Efficient iterative time preconditioners for harmonic balance RF circuit simulation

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Abstract

Efficient iterative time preconditioners for Krylov-based harmonic balance circuit simulators are proposed. Some numerical experiments assess their performance relative to the well-known block-diagonal frequency preconditioner and the previously proposed time preconditioners.

INTRODUCTION

The frequency-domain harmonic balance (HB) method proved an efficient alternative to the time-domain shooting method for computing periodic and quasi-periodic circuit steady states. Both methods were made efficient for large circuits by implementing the underlying Newton method using a matrix-implicit Krylov-subspace algorithm [1], [4], [8], [10], [12], [14], for the computation of the Newton direction at each iteration. Such methods require the use of an appropriate preconditioner in order to converge.

Some purely algebraic preconditioners based on incomplete factorizations were tested for harmonic balance simulation (see *e.g.*[15]), without much success. Therefore designing special purpose preconditioners soon appeared mandatory. A first approach was the frequency block-diagonal preconditioner proposed in [4], that is ideal for linear circuits and was found efficient for mildly nonlinear circuits. The need for efficient preconditioners to simulate nonlinear and strongly nonlinear circuits has given rise to some ongoing research.

In [5]-[7] two frequency-domain preconditioners are proposed. One factorizes an approximate Jacobian matrix obtained by neglecting the small harmonic of conductances and capitances. The other one is built in a similar way for an equivalent linear system already right preconditioned by the diagonal blocks of the harmonic balance Jacobian matrix.

Some preconditioners using factorized approximate time Jacobian operators based on averaging over time intervals during a signal period were proposed in [9], efficiently implemented using the theory of displacement structure.

The use of an approximate finite-difference time Jacobian matrices as preconditioner for the Harmonic Balance linear systems at each Newton iteration was proposed in [13]. The authors found the approximate Backward-Euler (BE) time Jacobian operator not very effective for preconditioning and suggested using higher-order differencing schemes.

The approximate BE and 2nd-order Backward Differentiation Formulae (BDF-2) operators were assessed in [11], where they perform better than the well-known block-diagonal frequency preconditioner [3], [4] as a right preconditioner, except for a small number of harmonics (typically less than 20). The author reports on page 32 costly increases in both the numbers of GMRES and Newton iterations when they are used for left preconditioning.

The connection between spectral and standard finite-difference schemes [13] and the ability of time-domain shooting methods to handle strongly nonlinear circuits [11] motivate the use of a finite-difference time preconditioner for the accurate HB method. The approach proposed in [11] and [13] is extended in the next section by removing the approximations suggested by the authors, using an iterative method. The following section reports some numerical experiments.

ITERATIVE TIME PRECONDITONERS

Following [13] the time-domain circuit equations are written

$$\frac{d}{dt}q(v(t))+i(v(t))+u(t)=0, \qquad (1)$$

where $u(t)$ is the vector of input sources, $v(t)$ is the vector of node voltages, and $i(v(t))$, $q(v(t))$ are the vectors of resistive node currents and node charges (or fluxes) respectively; all these vectors are of size N.

In harmonic balance formulation, any waveform $x(t)$ is represented as a truncated Fourier series

$$x(t)=\sum_{k=-K}^{k=K} X_k e^{j\omega_k t} \qquad (2)$$

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Substituting the Fourier representations of *v(t)* and *u(t)* in (2) and using orthogonality of the Fourier basis functions, one obtains the harmonic balance formulation of circuit equations

$$F(V) = \Omega\Gamma q(\Gamma^{-1}V) + \Gamma i(\Gamma^{-1}V) + \Gamma U = 0, \qquad (3)$$

where capital letters *U*, *V* and *F* indicate vectors of Fourier coefficients, Ω is a diagonal matrix expressing time derivation in frequency domain, and Γ is the inverse discrete Fourier transform matrix operating on vectors of size *NM* where *N* is the number of nodal unknowns and M=2K+1 is the number of Fourier components. In practice the action of the operator Γ and of its inverse are implemented using numerically efficient Fast Fourier Transforms.

Newton's method applied to (3) yields the iterations

$$J(V^n)(V^{n+1} - V^n) = -F(V^n) \qquad (4)$$

where *n* is the Newton iteration index and the Jacobian matrix is given by

$$J(V^n) = \Omega\Gamma C\Gamma^{-1} + \Gamma G\Gamma^{-1} \qquad (5)$$

In the above equation *C* is a block-diagonal matrix with blocks $C_j = dq(v(t_j))/dv$ and *G* is a block-diagonal matrix with blocks $G_j = di(v(t_j))/dv$, both evaluated at the current residual time vector $v^n = \Gamma^{-1}V^n$.

In order to solve (4) efficiently using Krylov subspace methods [12], [13], one needs a suitable preconditioner. Following [11], we build such a preconditioner by approximating the time counterpart of the frequency-domain Jacobian (given in (5)):

$$\Gamma^{-1}J(V^n)\Gamma = \Gamma^{-1}\Omega\Gamma C + G = \frac{dC}{dt} + G \qquad (6)$$

Using a Backward-Euler scheme for the time differentiation, one obtains the following preconditioner matrix

$$P = L - B \qquad (7)$$

where *L* denotes the block lower triangular part

$$L = \begin{bmatrix} \frac{C_1}{h} + G_1 & & \\ -\frac{C_1}{h} & \ddots & \\ & -\frac{C_{M-1}}{h} & \frac{C_M}{h} + G_M \end{bmatrix} \qquad (8)$$

and *B* contains only one extra upper triangular nonzero block

$$B = \begin{bmatrix} 0 & & \frac{C_M}{h} \\ 0 & \ddots & \\ & 0 & 0 \end{bmatrix} \qquad (9)$$

Other preconditioner matrices can be formed using different differentiation schemes such as the second-order Backward Differentiation Formula (BDF-2).

Algorithm

In this paper, unlike [11] and [13] that suggest to drop the extra-block matrix *B* and to use only *L* as a preconditioner, the full preconditioner matrix in (7) is retained. An iterative time preconditioner is built as in the following figure.

1. Transform to time domain : $z = \Gamma^{-1}Z$
2. Compute $y = L^{-1}z$
3. Using an unpreconditioned iterative linear solver, compute an approximate solution of $(I - L^{-1}B)x = y$, starting from *y*.
4. Transform back to frequency domain to get the preconditioned vector : $X = \Gamma x$.

Figure 1. Application of the iterative time preconditioner to a vector Z of size *NM* of frequency components

Some remarks can be made:

- First, excluding step 3 of the preconditioning algorithm one recovers a static (non-iterative) time preconditioner as proposed by [11] and [13]. Our iterative approach extends it by enhancing the quality of the preconditioning in step 3.
- Second, [11] and [13] conclude that the Backward-Euler static time preconditioner (with the lower-triangular matrix *L* only) is not very effective and they propose to use preconditioners derived from higher-order differencing schemes. Although using higher-order differencing schemes would reduce the corresponding truncation error, the main source of error is more likely orginated from dropping the extra-block part *B*. Higher-order schemes such as BDF-2 would require to drop more blocks, and the resulting static time preconditioner may not be more effective and it could even be worse. On the contrary, our iterative time preconditioner retains the extra blocks and using higher-order differencing schemes will certainly benefit from reducing the corresponding truncation error. However a BDF-2 iterative time preconditioner may not be more effective than a Backward-Euler one, because the corresponding linear system in step 3 of the algorithm is likely to be more badly conditioned and the initial iterate *y* provided by step 2 may be further from its solution.

- Third, the iterative nature of the preconditioner requires that the Krylov method used to solve the Newton iteration (4) could cope with a variable preconditioner. This is case for example with the flexible GMRES (FGMRES) method.
- Finally, assume one solves the linear system in step 3 exactly. Then, disregarding the error introduced by the time differencing scheme, the Newton iteration (4) requires only one iteration of the Krylov subspace method using this preconditioner. Indeed, this amounts to solve the time equivalent of the Newton iteration (4) using a finite-difference method. Therefore, our iterative time preconditioner provides a natural mixed frequency-time framework where the part of the solution process is transfered to the time domain and this transfer is controlled by the accuracy of the linear-system solver requested in step 3.

NUMERICAL EXPERIMENT

The iterative time preconditioner introduced in the previous section was implemented within the circuit simulator Eldo RF of Mentor Graphics Corporation©.

This section reports experiments with two types of circuits that are time-consuming for harmonic balance simulation, namely a divider-by-64 circuit and a third-order 8MHz phase-locked loop (PLL) circuit with ideal charge pump described in [2]. Results are presented in tables 1 and 3 respectively, where the static and iterative time preconditioners CPU-time performance is measured against that of the classical block-diagonal frequency preconditioner [4] and reported as a speed-up ratio. The number of Newton iterations to solve the harmonic balance formulation (3) and the total number of Krylov iterations performed during the Newton iterations (4) are also reported.

Table 1. Experiments with frequency divider by 64

Preconditioner	Speed-up ratio	Newton iterations	Krylov iterations
Block-diagonal frequency preconditioner	1.0	4	173
Static time preconditioner	1.67	4	25
Iterative time preconditioner	1.34	4	25

The frequency divider circuit has N=68 nodal unknowns and it was simulated using K=640 harmonics. The static time preconditioner outperforms the block-diagonal frequency preconditioner, with a number of Krylov iterations approximately divided by 7 and a speed-up ratio of 1.67. The discrepancy between these two numbers arises from the extra work performed at each Krylov iteration inside the preconditioner. The iterative time preconditioner yields no further improvement in terms of Krylov iterations and therefore a degradation in terms of speed-up ratio compared to the static version, due to the extra work perform in step 3 of the algorithm.

This divider circuit was designed by cascading D flip-flops set up as divide-by-2 cells. Table 2 reports the number of Newton iterations and the number of Krylov iterations required to compute the steady state using the iterative time preconditioner, for different division factors and number of harmonics.

Table 2. Experiments with frequency dividers

Frequency division factor	Number of harmonics	Newton iterations	Krylov iterations
2	30	3	15
4	40	4	21
8	80	4	21
16	160	4	21
32	320	4	22
64	640	4	25

The iterative time preconditioner clearly appears almost insensitive to the frequency division factor, except for a relatively small number of harmonics.

Table 3. Experiments with third-order 8MHz PLL with ideal charge pump

Preconditioner	Speed-up ratio	Newton iterations	Krylov iterations
Block-diagonal frequency preconditioner	1.0	10	1715
Iterative time preconditioner	4.53	12	86

The third-order 8MHz PLL has N=72 nodal unknowns and was simulated using K=100 harmonics. For this circuit, the static time preconditioner (not reported in table 3) is so poor that the Newton method does not converge. The enhancement provided by step 3 in the algorithm of the iterative time preconditioner not only enables the Newton method to converge, but also leads to a number of Krylov iterations approximately divided by 20 and a speed-up ratio of 4.53 when compared to the simulation using the block-diagonal frequency preconditioner.

CONCLUSIONS

An iterative time preconditioner is proposed for harmonic balance RF circuit simulation and was implemented in the commercial circuit simulator Eldo RF of Mentor Graphics Corporation©. In numerical experiments, using a divider and a PLL circuit, this new preconditioner is compared to the classical block-diagonal frequency preconditioner [4] and a static time preconditioner as proposed in [11] and [13].

For both circuits, the relatively small number of Krylov iterations per Newton iteration is an indicator of the effectiveness of our preconditioning approach. A reduction of this indicator ratio may also correspond to substantial savings in terms of memory if a non-restarted FGMRES solver is used.

However the efficiency of a particular preconditioner is always a trade-off between the reduction of Krylov iterations and the computational requirements of the preconditioner itself, and it is better measured by the CPU time speed-up ratio.

These test circuits were deliberately chosen because of the inadequacy of the block-diagonal frequency preconditioner. For linear and mildly nonlinear circuits, the latter is likely to outperform the iterative time preconditioner proposed here.

Further experiments of our iterative time preconditioner with proprietary circuits have shown speed-up up to an order of magnitude in some cases.

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