An efficient method MEGCR for solving systems with multiple right-hand sides in 3-D parasitic inductance extraction

Liu Yang, Xiaobo Guo, Zeyi Wang

Department of Computer Science and Technology,
Tsinghua Univ., Beijing 100084, China
E-mail: yangliu99@mails.tsinghua.edu.cn

Abstract - With the development of the VLSI circuits, the feature size has been decreased to the deep sub-micron level, and the working frequency has reached to 3GHz. In order to assure the correctness of the IC design with high performance, the parasitic interconnect inductance and resistance should be calculated quickly and accurately. In this paper, the multipole method with modified non-uniform cube partitioning is applied to the matrix-vector products in the GCR iteration. Based on the EGCR (Extended Generalized Conjugate Residual) method, this paper proposes a MEGCR method for solving systems with multiple right-hand sides (multiple RHS). Numerical results show that it runs faster than FastHenry tens' times with comparable accuracy.

Keyword - Inductance extraction, PEEC, Multipole method, Multiple right-hand sides

I. Introduction

As Very Large Scale Integration (VLSI) design enters the deep sub-micron (DSM) regime, scaled feature sizes, multilevel metal layer technology are helping to achieve faster, more complex, and more powerful integrated circuits (ICs). High performance ICs are operating in the GHz range with very fast signal transition. The parasitic inductance not only affects the signal delay, but may also cause voltage overshoot and reduced rise time, which can increase cross-talk noise on neighboring lines. Therefore, it is critical to extract the parasitic inductance quickly and accurately [1, 2] in designing the circuits with high performance.

Several models for extracting the 3-D parasitic inductance and resistance have been proposed and can be classified into two kinds. One is the volume integral equation model [1, 2], and the other is boundary element method [4, 5]. It should be noted that the latter is still situated in its initial stage and cannot be used in practice. Currently, compared with the boundary element methods, the volume element methods based on the volume integral equations are more mature for extracting the 3-D parasitic inductance and resistance. The main advantage of the volume element methods is its simplicity of implementation and applicability to the problems with non-homogeneity in the conductors and dielectrics. In recent years, many advanced numerical methods, such as the multipole acceleration [1, 2, 3], hierarchical refinement [2] etc, were employed in the volume element methods to greatly increase extraction speed of the 3-D parasitic inductance and resistance.

The software package FastHenry for inductance extraction is famous one with a simplified assumption that the current in each filament flows only along its axis direction. It uses the multipole method with uniform cube partitioning to increase the computational speed of matrix-vector product. Refs. [2, 3] point out that for the regions with crowded source points, the near-field computation with the uniform cube partitioning increases dramatically, but with non-uniform cube partitioning it does not change much. Therefore, we use the multipole method with modified non-uniform cube partitioning [3, 6] applied to the matrix-vector products.

In this paper, we investigate the approaches to solving the linear systems with the multiple right-hand sides (multiple RHS). For calculating the whole impedance of a multi-terminal-pair structure, the problem can be referred as solving linear systems with multiple right-hand sides. The seed method [7, 8, 9] is an effective method for solving these problems. This class of methods consists of selecting a single system as the seed system, generating a corresponding Krylov subspace by the Arnoldi or Lanczos process, and then projecting the residuals of the other systems onto the Krylov subspace. This procedure is repeated until all the systems are solved. Among the seed methods, the single seed method proposed by Smith et al. [7] is a very effective implementation. Chan and Wang [8] have given the theoretical analysis. Based on the main idea of the seed methods, F. J. Ligen proposed an EGCR method [9]. Compared with

other seed methods, the distinct advantage of the EGCR method is to enlarge the search space. But, we observed that when number of the RHSs increases, computation of Gram-Schmidt orthogonal procedure in the EGCR method increases dramatically. In this paper, an improved EGCR is proposed to decrease the computation of orthogonal procedure. Numerical results show that the computational speed of the new method is faster than the original EGCR method. Summarized the multipole method with modified non-uniform cube partitioning and multiple RHS technique, our extractor runs faster than FastHenry tens' times with comparable accuracy.

The remainder of this paper is organized as follows. Section 2 describes the discretization of the integral formulation for magneto-quasi-static analysis of a conductor system. Section 3 gives an efficient seed method, MEGCR (Modified Extended Generalized Conjugate Residual). Section 4 presents experimental results.

II. Integral formula

Inductance extraction is the process of computing the complex frequency-dependent impedance matrix of a multi-conductor system under the magneto-quasi-static approximation. For a problem with s terminal pairs, let $Z(w) \in C^{s \times s}$ denote this impedance matrix at frequency w, then

$$Z(w)I(w) = V(w) \quad (1)$$

where $I(w), V(w) \in C^s$ are vectors of the terminal current and voltage respectively. Generally, the column i of $Z(w)$ can be computed by setting entry i of $I(w)$ to one, the rest to zero, and then computing the resulting voltage vector $V(w)$. The i^{th} column of $Z(w)$ is then given by $V(w)$.

Given the magneto-quasi-static assumption, the current within a long thin conductor can be assumed to flow parallel to its surface, as there is no charge accumulation on the surface. In order to properly capture skin and proximity effects, the conductors need to be subdivided into a bundle of parallel filaments with rectangular cross-section within which the current is assumed to flow along the length of the filament. The discrete system in matrix form becomes [1]

$$(R + jwL)I_b = f_A - f_B \quad (2)$$

where $I_b \in C^b$ is the vector of b filament current,

$$R_{ii} = \frac{l_i}{s\, a_i}, L_{ij} = \frac{m}{4p\, a_i a_j} \int_{V_i} \int_{V_j} \frac{l_i \cdot l_j}{\|r - r'\|} dV' dV \quad (3)$$

where R_{ii} is the $b \times b$ diagonal matrix of filament dc resistance, and L_{ij} is the $b \times b$ dense, symmetric positive definite matrix of partial inductances.

In accordance with Kirchoff voltage law, node voltage of the discrete system is

$$MV_b = V_s \quad (4)$$

Mesh current satisfies

$$M^t I_m = I_b \quad (5)$$

where M is the mesh matrix of the circuit corresponding to the discrete system, superscript t means the transpose of M, and $I_m \in R^m$ is the mesh current vector. Substituting (4) and (5) into (2), formula (2) becomes as follows

$$MZM^t I_m = V_s \quad (6)$$

Solving equation (6) with GCR, we can calculate the inductance and resistance of the multi-conductor system.

III. Multiple right-hand sides

In most layout-to-circuit extractors lumped models of the interconnection are produced. This can be done by subdividing the conductors into small elements and replacing each element by a lumped RLC section. However, if the lines are long, or the frequency is very high, this will not suffice, and the coupled effects will have to be taken into account. In inductance extraction, for a s terminal-pair problem, the admittance matrix can be obtained by repeating computing the currents s times resulting from setting entry i of $V(w)$ to one, the rest to zero. This means we should solve the following linear systems

$$AX = B \quad (7)$$

where A is a complex symmetric positive definite matrix of order n, $B = [b^{(1)}, b^{(2)}, \mathrm{L}\ , b^{(s)}]$ is a rectangular matrix with s column vectors as the right-hand sides to be solved and s is of moderate size ($s << n$).

A. EGCR (Extended Generalized Conjugate Residual)

The single seed method proposed by Smith et al. [7] is a very effective implementation of the projection methods. The main idea of this method is to select one seed system and solve it by some Krylov subspace iterative methods. Then one performs a Galerkin projection of the residuals onto the Krylov subspace generated by

the seed to obtain approximate solutions for the unsolved systems. Based on the projection idea, various iterative methods have been introduced to solve the linear systems with multiple right-hand sides, such as GMRES, QMR, GCR, etc. Practical results show that the seed methods have a super-convergence behavior compared with the usual iterative methods. On the other hand, if the right-hand sides are close, it usually only takes very few restarts to solve all the systems [8].

The main idea of the seed methods is projecting the residual and generating a good initial guess. The search space of unsolved systems does not change. Ligen proposed an EGCR method [9] to solve systems with multiple right-hand sides. This method has a distinct advantage compared with other seed methods. The size of the search space is enlarged with the previously generated search vectors, and only less iteration will be needed to achieve convergence.

B. MEGCR (Modified Extended Generalized Conjugate Residual)

We implemented the EGCR method in our inductance extractor. The iterative number and computational time have much decrease compared with the usual iterative methods. On the other hand, experimental results show that the EGCR method has two disadvantages. Firstly, when the search space enlarges, the vector-inner products in the Gram-Schmidt orthogonal procedure would be increased quickly. It makes the computational time of each iteration step increases dramatically. Secondly, while the search space enlarges, a lot of the vector-inner products would make the machine error distort the orthogonality of the basic vectors in the search space. It affects not only the result accuracy, but also the number of iterations.

Next we will analyze the relationship between the complexity of the EGCR method and the number of RHSs. The process of GCR and EGCR method include three parts, the Gram-Schmidt orthogonal procedure, the matrix-vector products and computing approximation solution. There are only several vector-inner products in the process of computing approximation solution, and its computation is small compared with the other two parts. Therefore, the computation of this part can be ignored in the following discussion. We implement the multipole method with modified non-uniform cube partitioning applied to the matrix-vector products, and the complexity of matrix-vector product in every iteration step is $O(n\log_2 n)$ [6], and the computation is $kn\log_2 n$ where k is a constant. Convenient for discussion, we suppose $k=1$ in the following formulas (8)-(10). Suppose that the order of the matrix is n, and the number of RHSs is s. Suppose that the mean of iteration for each RHS with GCR method is m, and mean of iteration for each RHS with EGCR method can reduce to p. $p=f(s)$ is a function of the number of RHSs. Hence, the time complexity of GCR method is

$$T_{GCR} = s(mn\log_2 n + n(1+2+\mathrm{L}\ \ m)) = smn(\log_2 n + \frac{m+1}{2}) \qquad (8)$$

where the first part in (8) is the computation of the matrix-vector products, and the second part is the computation of the Gram-Schmidt orthogonal procedure. The time complexity of EGCR method is

$$T_{EGCR} = spn\log_2 n + n(1+2+\mathrm{L}\ \ +sp) = spn(\log_2 n + \frac{sp+1}{2}) \qquad (9)$$

For an example with n=1024, m=20, we give the ratio of average iterations p/m curves versus number of RHSs while the speed-up of solving time T_{GCR}/T_{EGCR}=1, 2, 4.

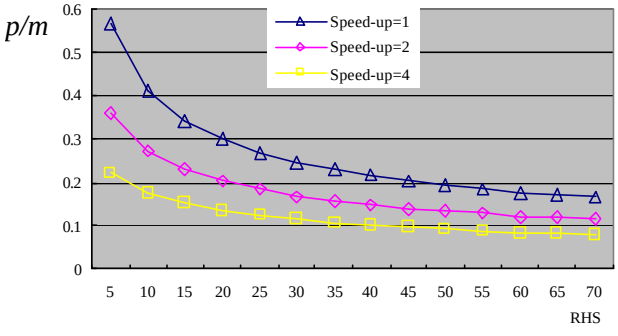


Figure 1. *p/m* curves versus number of RHSs

Fig. 1 shows that the ratio of average iteration number needs to be decreased to 0.1 when the number of RHSs is 35 and the speed-up of solving time is 4.

In order to increase efficiency of the EGCR method, computation of the Gram-Schmidt orthogonal procedure should be reduced. Therefore, we propose a modified EGCR method MEGCR. The main idea of MEGCR is to select one seed system and solve it by GCR method. Then we project the residuals onto the subspace generated by the seed to obtain an initial guess for the unsolved systems. The search space of unsolved systems is orthogonalized with the space generated by the seed. In EGCR method, the search space of unsolved systems is orthogonalized with the previously generated search vectors of solved

systems. When the number of RHSs increases, the size of the search space becomes larger and larger. It leads to the increment of the vector-inner products in every iteration step. In EGCR method, the complexity of the Gram-Schmidt orthogonal procedure is linear dependent with the number of RHSs. Therefore, we modify the Gram-Schmidt orthogonal procedure. The search space of unsolved systems is orthogonalized only with the space generated by the seed system instead of the space generated by all solved systems. When the number of RHSs increases, the size of the space generated by the seed system does not change, and the computation of the vector-inner products is constant in every iteration step. The complexity of orthogonal procedure in MEGCR is independent with the number of RHSs.

The modified EGCR method is summarized as follows

MEGCR method

1. Choose the first right-hand side as the seed system and choose an initial guess.
2. Solve the seed system using the basic GCR method, and generate the corresponding search space, denoted as $\{V\}$.
3. Assign the search space $\{V\}$ to the projected space $\{P\}$
4. For j = 2 to s do
   1) Project the residual of j^{th} RHS onto the projected space $\{P\}$ to obtain a good initial guess
   2) Construct the search space $\{W\}$ of j^{th} RHS orthogonalized with the space $\{V\}$
   3) Compute the approximate solution
   4) Merge the projected space $\{P\}$ with $\{W\}$ to generate the new projected space, denoted as $\{P\}$
5. End for

Suppose that the mean of iteration for each RHS with EGCR method is $q = g(s)$, where q is a function of s. Hence, the time complexity of MEGCR method is

$$T_{MEGCR} = sqn\log_2 n + sn(1+2+\mathrm{L}\ +2q)$$
$$= sqn(\log_2 n + 2q + 1) \qquad (10)$$

We give the ratio of average iterations p/q curves versus number of RHSs while $T_{EGCR} = T_{MEGCR}$. Fig. 2 shows that p/q needs to decrease to 0.4 when the number of RHSs is 30.

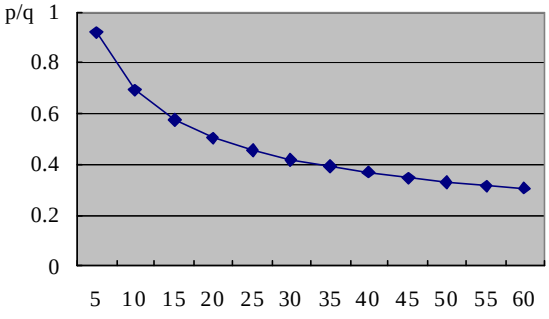


Figure 2. p/q curves versus number of RHSs

IV. Experimental results

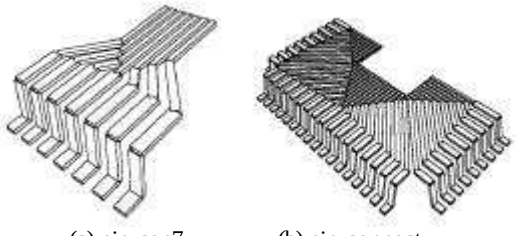
A code FIE (Fast Inductance Extraction) implementing the algorithm presented above is written in C++ language and run on a Sun Fire 880 workstation. On the website (http://rle-vlsi.mit.edu), the developers of FastHenry gave a few examples that indicate its excellent performance. The following cases are selected from those examples to demonstrate efficiency of our improvement.

A. Comparison of MEGCR and EGCR

We compare the iteration and solving time of GCR, EGCR and MEGCR. Table 1 shows that the number of iteration of MEGCR is comparable with EGCR, yet the solving time of MEGCR is less than that of EGCR. For the example pin-connect, the speed-up of MEGCR to EGCR is 3.22. We observed that the iteration step of EGCR becomes slower than that of MEGCR after the fifth right-hand side. The computational time of every iteration step in EGCR continues increasing, but it does not change in MEGCR.

B. Comparison of FIE and FastHenry

We compare our extractor with the famous inductance extractor FastHenry[1] by four examples, where two cases are shown in Fig. 3. The conductivity is 5.81e7; the working frequency is 1GHz. Table 2 shows that the results of FIE and FastHenry are comparable in accuracy, the greater discrepancy between the FIE and FastHenry is 3.03%. FIE is much faster than FastHenry. FIE takes 10.78 seconds while FastHenry takes 481.49 seconds for the sample pin-connect with 3270 unknowns. Hence the increase in speed is 44.67.



(a) pin-con7 (b) pin-connect

Figure 3. Two samples provided by FastHenry

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Table 1. Comparison of GCR, EGCR and MEGCR

		RHS	Iterations	Solving time(s)	Speed-up of MEGCR to GCR	Speed-up of MEGCR to EGCR
49pin	GCR	49	533	61.28	--	--
	EGCR	49	160	61	1.005	
	MEGCR	49	238	28.42	2.16	2.15
30pin	GCR	30	285	26.95	--	--
	EGCR	30	97	21.79	1.24	--
	MEGCR	30	98	10.33	2.47	2.11
Pin-con nect	GCR	35	175	10.91	--	--
	EGCR	35	105	25.05	0.44	--
	MEGCR	35	107	7.79	1.40	3.22

Table 2. Comparison of FIE and FastHenry

		Unknowns	RHS	Iterations	Total time	Discrepancy	Speed-up
49pin	FastHenry	4900	49	536	1226.19	2.49%	17.80
	FIE	4900	49	240	68.87		
Pin-con7	FastHenry	1920	7	108	131.58	0.70%	36.86
	FIE	1920	7	42	3.57		
Pin-connect	FastHenry	3270	35	174	481.49	0.56%	44.67
	FIE	3270	35	107	10.78		
30pin	FastHenry	3640	30	280	852.69	3.03%	31.13
	FIE	3640	30	102	26.54		