Towards Verification of Cyber-Physical Systems with UTP and Isabelle/HOL

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INTO-CPS

into-cps.au.dk
Outline

Background

Mechanising the UTP

Theory of Hybrid Systems

Conclusion
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Mechanising the UTP

Theory of Hybrid Systems

Conclusion
Cyber-Physical Systems

- networked, real-time, and embedded systems able to interact with their environment using sensors and actuators
- e.g. robots, self-driving cars
- trustworthiness an important precursor to their adoption
- need for techniques for verifying CPS via formal models
- INTO-CPS project building tools for MBD of CPSs
- emphasis on semantic integration of models
- how to compose heterogeneous “multi-models”
  - SysML for high-level system modelling
  - VDM-RT and Circus for modelling discrete controllers
  - Modelica, 20-sim, and Simulink for system dynamics
Agricultural Robot (AgroIntelli)

- **kinematics model** (torque, power transfer)
- **sensors, camera model**
- **controller** (decision making, network link)
- **environment model** (field, crops, obstacles)
Unifying Theories of Programming

- how to integrate heterogeneous modelling languages and tools?
- study the underlying computational theories in isolation
- UTP theories as building blocks of heterogeneous languages
- supermarket: select & compose theories to produce semantics
- our approach to the multi-model problem
Unifying Theories of Programming

- how to integrate heterogeneous modelling languages and tools?
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- our approach to the multi-model problem

“At present, the main available mechanised mathematical tools are programmed for use in isolation [...] it will be necessary to build within each tool a structured library of programming design aids which take the advantage of the particular strengths of that tool. To ensure the tools may safely be used in combination, it is essential that these theories be unified.” — Chapter 0, UTP book
UTP semantic stack

Tool-chain

- Compiler
- Simulator
- Model Checker
- Refinement Calculator
- Theorem Prover

Operational
(SOS rules...)

Axiomatic
(Hoare logic, WP sem...)

Denotational Semantics

UTP Theory (1)  UTP Theory (2)  UTP Theory (n)

Alphabetised Predicates
UTP background

- environment-free approach to encoding denotational semantics
- based on programs-as-predicates
- programming constructs denoted by characteristic predicates
- predicates encode the set of observable behaviours
- alphabetised relations over input \((x)\), output variables \((x')\)
- alphabet gives the domain of possible observations
- UTP theories encapsulate domains with a set of invariants
- e.g. \(time, time' : \mathbb{R}_{\geq 0}\) with \(time \leq time'\)
Example

- simple imperative programming language

\[
\begin{align*}
x &:= v \triangleq x' = v \land y' = y \\
P ; Q &\triangleq \exists x_0 \cdot P[x_0/x'] \land Q[x_0/x] \\
P \triangleleft b \triangleright Q &\triangleq (b \land P) \lor (\neg b \land Q) \\
P^* &\triangleq \nu X \cdot P ; X
\end{align*}
\]
Algebraic laws of programs

\[(P ; Q) ; R = P ; (Q ; R)\]
\[P ; \text{false} = \text{false} ; P = \text{false}\]
\[(P \lhd b \rhd Q) ; R = (P ; R) \lhd b \rhd (Q ; R)\]
\[\text{while } b \text{ do } P = (P ; \text{while } b \text{ do } P) \lhd b \rhd \equiv\]
\[(P \land b) ; Q = P ; (b' \land Q)\]
\[(x := e ; y := f) = (y := f ; x := e) \tag{1}\]
\[x := e ; P = P[e/x]\]

(1) \(x \neq y, x \notin \text{fv}(f), y \notin \text{fv}(e)\)
Our work

1. **mechanised theory engineering for the UTP framework**
   - formalising UTP theories and associated laws
   - transcribe and verify whiteboard-style proofs
   - heterogeneous program verification / refinement
Our work

1. **mechanised theory engineering for the UTP framework**
   - formalising UTP theories and associated laws
   - transcribe and verify whiteboard-style proofs
   - heterogeneous program verification / refinement

2. **creation of new theories to underlie Cyber-Physical Systems**
   - reactive processes (CSP)
   - timed reactive designs
   - hybrid relations
   - integration with math libraries (e.g. ODEs)
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Isabelle/UTP

- a semantic embedding of the UTP in Isabelle/HOL
  - Isabelle = highly extensible + trustworthy proof framework
- no explicit reliance on syntax – purely denotational approach
- tactics for automating proof steps via HOL proof methods
- large library of algebraic laws of programming
- more than 2000 supporting theorems, lemmas, and proofs
- integration with existing libraries (e.g. lattices, Kleene algebra)
INtegrated TOolchain for Cyber-Physical Systems

http://into-cps.au.dk/

Screenshot

```
Lemma Miracle_greatest:
  assumes "P is CSP"
  shows "P ⊆ Miracle"
proof -
  have "P = RH (pre(RP) ⊔ peri(RP) ⊔ post(RP))"
    by (metis CSP_reactive_tri_design assms)
  also have "... ⊆ RH(true ⊔ false)"
    by (rule RH_monotone, refl auto)
  also have "RH(true ⊔ false) = RH(true ⊔ false ⊔ false)"
    by (simp add: wait'_cond_def cond_def)
finally show ?thesis
  by (simp add: Miracle_def)
qed

Lemma Chaos_least:
  assumes "P is CSP"
  shows "Chaos ⊆ P"
proof
proof (prove)
  goal (1 subgoal):
  1. P ⊆ Miracle
```
Mechanising state spaces

- fundamental to the programs-as-predicates approach
- predicates are sets of **observations** of the state
- should combine efficient **proof automation** with **expressivity**
- deep vs. shallow embedding dichotomy
Mechanising state spaces

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- deep vs. shallow embedding dichotomy

- lenses as a uniform semantic interface for variables
- identify variables by the position they occupy in the state
- regions of the state can be variously composed and related
- using separation algebra style operators
- nameless and spatial representation of variables
Lenses

- $X : V \implies S$ for view type $V$ and (“bigger”) source type $S$
- allow to focus on $V$ independently of rest of $S$
Lenses

- $X : V \rightarrow S$ for view type $V$ and (“bigger”) source type $S$
- allow to focus on $V$ independently of rest of $S$
- signature consists of two functions:
  - $get : S \rightarrow V$
  - $put : S \rightarrow V \rightarrow S$
- characterised by intuitive laws

\[
\begin{align*}
  get(put s v) &= v \quad \text{(PutGet)} \\
  put(put s v') v &= put s v \quad \text{(PutPut)} \\
  put s (get s) &= s \quad \text{(GetPut)}
\end{align*}
\]
Lenses

- \( X : V \mapsto S \) for view type \( V \) and (“bigger”) source type \( S \)
- allow to focus on \( V \) independently of rest of \( S \)
- signature consists of two functions:
  - \( \text{get} : S \rightarrow V \)
  - \( \text{put} : S \rightarrow V \rightarrow S \)
- characterised by intuitive laws
- several models, example: record lenses

\[
( \text{forename} : \text{String}, \text{surname} : \text{String}, \text{age} : \text{Int} )
\]
Lenses

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\[
\begin{align*}
\{ \text{forename} : String, \text{surname} : String, \text{age} : Int \} \\
\text{lens 1} \quad \text{lens 2} \quad \text{lens 3}
\end{align*}
\]
Lenses

- $X : V \rightarrow S$ for view type $V$ and (“bigger”) source type $S$
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- several models, example: record lenses
Lens comparison

- how to compare the behaviour of two lenses?
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- lens independence \( (X \bowtie Y) \)
- \( X, Y \) are independent if they view spatially separate regions
- e.g. forename \( \bowtie \) surname
Sublens relation

- if lenses $X$ and $Y$ are not independent, how are they related?
- $X$ is a sublens of $Y$ ($X \preceq Y$) if $Y$’s view encompasses $X$’s
- can also induce an equivalence $X \approx Y \iff X \preceq Y \land Y \preceq X$
Lens sum

- $X \oplus Y$ parallel composes two independent lenses
- similar to the heap composition operator of separation algebra

\[ \begin{align*}
V_1 \times V_2 \to S \\
\V_1 \to X \\
\V_2 \to Y \\
X \oplus Y \to S
\end{align*} \]

- e.g. $X \preccurlyeq X \oplus Y$ and $X \oplus Y \approx Y \oplus X$
Mechanised alphabetised predicates

- alphabets are modelled as Isabelle types ($\alpha$)
- our basic predicate model is $\mathbb{P}\alpha$
- lenses $\tau \rightarrow \alpha$ model the variables
- variable sets using $\oplus$ for $\cup$
- predicate operators created by lifting Isabelle equivalents
- provides direct proof automation support from HOL libraries
- can denote meta-logical style operators:
  - $x \# P - P$ does not depend on lens $x$
  - $P[v/x]$ - assign $v$ to lens $x$
  - $P \oplus_p a$ - extend alphabet using lens $a : \alpha \rightarrow \beta$
- from this basis we prove the UTP laws of programming
# Laws of programming

## Theorem (Unital quantale)

*UTP relations form a unital quantale and thus a Kleene algebra (Armstrong, 2015)*
Laws of programming

**Theorem (Unital quantale)**

*UTP relations form a unital quantale and thus a Kleene algebra (Armstrong, 2015)*

**Theorem (Assignment laws)**

\[
\begin{align*}
x := e ; P &= P[e/x] \\
x := e ; x := f &= x := f \quad (x \# f) \\
x := e ; y := f &= y := f ; x := e \quad (x \Join y, x \# f, y \# e)
\end{align*}
\]
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Hybrid Systems in UTP

- support semantics for languages like Modelica and Simulink
- augment UTP’s relational calculus with continuous variables
  - modelled as partial contiguous functions $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$
Hybrid Systems in UTP

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- augment UTP’s relational calculus with continuous variables
  - modelled as partial contiguous functions $x : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$
- retain standard discrete operators defs – e.g. $P ; Q$, $x := v$
- combine discrete and continuous with coupling invariants
- combine with other theories, e.g. CSP and reactive designs
Hybrid Systems in UTP

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- inspirations:
  - Hybrid CSP (He, Zhan et al.) – DAEs and pre-emption
  - HRML (He) – tri-partite alphabet
  - Duration Calculus (Zhu et al.) – interval operator
  - Timed Reactive Designs (Hayes et al.)
Hybrid relational calculus

- kernel language of imperative hybrid programs
- operators given a semantics in the theory of hybrid relations
Hybrid relational calculus

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- operators given a semantics in the theory of hybrid relations
- discrete relational operators
  - sequential composition — $P ; Q$
  - assignment — $x := v$
  - if-then-else conditional — $P \triangleleft b \triangleright Q$
  - iteration — $P^*$ and $P^\omega$
Hybrid relational calculus

- kernel language of imperative hybrid programs
- operators given a semantics in the theory of hybrid relations
- discrete relational operators
  - sequential composition — $P ; Q$
  - assignment — $x := v$
  - if-then-else conditional — $P < b > Q$
  - iteration — $P^*$ and $P^\omega$
- continuous evolution operators
  - differential equations — $\langle \dot{x} = \mathcal{F}(x, \dot{x}) \rangle$
  - pre-emption — $P [ B ] Q$
  - interval (continuous invariant) — $[ [ P ]$
Example: Bouncing Ball

Bouncing ball in Modelica

```
model BouncingBall
  Real p(start=2, fixed=true), v(start=0, fixed=true);
  equation
  der(v) = -9.81;
  der(p) = v;
  when p <= 0 then
    reinit(v, -0.8*v);
  end when;
end BouncingBall;
```
Example: Bouncing Ball

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```model BouncingBall
    Real p(start=2, fixed=true), v(start=0, fixed=true);
equation
    der(v) = -9.81;
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    when p <= 0 then
        reinit(v, -0.8*v);
    end when;
end BouncingBall;
```

Bouncing ball in hybrid relational calculus

\[ p, v := 2, 0 ; (\langle \dot{p} = v; \dot{v} = -9.81 \rangle [ p \leq 0 ] v := -v \times 0.8) \]
UTP theory of hybrid relations

- model for hybrid relational calculus
- use timed trace model from Hayes et al.
UTP theory of hybrid relations

- model for hybrid relational calculus
- use timed trace model from Hayes et al.
- hybrid relation = set of possible continuous variable evolutions
  - e.g. $x := v ; \langle \dot{x} = F(x, \dot{x}) \rangle$ sets up initial value problem
UTP theory of hybrid relations

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- hybrid relation = set of possible continuous variable evolutions
  - e.g. $x := v ; \langle \dot{x} = F(x, \dot{x}) \rangle$ sets up initial value problem
- $\ell : \mathbb{R}_{\geq 0}$ length of the current observation
- open domain for continuous variables $\text{dom}(x) = [0, \ell)$
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- $\ell : \mathbb{R}_{\geq 0}$ length of the current observation
- open domain for continuous variables $\text{dom}(x) = [0, \ell)$
- no continuous variable sharing in sequence $P ; Q$
- continuous variables accompanied by relational “copy variables”
- continuous linking invariants: $x = \underline{x}(0)$ and $x' = \lim_{t \to \ell}(\underline{x}(t))$
UTP theory of hybrid relations

- model for hybrid relational calculus
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- open domain for continuous variables $\text{dom}(x) = [0, \ell)$
- no continuous variable sharing in sequence $P ; Q$
- continuous variables accompanied by relational “copy variables”
- continuous linking invariants: $x = x(0)$ and $x' = \lim_{t \to \ell}(x(t))$
- observational variables: $tr, tr' : \mathbb{T}T$ – the timed trace
Timed traces ($\mathcal{T}$)

- represent relative continuous evolution of a process
- finite set of discontinuities represent events (e.g. assignment)
- domain is right-open interval $[0, \ell)$
- range is a suitable topological space $\Sigma$ – the continuous state
- allows integration of non-continuous data (e.g. CSP events)
- continuous variables as lenses on $\Sigma$
Trace concatenation

- **Theorem:** closed under piecewise continuity and convergence
- **Partial order:** $x \leq y \iff \exists z \cdot y = x \wedge z$
- **Subtraction:** $x - y \triangleq \nu z \cdot y = x \wedge z$
Hybrid denotational semantics

\[
\begin{align*}
\llbracket P \rrbracket & \triangleq tr' > tr \land (\forall t \in [0, \ell) \bullet P[x(t)/x]) \\
\llbracket P \rrbracket & \triangleq \llbracket P \rrbracket \land \left( x = x(0) \land v' = \lim_{t \to \ell} x(t) \right)
\end{align*}
\]

\[
\llangle \dot{x} = F'(x, \dot{x}) \mid B(x) \rrangle \triangleq \exists F \bullet \llbracket F' \text{ has-deriv } F \text{ at } \tau \land x = F(\tau) \land B(x) \rrbracket
\]
Hybrid denotational semantics

\[
[P] \triangleq tr' > tr \land (\forall t \in [0, \ell) \bullet P[x(t)/\overline{x}])
\]

\[
\llbracket P \rrbracket \triangleq [P] \land \left( x = x(0) \land v' = \lim_{t \to \ell} x(t) \right)
\]

\[
\langle \dot{x} = \mathcal{F}'(x, \ddot{x}) \mid \mathcal{B}(x) \rangle \triangleq \exists \mathcal{F} \bullet \left[ \mathcal{F}' \text{ has-deriv } \mathcal{F} \text{ at } \tau \land x = \mathcal{F}(\tau) \land \mathcal{B}(x) \right]
\]

- current work focuses on defining CSP operators
- takes inspiration from Hybrid CSP and Duration Calculus
Mechanisation

- based on Isabelle/UTP and the Multivariate Analysis package
- proof support for hybrid relational calculi
- real numbers based on Cauchy sequences
- differential equations based on topological and metric spaces
- support for limits, ODEs, and their solutions
- proved key properties of healthiness conditions and signature
- continuous variables as lenses into $\Sigma$
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- UTP is a great platform for integrating diverse semantics
- we are applying it to production of formal semantics that underlie Cyber-Physical Systems
- all of which are being mechanised in Isabelle/UTP
- [https://github.com/isabelle-utp/utp-main](https://github.com/isabelle-utp/utp-main)
- enables integration of reactive and hybrid systems
- applying these techniques to INTO-CPS case studies
- long-term aim proof support for CPS
- through integration with other tools (e.g. CAS)
- and a new extension of Circus called CyPhyCircus
References

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▶ J. He. HRML: a hybrid relational modelling language. QRS 2015.