Efficient Mixed-Domain Analysis of Electrostatic MEMS

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Abstract

We present efficient computational methods for scattered point and meshless analysis of electrostatic microelectromechanical systems (MEMS). Electrostatic MEMS are governed by coupled mechanical and electrostatic energy domains. A self-consistent analysis of electrostatic MEMS is implemented by combining a finite cloud method based interior mechanical analysis with a boundary cloud method based exterior electrostatic analysis. Lagrangian descriptions are used for both mechanical and electrostatic analyses. Meshless finite cloud and boundary cloud methods combined with fast algorithms and Lagrangian descriptions are flexible, efficient and attractive alternatives compared to conventional finite element/boundary element methods for self-consistent electromechanical analysis. Numerical results are presented for an electrostatic comb drive device.

keywords: coupled electro-mechanical analysis, meshless, finite cloud method, boundary cloud method, Lagrangian electrostatics

1 Introduction

Although there are many microelectromechanical system designs that use piezoelectric, thermal, pneumatic, and magnetic actuation, the most popular approach in present day microsensor and microactuator designs is to use electrostatic forces to move micromachined parts - referred to as electrostatic MEMS. Computational analysis [1] of electrostatic MEMS requires a self-consistent solution of the coupled interior mechanical domain and the exterior electrostatic domain [2, 3, 4]. Conventional methods for coupled domain analysis, such as FEM/BEM, require mesh generation, mesh compatibility, re-meshing and interpolation of solution between the domains. Mesh generation can be difficult and time consuming for complex geometries. Furthermore, mesh distortion can occur for micromechanical structures that undergo large deformations. To overcome all these difficulties, we propose an efficient approach to perform static analysis of electrostatically actuated MEMS. The primary contributions of the paper are as follows: (1) We employ a meshless Finite Cloud Method (FCM)[5, 6] to solve the interior mechanical domain. The Finite Cloud Method is a true meshless method in which only points are needed to cover the structural domain and no connectivity information among the points is required. This method completely eliminates the meshing process and radically simplifies the analysis procedure. (2) A Boundary Cloud Method (BCM) [7, 8] is used to analyze the exterior electrostatic domain to compute the electrostatic forces acting on the surface of the structures. The BCM utilizes a meshless interpolation technique and a cell based integration. Besides the flexibility of the cell integration, the BCM is an excellent match to the FCM for coupled domain analysis since both of them have meshless interpolations. (3) A Lagrangian description [9] of the boundary integral equation is developed and implemented with BCM. Typically, the mechanical analysis is performed by a Lagrangian approach using the undeformed position of the structures. However, the electrostatic analysis is performed on the deformed position of the conductors. The Lagrangian description maps the electrostatic analysis to the undeformed position of the conductors. Thus, the electrostatic forces and mechanical deformations are all computed on the undeformed configuration of the structures. The Lagrangian description eliminates the requirement of geometry updates and re-computation of the interpolation functions.

2 Coupled Analysis of Electromechanical Systems

Computational analysis of electrostatically actuated MEMS requires a self-consistent solution of the coupled mechanical and electrical equations. Conventionally, a Finite Element Method (FEM) is employed to perform the mechanical analysis and a Boundary Element Method (BEM) is employed to compute the surface electrostatic forces. The mechanical analysis is performed by discretizing the structural or the mechanical domain into nodes and elements. A finite element analysis is then performed by applying the electrostatic pressure as Neumann boundary conditions to compute the structural displacements. Once the displacement is computed, the geometry of the structure or the conductor is updated. Electrostatic analysis is performed on the updated geometry by discretizing the surface of the conductor into panels or elements. A boundary element method is then used to compute the surface charge density on each panel. Once the surface charge densities are known, the new electrostatic pressure is computed. The mechanical and electrostatic analysis are repeated until an equilibrium state is computed. Algorithm 1 summarizes the key steps involved in the self-consistent solution of the coupled electromechanical problem.

There are several difficulties with the approach described in Algorithm 1: (1) The structural domain needs to be disctretized into elements. For structures with complex geometries mesh generation can be a complicated and time consuming task. (2) Typically, the boundary element mesh on the surface of the conductors does not match with the finite element mesh. In this case, the electrostatic pressure computed from the BEM analysis needs to the interpolated to the finite element mesh so that a mechanical analysis can be performed. The interpolation process can be cum-

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Algorithm 1 A procedure for self-consistent analysis of coupled electromechanical devices

repeat

1. Do mechanical analysis (on the undeformed geometry) to compute structural displacements
2. Update the geometry of the conductors using the computed displacements
3. Compute surface charge density by electrostatic analysis(on the deformed geometry)
4. Compute electrostatic forces (on the deformed geometry)
5. Transform electrostatic forces to the original undeformed configuration

until an equilibrium state is reached

bersome and can introduce significant error. One solution to this problem is to match the finite element nodes on the surface of the conductors with the boundary element nodes so that no interpolation is involved. However, this can be inefficient as a refinement of either the finite element mesh or the boundary element mesh would require that the other domain be remeshed. (3) The need to update the geometry of the conductors before an electrostatic analysis is performed during each iteration also presents several problems—First, flat surfaces of the conductors in the initial configuration can become curved due to conductor deformation. This requires the development of complex integration schemes on curved panels to perform electrostatic analysis. Second, when the structure undergoes large deformation, remeshing the surface may become necessary before an electrostatic analysis is performed. Third, interpolation functions, used in many numerical methods, need to be recomputed whenever the geometry changes. Each of these issues significantly increases the computational effort making the self-consistent analysis of electrostatic MEMS an extremely complex and challenging task.

The combination of the finite cloud method, boundary cloud method and the Lagrangian electrostatics approach overcomes the difficulties mentioned above. First, a finite cloud method does not require a mesh or elements and mechanical analysis can be performed by simply sprinkling points with out the need for connectivity information among the nodes or points. Second, using a boundary cloud method, exterior electrostatic analysis is performed by sprinkling points on the surface of the conductors and using a background cell structure for integration purpose. Unlike a boundary element method, the boundary cloud method method does not require panels or elements. Third, by combining the Lagrangian electrostatics formulation with the total Lagrangian mechanical formulation, coupled electromechanical analysis can be implemented using only the initial configuration. The use of Lagrangian techniques for both mechanical and electrostatic analysis eliminates the need for geometry updates there by simplifying the coupled electromechanical analysis. Algorithm 2 summarizes the Lagrangian approach for efficient scattered point and self-consistent analysis of coupled electromechanical devices.

3 FCM For Mechanical Analysis

Electrostatically actuated microstructures can undergo large deformations for certain geometric configurations and applied

Algorithm 2 A procedure for self-consistent analysis of coupled electromechanical devices by using a Lagrangian approach for both mechanical and electrostatic analysis

repeat

1. Do mechanical analysis (on the undeformed geometry) by FCM to compute structural displacements
2. Do electrostatic analysis(on the undeformed geometry) by BCM to compute surface charge density
3. Compute electrostatic pressure (on the undeformed geometry)

until an equilibrium state is reached

voltages. In this paper, we perform 2-D geometrically nonlinear analysis of microstructures. For electro-mechanical analysis, the governing equations for an elastic body using a Lagrangian description are given by

$$\nabla \cdot (\mathbf{FS}) + \mathbf{B} = 0 \qquad in \quad \Omega \tag{1}$$

$$\mathbf{u} = \mathbf{G} \qquad on \quad \Gamma_g \tag{2}$$

$$\mathbf{P} \cdot \mathbf{N} = \mathbf{H} \qquad on \quad \Gamma_h \tag{3}$$

where Ω is the mechanical domain, Γ_g is the portion of the boundary on which Dirichlet boundary conditions are specified and Γ_h is the portion of the boundary on which Neumann boundary conditions are specified,. The boundary of the mechanical domain is given by $\Gamma = \Gamma_g \cup \Gamma_h$. $\mathbf{F}$ is the deformation gradient, $\mathbf{S}$ is the second Piola-Kirchhoff stress, $\mathbf{B}$ is the body force vector per unit undeformed volume, $\mathbf{u}$ is the displacement, $\mathbf{G}$ is the prescribed displacement, $\mathbf{P}$ is the first Piola-Kirchhoff stress tensor given by $\mathbf{P} = \mathbf{FS}$, $\mathbf{N}$ is the unit outward normal vector in the initial configuration and $\mathbf{H}$ is the surface traction vector per unit undeformed area. For electromechanical analysis, $\mathbf{H} = P_e J \mathbf{F}^{-T}\mathbf{N}$, where P_e is the surface electrostatic pressure and J is the determinant of $\mathbf{F}$.

We use a meshless Finite Cloud Method to solve the mechanical equations given in Eq. (1-3). The Finite Cloud Method uses a fixed kernel technique to construct the interpolation functions and a point collocation technique to discretize the governing partial differential equations. In a 2-D fixed kernel approach, an approximation $u^a(x,y)$ to an unknown $u(x,y)$ is given by

$$u^a(x,y) = \int_\Omega C(x,y,x_k - s, y_k - t)\phi(x_k - s, y_k - t)u(s,t)\,ds\,dt \tag{4}$$

where ϕ is the kernel function centered at (x_k, y_k) which is usually taken as a cubic spline or a Gaussian function. $C(x,y,x_k - s, y_k - t)$ is the correction function given by

$$C(x,y,x_k - s, y_k - t) = \mathbf{P}^T(x_k - s, y_k - t)\mathbf{C}(x,y) \tag{5}$$

$\mathbf{P}^T = \{p_1, p_2, \ldots, p_m\}$ is an $1 \times m$ vector of basis functions. In two dimensions, a quadratic basis vector $\mathbf{P}^T = \{1, x_k - s, y_k - t, (x_k - s)^2, (x_k - s)(y_k - t), (y_k - t)^2\}, m = 6$ is employed. $\mathbf{C}(x,y)$ is an $m \times 1$ vector of unknown correction function coefficients. The correction function coefficients are computed by satisfying the consistency conditions (see [5, 6] for details). The discrete form of the approximation $u^a(x,y)$ is given by

$$u^a(x,y) = \sum_{I=1}^{NP} N_I(x,y)\hat{u}_I \tag{6}$$

where $\hat{u}_I$ is the nodal parameter for node I, and $N_I(x,y)$ is the fixed kernel meshless interpolation function (see [5, 6] for details). In the mechanical analysis, the displacements u and v are approximated by using Eq. (6). Consequently, the deformation gradient $\mathbf{F}$ and all the other mechanical quantities can be rewritten as functions of the approximated displacements u^a and v^a. After the interpolation functions are constructed, FCM uses a point collocation technique to discretize the governing equations.

4 Lagrangian Electrostatics

When electrostatic potentials are applied on micro-structures, electrostatic forces are generated on the surfaces of the microstructures. The structures undergo deformation because of electrostatic forces and the surface charge density on the structure redistributes. Typically, the new surface charge density is computed by updating the geometry of the microstructures and redoing an electrostatic analysis. The basic idea in Lagrangian electrostatics is to compute the new surface charge density without updating the geometry of the microstructures.

The 2D governing equation for electrostatic analysis can be written in a boundary integral form as [10]

$$\phi(p) = \int_{d\omega} G(p,q)\sigma(q)d\gamma_q + C \quad (7)$$

$$\int_{d\Omega} \sigma(q)d\gamma_q = C_T \quad (8)$$

where p is the source point, q is the field point which moves along the boundary of the conductors and G is the Green's function. In two dimensions, $G(p,q) = -ln|p-q|/(2\pi\varepsilon)$, where ε is the dielectric constant of the medium and $|p-q|$ is the distance between p and q. C_T is the total charge of the system and C is an unknown variable which can be used to compute the potential at infinity.

Equations (7) and (8) are defined in the deformed configuration of the conductors, i.e., the surface charge density is computed by solving the boundary integral equations on the deformed geometry of the conductors. We refer to this approach as the deformed configuration approach. The need to update the geometry of the structures in the deformed configuration approach presents several difficulties as stated in Section 2. In this paper, we employ a Lagrangian approach [9] to compute the surface charge density in the undeformed configuration of the conductors. The Lagrangian form of the boundary integral equations given in Eq. (7-8) is given by

$$\phi(P) = \int_{d\Omega} G(p(P), q(Q))\sigma(q(Q))J(Q)d\Gamma_Q + C \quad (9)$$

$$\int_{d\Omega} \sigma(q(Q))J(Q)d\Gamma_Q = C_T \quad (10)$$

where P and Q are the source and field points in the initial configuration corresponding to the source and field points p and q in the deformed configuration, $J(Q) = (\mathbf{T}(Q)\cdot\mathbf{C}(Q)\mathbf{T}(Q))^{\frac{1}{2}}$, $\mathbf{T}(Q)$ is the tangential unit vector at field point Q and $\mathbf{C}(Q)$ is the Green deformation tensor.

5 BCM For Electrostatic Analysis

A boundary cloud method is employed to solve the Lagrangian description of the electrostatic governing equations (Eq.(9-10)). In a boundary cloud method, the surface of the domain is discretized into scattered points. The points can be sprinkled randomly covering the boundary of the domain. Interpolation functions are constructed by centering a weighting function at each point or node. For the electro-mechanical problem, the potential ϕ is prescribed on the structures. The unknown surface charge density σ in the vicinity of the point t is approximated by either a Hermite-type interpolation [7] or a varying basis least-squares approximation [8]. In this paper, we employ a varying basis least-squares approach to approximate the unknown quantity, i.e.

$$\sigma(x,y) = \mathbf{p}_v^T(x,y)\mathbf{b}_t \quad (11)$$

where $\mathbf{p}_v$ is the varying base interpolating polynomial and $\mathbf{b}_t$ is the unknown coefficient vector for point t. For a point t, the unknown coefficient vector $\mathbf{b}_t$ is computed by using a least-squares approach (see [8] for details). The discrete form of the varying basis approximation for the unknowns is given by

$$\sigma(x,y) = \sum_{I=1}^{NP} \overline{N}_I(x,y)\hat{\sigma}_I \quad (12)$$

The boundary of the structure is discretized into NC cells for integration purpose. Each cell contains a certain number of nodes and the number of nodes can vary from cell to cell. Different from an element or a panel in boundary-element methods, the cell can be of any shape or size and the only restriction is that the union of all the cells equal the boundary of the domain. Substituting the varying basis approximation for the unknown charge density, the boundary integral equation for the electrostatic problem given in Eq. (9-10) can be rewritten in a matrix form as

$$\mathbf{M}\hat{\sigma} = \phi \quad (13)$$

where $\mathbf{M}$ is an $(NC+1)\times(NC+1)$ coefficient matrix and ϕ and $\hat{\sigma}$ are $(NC+1)\times 1$ right hand side and unknown vector, respectively. By substituting the potential on the conductors and the total charge into Eq. (13), the surface charge density can be computed from Eq. (12) and Eq. (13).

6 Numerical Results

An electrostatic comb drive discussed in [11] is considered in this section. The device is simulated with scattered point distributions by using the methods described in the previous sections. As shown in Figure 1, a center mass with 12 teeth is supported by fixed-fixed beams. A voltage is applied between the movable comb and the fixed teeth. The support beams are 1000 μm long, 2.5 μm wide and 4.5 μm thick. The center mass is 98 μm by 98 μm. Each comb tooth is 49 μm long, 2.8 μm wide and 4.5 μm thick. The gap between the movable teeth and the fixed teeth is 5.6 μm and the initial overlap at zero volts is 16.8 μm. The Young's modulus of the comb structure is 169 GPa and the Poisson's ratio is 0.3. The scattered point distribution for the device is shown in Figure 2. Figure 3 presents the computed displacement as a function of the applied voltage. Both linear [12] and nonlinear elastostatic theories are employed in this example. As shown in Figure 3, the comb structure starts to operate in a mechanically nonlinear regime for an applied voltage of 100 V or higher. The stiffness of the supporting beams increases quickly as the displacement of the center mass increases. Thus, for MEMS actuators where a large stroke is desired, the fixed-fixed type support is not advantageous due to the high stiffness of the support. For this reason, folded supporting beams are widely used in comb drive applications. The fixed-fixed support, however, provides a higher stability and a higher resistance to the external forces. Therefore, for applications such as force sensors the fixed-fixed support could still be an appropriate choice.

7 Conclusions

We have presented a new combined finite cloud/boundary cloud method for efficient analysis of microelectromechanical devices. The FCM/BCM approach requires only a scattered set of points and no connectivity information or a mesh is necessary. Even though the electrostatic analysis is coupled to the mechanical analysis through the same set of boundary nodes, the point distribution for electrostatic analysis can be refined without affecting the interior mechanical analysis. The Lagrangian electrostatics formulation combined with the well-known Lagrangian mechanical formulation allows coupled electromechanical analysis with only the initial configuration, there by eliminating the need for geometry updates and recalculation of interpolation functions. Compared to the conventional FEM/BEM approach, the hybrid FCM/BCM along with Lagrangian electrostatic and mechanical analysis radically simplifies self-consistent analysis of electrostatic MEMS.

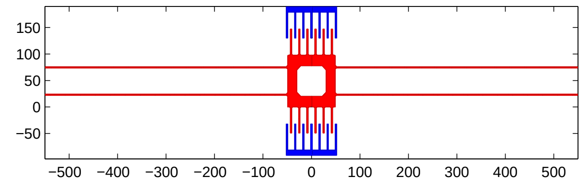

Figure 1: Electrostatic comb drive.

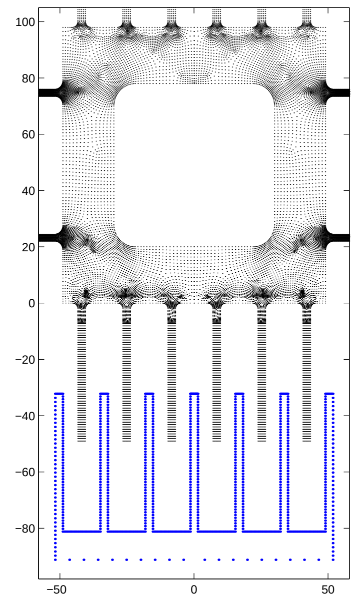

Figure 2: Scattered point distribution for the comb drive example

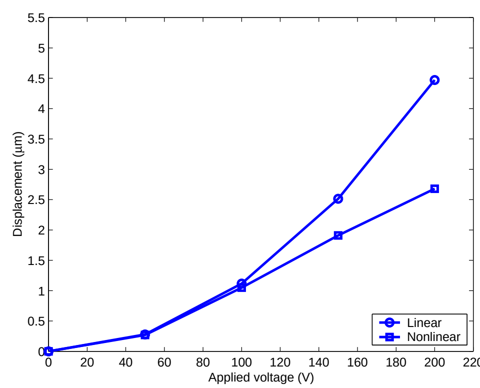

Figure 3: Comparison of the simulation results with the experimental data.

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