Closed Form Expressions for Extending Step Delay and Slew Metrics to Ramp Inputs

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Abstract: *Recent years have seen significant research in finding closed form expressions for the delay of an RC circuit that improves upon the Elmore delay model. However, several of these formulae assume a step excitation, leaving it to the reader to find a suitable extension to ramp inputs (we always assume a saturated ramp in this paper). The few works that do consider ramp inputs do not present a closed-form formula that works for a wide range of possible input slews. We propose the PERI (Probability distribution function Extension for Ramp Inputs) technique, that extends delay metrics for step inputs to the more general and realistic non-step inputs. Although there has been little work done in finding good slew - which is also referred as signal transition time - metrics, we also show how one can extend a slew metric for step inputs to the non-step case. We validate the efficacy of our approach through experimental results from several hundred RC dominated nets extracted from an industry ASIC design.*

Categories and Subject Descriptors
B.7.2 [Integrated Circuits]: Design Aids - Placement and Routing, Simulation, and Verification.

General Terms
Algorithms, Experimentation, and Performance.

Keywords
Interconnects, delay, slew, moments, PDF, skewness, Elmore, median, timing.

1. Introduction
Simple, closed-form formulae for delay are desirable for timing-driven performance optimization tasks such as floorplanning, placement, routing, and physical synthesis. Delay metrics provide an efficient means for estimating wire delay, which may be computed millions of times during the above mentioned physical design tasks, provided they are reasonably accurate. More than fifty years ago, by interpreting the impulse response of a linear circuit as a probability distribution function (PDF), Elmore proposed [4] using the mean of the impulse response to approximate the 50% delay of the circuit (the median of the impulse response under the probability interpretation) under a step excitation. This metric was resurrected in the early eighties when Rubenstein et al [12] published a simple closed-form formula for computing the mean of the impulse response of RC interconnect trees. Since the formula was simply expressed in terms of the Rs and Cs of the wire, the Elmore metric was widely adopted by the physical design community. In that paper, it was also established that the Elmore metric was the negative of the first *moment* of the impulse response. However, a general technique to compute higher order moments was discovered a few years later in [10]. In the same work, it was also shown how these moments can be used to approximate the poles of the circuit and hence the time domain waveform can be computed under arbitrary inputs. This technique, called AWE, was implemented in an interconnect analysis tool called RICE[11] which can approach SPICE-like accuracy. However, the delay is not a simple closed-form formula anymore and a non-linear equation needs to be solved for delay. Therefore, the Elmore metric continues to be used in the physical design tasks. It has been well established, however, that the Elmore metric can be orders of magnitude off in some cases and on average for far-end nodes, it is about 25-30% in error for step inputs. Hence, considerable research has been done to seek alternate delay metrics that are more accurate than the Elmore metric [13, 6, 8, 7, 1, 9].

In almost all of these works, the delay metric assumes a step excitation. In any practical application, the interconnect is driven by a non-linear device and the driving-point waveform is not a step. It is common practice for timing analyzers to replace the non-linear driver by an ideal voltage source generating a saturated ramp signal that has the same 10-90 transition time as the original waveform. Thus, any practical delay metric should be able to handle non-zero input slew[1]. Recognizing this, the authors in [8] propose adding a third dimension to the two-dimensional lookup table used for computing step delay. While being accurate, this method requires a carefully constructed table which is made harder by the fact that input slews can vary over a wide range, especially during the initial stages of design. In the above cited methods which compute delay directly as a function of moments, either through a lookup-table [8, 7] or an explicit formula [6, 1], a form of "moment massaging"

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[1] We use the term slew to refer to the transition time of the signal between two specified voltage thresholds.

was proposed in [7] to handle non-step inputs. The impulse response moments are modified to account for the non-zero input slews and these modified moments are used in the original step formula. The advantage of this approach is that the delay metric remains a closed-form formula even for ramp inputs. However, the formula is only valid for very fast input slews and has large errors for even moderately slow inputs, as we show later in the paper.

In this work, we propose a simple technique for extending any delay metric derived for a step input into a delay metric for a ramp input for RC trees that is valid over all input slews. A noteworthy feature of our method is that the delay metric reduces to the Elmore delay of the circuit under the limiting case of an infinitely slow ramp, a fact first proved in [5] to establish the Elmore delay as an upper bound. We also propose a method for converting a slew metric derived for step input to a slew metric valid under non-step inputs. In the limiting case of infinitely slow input, output slew computed using our formula equals the input slew which is correct since the interconnect essentially becomes a ramp-follower under this condition. Bakoglu proposed a simple slew metric based on the step response of an RC circuit [3]. Elmore [4], and more recently, Pileggi et al [5] noted that the standard deviation of the impulse response was a good estimator of output slew (Elmore called it the radius of gyration). This notion makes intuitive sense since standard-deviation measures the *spread* of the impulse response and that is in essence what slew measures. Nevertheless, the approach of this paper remains valid and can be used with any step slew metric.

This remainder of the paper is organized as follows. Section 2 presents pertinent background information relating probability and circuit theory. Section 3 shows our techniques for extending step delay and slew metrics to ramp inputs. Section 4 presents experimental results that both validate the effectiveness of our approach and show that our technique is much more effective than moment massaging. Finally, we conclude the paper in Section 5.

2. Background

We briefly review the relevant connections between probability and circuit theory; for details, please refer to [5]. First, consider an unimodal probability distribution function (PDF) $f(t)$ of a random variable t and its integral, the cumulative distribution function (CDF) $F(t)$ in Fig. 1. The *mean* of $f(t)$ is denoted by μ and is marked in the figure. Also marked are the 50% point or the *median* M and the *standard deviation* σ.

Now consider the impulse response at a node of an RC tree, $h(t)$ and its integral, $y(t) = \int_0^t h(t)dt$, which is the step response. We assume that there is no DC path to ground and all voltages are normalized to one. As shown in [12], the

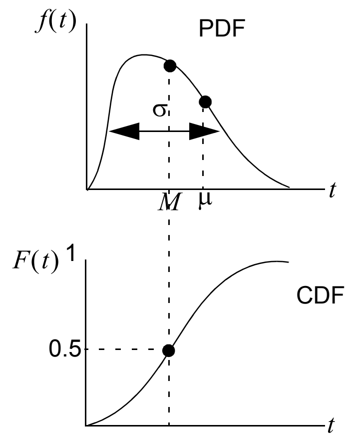

Figure 1 The PDF and CDF of a random variable t

impulse response $h(t)$ is greater than or equal to zero over all t and $y(\infty) = 1$. Any continuous function which satisfies these properties is a probability distribution function (PDF)[2], i.e., $h(t)$ and $y(t)$ respectively correspond to $f(t)$ and $F(t)$ in Fig. 1. The authors of [5] also showed that this PDF must be positively-skewed ($M \le \mu$) and unimodal.

The moments of the circuit are given by:

$$m_k = \frac{(-1)^k}{k!}\int_0^\infty t^k h(t)dt \qquad (1)$$

and they can be computed efficiently, e.g., by path tracing [11]. The Elmore delay, which is the mean μ of $h(t)$ is given by $-m_1$, i.e., the first moment of the impulse response. The standard deviation σ of $h(t)$ is given by the square root of the second *central moment* μ_2 of a PDF. The central moments can be expressed explicitly as functions of the circuit moments, e.g., $\mu_2 = 2m_2 - m_1^2$ and $\mu_3 = -6m_3 + 6m_1m_2 - 2m_1^3$. Hence, we have

$$\sigma = \sqrt{\mu_2} = \sqrt{2m_2 - m_1^2} \qquad (2)$$

The next section shows how we use this relationship between σ and the first two circuit moments to estimate output delay and slew.

3. The PERI Method

We now describe the PERI (PDF Extension to Ramp Inputs) technique for extending delay and slew metrics for step responses to ramps.

Suppose that an input $x(t)$ waveform is applied to the RC circuit. The corresponding circuit response at the output is given by the waveform $z(t) = x(t) \bullet h(t)$, where $\bullet$ is the convolution operator. The derivatives $x'(t)$ and $z'(t)$ are both PDFs since $x(t)$ and $z(t)$ are both CDFs. It is a property of convolution that $z'(t) = x'(t) \bullet h(t)$. This can be interpreted as the convolution of two PDFs to get the PDF at the output node.

Let S, I, and R represent the PDFs corresponding to the step response, the input waveform, and the output ramp response from convolving S and I. When referring to a particular property of a PDF, we use functional notation to specify the given PDF, e.g., $\mu(S)$ is the mean of the step response and $\sigma(R)$ is the standard deviation of the ramp output response.

The PERI technique derives from the following idea in probability theory: when two mutually independent PDFs are convolved, both the means and the central moments add (see [5] and the references therein). For our application, this means that

$$\mu(R) = \mu(S) + \mu(I), \ \mu_2(R) = \mu_2(S) + \mu_2(I), \text{ and } \mu_3(R) = \mu_3(S) + \mu_3(I). \quad (3)$$

In addition, as a direct result of Equation (2), we have that the sum of the squares of the standard deviations add:

$$\sigma^2(R) = \sigma^2(S) + \sigma^2(I). \quad (4)$$

3.1 PERI for Delay Metrics

It is the accepted practice to measure delay as the time difference between when the output crosses the 50% point and the time when the input crosses the 50% point. Thus, the delay due to a step input is given by the median M (as in Fig. 1) of the step response, $M(S)$. We assume that a delay metric for $M(S)$ is given, e.g., via D2M [1] or h-Gamma [8]. Our objective is to find an accurate estimate for the median $M(R)$ of the ramp output response. The estimated delay D under a ramp input is thus given by $D(R) = M(R) - M(I)$.

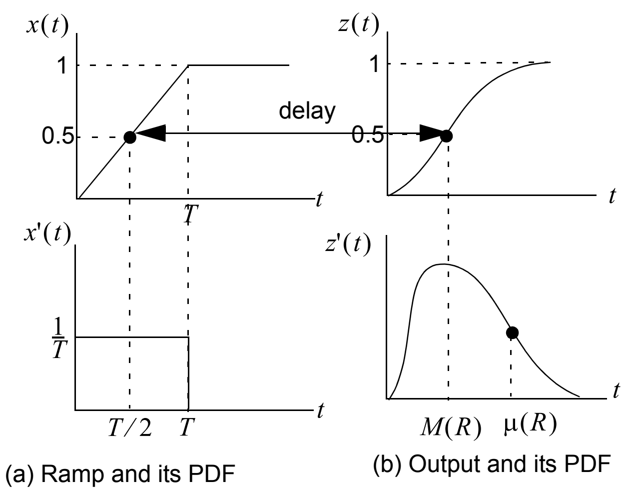

Figure 2 Ramp input and its corresponding output response of an RC circuit

For example, one might assume that the input waveform is a ramp input with slope T, as shown in Fig. 2(a) (though PERI can be applied to any symmetric input waveform with known mean and standard deviation). The PDF of this waveform is a uniform distribution with mean $\mu(I) = T/2$ and standard deviation $\sigma(I) = T/\sqrt{12}$. Thus, the delay of the output ramp is $D(R) = M(R) - T/2$ as shown in Fig. 2(b).

To derive PERI, we use the principle of the *skewness*, i.e., the relative difference between the mean and the median. Specifically, the *Pearson Skewness* [14] is defined as μ_3^2/σ^6, and the *Pearson Skewness Coefficient* [14] is defined as $(\mu - M)/\sigma$. The skewness and the skewness coefficient are proportional, i.e., $(\mu - M)/\sigma \propto \mu_3^2/\sigma^6$. Thus, we can write

$$\frac{\mu(S) - M(S)}{\sigma(S)} = K_S\left(\frac{\mu_3^2(S)}{\sigma^6(S)}\right) \quad (5)$$

$$\frac{\mu(R) - M(R)}{\sigma(R)} = K_R\left(\frac{\mu_3^2(R)}{\sigma^6(R)}\right) \quad (6)$$

Note that a symmetric input waveform has zero skewness, i.e., $\mu_3(I) = 0$. Hence from Equation (3), we have $\mu_3(S) = \mu_3(R)$. The fact that the input waveform has zero skewness also implies that $M(I) = \mu(I)$. Hence, since $D(R) = M(R) - M(I)$, we have $\mu(R) - M(R) = (\mu(S) + \mu(I)) - (D(R) + M(I))$ which simplifies to $\mu(S) - D(R)$. We can rewrite Equation (6) as

$$\frac{\mu(S) - D(R)}{\sigma(R)} = K_R\left(\frac{\mu_3^2(S)}{\sigma^6(R)}\right) \quad (7)$$

We now assume that the constants of proportionality K_S and K_R are equal. Note that this is the one assumption that introduces error into the PERI method. We believe that in general these values should be quite close as the relationship between skewness and the skewness coefficient should be similar across similar looking PDFs. Dividing Equation (7) by Equation (5) yields:

$$\frac{\mu(S) - D(R)}{\mu(S) - M(S)} = \left(\frac{\sigma(S)}{\sigma(R)}\right)^5 \quad (8)$$

If we let α denote the constant $\sigma^5(S)/\sigma^5(R)$, then solving Equation (8) for $D(R)$ yields our delay estimate for the ramp response:

$$D(R) = (1 - \alpha)\mu(S) + \alpha M(S) \quad (9)$$

Recall from Equation (4) that $\sigma^2(R) = \sigma^2(S) + \sigma^2(I)$ which implies that the constant α lies between zero and one. Thus, α reflects the amount of trade-off between the Elmore Delay ($\mu(S)$) and the step delay metric $M(S)$. For example, when the input response is a ramp input with slope T,

$$\alpha = \left(\frac{\sigma(S)}{\sigma(R)}\right)^5 = \left(\frac{\sigma^2(S)}{\sigma^2(S) + \sigma^2(I)}\right)^{5/2} = \left(\frac{2m_2 - m_1^2}{2m_2 - m_1^2 + \frac{T^2}{12}}\right)^{5/2} \quad (10)$$

Notice that for a step input ($T = 0$), $\alpha = 1$ and thus

$D(R)$ equals the step delay metric $M(S)$. On the other hand, for infinitely slow ramps, as $T \to \infty$, we have $\alpha \to 0$ and $D(R)$ approaches the Elmore Delay, which has been shown to be the upper limit on delay [5]. Thus, the PERI technique has the desired asymptotic behavior.

3.2 PERI for Slew Metrics

Slew is a single measure for how fast the output waveform rises to Vdd. Since the CDF may asymptotically approach Vdd without actually reaching it, a commonly used definition of slew is the 10/90 slew, i.e., the time difference between when the waveform crosses the 90% point and the 10% point. The slowly climbing tail of the output waveform may necessitate an even tighter slew definition, e.g., 30/70.

Elmore observed that the output slew is proportional to its standard deviation [4], and this was also noted by Gupta *et al.* [5]. For our convolution example, this means that $Slew(S) = C_S\sigma(S)$, $Slew(I) = C_I\sigma(I)$ and $Slew(R) = C_R\sigma(R)$, for constants C_S, C_I, and C_R. From Equation (4), we have:

$$\left(\frac{Slew(R)}{C_R}\right)^2 = \left(\frac{Slew(S)}{C_S}\right)^2 + \left(\frac{Slew(I)}{C_I}\right)^2. \qquad (11)$$

As we did for delay, we assume the constants of proportionality are identical. While in reality the constants of proportionality are not all the same, experimental results show that only a small error is incurred by this assumption. This assumption leads to our proposed estimation method for ramp slew:

$$Slew(R) = \sqrt{Slew^2(S) + Slew^2(I)}.^2 \qquad (12)$$

This result is conceptually quite simple. The output slew is the root-mean square of the step slew and input slew. This result can also be applied for any slew measurement, 10/90, 30/70, etc., as long as all three slews are measured using the same criteria. For example, in Fig. 2, $Slew(I) = 0.8T$ for 10/90 slew and $0.4T$ for 30/70 slew. Further, Equation (12) exhibits the right limiting behavior: as $Slew(I) \to \infty$, we have $Slew(R) \to \infty$, and as $Slew(I) \to 0$, we have $Slew(R) \to Slew(S)$.

3.3 A Simple Example

We now show how one can apply PERI to a simple example, the last node (node 5) in the randomly generated 5-segment RC ladder shown in Fig. 3. The first two circuit moments of this node are: $m_1 = -76.96$ and $m_2 = 5029.18$. Though we can use any delay metric for $M(S)$, to show the effectiveness of PERI, we use the actual delay $M(S) = 58.01$ ps, as computed by RICE. Since $\mu(S) = -m_1$, the ramp delay in Equation (9) reduces to

2 As pointed out by one of the anonymous reviewers, apparently such a formula has been part of the folklore among oscilloscope users.

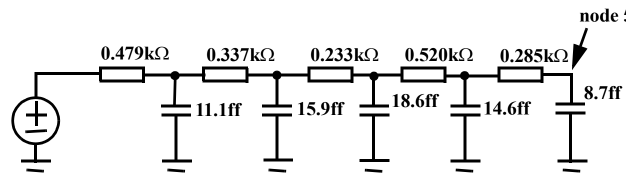

Figure 3 A simple 5-segment RC ladder.

$D(R) = 76.96 - 18.95\alpha$. One can see that when $\alpha = 0$, the ramp delay equals the Elmore delay, corresponding to an infinite input ramp. When $\alpha = 1$, the ramp delay equals the step delay, and for all other values of α, the ramp delay lies between these two extremes.

For example, if we assume a saturated ramp input of $T = 100$ ps, then Equation (10) yields $\alpha = 0.6319$, giving us $D(R) = 64.99$ ps. The actual delay value is 63.90. Alternatively, a $T = 200$ ps input ramp yields $\alpha = 0.2281$ and a corresponding delay of $D_T = 72.64$ps. The actual delay is 71.59 ps. Thus, for both input slews, the PERI delay method (which can be computed quickly from just two circuit moments) has relative error of less than two percent.

To find the ramp output slew at node 5, we use the actual value for the step slew. If one assumes $T = 100$ ps, the 10/90 slew is $Slew(I) = 80$ ps, and we find using RICE that $Slew(S) = 142.2$ ps. Applying Equation (12) yields $Slew(R) = 163.2$ ps. The actual 10/90 output slew is 165.2 ps, which is a less than two percent relative error.

4. Experimental Results

For our experiments, we extracted 432 routed nets from an industrial ASIC part in 0.20um technology. The nets were chosen to be those with maximum sink delay of at least 10 ps and with a ratio of the delay to the closest sink to the delay to the furthest sink to be less than 0.25. This criteria ensures that the nets are reasonably challenging by having both near and far end sinks. A distribution of the nets by number of sinks is given Table .

Number of sinks	1-2	3-4	5-8	9-13	14-19	Total
Number of Nets	169	122	46	54	41	432

Table 1 Net sink distribution.

We classified the sinks into three categories: i) far-end sinks: sinks that had a delay greater than or equal to 75% of the maximum delay (where the maximum is taken over all sinks in the net), ii) near-end sinks: sinks that had delay less than or equal to 25% of the maximum delay, and ii) mid-end sinks: the rest of the sinks. Of the 2244 total sinks, 367 are near-end, 670 are mid-end, and 1187 are far-end sinks. For every sink we measured the delay and 10/90 slew using fourth order RICE approximation as our golden calculator, since experience suggests the resulting waveform is indistinguishable from that obtained using SPICE simulations. For each net we randomly chose a saturated ramp with a slew between 100 ps and 1000 ps distributed randomly.

4.1 Experiments for an Ideal Delay Metric

Our first experiments examine how well PERI performs when we use an ideal step delay and slew metric, i.e., 4-pole RICE. The purpose is to isolate the error caused by using PERI to approximate the ramp response from the chosen delay or slew metric. We measure the ratio of the PERI delay $D(R)$ to the ideal delay, and report the average, minimum and maximum values, along with the standard deviation of these values in Table 2.

For comparison purposes, we implemented another algorithm for comparison that uses a linear interpolation technique. Although not published, the idea behind linear interpolation is a "folklore" algorithm. Basically, this algorithm uses the same principle as in Equation (9) to trade off between Elmore delay and a delay metric (e.g., $\ln 2$ times Elmore delay), but uses a much simpler formula for α:

$$\alpha = max(0, 1 - \frac{T}{10\mu(S)}) \quad (13)$$

When $T = 0$, then $\alpha = 1$, resulting in the step delay metric formula, and when T is greater than or equal to ten times the Elmore delay, $\alpha = 0$, yielding the Elmore delay. We know that as $T \to \infty$, the delay approaches Elmore delay. We assume ten times Elmore to be sufficiently close to a saturated ramp. Of course, one could use any other constants as well. Results are also shown in Table 2.

Note that from the last line in the table that on average PERI is only 0.3% away from optimal, though the range of errors may potentially be large. By comparison, the average for linear interpolation is 3.2% and its standard deviation is higher than that of PERI. PERI is extremely effective for far-end sinks as one can see from the low standard deviation and that minimum and maximum values are close to one. Linear interpolation is also effective at the far end, though the standard deviation is higher and the worst-case behavior is more extreme.

4.2 Experiments for an Ideal Slew Metric

We now repeat the same experiments, except using an ideal slew metric, i.e., 4-pole RICE. For comparison, we use a linear interpolation slew method. Industry folklore typically treats slew as additive with respect to the input slew, i.e., $Slew(R) = T + \Delta$, where Δ represents the *slew degradation* of the interconnect. To get a linear interpolation ramp extension for slew, we use the same α as in Equation (13) and choose $\Delta = \alpha Slew(S)$. One can also view the formula as $Slew(R) = Slew(I) + \alpha Slew(S)$ and get an additive form (instead of root-mean square) of Equation (12). This formula has the correct limiting behavior $Slew(R) = Slew(I)$ as $T \to \infty$ and $Slew(R) = Slew(S)$ for $T = 0$.

Table 3 presents slew results in the same manner as Table 2, this time measuring 10/90 slew accuracy. Observe from the last row that the average error for PERI only 1% (versus 11.2% for Linear Interpolation) and that the largest relative error over all 2244 instances is just 8.6% (versus 28.6% for Linear Interpolation). Indeed, by comparing the PERI results in Table 3 to Table 2, one can observe much smaller standard deviation for slew (0.016) than for delay (0.164), meaning the overall error introduced by PERI for slew is minimal. To us, it is clear that PERI's simple root mean square formula for output slew is preferred to the traditional idea of additive slew degradation.

4.3 Comparing PERI to Moment Massaging

So far we have compared PERI to a folklore Linear Interpolation method for extending step metrics to ramps. Another approach which accomplishes this is the "moment massaging" technique that was suggested in [1]. Moment massaging converts the circuit moments of the impulse response to modified moments of the ramp response. For example the first two ramp moments are $n_1 = m_1 - T/2$ and $n_2 = m_2 - (m_1 T)/2 + T^2/6$. Then a delay and/or slew metric can be computed using these new circuit moments, assuming the delay/slew metrics are computed from the circuit moments.

To compare to moment massaging, we cannot directly apply RICE for the step delay and choose the D2M delay metric [1] for our delay experiments. We re-ran the same experiments as in Section 4.1 using D2M instead of RICE for the step delay basis and summarize the results in Table 4. Observe from the last row that the average relative error for D2M is about 8.5% which is significantly better than the 153% relative error for moment massaging. Indeed, it is clear that moment massaging is not an effective technique for extending step metrics to ramps. Also note that like D2M, PERI has less than 1% error for far-end sinks.Next, we compare PERI to moment massaging for slew. Although there really have not been any higher-order slew metrics proposed in the CAD literature, Bakoglu suggested an Elmore-like metric for 10/90 slew, namely 2.2 (or $\log(9)$) times the RC delay [3]. The results are shown in Table 5. Despite being a first-order slew approximation, the PERI results using the Bakoglu metric are surprisingly accurate. From the last row in the table, one observe a 4.0% relative error versus RICE for PERI, while moment massaging sports a 31.8% relative error. As compared with Table 4, we see that most of the error induced for PERI is from the slew metric, not from the PERI extension to a ramp response.

4.4 PERI Performance As Input Slew Changes

For the final set of experiments, we wish to demonstrate how PERI performs as a function of the input slew. Consider the seven node RC circuit in Fig. 4, which was also presented in [1][7]. 10/90 input slew values of 200, 500, 1000, 2000 , and 4000 picoseconds were chosen and delays and slew were computed for each node by RICE. We also computed the RICE delay and slew using a step input, and use this value to as the step basis for computing delays and slews via PERI and linear interpolation. The results are shown in Table 6 and Table 7.

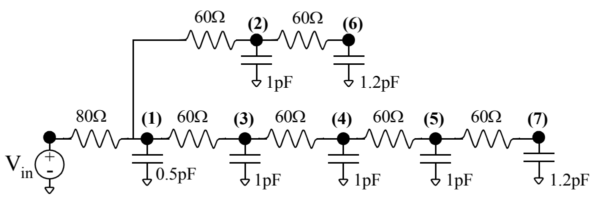

Figure 4 An example 7-node RC circuit.

Once again, observe that PERI delays and slews are quite comparable to RICE. The accuracy of both PERI and LI is best when the input slew is either very small (250 ps) or very large (4000 ps). For the mid-rang input slew values, PERI deviates a bit more from RICE, especially for the near-end node. The delays and slews from PERI are more accurate (as measured by RICE) than Linear Interpolation. The advantage for PERI versus Linear Interpolation is more pronounced for slew than for delay. For a closed-form metric approach, the PERI technique are quite promising

5. Conclusions

We presented PERI, a technique for extending the step delay and slew metrics to handle the more realistic case of ramp inputs. PERI derives from a key result in statistics: that under convolution the means and the central moments of PDFs are additive. We also make use of the fact that the skewness of a PDF is proportional to the relative separation of the mean from the median. For output slew, we show that a simple root mean square formula is extremely effective for measuring the output ramp slew. The key advantages of PERI are that it is simple to use, requiring nothing more than two circuit moments, and that it can be used in conjunction with any step input delay and slew metrics. Our experiments validate the effectiveness of PERI using ideal delay and slew metrics on challenging, actual industry Steiner routes. Further, we show that PERI is clearly more effective than either linear interpolation or moment massaging.

Our future work seeks to find a simple, yet accurate slew metric for a step input, since unlike delay metrics, slew metrics for RC trees has not yet been explored significantly in the literature.

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Slew extension method	Linear Interpolation				PERI			
error measure	avg	SD	max	min	avg	SD	max	min
far-end sinks (1187)	1.007	0.010	0.996	0.936	1.006	0.005	1.021	1.000
mid-end sinks (670)	0.976	0.056	1.015	0.665	0.984	0.038	1.040	0.781
near-end sinks (367)	1.249	0.600	3.818	0.601	1.025	0.398	2.250	0.333
all sinks (2224)	1.032	0.264	3.818	0.601	1.003	0.164	2.250	0.333

Table 2 Delay ratios of PERI ramp output to exact output over all sinks from 432 nets. Average, minimum, and maximum ratios, along with standard deviation is reported.

Slew Extension Method	Linear Interpolation				PERI			
error measure	avg	SD	max	min	avg	SD	max	min
far-end sinks (1187)	1.122	0.057	1.254	1.013	0.999	0.004	1.004	0.983
mid-end sinks (670)	1.100	0.045	1.240	1.015	0.987	0.014	1.003	0.934
near-end sinks (367)	1.100	0.095	1.286	0.960	0.968	0.022	1.014	0.914
all sinks (2224)	1.112	0.063	1.286	0.960	0.990	0.016	1.014	0.914

Table 3 10/90 Slew ratios of PERI ramp output to exact output over all sinks from 432 nets. Average, minimum, and maximum ratios, along with standard deviation is reported.

Step Delay Basis	D2M (2-moment delay metric)							
Ramp Extension Method	moment massaging				PERI			
error measure	avg	SD	max	min	avg	SD	max	min
far end sinks (1187)	1.291	0.162	1.801	1.027	1.003	0.008	1.034	0.983
mid end sinks (670)	1.618	0.294	2.805	1.115	1.140	0.096	1.472	0.976
near end sinks (367)	8.225	10.105	52.46	1.259	1.246	0.336	2.250	0.500
all sinks (2224)	2.534	4.828	52.45	1.027	1.085	0.174	2.250	0.500

Table 4 Delay comparisons of PERI to moment massaging using the D2M delay metric.

Step Slew Basis	Bakoglu Slew Metric							
Ramp Extension Method	moment massaging				PERI			
error measure	avg	SD	max	min	avg	SD	max	min
far-end sinks	1.362	0.120	1.726	1.043	1.134	0.058	1.247	0.978
mid-end sinks	1.115	0.140	1.772	0.840	0.892	0.076	1.207	0.748
near-end sinks	1.543	0.383	2.860	0.656	1.006	0.214	1.898	0.656
all sinks	1.318	0.244	2.860	0.656	1.040	0.150	1.898	0.656

Table 5 10/90 Slew comparisons of PERI to moment massaging using Bakoglu’s slew metric.

input slew	250			500			1000			2000			4000		
node	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**
1	213	207	210	229	235	272	261	321	371	325	465	466	454	541	530
2	386	383	383	397	407	409	420	479	499	465	604	597	555	674	662
3	487	484	482	497	505	498	517	572	578	558	702	699	639	788	777
4	708	707	705	716	724	716	730	781	761	760	896	884	819	980	968
5	851	851	849	858	867	859	870	921	900	895	1030	1014	945	1111	1099
6	463	461	461	473	484	487	493	555	570	533	678	668	613	746	733
7	925	925	923	931	941	933	944	994	974	966	1102	1086	1013	1183	1171

Table 6 Delay comparisons for each node in Fig. 4 using RICE as the ideal delay metric (ps).

input slew	250			500			1000			2000			4000		
node	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**	**LI**	**PERI**	**RICE**
1	1772	1614	1659	1949	1671	1758	2305	1882	2003	3017	2558	2624	4439	4306	4228
2	1894	1725	1733	2082	1779	1816	2457	1978	2046	3208	2629	2659	4709	4349	4267
3	2168	1996	2003	2356	2042	2079	2733	2218	2293	3487	2814	2872	4995	4463	4395
4	2354	2173	2164	2549	2215	2219	2941	2379	2412	3725	2942	2969	5291	4545	4468
5	2416	2230	2223	2617	2271	2267	3019	2431	2449	3823	2985	2998	5430	4573	4491
6	1918	1743	1748	2110	1796	1826	2497	1994	2054	3270	2641	2665	4812	4356	4272
7	2426	2237	2233	2630	2279	2276	3038	2438	2456	3853	2990	3004	5482	4576	4495

Table 7 Slew comparisons for each node in Fig. 4 using RICE as the ideal slew metric (ps).