 \ (R2 \ u) \ \sigma_f \\
\mathbf{CSP_R_wait_false_thm} \\
& \vdash \forall u : REA_PROCESS \bullet R \ u \ \omega_f = R1 \ (R2 \ u) \ \omega_f \\
\mathbf{CSP_R_design_not_wait_converge_thm1} \\
& \vdash \forall P, Q : REA_PREDICATE \\
& \quad | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \quad \quad \wedge P.1 \in WF_Skip_{REA} \\
& \quad \bullet (R (P \vdash_D \ Q) \ \omega_f) \ \sigma_t \\
& \quad \quad = CSP1 \\
& \quad \quad (R1 \ (R2 \ (((P \ \omega_f) \ \sigma_t) \Rightarrow_R (Q \ \omega_f) \ \sigma_t))) \\
\mathbf{CSP_R_design_not_wait_diverge_thm1} \\
& \vdash \forall P, Q : REA_PREDICATE \\
& \quad | (P, Q) \in WF_DES_PREDICATE_PAIR \\
& \quad \quad \wedge P.1 \in WF_Skip_{REA} \\
& \quad \bullet (R (P \vdash_D \ Q) \ \omega_f) \ \sigma_f \\
& \quad \quad = R1 \ (\neg_R (OKAY \wedge_R R2 \ ((P \ \omega_f) \ \sigma_f))) \\
\mathbf{CSP_R2_Condition_thm} \\
& \vdash \forall g : REA_PREDICATE \\
& \quad | unrestALPHABET_CSP \ g \wedge unrestALPHABET_CSP' \ g \\
& \quad \bullet R2 \ g = g \\
\mathbf{CSP_R1_}\neg_R\mathbf{OKAY_is_CSP1_thm} \\
& \vdash R1 \ (\neg_R \ OKAY)
\end{aligned}$$

$$\begin{aligned}
&= =_R (ALPHABET_OWTR, \textit{okay}, \textit{Val} (\textit{Bool} \textit{false})) \\
&\quad \wedge_R =_{+R} \\
&\quad (ALPHABET_OWTR, \\
&\quad \quad \textit{Rel} ((- \leq_R -), \textit{Var} \textit{tr}, \textit{Var} (\textit{dash} \textit{tr})), \\
&\quad \quad \textit{Val} (\textit{Bool} \textit{true})) \\
\mathbf{CSP_not_wait_false_not_okay_dash} &\Rightarrow_R \mathbf{okay_dash_thm} \\
\vdash \forall A : \mathbf{CSP_PROCESS} & \\
\bullet ((A \ \omega_f) \ \sigma_f) &\Rightarrow_R (A \ \omega_f) \ \sigma_t = \textit{True}_R \ A.1
\end{aligned}$$

14 THE Z THEORY utp-circus

14.1 Parents

utp-csp

14.2 Global Variables

Z_VAR_NAME	$\mathbb{P} \text{ NAME}$
VAR_DECLS	$(\mathbb{Z} \leftrightarrow \text{NAME}) \leftrightarrow \mathbb{Z} \leftrightarrow \text{EXPRESSION}$
CIRCUS_PREDICATE	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
CIRCUS_CONDITION	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Skip_C	$\mathbb{P} (\mathbb{P} \text{ NAME})$
WF_Guard_C	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_if_{C-fi}	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Semi_C	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_Var_C	$\mathbb{P} (\text{NAME} \times \text{VALUE} \times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})))$
WF_Assign_C	$\mathbb{P} (\mathbb{P} \text{ NAME} \times (\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION}))$
WF_SpecStatement_C	\mathbb{P} $(\mathbb{P}$ NAME $\times \mathbb{P}$ NAME $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})))$
WF_Condition_C	$\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_SchemaExp_C	$(\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION})$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
WF_param_C	\mathbb{P} $(\text{NAME}$ $\times \text{EXPRESSION}$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times \text{EXPRESSION})$

<i>WF_val_C</i>	\mathbb{P} $(NAME$ $\times VALUE$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times EXPRESSION)$
<i>WF_res_C</i>	$\mathbb{P} (NAME \times VALUE \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)) \times NAME)$
<i>WF_vres_C</i>	$\mathbb{P} (NAME \times VALUE \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)) \times NAME)$
<i>WF_PREFIXING_C</i>	$\mathbb{P} NAME \times NAME \times EXPRESSION \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>WF_PREFIXING_{CinR}</i>	\mathbb{P} $(\mathbb{P}$ $NAME$ $\times NAME$ $\times NAME$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)))$
<i>Stop</i>	$\mathbb{P} NAME \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>Skip</i>	$\mathbb{P} NAME \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>Chaos</i>	$\mathbb{P} NAME \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- ;_C -)</i>	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- &_C -)</i>	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- ⌈_C -)</i>	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- ∩_C -)</i>	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- →_C -)</i>	$(\mathbb{P} NAME \times NAME \times EXPRESSION)$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>(- →_{CSync} -)</i>	$(\mathbb{P} NAME \times NAME) \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$

$(- \rightarrow_{Cout} -)$	$(\mathbb{P} NAME \times NAME \times EXPRESSION)$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>var_C</i>	$NAME \times VALUE \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>NonRef</i>	$NAME \times NAME \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)) \leftrightarrow VALUE$
<i>Traces</i>	$NAME \times NAME \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)) \leftrightarrow VALUE$
<i>do_I</i>	$\mathbb{P} NAME \times NAME \times NAME \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
$(- \rightarrow_{CinR} -)$	$(\mathbb{P} NAME \times NAME \times NAME \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
$(- \rightarrow_{Cin} -)$	$(\mathbb{P} NAME \times NAME \times NAME) \times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MTrInter</i>	$VALUE \times VALUE \leftrightarrow VALUE$
<i>MTrPar</i>	$VALUE \times VALUE \leftrightarrow VALUE$
<i>MTrParPred</i>	$VALUE \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MSync</i>	$VALUE \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>BranchesWaiting</i>	$\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>BranchesNotWaiting</i>	$\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MRefPar</i>	$VALUE \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MSt</i>	$\mathbb{P} NAME \times \mathbb{P} NAME \times \mathbb{P} NAME \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MWtRefStPar</i>	$VALUE \times \mathbb{P} NAME \times \mathbb{P} NAME \times \mathbb{P} NAME$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MPar</i>	$VALUE \times \mathbb{P} NAME \times \mathbb{P} NAME \times \mathbb{P} NAME$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>DivPar</i>	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times VALUE$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
$(- \llbracket_C - \rrbracket_C -)$	$(\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\times (\mathbb{P} NAME \times VALUE \times \mathbb{P} NAME)$ $\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$ $\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
<i>MTrInterPred</i>	$\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$

<i>MRefInter</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>MWtRefStInter</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} \text{ NAME} \times \mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>MInter</i>	$\mathbb{P} \text{ NAME} \times \mathbb{P} \text{ NAME} \times \mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>DivInter</i>	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>(- [[_C -]]_C -)</i>	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} \text{ NAME})$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>(- \setminus_C -)</i>	$(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})) \times \text{VALUE}$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>CIRCUS_FUNCTION</i>	
<i>μ_C</i>	\mathbb{P} $(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $(\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>Assign_C</i>	$\mathbb{P} \text{ NAME} \times (\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION})$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>SpecStatement_C</i>	\mathbb{P} NAME $\times \mathbb{P}$ NAME $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>{_C - }_C</i>	$\mathbb{P} \text{ NAME} \times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>(_C -)_C</i>	$\mathbb{P} \text{ NAME} \times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>Typing</i>	$((\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION})) \times \mathbb{P} \text{ NAME}$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
<i>SchemaExp_C</i>	$((\mathbb{Z} \leftrightarrow \text{NAME}) \times (\mathbb{Z} \leftrightarrow \text{EXPRESSION}))$ $\times (\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}))$ $\leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

	($\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\leftrightarrow \mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
TrueGuards	($\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\times (\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
NonDivActions	
	($\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\times (\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
ExecActions	($\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\times (\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
(if_C - fi_C)	
	($\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\times (\mathbb{Z} \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
(ifb_C - \mapsto_C - else_C - \mapsto_C - fib_C)	
	($\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$)
	$\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
param_C	NAME
	$\times EXPRESSION$
	$\times (\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE))$
	$\times EXPRESSION$
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
val_C	NAME \times VALUE \times ($\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$) \times EXPRESSION
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
res_C	NAME \times VALUE \times ($\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$) \times NAME
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
vres_C	NAME \times VALUE \times ($\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$) \times NAME
	$\leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
C1	$\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
C1_healthy	$\mathbb{P} NAME \leftrightarrow \mathbb{P} (NAME \leftrightarrow VALUE)$
C2	$\mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE) \leftrightarrow \mathbb{P} NAME \times \mathbb{P} (NAME \leftrightarrow VALUE)$
C2_healthy	$\mathbb{P} NAME \leftrightarrow \mathbb{P} (NAME \leftrightarrow VALUE)$

C3 $\mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE}) \leftrightarrow \mathbb{P} \text{ NAME} \times \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
C3_healthy $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$
CIRCUS_ACTION
 $\mathbb{P} \text{ NAME} \leftrightarrow \mathbb{P} (\text{NAME} \leftrightarrow \text{VALUE})$

14.3 Fixity

fun 0 rightassoc
 $(\text{ifb}_C - \mapsto_C - \text{else}_C - \mapsto_C - \text{fib}_C)$
 $(\text{if}_C - \text{fi}_C)$
 $(- \ \&_C \ -)$
 $(- \ ;_C \ -)$
 $(- \ \backslash_C \ -)$
 $(- \ \cap_C \ -)$
 $(- \ \boxtimes_C \ -)$
 $(- \ \rightarrow_{\text{CinR}} \ -)$
 $(- \ \rightarrow_{\text{Cin}} \ -)$
 $(- \ \rightarrow_{\text{Cout}} \ -)$
 $(- \ \rightarrow_{\text{CSync}} \ -)$
 $(- \ \rightarrow_C \ -)$
 $(- \ \llbracket_C \ - \rrbracket_C \ -)$
 $(- \ \llbracket\llbracket_C \ - \rrbracket\rrbracket_C \ -)$
 $(\{_C \ - \ \}_C)$
 $(\langle_C \ - \ \rangle_C)$

14.4 Axioms

CIRCUS_PREDICATE

$\vdash \text{CIRCUS_PREDICATE} \in \mathbb{P} \text{ REA_PREDICATE}$
 $\wedge \text{CIRCUS_PREDICATE}$
 $= \{c : \text{REA_PREDICATE} \mid \text{unrestALPHABET_CSP } c \wedge \text{unrestALPHABET_CSP}' c\}$

CIRCUS_CONDITION

$\vdash \text{CIRCUS_CONDITION} \in \mathbb{P} \text{ REA_PREDICATE}$
 $\wedge \text{CIRCUS_CONDITION}$
 $= \{c : \text{REA_PREDICATE} \mid \text{unrestALPHABET_CSP } c$
 $\wedge \text{unrestALPHABET_CSP}' c\}$

	$\wedge c -_R \text{ALPHABET_OWTR} \in \text{REL_CONDITION}\}$
Stop	$\begin{aligned} &\vdash \text{Stop} \in \text{WF_Skip}_C \rightarrow \text{CSP_PROCESS} \\ &\wedge (\forall a : \text{WF_Skip}_C \\ &\quad \bullet \text{Stop } a \\ &\quad = R \\ &\quad (\text{True}_R a \\ &\quad \quad \vdash_D =_R (a, \text{dash } tr, \text{Var } tr) \\ &\quad \quad \wedge_R =_R \\ &\quad \quad (a, \\ &\quad \quad \quad \text{dash wait}, \\ &\quad \quad \quad \text{Val } (\text{Bool true})))) \end{aligned}$
Skip	$\begin{aligned} &\vdash \text{Skip} \in \text{WF_Skip}_C \rightarrow \text{CSP_PROCESS} \\ &\wedge (\forall a : \text{WF_Skip}_C \\ &\quad \bullet \text{Skip } a \\ &\quad = R \\ &\quad (\text{True}_R a \\ &\quad \quad \vdash_D =_R (a, \text{dash } tr, \text{Var } tr) \\ &\quad \quad \wedge_R =_R \\ &\quad \quad (a, \\ &\quad \quad \quad \text{dash wait}, \\ &\quad \quad \quad \text{Val } (\text{Bool false})) \\ &\quad \quad \wedge_R \Pi_R (a \setminus \text{ALPHABET_OWTR}))) \end{aligned}$
Chaos	$\begin{aligned} &\vdash \text{Chaos} \in \text{WF_Skip}_C \rightarrow \text{CSP_PROCESS} \\ &\wedge (\forall a : \text{WF_Skip}_C \\ &\quad \bullet \text{Chaos } a = R (\text{False}_R a \vdash_D \text{True}_R a)) \end{aligned}$
- ;_C -	$\begin{aligned} &\vdash (- ;_C -) \in \text{WF_Semi}_C \rightarrow \text{CSP_PROCESS} \\ &\wedge (\forall u1_u2 : \text{WF_Semi}_C \\ &\quad \bullet u1_u2.1 ;_C u1_u2.2 = u1_u2.1 ;_R u1_u2.2) \end{aligned}$
- &_C -	$\begin{aligned} &\vdash (- \&_C -) \in \text{WF_Guard}_C \rightarrow \text{CSP_PROCESS} \\ &\wedge (\forall g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\ &\quad (g, a) \in \text{WF_Guard}_C \\ &\quad \bullet g \&_C a \\ &\quad = R \\ &\quad ((g \Rightarrow_R \neg_R ((a \omega_f) \sigma_f)) \\ &\quad \quad \vdash_D (g \wedge_R (a \omega_f) \sigma_t) \\ &\quad \quad \vee_R \neg_R g \\ &\quad \quad \wedge_R =_R (a.1, \text{dash } tr, \text{Var } tr) \\ &\quad \quad \wedge_R =_R \end{aligned}$

$$\begin{array}{lcl}
& & (a.1, \\
& & \quad dash \ wait, \\
& & \quad Val \ (Bool \ true))) \\
- \boxtimes_C - & \vdash & (- \boxtimes_C -) \in WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS \\
& & \wedge (\forall aa : WF_CSP_PROCESS_PAIR \\
& & \quad \bullet aa.1 \boxtimes_C aa.2 \\
& & \quad = R \\
& & \quad ((\neg_R ((aa.1 \ \omega_f) \ \sigma_f) \\
& & \quad \quad \wedge_R \neg_R ((aa.2 \ \omega_f) \ \sigma_f)) \\
& & \quad \vdash_D (((aa.1 \ \omega_f) \ \sigma_t) \\
& & \quad \quad \wedge_R (aa.2 \ \omega_f) \ \sigma_t) \triangleleft_R \\
& & \quad =_{+R} \\
& & \quad (aa.1.1, \\
& & \quad \quad Var \ (dash \ tr), \\
& & \quad \quad Var \ tr) \\
& & \quad \wedge_R =_R \\
& & \quad (aa.1.1, \\
& & \quad \quad dash \ wait, \\
& & \quad \quad Val \ (Bool \ true)) \triangleright_R \\
& & \quad ((aa.1 \ \omega_f) \ \sigma_t) \\
& & \quad \quad \vee_R (aa.2 \ \omega_f) \ \sigma_t)) \\
- \cap_C - & \vdash & (- \cap_C -) \in WF_CSP_PROCESS_PAIR \rightarrow CSP_PROCESS \\
& & \wedge (\forall aa : WF_CSP_PROCESS_PAIR \\
& & \quad \bullet aa.1 \cap_C aa.2 = aa.1 \vee_R aa.2) \\
- \rightarrow_C - & \vdash & (- \rightarrow_C -) \in WF_PREFIXING_C \rightarrow CSP_PROCESS \\
& & \wedge (\forall a_a : WF_PREFIXING_C \\
& & \quad \bullet a_a.1 \rightarrow_C a_a.2 \\
& & \quad = R \\
& & \quad (True_R \ a_a.1.1 \\
& & \quad \quad \vdash_D (do_C \ a_a.1) \\
& & \quad \quad \wedge_R \Pi_R \\
& & \quad \quad (a_a.1.1 \setminus ALPHABET_OWTR)) \\
& & \quad ;_C \ a_a.2) \\
- \rightarrow_{CSync} - & \vdash & (- \rightarrow_{CSync} -) \\
& & \in (ALPHABET \times VAR_NAME) \times CSP_PROCESS \\
& & \rightarrow CSP_PROCESS \\
& & \wedge (\forall a : ALPHABET; c : VAR_NAME; A : CSP_PROCESS \\
& & \quad \bullet (a, c) \rightarrow_{CSync} A = (a, c, Val \ Sync) \rightarrow_C A)
\end{array}$$

$- \rightarrow_{Cout} -$	$\vdash (- \rightarrow_{Cout} -) \in WF_PREFIXING_C \rightarrow CSP_PROCESS$ $\wedge (\forall a_a : WF_PREFIXING_C$ $\bullet a_a.1 \rightarrow_{Cout} a_a.2 = a_a.1 \rightarrow_C a_a.2)$
<i>var_C</i>	$\vdash var_C \in WF_Var_C \rightarrow CSP_PROCESS$ $\wedge (\forall n : VAR_NAME; t : SET_VAL; a : CSP_PROCESS$ $\mid (n, t, a) \in WF_Var_C$ $\bullet var_C (n, t, a)$ $= varT_R (a.1, n, t)$ $;_R a ;_R endT_R (a.1, n, t))$
<i>NonRef</i>	$\vdash NonRef$ $\in VAR_NAME \times VAR_NAME \times CIRCUS_CONDITION$ $\rightarrow SET_EVENT_VAL$ $\wedge (\forall c : VAR_NAME;$ $x : VAR_NAME;$ $p : CIRCUS_CONDITION$ $\bullet NonRef (c, x, p)$ $= Set$ $\{v : VALUE$ $\mid /_R (p, Val v, x) = True_R p.1$ $\bullet Pair (Channel c, v)\})$
<i>Traces</i>	$\vdash Traces$ $\in VAR_NAME \times VAR_NAME \times CIRCUS_CONDITION$ $\rightarrow SET_SEQ_EVENT_VAL$ $\wedge (\forall c : VAR_NAME;$ $x : VAR_NAME;$ $p : CIRCUS_CONDITION$ $\bullet Traces (c, x, p)$ $= Set$ $\{v : VALUE$ $\mid /_R (p, Val v, x) = True_R p.1$ $\bullet Seq \langle Pair (Channel c, v) \rangle\})$
<i>do_I</i>	$\vdash do_I$ $\in ALPHABET_REA$ $\times VAR_NAME$ $\times VAR_NAME$ $\times CIRCUS_CONDITION$ $\rightarrow CSP_PROCESS$ $\wedge (\forall a : ALPHABET_REA;$

$$\begin{aligned}
& c : VAR_NAME; \\
& x : VAR_NAME; \\
& p : CIRCUS_CONDITION \\
& \bullet do_I (a, c, x, p) \\
& = (=_R (a, dash\ tr, Var\ tr) \\
& \quad \wedge_R =_{+R} \\
& \quad (a, \\
& \quad \quad Fun_2 \\
& \quad \quad ((- \cap_R -), \\
& \quad \quad \quad Val (NonRef (c, x, p)), \\
& \quad \quad \quad Var (dash\ ref)), \\
& \quad \quad Val (Set\ \emptyset))) \triangleleft_R \\
& =_R (a, dash\ wait, Val (Bool\ true)) \triangleright_R \\
& =_{+R} \\
& (a, \\
& \quad Rel \\
& \quad ((- \in_R -), \\
& \quad \quad Fun_2 \\
& \quad \quad ((- SeqDif_R -), \\
& \quad \quad \quad Var (dash\ tr), \\
& \quad \quad \quad Var\ tr), \\
& \quad \quad Val (Traces (c, x, p))), \\
& \quad Val (Bool\ true))) \\
- \rightarrow_{CinR} - & \quad \vdash (- \rightarrow_{CinR} -) \\
& \in (ALPHABET_REA \\
& \quad \times VAR_NAME \\
& \quad \times VAR_NAME \\
& \quad \times CIRCUS_CONDITION) \\
& \times CSP_PROCESS \\
& \rightarrow CSP_PROCESS \\
& \wedge (\forall a : ALPHABET_REA; \\
& \quad c : VAR_NAME; \\
& \quad x : VAR_NAME; \\
& \quad p : CIRCUS_CONDITION; \\
& \quad A : CSP_PROCESS \\
& \bullet (a, c, x, p) \rightarrow_{CinR} A \\
& = var_C \\
& (x,
\end{aligned}$$

$$\begin{array}{l}
\text{Set } \{v : \text{VALUE}\}, \\
R \\
\quad (\text{True}_R \text{ A.1} \\
\quad \quad \vdash_D \text{do-I } (a, c, x, p) \\
\quad \quad \quad \wedge_R \Pi_R \\
\quad \quad \quad (A.1 \setminus \{x, \text{dash } x\})) \\
\quad ;_C A)) \\
- \rightarrow_{Cin} - \quad \vdash (- \rightarrow_{Cin} -) \\
\quad \in (\text{ALPHABET_REA} \times \text{VAR_NAME} \times \text{VAR_NAME}) \\
\quad \quad \times \text{CSP_PROCESS} \\
\quad \quad \rightarrow \text{CSP_PROCESS} \\
\quad \wedge (\forall a : \text{ALPHABET_REA}; \\
\quad \quad c : \text{VAR_NAME}; \\
\quad \quad x : \text{VAR_NAME}; \\
\quad \quad A : \text{CSP_PROCESS} \\
\quad \quad \bullet (a, c, x) \rightarrow_{Cin} A \\
\quad \quad = (a, c, x, \text{True}_R \text{ A.1}) \rightarrow_{CinR} A) \\
\mathbf{MTrInter} \quad \vdash \text{MTrInter} \\
\quad \in \text{SEQ_EVENT_VAL} \times \text{SEQ_EVENT_VAL} \\
\quad \quad \rightarrow \text{SET_SEQ_EVENT_VAL} \\
\quad \wedge (\forall s1, s2 : \text{SEQ_EVENT_VAL} \\
\quad \quad \bullet \text{MTrInter } (s1, s2) \\
\quad \quad = \text{Set} \\
\quad \quad \quad \{s : (\text{Seq } \sim) s1 \parallel_Z (\text{Seq } \sim) s2 \\
\quad \quad \quad \bullet \text{Seq } s\}) \\
\mathbf{MTrPar} \quad \vdash \text{MTrPar} \\
\quad \in \text{PAIR_SEQ_EVENT_VAL} \times \text{SET_EVENT_VAL} \\
\quad \quad \rightarrow \text{SET_SEQ_EVENT_VAL} \\
\quad \wedge (\forall ps : \text{PAIR_SEQ_EVENT_VAL}; cs : \text{SET_EVENT_VAL} \\
\quad \quad \bullet \text{MTrPar } (ps, cs) \\
\quad \quad = \text{Set} \\
\quad \quad \quad \{s \\
\quad \quad \quad \quad : (\text{Seq } \sim) ((\text{Pair } \sim) ps).1 \parallel_Z \\
\quad \quad \quad \quad (\text{Set } \sim) cs \parallel_Z \\
\quad \quad \quad \quad (\text{Seq } \sim) ((\text{Pair } \sim) ps).2 \\
\quad \quad \quad \bullet \text{Seq } s\}) \\
\mathbf{MTrParPred} \quad \vdash \text{MTrParPred} \in \text{SET_EVENT_VAL} \rightarrow \text{REL_PREDICAT} \\
\quad \wedge (\forall cs : \text{SET_EVENT_VAL}
\end{array}$$

• $MTrParPred\ cs$
 $= =_{+R}$
 $(\{tr, dash\ tr\},$
 Rel
 $((- \in_R -),$
 Fun_2
 $((- SeqDif_R -),$
 $Var\ (dash\ tr),$
 $Var\ tr),$
 Fun_2
 $(MTrPar,$
 Fun_2
 $(MkPair,$
 Fun_2
 $((- SeqDif_R -),$
 $Var\ (one\ tr),$
 $Var\ tr),$
 Fun_2
 $((- SeqDif_R -),$
 $Var\ (two\ tr),$
 $Var\ tr)),$
 $Val\ cs)),$
 $Val\ (Bool\ true)))$
 $MSync$ $\vdash MSync \in SET_EVENT_VAL \rightarrow REL_PREDICATE$
 $\wedge (\forall cs : SET_EVENT_VAL$
 • $MSync\ cs$
 $= =_{+R}$
 $(\{one\ tr, two\ tr\},$
 $Fun_2\ ((- \vdash_R -), Var\ (one\ tr), Val\ cs),$
 $Fun_2\ ((- \vdash_R -), Var\ (two\ tr), Val\ cs)))$
 $BranchesWaiting$
 $\vdash BranchesWaiting \in REL_PREDICATE$
 $\wedge BranchesWaiting$
 $= =_R (\{one\ wait\}, one\ wait, Val\ (Bool\ true))$
 $\vee_R =_R$
 $(\{two\ wait\}, two\ wait, Val\ (Bool\ true))$
 $BranchesNotWaiting$
 $\vdash BranchesNotWaiting \in REL_PREDICATE$

$$\begin{array}{l}
\wedge \text{BranchesWaiting} \\
= =_R (\{one\ wait\}, one\ wait, Val\ (Bool\ false)) \\
\wedge_R =_R \\
(\{two\ wait\}, two\ wait, Val\ (Bool\ false)) \\
\mathbf{MRefPar} \quad \vdash MRefPar \in SET_EVENT_VAL \rightarrow REL_PREDICATE \\
\wedge (\forall cs : SET_EVENT_VAL \\
\bullet MRefPar\ cs \\
= =_{+R} \\
(\{dash\ ref\}, \\
Rel \\
((-\subseteq_R -), \\
Var\ (dash\ ref), \\
Fun_2 \\
((-\cup_R -), \\
Fun_2 \\
((-\cap_R -), \\
Fun_2 \\
((-\cup_R -), \\
Var\ (one\ ref), \\
Var\ (two\ ref)), \\
Val\ cs), \\
Fun_2 \\
((-\backslash_R -), \\
Fun_2 \\
((-\cap_R -), \\
Var\ (one\ ref), \\
Var\ (two\ ref)), \\
Val\ cs))))), \\
Val\ (Bool\ true))) \\
\mathbf{MSt} \quad \vdash MSt \in ALPHABET \times ALPHABET \times ALPHABET \rightarrow REL_PREDICATE \\
\wedge (\forall ns1, ns2, st : ALPHABET \\
\bullet st = \emptyset \\
\Rightarrow MSt\ (st, ns1, ns2) = True_R \emptyset \wedge st \neq \emptyset \\
\Rightarrow (\exists n : st \\
\bullet n \in ns1 \\
\Rightarrow MSt\ (st, ns1, ns2) \\
= =_R \\
(\{dash\ n, one\ n\},
\end{array}$$

$$\begin{aligned}
& \text{dash } n, \\
& \text{Var } (\text{one } n)) \\
& \wedge_R \text{MSt } (st \setminus \{n\}, ns1, ns2) \\
& \wedge n \in ns2 \\
& \Rightarrow \text{MSt } (st, ns1, ns2) \\
& = =_R \\
& (\{\text{dash } n, \text{two } n\}, \\
& \text{dash } n, \\
& \text{Var } (\text{two } n)) \\
& \wedge_R \text{MSt } (st \setminus \{n\}, ns1, ns2) \\
& \wedge n \notin ns1 \cup ns2 \\
& \Rightarrow \text{MSt } (st, ns1, ns2) \\
& = =_R (\{\text{dash } n, n\}, \text{dash } n, \text{Var } n) \\
& \wedge_R \text{MSt } (st \setminus \{n\}, ns1, ns2))) \\
\mathbf{MWtRefStPar} & \vdash \text{MWtRefStPar} \\
& \in \text{SET_EVENT_VAL} \times \text{ALPHABET} \times \text{ALPHABET} \times \text{ALPHABET} \\
& \rightarrow \text{REL_PREDICATE} \\
& \wedge (\forall ns1, ns2, st : \text{ALPHABET}; cs : \text{SET_EVENT_VAL} \\
& \bullet \text{MWtRefStPar } (cs, st, ns1, ns2) \\
& = \text{BranchesWaiting} \\
& \wedge_R \text{MRefPar } cs \triangleleft_R \\
& =_R \\
& (\{\text{dash } wait\}, \\
& \text{dash } wait, \\
& \text{Val } (\text{Bool } true)) \triangleright_R \\
& \text{BranchesNotWaiting} \\
& \wedge_R \text{MSt } (st, ns1, ns2)) \\
\mathbf{MPar} & \vdash \text{MPar} \\
& \in \text{SET_EVENT_VAL} \times \text{ALPHABET} \times \text{ALPHABET} \times \text{ALPHABET} \\
& \rightarrow \text{REL_PREDICATE} \\
& \wedge (\forall ns1, ns2, st : \text{ALPHABET}; cs : \text{SET_EVENT_VAL} \\
& \bullet \text{MPar } (cs, st, ns1, ns2) \\
& = \text{MTrParPred } cs \\
& \wedge_R \text{MSync } cs \\
& \wedge_R \text{MWtRefStPar } (cs, st, ns1, ns2)) \\
\mathbf{DivPar} & \vdash \text{DivPar} \\
& \in \text{CSP_PROCESS} \times \text{CSP_PROCESS} \times \text{SET_EVENT_VAL} \\
& \rightarrow \text{REL_PREDICATE}
\end{aligned}$$

$$\begin{aligned}
& \wedge (\forall a1, a2 : CSP_PROCESS; cs : SET_EVENT_VAL \\
& \bullet DivPar (a1, a2, cs) \\
& = \exists_{-R} \\
& \quad (\{dash (one \ tr), dash (two \ tr)\}, \\
& \quad \quad (((a1 \ \omega_f) \ \sigma_f) \\
& \quad \quad \quad ;_C =_R \\
& \quad \quad \quad (a1.1, dash (one \ tr), Var \ tr)) \\
& \quad \quad \wedge_R ((a2 \ \omega_f) \\
& \quad \quad \quad ;_C =_R \\
& \quad \quad \quad (a2.1, \\
& \quad \quad \quad \quad dash (two \ tr), \\
& \quad \quad \quad \quad Var \ tr)) \\
& \quad \quad \wedge_R MSync \ cs)) \\
- \llbracket_C - \rrbracket_C - & \vdash (- \llbracket_C - \rrbracket_C -) \\
& \in CSP_PROCESS \\
& \quad \times (ALPHABET \times SET_EVENT_VAL \times ALPHABET) \\
& \quad \times CSP_PROCESS \\
& \quad \rightarrow CSP_PROCESS \\
& \wedge (\forall a1, a2 : CSP_PROCESS; \\
& \quad cs : SET_EVENT_VAL; \\
& \quad ns1, ns2 : ALPHABET \\
& \mid (a1, a2) \in WF_CSP_PROCESS_PAIR \\
& \quad \wedge (ns1 \cap dashed = ns2 \cap dashed \\
& \quad \wedge ns2 \cap dashed = \emptyset) \\
& \quad \wedge ns1 \cap ns2 = \emptyset \\
& \bullet a1 \llbracket_C (ns1, cs, ns2) \rrbracket_C a2 \\
& = R \\
& \quad ((\neg_R (DivPar (a1, a2, cs)) \\
& \quad \quad \wedge_R \neg_R (DivPar (a2, a1, cs))) \\
& \quad \vdash_D (((((a1 \ \omega_f) \ \sigma_t) \\
& \quad \quad ;_C \ U \ (one, out_a \ a1.1)) \\
& \quad \quad \wedge_R ((a2 \ \omega_f) \ \sigma_t) \\
& \quad \quad ;_C \ U \ (two, out_a \ a2.1)) \\
& \quad +_R \ \{tr\} \\
& \quad \cup (a1.1 \\
& \quad \quad \setminus (ALPHABET_OWTR \\
& \quad \quad \cup dashed)))
\end{aligned}$$

$$\begin{aligned}
& \text{;}_C \text{ } MPar \\
& \quad (cs, \\
& \quad \quad a1.1 \\
& \quad \quad \quad \setminus (ALPHABET_OWTR \\
& \quad \quad \quad \cup dashed), \\
& \quad \quad ns1, \\
& \quad \quad ns2))) \\
MTrInterPred & \vdash MTrInterPred \in REL_PREDICATE \\
& \wedge MTrInterPred \\
& = =_{+R} \\
& \quad (\{tr, dash \ tr\}, \\
& \quad \quad Rel \\
& \quad \quad ((- \in_R -), \\
& \quad \quad \quad Fun_2 \\
& \quad \quad \quad ((- SeqDif_R -), \\
& \quad \quad \quad \quad Var (dash \ tr), \\
& \quad \quad \quad \quad Var \ tr), \\
& \quad \quad \quad Fun_2 \\
& \quad \quad \quad (MTrInter, \\
& \quad \quad \quad \quad Fun_2 \\
& \quad \quad \quad \quad ((- SeqDif_R -), \\
& \quad \quad \quad \quad \quad Var (one \ tr), \\
& \quad \quad \quad \quad \quad Var \ tr), \\
& \quad \quad \quad \quad Fun_2 \\
& \quad \quad \quad \quad ((- SeqDif_R -), \\
& \quad \quad \quad \quad \quad Var (two \ tr), \\
& \quad \quad \quad \quad \quad Var \ tr))), \\
& \quad \quad Val (Bool \ true)) \\
MRefInter & \vdash MRefInter \in REL_PREDICATE \\
& \wedge MRefInter \\
& = =_{+R} \\
& \quad (\{dash \ ref\}, \\
& \quad \quad Rel \\
& \quad \quad ((- \subseteq_R -), \\
& \quad \quad \quad Var (dash \ ref), \\
& \quad \quad \quad Fun_2 \\
& \quad \quad \quad ((- \cap_R -), \\
& \quad \quad \quad \quad Var (one \ ref),
\end{aligned}$$

$$\begin{array}{l}
\text{Var } (two \text{ ref}))), \\
\text{Val } (Bool \text{ true})) \\
\mathbf{MWtRefStInter} \quad \vdash \text{MWtRefStInter} \\
\quad \in ALPHABET \times ALPHABET \times ALPHABET \rightarrow REL_PREDICATE \\
\quad \wedge (\forall ns1, ns2, st : ALPHABET \\
\quad \bullet \text{MWtRefStInter } (st, ns1, ns2) \\
\quad = \text{BranchesNotWaiting} \\
\quad \wedge_R \text{MRefInter} \triangleleft_R \\
\quad =_R \\
\quad \quad (\{dash \text{ wait}\}, \\
\quad \quad \quad dash \text{ wait}, \\
\quad \quad \quad \text{Val } (Bool \text{ true})) \triangleright_R \\
\quad \quad \text{BranchesWaiting} \wedge_R \text{MSt } (st, ns1, ns2)) \\
\mathbf{MInter} \quad \vdash \text{MInter} \\
\quad \in ALPHABET \times ALPHABET \times ALPHABET \rightarrow REL_PREDICATE \\
\quad \wedge (\forall ns1, ns2, st : ALPHABET \\
\quad \bullet \text{MInter } (st, ns1, ns2) \\
\quad = \text{MTrInterPred} \\
\quad \wedge_R \text{MWtRefStInter } (st, ns1, ns2)) \\
\mathbf{DivInter} \quad \vdash \text{DivInter} \in CSP_PROCESS \times CSP_PROCESS \rightarrow REL_PREDICATE \\
\quad \wedge (\forall a1, a2 : CSP_PROCESS \\
\quad \bullet \text{DivInter } (a1, a2) \\
\quad = \exists_{-R} \\
\quad \quad (\{dash \text{ (one tr)}, dash \text{ (two tr)}\}, \\
\quad \quad \quad (((a1 \ \omega_f) \ \sigma_f) \\
\quad \quad \quad \quad ;_C =_R \\
\quad \quad \quad \quad (a1.1, dash \text{ (one tr)}, Var \text{ tr})) \\
\quad \quad \quad \wedge_R (a2 \ \omega_f) \\
\quad \quad \quad \quad ;_C =_R \\
\quad \quad \quad \quad (a2.1, \\
\quad \quad \quad \quad \quad dash \text{ (two tr)}, \\
\quad \quad \quad \quad \quad Var \text{ tr}))) \\
- \llbracket_C - \rrbracket_C - \quad \vdash (- \llbracket_C - \rrbracket_C -) \\
\quad \in CSP_PROCESS \\
\quad \times (ALPHABET \times ALPHABET) \\
\quad \times CSP_PROCESS
\end{array}$$

$$\begin{aligned}
& \rightarrow CSP_PROCESS \\
& \wedge (\forall a1, a2 : CSP_PROCESS; ns1, ns2 : ALPHABET \\
& \quad | (a1, a2) \in WF_CSP_PROCESS_PAIR \wedge ns1 \cap ns2 = \emptyset \\
& \quad \bullet a1 \llbracket_C (ns1, ns2) \rrbracket_C a2 \\
& \quad = R \\
& \quad ((\neg_R (DivInter (a1, a2)) \\
& \quad \quad \wedge_R \neg_R (DivInter (a2, a1))) \\
& \quad \quad \vdash_D ((((((a1 \omega_f) \sigma_t) \\
& \quad \quad \quad ;_C U (one, out_a a1.1)) \\
& \quad \quad \quad \wedge_R ((a2 \omega_f) \sigma_t) \\
& \quad \quad \quad ;_C U (two, out_a a1.1)) \\
& \quad \quad \quad +_R \{tr\} \\
& \quad \quad \quad \cup (a1.1 \\
& \quad \quad \quad \quad \setminus (ALPHABET_OWTR \\
& \quad \quad \quad \quad \cup dashed))) \\
& \quad \quad \quad \wedge_R \Pi_R (a1.1 \setminus ALPHABET_OWTR)) \\
& \quad \quad ;_C MInter \\
& \quad \quad (a1.1 \setminus ALPHABET_OWTR \setminus dashed, \\
& \quad \quad \quad ns1, \\
& \quad \quad \quad ns2))) \\
- \setminus_C - & \vdash (- \setminus_C -) \in CSP_PROCESS \times SET_EVENT_VAL \rightarrow CSP_PROCESS \\
& \wedge (\forall a : CSP_PROCESS; cs : SET_EVENT_VAL \\
& \quad \bullet \exists s : VAR_NAME \\
& \quad \quad | s \notin a.1 \\
& \quad \bullet a \setminus_C cs \\
& \quad = R \\
& \quad (\exists_{-R} \\
& \quad \quad (\{s\}, \\
& \quad \quad \quad /_R \\
& \quad \quad \quad (/_R \\
& \quad \quad \quad \quad (a \oplus_R \{s\}, \\
& \quad \quad \quad \quad \quad Var s, \\
& \quad \quad \quad \quad \quad dash tr), \\
& \quad \quad \quad Fun_2 \\
& \quad \quad \quad ((- \cup_R -), \\
& \quad \quad \quad \quad Val cs, \\
& \quad \quad \quad \quad \quad Var ref), \\
& \quad \quad \quad dash ref)
\end{aligned}$$

$$\begin{array}{l}
\wedge_R =_{+R} \\
(\{tr, \text{dash } tr, s\}, \\
\text{Fun}_2 \\
((- \text{SeqDif}_R -), \\
\text{Var } (\text{dash } tr), \\
\text{Var } tr), \\
\text{Fun}_2 \\
((- \setminus_R -), \\
\text{Val} \\
(\text{Set} \\
\text{EVENT_VAL}), \\
\text{Val } cs)))) \\
;_C \text{ Skip } a.1) \\
\mu_C \vdash \mu_C \in \text{CIRCUS_FUNCTION} \rightarrow \text{CSP_PROCESS} \\
\wedge (\forall f : \text{CIRCUS_FUNCTION} \bullet \mu_C f = \mu_R f) \\
\text{Assign}_C \vdash \text{Assign}_C \in \text{WF_Assign}_C \rightarrow \text{CSP_PROCESS} \\
\wedge (\forall a : \text{ALPHABET}; \\
ns : \text{seq VAR_NAME}; \\
exps : \text{seq EXPRESSION} \\
| (a, ns, exps) \in \text{WF_Assign}_C \\
\bullet \text{Assign}_C (a, ns, exps) \\
= R \\
(\text{True}_R a \\
\vdash_D \text{Assign}_R (a, ns, exps) \\
\wedge_R =_R (a, \text{dash } tr, \text{Var } tr) \\
\wedge_R =_R \\
(a, \\
\text{dash } wait, \\
\text{Val } (\text{Bool } false)))) \\
\text{SpecStatement}_C \vdash \text{SpecStatement}_C \in \text{WF_SpecStatement}_C \rightarrow \text{CSP_PROCESS} \\
\wedge (\forall a : \text{ALPHABET}; \\
f : \mathbb{F} \text{ VAR_NAME}; \\
preC : \text{CIRCUS_CONDITION}; \\
postC : \text{CIRCUS_PREDICATE} \\
| (a, f, preC, postC) \in \text{WF_SpecStatement}_C \\
\bullet \text{SpecStatement}_C (a, f, preC, postC) \\
= R
\end{array}$$

$$\begin{array}{l}
\begin{array}{l}
(preC \\
\vdash_D postC \\
\wedge_R =_R (a, dash\ tr, Var\ tr) \\
\wedge_R =_R \\
(a, \\
dash\ wait, \\
Val\ (Bool\ false)) \\
\wedge_R \Pi_R \\
(a \\
\setminus (ALPHABET_OWTR \\
\cup (f \\
\cup \{n : f \\
\bullet dash \\
n\}))))))
\end{array} \\
\{C - \}_C \quad \vdash (\{C - \}_C) \in WF_Condition_C \rightarrow CSP_PROCESS \\
\quad \wedge (\forall a : ALPHABET; g : CIRCUS_CONDITION \\
\quad | (a, g) \in WF_Condition_C \\
\quad \bullet \{C (a, g) \}_C \\
\quad = SpecStatement_C (a, \{\}, g, True_R\ a)) \\
\langle C - \rangle_C \quad \vdash (\langle C - \rangle_C) \in WF_Condition_C \rightarrow CSP_PROCESS \\
\quad \wedge (\forall a : ALPHABET; g : CIRCUS_CONDITION \\
\quad | (a, g) \in WF_Condition_C \\
\quad \bullet \langle C (a, g) \rangle_C = R (True_R\ a \vdash_D g)) \\
\textbf{Typing} \quad \vdash Typing \in VAR_DECLS \times ALPHABET \leftrightarrow REL_PREDICATE \\
\quad \wedge (\forall decls : VAR_DECLS; a : ALPHABET \\
\quad | ran\ decls.1 \subseteq a \\
\quad \bullet \# decls.1 = 1 \\
\quad \wedge (\exists n : Z_VAR_NAME; e : EXPRESSION \\
\quad | n = head\ decls.1 \wedge e = head\ decls.2 \\
\quad \bullet Typing (decls, a) \\
\quad = =_{+R} \\
\quad (a, \\
\quad Rel\ ((- \in_R -), Var\ n, e), \\
\quad Val\ (Bool\ true))) \\
\quad \vee \# decls.1 > 1 \\
\quad \wedge (\exists n : Z_VAR_NAME; e : EXPRESSION \\
\quad | n = head\ decls.1 \wedge e = head\ decls.2 \\
\quad \bullet Typing (decls, a)
\end{array}$$

$$\begin{aligned}
&= =_{+R} \\
&\quad (a, \\
&\quad \quad Rel \ ((- \in_R -), Var \ n, e), \\
&\quad \quad Val \ (Bool \ true)) \\
&\quad \wedge_R \ Typing \\
&\quad \quad ((tail \ decls.1, tail \ decls.2), \\
&\quad \quad \quad a))) \\
\mathbf{SchemaExp}_C &\vdash SchemaExp_C \in WF_SchemaExp_C \rightarrow CSP_PROCESS \\
&\quad \wedge (\forall \ decls : VAR_DECLS; p : REL_PREDICATE \\
&\quad \quad | (decls, p) \in WF_SchemaExp_C \\
&\quad \quad \bullet \exists f : \mathbb{F} \ Z_VAR_NAME \\
&\quad \quad \quad | f \subseteq undashed \\
&\quad \quad \quad \wedge ran \ (decls.1 \upharpoonright dashed) = dash \ (f) \\
&\quad \quad \bullet SchemaExp_C \ (decls, p) \\
&\quad \quad = SpecStatement_C \\
&\quad \quad \quad (ran \ decls.1 \cup ALPHABET_OWTR, \\
&\quad \quad \quad \quad f, \\
&\quad \quad \quad \quad \exists_{-R} \\
&\quad \quad \quad \quad \quad (ran \ decls.1 \setminus undashed, \\
&\quad \quad \quad \quad \quad \quad Typing \ (decls, p.1) \wedge_R p), \\
&\quad \quad \quad \quad Typing \ (decls, p.1) \wedge_R p)) \\
\mathbf{TrueGuards} &\vdash TrueGuards \in GUARDED_ACTIONS \rightarrow REL_PREDICATE \\
&\quad \wedge (\forall \ gactions : GUARDED_ACTIONS \\
&\quad \quad \bullet \# \ gactions.1 = 1 \\
&\quad \quad \quad \wedge (\exists \ g : CIRCUS_CONDITION; a : CSP_PROCESS \\
&\quad \quad \quad \quad | g = head \ gactions.1 \wedge a = head \ gactions.2 \\
&\quad \quad \quad \quad \bullet TrueGuards \ gactions = g) \\
&\quad \quad \vee \# \ gactions.1 > 1 \\
&\quad \quad \quad \wedge (\exists \ g : CIRCUS_CONDITION; a : CSP_PROCESS \\
&\quad \quad \quad \quad | g = head \ gactions.1 \wedge a = head \ gactions.2 \\
&\quad \quad \quad \quad \bullet TrueGuards \ gactions \\
&\quad \quad \quad \quad = g \\
&\quad \quad \quad \quad \vee_R \ TrueGuards \\
&\quad \quad \quad \quad \quad (tail \ gactions.1, \\
&\quad \quad \quad \quad \quad \quad tail \ gactions.2))) \\
\mathbf{NonDivActions} &\vdash NonDivActions \in GUARDED_ACTIONS \rightarrow REL_PREDICATE \\
&\quad \wedge (\forall \ gactions : GUARDED_ACTIONS
\end{aligned}$$

$$\begin{aligned}
& \bullet \# \text{ gactions}.1 = 1 \\
& \quad \wedge (\exists g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \quad | g = \text{head gactions}.1 \wedge a = \text{head gactions}.2 \\
& \quad \quad \bullet \text{NonDivActions gactions} \\
& \quad \quad \quad = g \Rightarrow_R \neg_R ((a \omega_f) \sigma_f)) \\
& \vee \# \text{ gactions}.1 > 1 \\
& \quad \wedge (\exists g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \quad | g = \text{head gactions}.1 \wedge a = \text{head gactions}.2 \\
& \quad \quad \bullet \text{NonDivActions gactions} \\
& \quad \quad \quad = (g \Rightarrow_R \neg_R ((a \omega_f) \sigma_f)) \\
& \quad \quad \quad \wedge_R \text{NonDivActions} \\
& \quad \quad \quad \quad (\text{tail gactions}.1, \\
& \quad \quad \quad \quad \text{tail gactions}.2))) \\
\text{ExecActions} \vdash & \text{ExecActions} \in \text{GUARDED_ACTIONS} \rightarrow \text{REL_PREDICATE} \\
& \wedge (\forall \text{gactions} : \text{GUARDED_ACTIONS} \\
& \quad \bullet \# \text{ gactions}.1 = 1 \\
& \quad \quad \wedge (\exists g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \quad \quad | g = \text{head gactions}.1 \wedge a = \text{head gactions}.2 \\
& \quad \quad \quad \bullet \text{ExecActions gactions} = g \wedge_R (a \omega_f) \sigma_t) \\
& \vee \# \text{ gactions}.1 > 1 \\
& \quad \wedge (\exists g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \quad | g = \text{head gactions}.1 \wedge a = \text{head gactions}.2 \\
& \quad \quad \bullet \text{ExecActions gactions} \\
& \quad \quad \quad = (g \wedge_R (a \omega_f) \sigma_t) \\
& \quad \quad \quad \vee_R \text{ExecActions} \\
& \quad \quad \quad \quad (\text{tail gactions}.1, \\
& \quad \quad \quad \quad \text{tail gactions}.2))) \\
\text{if}_C - \text{fi}_C \vdash & (\text{if}_C - \text{fi}_C) \in \text{GUARDED_ACTIONS} \rightarrow \text{CSP_PROCESS} \\
& \wedge (\forall \text{gactions} : \text{GUARDED_ACTIONS} \\
& \quad \bullet \text{if}_C \text{ gactions fi}_C \\
& \quad \quad = R \\
& \quad \quad ((\text{TrueGuards gactions} \\
& \quad \quad \quad \wedge_R \text{NonDivActions gactions}) \\
& \quad \quad \quad \vdash_D \text{ExecActions gactions})) \\
\text{ifb}_C - \mapsto_C - \text{else}_C - \mapsto_C - \text{fib}_C \vdash & (\text{ifb}_C - \mapsto_C - \text{else}_C - \mapsto_C - \text{fib}_C) \\
& \in \text{CIRCUS_CONDITION} \\
& \quad \times \text{CSP_PROCESS}
\end{aligned}$$

$$\begin{aligned}
& \times \text{CIRCUS_CONDITION} \\
& \times \text{CSP_PROCESS} \\
& \rightarrow \text{CSP_PROCESS} \\
& \wedge (\forall g1, g2 : \text{CIRCUS_CONDITION}; \\
& \quad a1, a2 : \text{CSP_PROCESS} \\
& \quad | (\langle g1, g2 \rangle, \langle a1, a2 \rangle) \in \text{GUARDED_ACTIONS} \\
& \quad \bullet \text{ifb}_C g1 \mapsto_C a1 \text{ else}_C g2 \mapsto_C a2 \text{ fib}_C \\
& \quad = R \\
& \quad (((g1 \vee_R g2) \\
& \quad \quad \wedge_R (g1 \Rightarrow_R \neg_R ((a1 \omega_f) \sigma_f)) \\
& \quad \quad \wedge_R g2 \Rightarrow_R \neg_R ((a2 \omega_f) \sigma_f)) \\
& \quad \quad \vdash_D (g1 \wedge_R \neg_R ((a1 \omega_f) \sigma_t)) \\
& \quad \quad \vee_R g2 \wedge_R \neg_R ((a2 \omega_f) \sigma_t))) \\
\text{param}_C & \vdash \text{param}_C \in \text{WF_param}_C \rightarrow \text{CSP_PROCESS} \\
& \quad \wedge (\forall x : \text{VAR_NAME}; \\
& \quad \quad T : \text{EXPRESSION}; \\
& \quad \quad a : \text{CSP_PROCESS}; \\
& \quad \quad e : \text{EXPRESSION} \\
& \quad | (x, T, a, e) \in \text{WF_param}_C \\
& \quad \bullet \text{param}_C (x, T, a, e) = /_R (a, e, x)) \\
\text{val}_C & \vdash \text{val}_C \in \text{WF_val}_C \rightarrow \text{CSP_PROCESS} \\
& \quad \wedge (\forall x : \text{VAR_NAME}; \\
& \quad \quad t : \text{SET_VAL}; \\
& \quad \quad a : \text{CSP_PROCESS}; \\
& \quad \quad e : \text{EXPRESSION} \\
& \quad | (x, t, a, e) \in \text{WF_val}_C \\
& \quad \bullet \text{val}_C (x, t, a, e) \\
& \quad \quad = \text{var}_C (x, t, \text{Assign}_R (a.1, \langle x \rangle, \langle e \rangle) ;_C a)) \\
\text{res}_C & \vdash \text{res}_C \in \text{WF_res}_C \rightarrow \text{CSP_PROCESS} \\
& \quad \wedge (\forall x : \text{VAR_NAME}; \\
& \quad \quad t : \text{SET_VAL}; \\
& \quad \quad a : \text{CSP_PROCESS}; \\
& \quad \quad y : \text{VAR_NAME} \\
& \quad | (x, t, a, y) \in \text{WF_res}_C \\
& \quad \bullet \text{res}_C (x, t, a, y) \\
& \quad \quad = \text{var}_C \\
& \quad \quad \quad (x, t, a ;_R \text{Assign}_R (a.1, \langle y \rangle, \langle \text{Var } x \rangle))) \\
\text{vres}_C & \vdash \text{vres}_C \in \text{WF_vres}_C \rightarrow \text{CSP_PROCESS}
\end{aligned}$$

$$\begin{aligned}
& \wedge (\forall x : VAR_NAME; \\
& \quad t : SET_VAL; \\
& \quad a : CSP_PROCESS; \\
& \quad y : VAR_NAME \\
& \quad | (x, t, a, y) \in WF_vres_C \\
& \quad \bullet vres_C (x, t, a, y) \\
& \quad = var_C \\
& \quad \quad (x, \\
& \quad \quad t, \\
& \quad \quad Assign_R (a.1, \langle x \rangle, \langle Var\ y \rangle) \\
& \quad \quad \quad ;_R a \\
& \quad \quad \quad ;_R Assign_R \\
& \quad \quad \quad (a.1, \langle y \rangle, \langle Var\ x \rangle))) \\
C1 & \quad \vdash C1 \in CSP_PROCESS \rightarrow CSP_PROCESS \\
& \quad \quad \wedge (\forall a : CSP_PROCESS \bullet C1\ a = a ;_R Skip\ a.1) \\
C1_healthy & \quad \vdash C1_healthy \in \mathbb{P}\ CSP_PROCESS \\
& \quad \quad \wedge C1_healthy = \{a : CSP_PROCESS \mid a = C1\ a\} \\
C2 & \quad \vdash C2 \in CSP_PROCESS \rightarrow CSP_PROCESS \\
& \quad \quad \wedge (\forall a : CSP_PROCESS \\
& \quad \quad \bullet C2\ a = a \llbracket_C (a.1, \emptyset) \rrbracket_C Skip\ a.1) \\
C2_healthy & \quad \vdash C2_healthy \in \mathbb{P}\ CSP_PROCESS \\
& \quad \quad \wedge C2_healthy = \{a : CSP_PROCESS \mid a = C2\ a\} \\
C3 & \quad \vdash C3 \in CSP_PROCESS \rightarrow CSP_PROCESS \\
& \quad \quad \wedge (\forall a : CSP_PROCESS \\
& \quad \quad \bullet C3\ a \\
& \quad \quad = R \\
& \quad \quad \quad ((\neg_R ((a\ \sigma_f)\ \omega_f) ;_R True_R\ a.1) \\
& \quad \quad \quad \vdash_D (a\ \sigma_t)\ \omega_f)) \\
C3_healthy & \quad \vdash C3_healthy \in \mathbb{P}\ CSP_PROCESS \\
& \quad \quad \wedge C3_healthy = \{a : CSP_PROCESS \mid a = C3\ a\}
\end{aligned}$$

14.5 Definitions

$$\begin{aligned}
Z_VAR_NAME & \quad \vdash Z_VAR_NAME = \{n : NAME \mid n \notin ALPHABET_OWT\} \\
VAR_DECLS & \quad \vdash VAR_DECLS \\
& = \{vars : seq\ Z_VAR_NAME; types : seq\ EXPRESSION \\
& \quad | dom\ vars \in (\mathbb{F}\ -) \\
& \quad \wedge dom\ types \in (\mathbb{F}\ -)\}
\end{aligned}$$

$$\begin{aligned}
& \wedge \#(\text{ran vars}) = \#(\text{ran types}) \\
& \wedge \#(\text{ran types}) > 0\} \\
\mathbf{WF_Skip}_C & \vdash \mathbf{WF_Skip}_C = \{a : \mathbf{WF_Skip}_R \mid \text{ALPHABET_OWTR} \subseteq a\} \\
\mathbf{WF_Guard}_C & \vdash \mathbf{WF_Guard}_C \\
& = \{g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \mid g.1 = a.1\} \\
\mathbf{WF_if}_{C_fiC} & \vdash \mathbf{WF_if}_{C_fiC} \\
& = \{g : \text{CIRCUS_CONDITION}; a : \text{CSP_PROCESS} \\
& \quad \mid g.1 = a.1\} \\
\mathbf{WF_Semi}_C & \vdash \mathbf{WF_Semi}_C \\
& = \{a1, a2 : \text{CSP_PROCESS} \\
& \quad \mid (a1, a2) \in \mathbf{WF_Semi}_R\} \\
\mathbf{WF_Var}_C & \vdash \mathbf{WF_Var}_C \\
& = \{n : \text{VAR_NAME}; t : \text{SET_VAL}; a : \text{CSP_PROCESS} \\
& \quad \mid n \notin \text{ALPHABET_OWTR} \\
& \quad \wedge n \in \text{undashed} \\
& \quad \wedge \{n, \text{dash } n\} \subseteq a.1\} \\
\mathbf{WF_Assign}_C & \vdash \mathbf{WF_Assign}_C \\
& = \{a : \text{ALPHABET}; \\
& \quad ns : \text{seq VAR_NAME}; \\
& \quad \text{exps} : \text{seq EXPRESSION} \\
& \quad \mid (\forall n : \text{ran } ns \bullet n \in a \wedge n \in \text{undashed}) \\
& \quad \wedge (\forall e : \text{ran } \text{exps} \bullet \text{FV } e \subseteq a \wedge \text{FV } e \subseteq \text{undashed}) \\
& \quad \wedge (\# ns = \# \text{exps} \\
& \quad \wedge \# \text{exps} \neq 0) \\
& \quad \wedge a \in \text{homogeneous} \\
& \quad \wedge \text{ALPHABET_OWTR} \subseteq a\} \\
\mathbf{WF_SpecStatement}_C & \\
& \vdash \mathbf{WF_SpecStatement}_C \\
& = \{a : \text{ALPHABET}; \\
& \quad f : \mathbb{F} \text{ VAR_NAME}; \\
& \quad \text{preC} : \text{CIRCUS_CONDITION}; \\
& \quad \text{postC} : \text{CIRCUS_PREDICATE} \\
& \quad \mid f \subseteq a \\
& \quad \wedge f \subseteq \text{undashed} \\
& \quad \wedge a \in \text{homogeneous} \\
& \quad \wedge (\text{preC}.1 = \text{postC}.1 \\
& \quad \wedge \text{postC}.1 = a)
\end{aligned}$$

$$\wedge \text{ALPHABET_OWTR} \subseteq a\}$$

WF_Condition_C

$$\vdash \text{WF_Condition}_C$$

$$= \{a : \text{ALPHABET}; g : \text{CIRCUS_CONDITION}$$

$$| a \in \text{homogeneous} \wedge a = g.1\}$$

WF_SchemaExp_C

$$\vdash \text{WF_SchemaExp}_C$$

$$= \{\text{decls} : \text{VAR_DECLS}; p : \text{REL_PREDICATE}$$

$$| p.1 \setminus \text{ALPHABET_OWTR} = \text{ran decls}.1\}$$

WF_param_C

$$\vdash \text{WF_param}_C$$

$$= \{x : \text{VAR_NAME};$$

$$T : \text{EXPRESSION};$$

$$a : \text{CSP_PROCESS};$$

$$e : \text{EXPRESSION}$$

$$| x \in a.1 \wedge \text{FV } e \subseteq a.1\}$$

WF_val_C

$$\vdash \text{WF_val}_C$$

$$= \{x : \text{VAR_NAME};$$

$$t : \text{SET_VAL};$$

$$a : \text{CSP_PROCESS};$$

$$e : \text{EXPRESSION}$$

$$| x \notin \text{FV } e \wedge x \in a.1 \wedge \text{FV } e \subseteq a.1\}$$

WF_res_C

$$\vdash \text{WF_res}_C$$

$$= \{x : \text{VAR_NAME};$$

$$t : \text{SET_VAL};$$

$$a : \text{CSP_PROCESS};$$

$$y : \text{VAR_NAME}$$

$$| \{x, y\} \subseteq a.1\}$$

WF_vres_C

$$\vdash \text{WF_vres}_C$$

$$= \{x : \text{VAR_NAME};$$

$$t : \text{SET_VAL};$$

$$a : \text{CSP_PROCESS};$$

$$y : \text{VAR_NAME}$$

$$| x \neq y \wedge \{x, y\} \subseteq a.1\}$$

WF_PREFIXING_C

$$\vdash \text{WF_PREFIXING}_C$$

$$= \{a_n_e : \text{WF_PREFIXING}; A : \text{CSP_PROCESS}$$

$$| a_n_e.1 = A.1\}$$

WF_PREFIXING_{CinR}

$$\begin{aligned}
& \vdash \text{WF_PREFIXING}_{CinR} \\
& = \{a : \text{ALPHABET_REA}; \\
& \quad c : \text{VAR_NAME}; \\
& \quad x : \text{VAR_NAME}; \\
& \quad p : \text{CIRCUS_CONDITION}; \\
& \quad A : \text{CSP_PROCESS} \\
& \quad | a = A.1 \wedge \{x, \text{dash } x\} \subseteq a\} \\
\mathbf{CIRCUS_FUNCTION} \\
& \vdash \text{CIRCUS_FUNCTION} \\
& = \{f : \text{CSP_PROCESS} \leftrightarrow \text{CSP_PROCESS} \\
& \quad | \exists a : \text{ALPHABET} \\
& \quad \bullet \forall u_dom : \text{dom } f; u_ran : \text{ran } f \\
& \quad \bullet a = u_dom.1 \wedge u_dom.1 = u_ran.1\} \\
\mathbf{GUARDED_ACTIONS} \\
& \vdash \text{GUARDED_ACTIONS} \\
& = \{\text{conditions} : \text{seq } \text{CIRCUS_CONDITION}; \\
& \quad \text{actions} : \text{seq } \text{CSP_PROCESS} \\
& \quad | \text{dom conditions} \in (\mathbb{F} \text{ } _) \\
& \quad \wedge \text{dom actions} \in (\mathbb{F} \text{ } _) \\
& \quad \wedge (\# (\text{ran conditions}) = \# (\text{ran actions}) \\
& \quad \wedge \# (\text{ran actions}) > 0) \\
& \quad \wedge (\exists a : \text{ALPHABET} \\
& \quad \bullet (\forall c : \text{ran conditions} \bullet c.1 = a) \\
& \quad \wedge (\forall A : \text{ran actions} \bullet A.1 = a))\} \\
\mathbf{CIRCUS_ACTION} \\
& \vdash \text{CIRCUS_ACTION} \\
& = \{c : \text{CSP_PROCESS} \\
& \quad | c \in C1_healthy \\
& \quad \wedge c \in C2_healthy \\
& \quad \wedge c \in C3_healthy\}
\end{aligned}$$

14.6 Theorems

$C_ALPHABET_WF_Skip_C$ -thm

$$\vdash \forall a : \text{WF_Skip}_C \bullet a \in \text{WF_Skip}_R \wedge a \in \text{ALPHABET_REA}$$

$C_PREDICATE_Stop_C$ -thm

$$\vdash \forall a : \text{WF_Skip}_C \bullet \text{Stop } a \in \text{CSP_PROCESS}$$

$CIRCUS_ACTION_ \in \text{CSP_PROCESS}$ -thm

$$\begin{aligned}
& \vdash \forall a : CIRCUS_ACTION \bullet a \in CSP_PROCESS \\
C_PREDICATE_Stop_C_alphabet_thm \\
& \vdash \forall a : WF_Skip_C \bullet (Stop\ a).1 = a \\
C_Stop_C_diverge_not_wait_thm \\
& \vdash \forall a : WF_Skip_C \\
& \quad \bullet (Stop\ a\ \omega_f)\ \sigma_f \\
& \quad =_R (a, okay, Val\ (Bool\ false)) \\
& \quad \wedge_R =_{+R} \\
& \quad \quad (ALPHABET_OWTR, \\
& \quad \quad Rel\ ((- \leq_R -), Var\ tr, Var\ (dash\ tr)), \\
& \quad \quad Val\ (Bool\ true)) \\
C_Stop_C_converge_not_wait_thm \\
& \vdash \forall a : WF_Skip_C \\
& \quad \bullet (Stop\ a\ \omega_f)\ \sigma_t \\
& \quad = CSP1 \\
& \quad \quad (=_R (ALPHABET_OWTR, dash\ tr, Var\ tr) \\
& \quad \quad \wedge_R =_R (a, dash\ wait, Val\ (Bool\ true))) \\
C_true_guard_thm1 \\
& \vdash \forall a : WF_Skip_C; A : CSP_PROCESS \\
& \quad | (True_R\ a, A) \in WF_Guard_C \\
& \quad \bullet True_R\ a \ \&_C\ A = A \\
C_true_guard_thm \\
& \vdash \forall a : WF_Skip_C; A : CIRCUS_ACTION \\
& \quad | (True_R\ a, A) \in WF_Guard_C \\
& \quad \bullet True_R\ a \ \&_C\ A = A \\
C_false_guard_thm1 \\
& \vdash \forall a : WF_Skip_C; A : CSP_PROCESS \\
& \quad | (False_R\ a, A) \in WF_Guard_C \\
& \quad \bullet False_R\ a \ \&_C\ A = Stop\ a \\
C_false_guard_thm \\
& \vdash \forall a : WF_Skip_C; A : CIRCUS_ACTION \\
& \quad | (False_R\ a, A) \in WF_Guard_C \\
& \quad \bullet False_R\ a \ \&_C\ A = Stop\ a \\
C_assumption_unit_thm \\
& \vdash \forall a : WF_Skip_C \bullet \{_C\ (a, True_R\ a)\}_C = Skip\ a \\
C_assumption_zero_thm \\
& \vdash \forall a : WF_Skip_C \bullet \{_C\ (a, False_R\ a)\}_C = Chaos\ a \\
C_∩_idem_thm1
\end{aligned}$$

$$\begin{array}{l}
\vdash \forall A : \text{CSP_PROCESS} \bullet A \cap_C A = A \\
\textbf{C_}\cap_C\textbf{-idem_thm} \\
\vdash \forall A : \text{CIRCUS_ACTION} \bullet A \cap_C A = A \\
\textbf{C_}\cap_C\textbf{-com_thm1} \\
\vdash \forall A1, A2 : \text{CSP_PROCESS} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet A1 \cap_C A2 = A2 \cap_C A1 \\
\textbf{C_}\cap_C\textbf{-com_thm} \\
\vdash \forall A1, A2 : \text{CIRCUS_ACTION} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet A1 \cap_C A2 = A2 \cap_C A1 \\
\textbf{C_CIRCUS_ACTION_}\in\textbf{-REL_PREDICATE_thm} \\
\vdash \forall A : \text{CIRCUS_ACTION} \bullet A \in \text{REL_PREDICATE} \\
\textbf{C_ALPHABET_OWTR_}\subseteq\textbf{-CIRCUS_ACTION_thm} \\
\vdash \forall A : \text{CIRCUS_ACTION} \bullet \text{ALPHABET_OWTR} \subseteq A.1 \\
\textbf{C_}\boxtimes_C\textbf{-com_thm1} \\
\vdash \forall A1, A2 : \text{CSP_PROCESS} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet A1 \boxtimes_C A2 = A2 \boxtimes_C A1 \\
\textbf{C_}\boxtimes_C\textbf{-com_thm} \\
\vdash \forall A1, A2 : \text{CIRCUS_ACTION} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet A1 \boxtimes_C A2 = A2 \boxtimes_C A1 \\
\textbf{C_}\cap_C\textbf{-refinement_thm1} \\
\vdash \forall A1, A2 : \text{CSP_PROCESS} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet (A1 \cap_C A2) \sqsubseteq_R A1 = \text{True}_R \emptyset \\
\textbf{C_}\cap_C\textbf{-refinement_thm} \\
\vdash \forall A1, A2 : \text{CIRCUS_ACTION} \\
\quad | (A1, A2) \in \text{WF_CSP_PROCESS_PAIR} \\
\quad \bullet (A1 \cap_C A2) \sqsubseteq_R A1 = \text{True}_R \emptyset \\
\textbf{C_Stop}_C\textbf{-}\boxtimes_C\textbf{-unit_thm1} \\
\vdash \forall a : \text{ALPHABET}; A : \text{CSP_PROCESS} \\
\quad | a \in \text{WF_Skip}_C \wedge a = A.1 \\
\quad \bullet \text{Stop } a \boxtimes_C A = A \\
\textbf{C_Stop}_C\textbf{-}\boxtimes_C\textbf{-unit_thm} \\
\vdash \forall a : \text{ALPHABET}; A : \text{CIRCUS_ACTION} \\
\quad | a \in \text{WF_Skip}_C \wedge a = A.1
\end{array}$$

$\bullet \text{ Stop } a \boxtimes_C A = A$
 $C_{\boxtimes_C} \text{idem_thm}$
 $\vdash \forall A : \text{CSP_PROCESS} \bullet A \boxtimes_C A = A$
 $C_{\text{Chaos_alphabet_thm}}$
 $\vdash \forall a : \text{ALPHABET} \mid a \in \text{WF_Skip}_C \bullet (\text{Chaos } a).1 = a$
 $C_{\text{Guard} \in \text{CSP_PROCESS_thm}}$
 $\vdash \forall g : \text{CIRCUS_CONDITION}; A : \text{CSP_PROCESS}$
 $\mid (g, A) \in \text{WF_Guard}_C$
 $\bullet g \&_C A \in \text{CSP_PROCESS}$
 $C_{\text{Guard}_C \cap_C \text{dist_thm}}$
 $\vdash \forall g : \text{CIRCUS_CONDITION}; A1, A2 : \text{CSP_PROCESS}$
 $\mid (g, A1) \in \text{WF_Guard}_C \wedge (g, A2) \in \text{WF_Guard}_C$
 $\bullet g \&_C A1 \cap_C A2 = (g \&_C A1) \cap_C g \&_C A2$
 $C_{\text{Guard} \cap_C \text{elim_thm}}$
 $\vdash \forall g : \text{CIRCUS_CONDITION}; A1, A2 : \text{CSP_PROCESS}$
 $\mid (A1, A2) \in \text{WF_CSP_PROCESS_PAIR}$
 $\wedge (g, A1) \in \text{WF_Guard}_C$
 $\bullet (g \&_C A1 \cap_C A2) \sqsubseteq_R g \&_C A1 = \text{True}_R \emptyset$
 $C_{\text{Semi}_C \cap_C \text{left_dist_thm}}$
 $\vdash \forall A1, A2, A3 : \text{CSP_PROCESS}$
 $\mid A1.1 = A2.1 \wedge A2.1 = A3.1 \wedge (A1, A2) \in \text{WF_Semi}_C$
 $\bullet A1 ;_C A2 \cap_C A3 = (A1 ;_C A2) \cap_C A1 ;_C A3$
 $C_{\text{Semi}_C \cap_C \text{left_dist_thm1}}$
 $\vdash \forall A1, A2, A3 : \text{CIRCUS_ACTION}$
 $\mid A1.1 = A2.1 \wedge A2.1 = A3.1 \wedge (A1, A2) \in \text{WF_Semi}_C$
 $\bullet A1 ;_C A2 \cap_C A3 = (A1 ;_C A2) \cap_C A1 ;_C A3$
 $C_{\rightarrow_C \cap_C \text{dist_thm}}$
 $\vdash \forall a : \text{ALPHABET};$
 $n : \text{VAR_NAME};$
 $e : \text{EXPRESSION};$
 $A1, A2 : \text{CSP_PROCESS}$
 $\mid A1.1 = A2.1 \wedge ((a, n, e), A1) \in \text{WF_PREFIXING}_C$
 $\bullet (a, n, e) \rightarrow_C A1 \cap_C A2$
 $= ((a, n, e) \rightarrow_C A1) \cap_C (a, n, e) \rightarrow_C A2$
 $C_{\rightarrow_{\text{CSync}} \cap_C \text{dist_thm}}$
 $\vdash \forall a : \text{ALPHABET}; n : \text{VAR_NAME}; A1, A2 : \text{CSP_PROCESS}$
 $\mid A1.1 = A2.1$
 $\wedge a = A1.1$

$$\begin{aligned}
& \wedge a \in WF_Skip_{REA} \\
& \wedge FV (Val Sync) \subseteq a \\
& \bullet (a, n) \rightarrow_{CSync} A1 \cap_C A2 \\
& = ((a, n) \rightarrow_{CSync} A1) \cap_C (a, n) \rightarrow_{CSync} A2 \\
\mathbf{C_} \rightarrow_{COut} \cap_{C_dist_thm} \\
& \vdash \forall a : ALPHABET; \\
& \quad n : VAR_NAME; \\
& \quad e : EXPRESSION; \\
& \quad A1, A2 : CSP_PROCESS \\
& \quad | A1.1 = A2.1 \wedge ((a, n, e), A1) \in WF_PREFIXING_C \\
& \bullet (a, n, e) \rightarrow_{Cout} A1 \cap_C A2 \\
& = ((a, n, e) \rightarrow_{Cout} A1) \cap_C (a, n, e) \rightarrow_{Cout} A2 \\
\mathbf{C_REA_PREDICATE_do_I_thm} \\
& \vdash \forall a : ALPHABET_REA; \\
& \quad c, x : VAR_NAME; \\
& \quad p : CIRCUS_CONDITION \\
& \bullet do_I (a, c, x, p) \in REA_PREDICATE \\
\mathbf{C_REA_PREDICATE_do_I_alphabet_thm} \\
& \vdash \forall a : ALPHABET_REA; \\
& \quad c, x : VAR_NAME; \\
& \quad p : CIRCUS_CONDITION \\
& \bullet (do_I (a, c, x, p)).1 = a \\
\mathbf{C_CSP_PROCESS_Var_C_thm} \\
& \vdash \forall n : VAR_NAME; t : SET_VAL; a : CSP_PROCESS \\
& \quad | (n, t, a) \in WF_Var_C \\
& \bullet var_C (n, t, a) \in CSP_PROCESS \\
\mathbf{C_CSP_PROCESS_Var_C_alphabet_thm} \\
& \vdash \forall n : VAR_NAME; t : SET_VAL; a : CSP_PROCESS \\
& \quad | (n, t, a) \in WF_Var_C \\
& \bullet (var_C (n, t, a)).1 \\
& = in_a (a.1 \setminus \{n\}) \cup out_a (a.1 \setminus \{dash\ n\}) \\
\mathbf{C_CSP_PROCESS_PrefixInR_C_thm} \\
& \vdash \forall a : ALPHABET_REA; \\
& \quad c : VAR_NAME; \\
& \quad x : VAR_NAME; \\
& \quad p : CIRCUS_CONDITION; \\
& \quad A : CSP_PROCESS \\
& \quad | (a, c, x, p, A) \in WF_PREFIXING_{CinR}
\end{aligned}$$

$\bullet (a, c, x, p) \rightarrow_{CinR} A \in CSP_PROCESS$
 $C_CSP_PROCESS_PrefixInR_alphabet_thm$
 $\vdash \forall a : ALPHABET_REA;$
 $c : VAR_NAME;$
 $x : VAR_NAME;$
 $p : CIRCUS_CONDITION;$
 $A : CSP_PROCESS$
 $| (a, c, x, p, A) \in WF_PREFIXING_{CinR}$
 $\bullet ((a, c, x, p) \rightarrow_{CinR} A).1$
 $= in_a (A.1 \setminus \{x\}) \cup out_a (A.1 \setminus \{dash\ x\})$
 $C_ \rightarrow_{CinR} \cap C_dist_thm$
 $\vdash \forall a : ALPHABET_REA;$
 $c, x : VAR_NAME;$
 $p : CIRCUS_CONDITION;$
 $A1, A2 : CSP_PROCESS$
 $| A1.1 = A2.1$
 $\wedge (a, c, x, p, A1) \in WF_PREFIXING_{CinR}$
 $\bullet (a, c, x, p) \rightarrow_{CinR} A1 \cap_C A2$
 $= ((a, c, x, p) \rightarrow_{CinR} A1)$
 $\cap_C (a, c, x, p) \rightarrow_{CinR} A2$
 $C_ \rightarrow_{Cin} \cap C_dist_thm$
 $\vdash \forall a : ALPHABET_REA;$
 $c, x : VAR_NAME;$
 $A1, A2 : CSP_PROCESS$
 $| \{x, dash\ x\} \subseteq a \wedge A1.1 = A2.1 \wedge A1.1 = a$
 $\bullet (a, c, x) \rightarrow_{Cin} A1 \cap_C A2$
 $= ((a, c, x) \rightarrow_{Cin} A1) \cap_C (a, c, x) \rightarrow_{Cin} A2$
 $C_Guard_not_wait_diverge_thm$
 $\vdash \forall g : CIRCUS_CONDITION; A : CSP_PROCESS$
 $| (g, A) \in WF_Guard_C$
 $\bullet ((g \&_C A) \omega_f) \sigma_f = CSP1 (g \wedge_R (A \omega_f) \sigma_f)$
 $C_Guard_C_not_wait_converge_thm$
 $\vdash \forall g : CIRCUS_CONDITION; A : CSP_PROCESS$
 $| (g, A) \in WF_Guard_C$
 $\bullet ((g \&_C A) \omega_f) \sigma_t$
 $= CSP1$
 $(R1$
 $((g \wedge_R (A \omega_f) \sigma_t)$

$$\begin{aligned}
& \vee_R \neg_R g \\
& \wedge_R =_R (A.1, \text{dash } tr, \text{Var } tr) \\
& \wedge_R =_R \\
& \quad (A.1, \\
& \quad \quad \text{dash wait}, \\
& \quad \quad \text{Val } (\text{Bool true}))))
\end{aligned}$$

C_Guard_alphabet_thm

$$\begin{aligned}
& \vdash \forall g : \text{CIRCUS_CONDITION}; A : \text{CSP_PROCESS} \\
& \quad | (g, A) \in \text{WF_Guard}_C \\
& \quad \bullet (g \ \&_C \ A).1 = A.1
\end{aligned}$$

C_Guard_C-expansion_thm

$$\begin{aligned}
& \vdash \forall g1, g2 : \text{CIRCUS_CONDITION}; A : \text{CSP_PROCESS} \\
& \quad | (g1 \vee_R g2, A) \in \text{WF_Guard}_C \wedge g1.1 = g2.1 \\
& \quad \bullet (g1 \vee_R g2) \ \&_C \ A = (g1 \ \&_C \ A) \boxtimes_C g2 \ \&_C \ A
\end{aligned}$$

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