Coupled Simulation of Circuit and Piezoelectric Laminates

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Abstract

In this paper, an algorithm for the coupled simulation of circuit and piezoelectric laminate devices is presented. A finite element solver for piezoelectric laminates is included in the SPICE framework as a capacitor. The charge of this capacitor is a function of both the terminal voltage and the mechanical strain in the piezoelectric material. The coupled simulator allows simulation of novel micropower generation circuits based on piezoelectric laminates.

1. Introduction

Piezoelectric materials, such as lead zirconate titanate ceramics (PZT), provide a desired transformation mechanism between the electrical and the mechanical domains. When a mechanical force is applied to a piezoelectric material, an electric voltage is generated. Conversely applying an electric voltage to this material produces a mechanical strain. Because of these bi-directional effects, piezoelectric materials are widely used for making sensors and actuators.

The recent technological trend toward smaller and more efficient mechanical devices has allowed micro-electro-mechanical systems (MEMS) to emerge as a new interdisciplinary engineering domain between electrical and mechanical fields. In many MEMS applications, the piezoelectric material in thin-film form is used for sensing and/or actuation. The use of thin PZT films for micro power supply is a new and exciting application [2 - 5]. In this application, the piezoelectric laminate is mechanically forced to vibrate and work as a generator to transform the mechanical energy into electrical charge. This generator is then combined with an electrical circuit that extracts the charge and converts it into a useful source of power. Our objective is to accurately simulate the performance of the piezoelectric generator and the associated energy harvesting circuit.

The finite element method has been widely applied to model piezoelectric devices [1]. As piezoelectric devices become increasingly diverse and sophisticated, the need arises for increasingly accurate and efficient modeling of their behavior for design purposes. There is a growing interest in combining the finite element simulation of piezoelectric devices with circuits. In most applications, the piezoelectric devices are connected to a circuit and a strong interaction exists between the piezoelectric device and the circuit to which it is connected. In such a situation one needs to simulate the piezoelectric device in conjunction with the electric circuit.

In the framework of ANSYS, Wang [6] presented a coupled simulation method for piezoelectric transducers with an attached electric circuit. In order to make the electric network equations compatible with the piezoelectric finite element formulations, a nonstandard circuit simulation method was used. In this approach, charge conservation is applied to each circuit node to establish an equivalent capacitance matrix for each circuit element. This is different from the nodal analysis in SPICE, where KCL is applied to each circuit node. The disadvantage of the approach in [6] is that one cannot take advantage of an existing and established standard circuit solver, such as SPICE.

In this paper, a simple algorithm for the coupled simulation of circuit and piezoelectric laminate devices is presented. This method is similar to the method proposed in [7] for mixed-level device and circuit simulation. Using this method, the finite element solver for a piezoelectric laminate can be implemented as a device model in the framework of SPICE. A brief review of the piezoelectric finite element formulation is presented in Section 2. The algorithms used to couple the finite element analysis as a device model to SPICE are described in 3. Numerical examples and conclusions are presented in Section 4 and Section 5, respectively.

2. Finite Element Formulation for Piezoelectric materials

When the electrical potential is chosen as an additional nodal degree-of-freedom besides the mechanical degree-of-freedoms, the general finite element equations for an element in a piezoelectric material can be expressed as [8]

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$$[\mathbf{m}_{uu}]\{\ddot{\mathbf{u}}_e\}+[\mathbf{c}_{uu}]\{\dot{\mathbf{u}}_e\}+[\mathbf{k}_{uu}]\{\mathbf{u}_e\}+[\mathbf{k}_{u\phi}]\ \{\boldsymbol{\varphi}_e\}=\{\mathbf{f}_e\} \quad (1)$$

$$[\mathbf{k}_{u\phi}]^T\cdot\{\mathbf{u}_e\}+[\mathbf{k}_{\phi\phi}]\cdot\{\boldsymbol{\varphi}_e\}=\{\mathbf{q}_e\} \quad (2)$$

where $\{\mathbf{u}_e\}$, $\{\dot{\mathbf{u}}_e\}$ and $\{\ddot{\mathbf{u}}_e\}$ are the nodal displacement, velocity and acceleration, respectively. $\{\boldsymbol{\varphi}_e\}$ is the nodal potential. $[\mathbf{m}_{uu}]$ is the mass matrix, $[\mathbf{c}_{uu}]$ is the damping matrix, $[\mathbf{k}_{uu}]$ is the mechanical stiffness matrix, $[\mathbf{k}_{u\varphi}]$ is the piezoelectric matrix and $[\mathbf{k}_{\phi\phi}]$ is the dielectric matrix. $\{\mathbf{f}_e\}$ is the equivalent nodal force vector and $\{\mathbf{q}_e\}$ is the equivalent nodal charge vector. Details about these matrices and vectors are available in [8].

For piezoelectric laminates, when a plate or shell element is used, the above equations can be simplified. By assuming the electric field has only one nonzero component in the thickness direction [9] and introducing the electrical potential as an additional degree-of-freedom on an element level, Eqs. (1) and (2) can be expressed as

$$[\mathbf{m}_{uu}]\{\ddot{\mathbf{u}}_e\}+[\mathbf{c}_{uu}]\{\dot{\mathbf{u}}_e\}+[\mathbf{k}_{uu}]\{\mathbf{u}_e\}+\{\mathbf{k}_{u\phi}\}\phi_e=\{\mathbf{f}_e\} \quad (3)$$

$$\{\mathbf{k}_{u\phi}\}^T\cdot\{\mathbf{u}_e\}+k_{\phi\phi}\cdot\phi_e=q_e \quad (4)$$

where q_e is an element charge and ϕ_e is the element voltage. Compared to Eqs. (1) and (2), vector $\{\boldsymbol{\varphi}_e\}$ and $\{\mathbf{q}_e\}$ degenerate into scalars, Matrix $[\mathbf{k}_{uu}]$ degenerates into a vector and matrix $[\mathbf{k}_{\phi\phi}]$ degenerates into a scalar.

3. Coupling Algorithm

In this section, we present an algorithm to couple the finite element simulation of piezoelectric laminates with that of circuits. We assume that the frequency of mechanical excitation applied to the laminates is much lower than their lowest natural frequency of vibration. In this case, the inertia effects can be neglected and from Eqs. (3) and (4)

$$[\mathbf{k}_{uu}]\cdot\{\mathbf{u}_e\}+[\mathbf{k}_{u\phi}]\ \cdot\phi_e=\{\mathbf{f}_e\} \quad (5)$$

$$[\mathbf{k}_{u\phi}]^T\cdot\{\mathbf{u}_e\}+k_{\phi\phi}\cdot\phi_e=q_e \quad (6)$$

Eq. (6) shows that the element sandwiched by electrodes responds to the external circuit as a capacitor. The charge of this capacitor is a function of the applied voltage and the displacements of the laminate.

When connected to a circuit, the current through the piezoelectric laminate device can be expressed as a time derivative of the total charge with respect to time:

$$i=\frac{dQ}{dt} \quad (7)$$

$$Q=\sum_i q_e^{(i)} \quad (8)$$

where $q_e^{(i)}$ is the charge in element i.

Applying backward Euler discretization to Eq. (7) at time point t_n we have

$$i_n=\alpha_1 Q_{n-1}+\alpha_0 Q_n \quad (9)$$

where α_j's are the discretization coefficients. The nonlinear equations are solved using Newton's method. For a Newton iteration on the current i_n we have

$$i_n^{k+1}=i_n^k+\left.\left(\frac{\partial Q_n}{\partial V_n}+\sum_{j=1}^{N}\frac{\partial Q_n}{\partial u_{j,n}}\frac{\partial u_{j,n}}{\partial V_n}\right)\right|_{V_n^k,u_{j,n}^k}\left(V_n^{k+1}-V_n^k\right) \quad (10)$$

where V is the terminal voltage and k is the iteration number. This equation can be finally written in the following form

$$i_n^{k+1}=I_{eq}^{k+1}+G_{eq}^{k+1}\phi_n^{k+1} \quad (11)$$

where

$$I_{eq}^{k+1}=i_n^k-\alpha_0\left.\left(\frac{\partial Q_n}{\partial V_n}+\sum_{j=1}^{N}\frac{\partial Q_n}{\partial u_{j,n}}\frac{\partial u_{j,n}}{\partial V_n}\right)\right|_{V_n^k,u_{j,n}^k}\phi_n^k \quad (12)$$

$$G_{eq}^{k+1}=\alpha_0\left.\left(\frac{\partial Q_n}{\partial V_n}+\sum_{j=1}^{N}\frac{\partial Q_n}{\partial u_{j,n}}\frac{\partial u_{j,n}}{\partial V_n}\right)\right|_{V_n^k,u_{j,n}^k} \quad (13)$$

Eq. (12) indicates that i_n^{k+1} can be expressed as a current source I_{eq}^{k+1} in parallel with a conductor G_{eq}^{k+1}, as shown in Fig. 1.

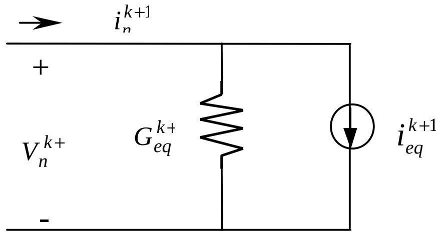
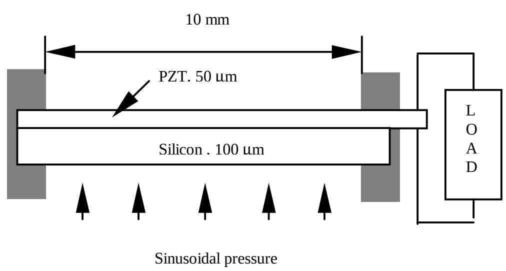
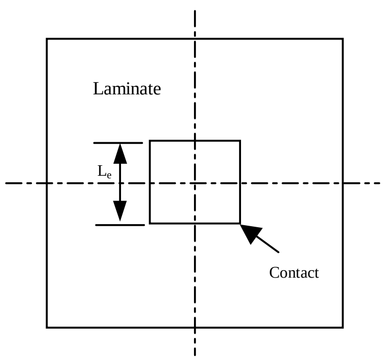


Figure 1. The equivalent circuit representation for a piezoelectric laminate.



(a)



(b)

Figure 2. Schematic of (a) piezoelectric micro power generator, (b) laminate and the contact electrode.

4. Numerical Examples

In this section, the coupled simulator is applied to different examples to demonstrate its capabilities.

Consider a clamped square piezoelectric laminate as a generator under a sinusoidal pressure. The laminate consists of a 100μm thick layer of silicon and a 50μm thick layer of PZT. The side length of the laminate is 10mm. The sinusoidal pressure $p = p_{\max} \sin(2\pi f t)$ has an amplitude $p_{\max}$ of 50kPa and a frequency f of 100Hz. A schematic of this laminate is shown in Fig. 2.

A 4-node bilinear isoparametric Mindlin plate element is used in this demonstration [11]. This element has three mechanical degrees-of-freedom (one deflection and two rotations) at each node. The electrical potential is chosen as an additional degree-of-freedom at an element level. This finite element model has been implemented as a device model in SPICE3 using the method discussed in Section 3.

Fig. 3 shows the effect of the electrode size on the open-circuit voltage, where Le is the side length of the electrode, as shown in Fig. 2 (b). The open-circuit voltage decreases as the electrode size increases. This change corresponds to a decrease of the average strain in the electrode-covered PZT when the electrode size increases.

Fig. 4 shows the output voltage when a resistive load is attached at the terminals. The voltage decreases with a decrease in the resistance value. A small change of the voltage phase can also be observed due to the capacitive nature of the piezoelectric laminate. The output power, on the other hand, reaches a maximum when R is about 150kΩ, as shown in Fig. 5. Similar results have been reported in [4] by experiments.

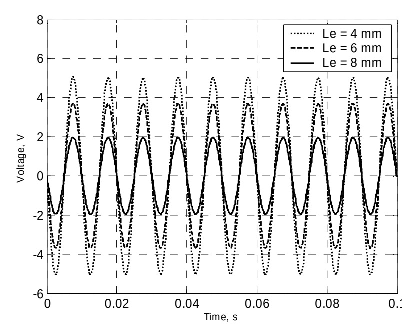


Figure 3. The open circuit voltage changes with electrode size.

Next consider connecting a bridge rectifier as shown in Fig. 6. The four diodes are assumed to be identical. The load capacitance is 10μF and the resistance is 1MΩ. Figure 8 shows the voltage across the resistor or capacitor. As expected, this voltage increases as the capacitor is charged. Similar results have been reported in [5], again based on experimental data.

5. Conclusions

An algorithm for the coupled simulation of circuit and piezoelectric laminates has been developed. A finite element solver for the piezoelectric laminates has been implemented in the framework of SPICE as a new device model. The piezoelectric laminate behaves as a capacitor whose charge is a function of both the terminal voltage and the mechanical displacement of the piezoelectric material. Several examples demonstrate the application of the coupled simulator for designing energy harvesting circuits and micro power generation systems.

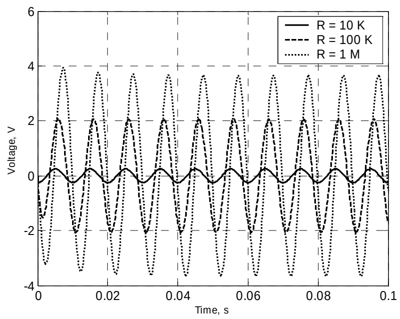


Figure 4. Output voltage change with load resistance.

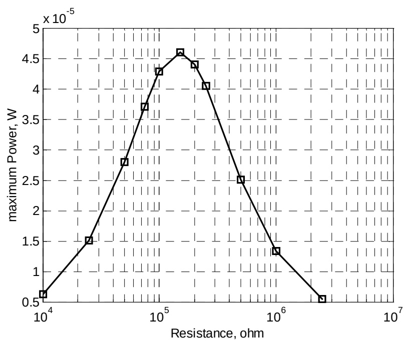


Figure 5. Output power change with load resistance.

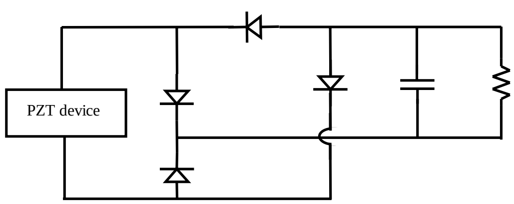


Figure 6. PZT device connected to a simple rectifier.

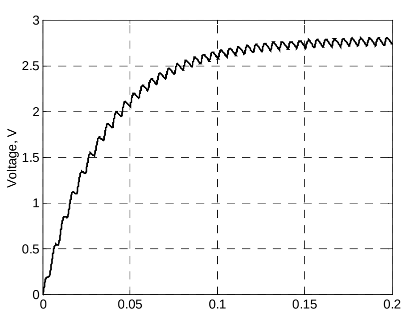


Figure 7. Time evolution of the voltage across the load resistor.

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