Model Order Reduction of Nonuniform Transmission Lines Using Integrated Congruence Transform

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ABSTRACT

This paper presents a new algorithm based on Integrated Congruence transform for the analysis of both uniform and nonuniform transmission lines. The key advantage of the proposed algorithm is that constructing a spanning orthonormal basis for the space-dependent moments is done without computing these moments explicitly. The proposed algorithm thus carries the numerical efficiency of Krylov-based projection techniques of lumped RLC networks to the domain of the distributed transmission line networks. The proposed algorithm can be used to construct an orthogonal basis for any set of moments related through a differential operator.

Categories and Subject Descriptors

I.6.5 [**Model Development**]: Modelling Methodology

General Terms

Algorithms, Theory

Keywords

Circuit Simulation, Model-Order Reduction, Integrated Congruence Transform, Nonuniform Transmission Lines, Signal Integrity, High-Speed Circuits

1. INTRODUCTION

The ever increasing quest for higher operating frequencies and smaller feature sizes in electronic circuits have made interconnects and transmission lines (TL's) a dominant factor in determining circuit performance and reliability in deep submicron designs.

Since TL's are distributed by nature, an accurate model has to be able to capture their distributed nature. Describing distributed elements is usually achieved through Telegrapher's Equations. However, difficulties arise in the presence of nonlinear terminations as these equations do not have a direct time-domain representation [1, 2]. In addition, guaranteeing passivity of the models has been a major concern [3, 4].

Analysis of nonuniform TL's presents an even more challenging task. One way to obtain time-domain macromodel is to divide the line into sections and treat each section as uniform TL. Obtaining passive reduced-order time-domain macromodel can then be achieved via various approaches in the literature that handle uniform TL's, e.g. [5, 6]. However, this is likely to produce inaccuracies as it introduces spurious reflections at the boundaries of each section.

Another approach that is used to obtain passive reduced-order macromodel for nonuniform TL is based on the Integrated Congreunce Transform (ICT) [7]. The main idea in this approach is to construct an orthogonal basis that spans the space of the first few derivatives (moments) w.r.t. to the frequency variable of the terminal voltages and currents, and use this basis to cast Telegrapher's partial differential equations into a set of ordinary differential equations (ODE) that can be linked easily to time-domain simulators, such as SPICE [8]. However, constructing the orthogonal basis requires explicit computation of the moments. This explicit computation of the moments presents a numerical disadvantage since it makes higher order moments obtained from a single point expansion of practically no value in enhancing the accuracy of the reduced model to match wider frequency range. By contrast, Krylov projection model order-reduction (MOR) algorithms of lumped RLC components [9–12] draw their numerical robustness from the ability to construct a spanning orthogonal basis for the moments without computing the moments explicitly.

This process is usually referred to as Implicit Moments Matching (IMM). The main obstacle in performing IMM for networks with distributed elements is that the moments are related through a *differential* operator rather than an *algebraic* operator as in the case of lumped RLC networks.

This paper presents a new algorithm based on ICT that does not compute the moments explicitly in order to construct their spanning orthogonal basis. The proposed algorithm thus represents the natural extension of Krylov-based projection techniques for lumped RLC networks to the domain of distributed networks, where the moments are related through a *differential* operator rather than *algebraic* operator. The proposed algorithm is developed through defining a weighted inner-product and norm mappings on the space of the moments and then expanding the moments in polynomials that are orthonormal w.r.t. to the same weighting function. Although the proposed algorithm can handle both uniform and nonuniform TL's, this paper focuses on its application for the nonuniform case.

The paper is organized as follows. Section 2 presents a brief background on Integrated Congreunce Transform. Section 3 describes a general outline of the proposed algorithm. Section 4 presents further implementation details for handling nonuniform transmission lines. Section 5 presents some numerical results.

2. REVIEW OF MODEL-ORDER REDUCTION BASED ON INTEGRATED CONGRUENCE TRANSFORM

Denote by $V(z,s)$ and $I(z,s) \in \mathbb{C}^m$ the Laplace-domain voltages and currents at an arbitrary point z along an m-conductor nonuniform transmission line of length d. $V(z,s)$ and $I(z,s)$ can be related through he Telegrapher's equations as follows

$$\begin{aligned}\frac{dV(z,s)}{dz} &= -\left(R(z)+sL(z)\right)I(z,s)\\ \frac{dI(z,s)}{dz} &= -\left(G(z)+sC(z)\right)V(z,s)\end{aligned} \tag{1}$$

where $R(z), L(z), G(z),$ and $C(z)$ are the per unit length (p.u.l.) z-dependent resistance, inductance, conductance and capacitance per-unit-length matrices, respectively. (1) can be rewritten in the following form,

$$T\frac{dX(z,s)}{dz} = -\left(N(z)+sM(z)\right)X(z,s) \tag{2}$$

where

$$\begin{aligned}N(z) &= \begin{pmatrix} R(z) & 0 \\ 0 & G(z)\end{pmatrix}\\ M(z) &= \begin{pmatrix} L(z) & 0 \\ 0 & C(z)\end{pmatrix}\\ T &= \begin{pmatrix} 0 & U_m \\ U_m & 0\end{pmatrix},\\ X(z,s) &= \begin{pmatrix} I(z,s) \\ V(z,s)\end{pmatrix}\end{aligned} \tag{3}$$

and U_m is an $m \times m$ identity matrix. $X(z,s)$ may be expressed in a Taylor's series expansion around some frequency point, s_0, in the frequency-domain

$$X(z,s) = \sum_{i=0}^{\infty} U^{(i)}(z,s_0)(s-s_0)^i \tag{4}$$

where it can be observed from (4) that the moments $U^{(i)}(z,s_0)$ are z-dependent. The fundamental idea in MOR using ICT is to construct an orthonormal basis for the first few moments. In fact, this idea is similar to that of MOR of lumped component networks, except that in this case the moments are functions of the spatial variable z whereas in the lumped component case the moments are constant vectors and can be represented as elements of the Euclidian space $\mathbb{C}^{2m}$. A classical method to generate an orthogonal basis for a set of elements is the Modified Gram-Schmidt (MGS) process. Using MGS to construct an orthonormal basis for the moments $U^{(i)}(z,s_0)$, however, requires a proper definition of the "inner-product" and "norm" mappings on the space of these moments. For that purpose, the following two mappings have been adopted [7]

$$\begin{aligned}\langle u(z)|v(z)\rangle &= \int_0^1 u(z)^T v(z)dz\\ \|u(z)\| &= \int_0^1 u(z)^T u(z)dz\end{aligned} \tag{5}$$

where $u(z)$ and $v(z) \in \mathcal{L}^{2m}$ and $\mathcal{L}^{2m}$ is the space of all z-dependent vectors of length $2m$. The algorithm then proceeds by first computing the moments explicitly, then applying MGS using the definitions of the inner-product and norm as given in (5) to form an orthonormal basis $Q(z)$ for the moments that is used in an ICT to reduce the system in (2) to the following form

$$\hat{Y}(s) = \hat{b}^T\left(s\hat{M}+\hat{N}\right)^{-1}\hat{b} \tag{6}$$

where

$$\begin{aligned}\hat{M} &= d\int_0^1 Q(z)^T M(z)Q(z)dz\\ \hat{N} &= \hat{N}_1+\hat{N}_2\\ \hat{N}_1 &= d\int_0^1 Q(z)^T N(z)Q(z)dz\\ \hat{N}_2 &= \hat{T}-P\\ P &= Q_i(1)^T Q_v(1) - Q_i(0)^T Q_v(0)\\ \hat{T} &= \int_0^1 Q(z)^T T\frac{dQ(z)}{dz}dz,\\ \hat{b} &= \begin{bmatrix} Q_i(0) \\ -Q_i(1)\end{bmatrix}^T\end{aligned} \tag{7}$$

with $Q_i(z)$ and $Q_v(z)$ denoting those parts of the basis corresponding to the current and voltage portions, respectively.

Using the moments explicitly to construct the orthonormal basis undermines the numerical robustness of the algorithm. This is due to the fact that higher-order moments obtained from a single point expansion tend to add no further accuracy to the resulting reduced-order model in order to match wider frequency range. This problem has been addressed extensively in the context of MOR of lumped RLC networks [9]. RLC lumped networks have their moments related through an algebraic operator. Distributed networks, on the other hand, have their moments related through a differential operator and general orthogonlization processes such as MGS has to take that into account in order to avoid computing the moments explicitly.

3. OUTLINE OF THE PROPOSED ALGORITHM

The objective of the proposed algorithm is to develop an orthogonalization process that generates an orthonormal basis for a set of moments related through a differential operator, but without having to compute these moments explicitly. To this end, we substitute from (2) into (4) and equate similar powers of s to obtain

$$\begin{aligned}\mathcal{D}U^{(0)}(z,s_0) &= 0\\ \mathcal{D}U^{(i)}(z,s_0) &= -M(z)U^{(i-1)}(z)\end{aligned} \tag{8}$$

where $\mathcal{D}$ is an operator given by

$$\mathcal{D} \equiv \frac{d}{dz} + s_0 M(z) + N(z) \tag{9}$$

It is to be noted here that the moments are related through a differential operator, while the moments in the case of lumped RLC networks are related through an algebraic operator. Computing an orthognal basis for the moments $U^{(i)}(z,s_0)$ without having these moments explicitly will be approached by adopting a different version for the inner-product and norm mappings. Those mappings will be defined as follows,

$$\begin{aligned}\langle u(z)|v(z)\rangle &= \int_a^b u(z)^T v(z)w(z)dz\\ \|u(z)\| &= \int_a^b u(z)^T u(z)w(z)dz\end{aligned} \tag{10}$$

where $w(z)$ is a positive weighting scalar function and a and b are arbitrary integration limits. It is obvious from (10) that if $u(z)$ and $v(z)$

are expressed as a series summation of some polynomials, $g_i(z)$,

$$u(z) = \sum_{i=0}^{N} U_i g_i(z) \qquad v(z) = \sum_{i=0}^{N} V_i g_i(z) \tag{11}$$

where these polynomials $g_i(z), i = 0, \cdots, N$ are orthogonal in the interval $[a,b]$ w.r.t. to the weighting function $w(z)$, then both the inner-product and norm mappings reduce to a summation of the Euclidian inner-products of the individual coefficients. The particular choice of $w(z), a, b$ and $g_i(z)$ comes as an implementation issue and will be discussed later in the following section.

Next constructing the orthogonal basis without computing the moments explicitly will be demonstrated. For that purpose, we assume that the basis $Q(z)$ consists of k elements $\{q_0(z,s_0), \cdots, q_{k-1}(z,s_0)\}$. The process of computing $Q(z)$ will be demonstrated inductively through two steps.

- **Step 1. Computing** $q_0(z,s_0)$**:** In this step, the following system of differential equations

$$T\frac{d\tilde{q}_0(z,s_0)}{dz} + [N(z) + s_0 M(z)]\,\tilde{q}_0(z,s_0) = 0 \tag{12}$$

  is solved using a set of initial conditions that are obtained as specified in Appendix C of [7]. $q_0(z,s_0)$ can be obtained from $\tilde{q}_0(z,s_0)$ using the following normalization step

$$q_0(z,s_0) = \frac{1}{\|q_0(z,s_0)\|}\tilde{q}_0(z,s_0) \tag{13}$$

- **Step 2. Computing** $q_i(z,s_0)$**:** In this step, the first i elements $\{q_0(z,s_0), \cdots, q_{i-1}(z,s_0)\}$ are assumed to be readily computed and the objective becomes that of computing the next element $q_i(z,s_0)$. The first step in achieving this is carried out through solving the following system of differential equations

$$T\frac{d\tilde{q}_i(z,s_0)}{dz} + [N(z) + s_0 M(z)]\,\tilde{q}_i(z,s_0) = M(z)\sum_{h=0}^{H}\theta_{i-1,h}g_h(z) \tag{14}$$

  where $\theta_{i-1,h}$'s are the coefficients of $q_{i-1}(z,s_0)$ as expressed in terms of the summation of the series of the orthogonal polynomials, $g_h(z)$, and $\tilde{q}_i(z,s_0)$ is the response of the system of differential equations in (2) under the forcing input given by $M(z)q_i(z,s_0)$. Next, $\tilde{q}_i(z,s_0)$ is orthogonalized w.r.t. to the previous elements in the basis using the inner-product mapping defined in (10) to produce $\hat{q}_i(z,s_0)$, i.e.,

$$\hat{q}_i(z,s_0) = \tilde{q}_i(z,s_0) - \sum_{j=0}^{i-1}\left\langle \tilde{q}_i(z,s_0)|q_j(z)\right\rangle q_j(z) \tag{15}$$

  $q_i(z)$ can then be obtained by normalizing $\hat{q}_i(z)$, i.e., $q_i(z) = \hat{q}_i(z)/\|\hat{q}_i(z)\|$.

Based on the above two steps, the main theoretical result of this section can be stated as follows.

THEOREM 1. *Let $Q(z)$ be the set of elements generated by executing (step 1) once and (step 2) for k times as shown above. Then $Q(z)$ is orthonormal and spans the Hilbert subspace spanned by the moments $\{U^{(0)}(z,s_0), \cdots, U^{(k)}(z,s_0)\}$.*

Proof has been omitted due to lack of space.

4. IMPLEMENTATION DETAILS

This section presents further implementation details regarding the computation of the orthogonal basis $Q(z)$. In particular, Chebychev polynomials of the first kind are adopted as our choice for the orthogonal polynomials $g_h(z)$. Using Chebyshev polynomial requires scaling the spatial variable z by replacing it with $\bar{z}$ where $\bar{z} = (2/d)z - 1$. Thus, with regards to (10), we have $a = -1$, $b = 1$ and $w(z) = 1/\sqrt{1-z^2}$. Let $u(\bar{z})$ and $v(\bar{z}) \in \mathcal{L}^{2m}$ be represented as a series of Chebyshev polynomials, i.e.,

$$u(\bar{z}) = \sum_{h=0}^{H} U_h T_h(\bar{z}) \quad v(\bar{z}) = \sum_{h=0}^{H} V_h T_h(\bar{z}) \tag{16}$$

where $T_h(\bar{z})$ is the h^{th} Chebyshev polynomial of the first kind. In that case, the inner-product and norm in (10) reduce to

$$\begin{aligned} \langle u(\bar{z})|v(\bar{z})\rangle &= \pi {U_0}^T V_0 + \frac{\pi}{2}\sum_{h=H}^{N} {U_h}^T V_h, \\ \|u(\bar{z})\| &= \pi {U_0}^T U_0 + \frac{\pi}{2}\sum_{h=1}^{H} {U_h}^T U_h \end{aligned} \tag{17}$$

Computing the k^{th} element of the basis $Q(\bar{z})$ proceeds by first representing all the z-dependent quantities in (14) as a summation of H Chebyshev polynomials

$$\begin{aligned} N(z) &= \sum_{h=0}^{H} N_h T_h(\bar{z}) \\ M(z) &= \sum_{h=0}^{H} M_h T_h(\bar{z}) \\ q_{k-1}(z) &= \sum_{h=0}^{H} \theta_{k-1,h} T_h(\bar{z}) \\ \tilde{q}_k(z) &= \sum_{h=0}^{H} \tilde{\theta}_{k,h} T_h(\bar{z}) \end{aligned} \tag{18}$$

Substituting from (18) into (14), taking the integral of both sides from -1 to $\bar{z}$, and using the following relations

$$\begin{aligned} T_m(\bar{z})T_n(\bar{z}) &= \frac{1}{2}(T_{m+n}(\bar{z}) + T_{|m-n|}(\bar{z})) \\ \int_{-1}^{\bar{z}} T_0(\bar{z})d\bar{z} &= T_0(\bar{z}) + T_1(\bar{z}) \\ \int_{-1}^{\bar{z}} T_1(\bar{z})d\bar{z} &= \frac{1}{4}(T_2(\bar{z}) - T_0(\bar{z})) \\ \int_{-1}^{\bar{z}} T_h(\bar{z})d\bar{z} &= \frac{1}{2}\left(\frac{T_{h+1}(\bar{z})}{h+1} + \frac{T_{h-1}(\bar{z})}{h-1}\right) + \frac{(-1)^{h+1}}{h^2-1} \end{aligned}$$

and equating coefficients of similar Chebyshev polynomials on both sides yields

$$\left[A + \frac{2}{d}\left(U_{H+1}\otimes T\right)\right]\tilde{\Phi}_i = A_M \Phi_{i-1} \tag{19}$$

where

$$\begin{aligned} A &= \frac{1}{2}(K_1 + K_2 + K_3) \\ A_M &= \frac{1}{2}(T_{M1} + T_{M2} + T_{M3}) \\ \Phi_{i-1} &= \left[{\theta_{i-1,H}}^T \cdots {\theta_{i-1,0}}^T\right]^T \\ \tilde{\Phi}_i &= \left[\tilde{\theta}_{i,H}^T \cdots \tilde{\theta}_{i,0}^T\right]^T \end{aligned} \tag{20}$$

$$K_1 = \begin{bmatrix} 0 & \cdots & 0 & \frac{1}{2H} D_H & \frac{1}{2H} D_{H-1} \\ 0 & \cdots & \frac{1}{2(H-1)} D_H & \frac{1}{2(H-1)} D_{H-1} & \frac{1}{2(H-1)} (D_{H-2} - D_H) \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \ddots & D_2 - \frac{1}{2} D_4 & D_1 - \frac{1}{2} D_3 & D_0 - \frac{1}{2} D_2 \\ D_H & \cdots & \cdots & D_1 - \frac{1}{4} D_2 + \sum_{h=2}^{H-1} D_h \frac{(-1)^{h+1}}{h^2-1} & D_0 - \frac{1}{4} D_1 + \sum_{h=2}^{H} D_h \frac{(-1)^{h+1}}{h^2-1} \end{bmatrix} \tag{21}$$

$$K_2 = \begin{bmatrix} \frac{1}{2H} D_1 & 0 & \cdots & \cdots & 0 & 0 \\ \frac{1}{2(H-1)} (D_2 - D_0) & \frac{1}{2(H-1)} D1 & 0 & \cdots & 0 & 0 \\ \vdots & \cdots & \cdots & \ddots & \vdots & \vdots \\ \frac{1}{4} (D_{H-1} - D_{H-3}) & \cdots & \frac{1}{4} D_1 & \frac{1}{4} D_0 & 0 & \\ -\frac{1}{2} D_{H-2} & \cdots & -\frac{1}{2} D_1 & -\frac{1}{2} D_0 & 0 & 0 \\ -\frac{1}{4} D_{H-1} + \sum_{h=2}^{H} D_{H-h} \frac{(-1)^{(h+1)}}{h^2-1} & \cdots & \cdots & -\frac{1}{4} D_1 - \frac{1}{3} D_0 & -\frac{1}{4} D_0 & 0 \end{bmatrix} \tag{22}$$

$$K_3 = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \frac{1}{2H} (D_{H-2} - D_H) & \frac{1}{2H} D_{H-1} \\ 0 & \cdots & \ddots & \ddots & \cdots & \vdots \\ 0 & \cdots & -\frac{1}{4} D_0 & -\frac{1}{4} D_1 & \frac{1}{4} (D_0 - D_2) & \frac{1}{4} (D_1 - D_3) \\ 0 & \cdots & 0 & -\frac{1}{2} D_0 & -\frac{1}{2} D_1 & D_0 - \frac{1}{2} D_2 \\ D_0 & \cdots & \cdots & \sum_{h=2}^{H} D_{h-2} \frac{(-1)^{h+1}}{h^2-1} & -\frac{1}{4} D_0 + \sum_{h=2}^{H} D_{h-1} \frac{(-1)^{h+1}}{h^2-1} & D_0 - \frac{1}{4} D_1 + \sum_{h=2}^{H} D_h \frac{(-1)^{h+1}}{h^2-1} \end{bmatrix} \tag{23}$$

and U_{H+1} is $(H+1) \times (H+1)$ identity matrix and $\otimes$ denotes the Kronecker product operator. In (20), the definitions of $K_1, K2,$ and K_3 are given by (21)- (23) where $D_j = N_j + s_0 * M_j$. Similar formulae for $T_{M1}, T_{M2},$ and T_{M3} can be obtained by substituting D_k by M_K for all possible values of the index k.

The reduced system is obtained in the form

$$\hat{Y}(s) = \hat{b}^T \left(s\hat{M} + \hat{N} \right)^{-1} \hat{b} \tag{24}$$

where

$$\begin{aligned} \hat{M} &= \frac{d}{2} \int_{-1}^{1} Q(z)^T M(z) Q(z) dz, \\ \hat{N} &= \hat{N}_1 + \hat{N}_2, \\ \hat{N}_1 &= \frac{d}{2} \int_{-1}^{1} Q(z)^T N(z) Q(z) dz, \\ \hat{N}_2 &= \hat{T} - P, \\ P &= Q_i(1)^T Q_v(1) - Q_i(-1)^T Q_v(-1), \\ \hat{T} &= \int_{-1}^{1} Q(z)^T T \frac{dQ(z)}{dz} dz \\ \hat{b} &= \begin{bmatrix} Q_i(-1) \\ -Q_i(1) \end{bmatrix} \end{aligned} \tag{25}$$

5. NUMERICAL RESULTS

5.1 Example 1

The proposed algorithm was implemented to obtain a reduced-order model for a uniform 3-conductor TL network of length $d = 10cm$. Figure 1 shows a graphical comparison between the exact response (obtained through an Ordinary Differential Equation (ODE) solver for the Telegrapher's equations in (1)) and the response obtained from the proposed algorithm after running it for $k = 65$ iterations. Also shown on the same graph, the response obtained by using 65 moments explicitly to construct the basis as in [7]. It is clear that for the same size of the reduced system, the one based on the proposed algorithm could match up double the frequency range matched through using the moments explicitly.

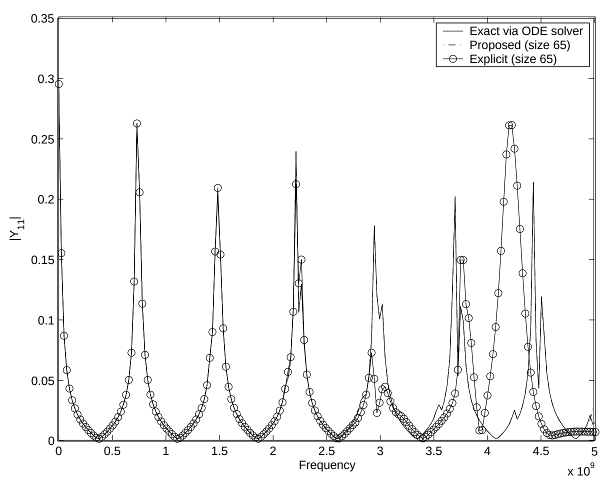


Figure 1: A comparison for the $|Y_{11}|$ parameter between the proposed algorithm the algorithm in [7] using explicit moments computation.

5.2 Example 2

The proposed algorithm was used to simulate a nonuniform TL network consisting of 3 coupled conductors. First the exact frequency-domain response of the system was obtained by solving the Telegrapher's equations in (1) using a numerical ODE solver. Figures 2(a) and 2(b) show a comparison between the exact response (through ODE solution) and the response obtained from the proposed algorithm for a sample of the Y-parameters. Figure 2(c) shows a comparison for the time-domain response, due to an input step signal of rise-time 0.1 ns. The comparison is between a lumped

RLC approximation of the line and the proposed algorithm. The lumped RLC model is obtained by dividing the nonuniform line into 50 sections and considering each section as a uniform line. Each uniform section is then represented by a number of lumped RLC sections according to its p.u.l. delay.

5.3 Example 3

A similar example to the previous one but with length, $d = 50cm$ has been employed to test the proposed algorithm. Figures 3(a) and 3(b) display a frequency-domain comparison for a sample of the Y-parameters. Here the frequency-domain response obtained from the proposed algorithm was computed using only single point expansion at 1.75 GHz. Figure 3(c) displays a comparison for the time-domain response due to an input pulse of rise/fall time of 0.1 ns and a pulse-width of 8 ns.

Table 1 displays a CPU comparison for the time taken to simulate both lumped and reduced models for both examples. The time-domain simulation was performed using the Numerical Algorithms Group (NAG) [13] sparse differential-algebraic equations solver on a Pentium III machine.

	Lumped	Proposed	Speedup
10cm Line	26.3 sec	0.93 sec	28
50cm Line	693 sec	5.9 sec	117

Table 1: CPU time comparison

6. CONCLUSION

This paper presented a new algorithm for passive model-reduction based on integrated congruent transform. The proposed algorithm does not require an explicit computation of the moments in order to construct a spanning orthogonal basis for them. It thus shows numerical superiority in that larger frequency range can be matched based on single point expansion.

7. REFERENCES

[1] Clayton R. Paul, *Analysis of Multiconductor Transmission Lines*, Wiley, New York, 1994.

[2] R. Achar and M. Nakhla, “Simulation of high-speed interconnects”, *Proceedings IEEE*, vol. 89, n. 5, pp. 693–728, May 2001.

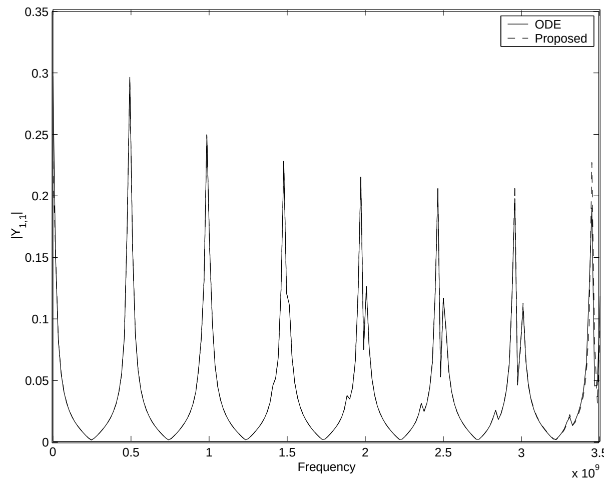
[3] A Odabasioglu, M. Celik and L. T. Pileggi, “PRIMA: passive reduced-order interconnect macromodeling algorithm”, *IEEE Trans. Computer-Aided Design*, vol. 17, n. 8, pp. 645–654, August 1998.

[4] K. J. Kerns and A. T. Yang, “Preservation of passivity during RLC network network reduction via split congruence transformations”, *IEEE Trans. Computer-Aided Design*, vol. 17, pp. 582–591, July 1998.

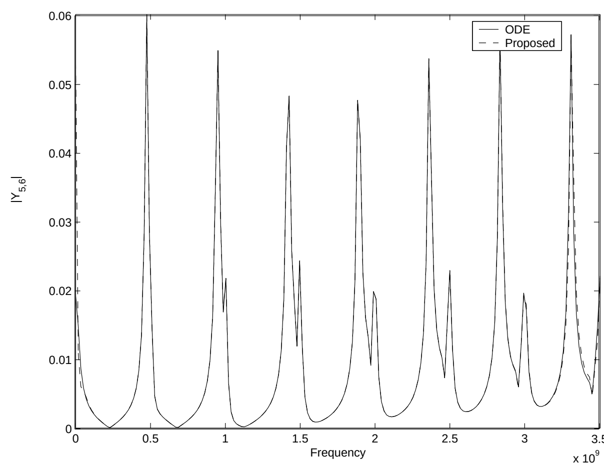
[5] A. Dounavis, X. Li, M. Nakhla and R. Achar, “Passive closed-loop transmission line model for general purpose circuit simulators”, *IEEE Trans. Microwave Theory Tech.*, vol. 47, n. 12, pp. 2450–2459, December 1999.

[6] A. Dounavis, R. Achar and M. Nakhla, “Efficient passive circuit models for distributed networks with frequency-dependent parameters”, *IEEE Trans. Adv. Packag.*, vol. 23, pp. 382–392, August 2000.

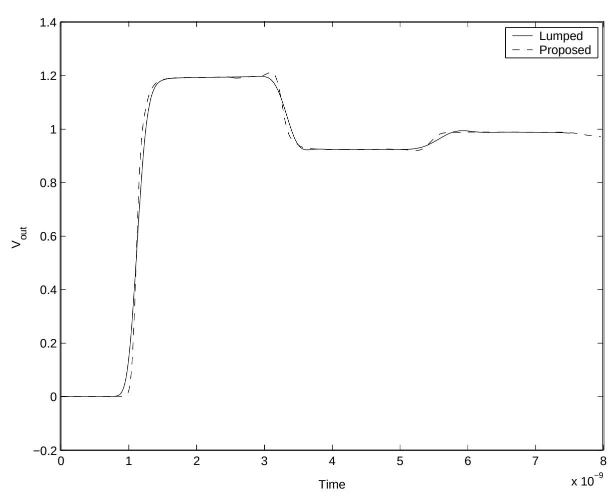
[7] Q. Yu, J. M. L. Wang and E. S. Kuh, “Passive multipoint moment matching model order reduction algorithm on



(a) Frequency Response at $Y_{1,1}$

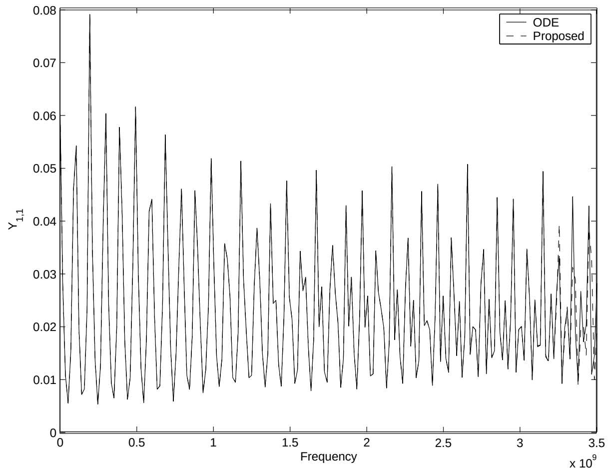


(b) Frequency Response at $Y_{5,6}$

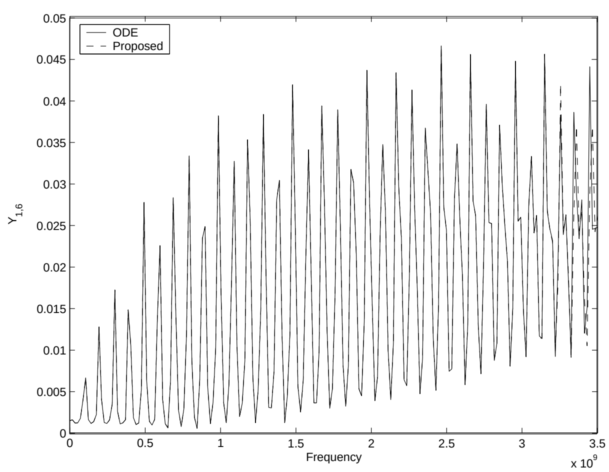


(c) Time Domain Response

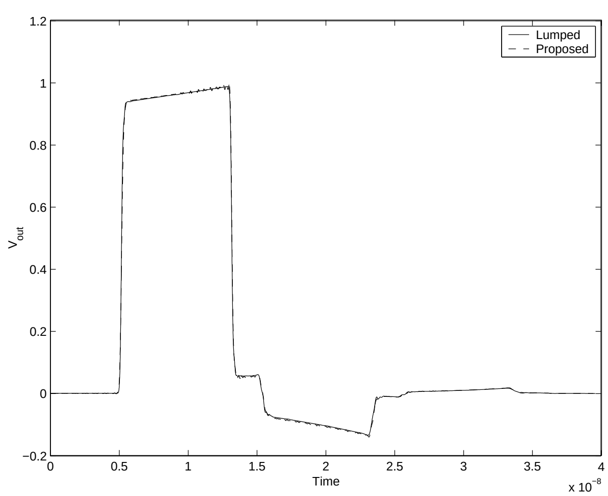
Figure 2: Numerical results for a 10 cm nonuniform transmission line.



(a) Frequency Response at $Y_{1,1}$



(b) Frequency Response at $Y_{1,6}$



(c) Time Domain Response

Figure 3: Numerical results for a 50 cm nonuniform transmission line

multiport distributed interconnect networks”, *IEEE Trans. Circuits Syst.*, vol. 46, n. 1, pp. 140–160, January 1999.

[8] K. Kundert, *The Designer’s Guide to SPICE and Spectre*, Kluwer Academic Publishers, Norwell, MA., 1995.

[9] P. Feldmann and R. W. Freund, “Efficient linear cicuit analysis by Padé via Lanczos process”, *IEEE Trans. Computer-Aided Design*, vol. 14, pp. 639–649, May 1995.

[10] M. Sylveria, M. Kamon, I. Elfadel and J. White, “A coordinate-transformed Arnoldi algorithm for generating guaranteed stable reduced-order models of arbitrary RLC circuits”, in *Proc. ACM ICCAD*, pp. 288–294, November 1996.

[11] A. Odabasioglu, M. Celik and L.T. Pileggi, “Practical considerations for passive reduction of RLC circuits”, in *Proc. Design Automation Conf.*, pp. 214–219, 1999.

[12] M. Kamon, F. Wang and J. White, “Generating nearly optimally compact models from Krylov subspace based reduced-order models”, *IEEE Trans. Circuits Syst. II*, vol. 47, n. 4, pp. 239–248, April 2000.

[13] Numerical Algorithms Group Limited, Oxford, UK, *NAG Fortran Library Mark 19*, July 1999.