An Optimum Fitting Algorithm for Generation of Reduced-Order Models

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Abstract - ***The paper presents a new approach to the problem of automatic order reducing of rational transfer function (TF). The approach is directed to obtain low-order models under criterion of minimal integral square error. The developed computational scheme allows to determine reduced model of the minimal order with the given error tolerance.***

I. Introduction

The approximation of rational transfer function (TF) is one of the important problems for different practical applications in the circuit theory and the control theory. In particular, this problem arises in automated macromodel generation. In this case the purpose is to approximate the original Laplace-domain rational transfer function by low-order rational TF. In spite of detailed study of the main approaches and presence of computational procedures this problem remains open for many practical applications.

The present paper is devoted to solving the problem of automation of linear macromodels generation.

The problem of automatic generation of linear macromodels puts forward the following requirements to the reducing algorithm:

1) it is desired to provide approximating TF over all the frequencies because it is insufficient if the algorithm processes the given set of frequency points distributed in the specified frequency range.

2) to generate the macromodel with fixed order the reducing algorithm has to determine the parameters of reduced TF (numerator and denominator coefficients) which minimize the error of approximation.

3) to determine the order of macromodel the reducing algorithm has to minimize the macromodel order under given error tolerance.

The most popular model-order reduction scheme in circuit simulation is AWE method based on Pade approximation [1]. AWE is a generalized approach of approximating the dominant pole/zeros for linear circuit. Some techniques directed to solve large dimension problems arising in VLSI/ULSI design have been implemented (see for instance [2-9]). At the present time state-of-art in this research direction can be conditionally characterized by successful exploitation of Pade via Lanczos method [5]. Pade based techniques determine reduced order rational TF which agree with the initial TF and some number of its derivatives at the given frequency points. To extend this technique for wide frequency region the multipoint algorithms have been developed (see for instance [9-10]). However methods of this group neither predict the order and error of reduced system nor minimize them [11].

In contrast to this group of algorithms the other group based on balance realization [11-13] does not need the specification of frequency points and held TF approximations in a whole frequency region. The possible application of balance realization methods to circuit simulation problems have been discussed in some published works (see for instance [2, 11, 14]). This is a promising approach to generate linear macromodels but it is connected with fixing a number of largest eigenvalues of original system [14].

The method [15] minimizes the error of approximation by varying both numerator and denominator coefficients. It solves nonlinear least square problem using the Levenberg-Marquardt algorithm. The error norm is defined in the given set of frequency points but the problem of the points selection is not discussed. The problem of order minimization at the given error tolerance is not also discussed.

The open problem of model reducing technology is the correct determination of order of simplified models.

The approach presented in this paper in our opinion can be considered as alternative and more appropriate for automatic model generation. This technique is quite universal because there are no restrictions on frequency region of model application. The developed computational scheme uses least-square minimization. In comparison with [15] our approach is based on minimization of the integral error norm in time domain.

The proposed algorithm allows to compute both approximating coefficients and order of TF simultaneously under control by specified error tolerance.

The definition of the problem is given in section 2. The computational algorithm is briefly explained in section 3. The examples of application of the developed algorithm and program are presented in section 4.

II. Mathematical Formulation of the Problem

Let the transfer function (TF) be defined in Laplace form as follows:

$$H(s) = \frac{P(s)}{Q(s)} = \frac{a_0 + \sum_{i=1}^{m} a_i \cdot s^i}{b_0 + \sum_{i=1}^{n} b_i \cdot s^i} \qquad (1)$$

Here m and n are the original orders of numerator and denominator respectively.

The approximate function $\tilde{H}$ must have the similar form of the lower order with the same values a_0, b_0:

$$\tilde{H}(s) = \frac{\tilde{P}(s)}{\tilde{Q}(s)} = \frac{a_0 + \sum_{i=1}^{\tilde{m}} \tilde{a}_i \cdot s^i}{b_0 + \sum_{i=1}^{\tilde{n}} \tilde{b}_i \cdot s^i} \qquad (2)$$

Here $m \le n, \tilde{m} \le \tilde{n}$. Let $h(t), \tilde{h}(t)$ be originals of $H(s)/s, \tilde{H}(s)/s$, and have the following generalized form:

$$h(t) = \frac{a_0}{b_0} + \sum_{i=1}^{K} d_i \cdot t^{k_i} \cdot \exp(r_i \cdot t) \qquad (3)$$

where r_i are poles of TF, k_i are corresponding multiplicity.

It is proposed to define the normalized error of approximation by the following way:

$$w = \frac{\int_0^{\infty} [h(t) - \tilde{h}(t)]^2 dt}{\int_0^{\infty} [h(t) - h(\infty)]^2 dt}, \qquad (4)$$

Values a_0, b_0 are saved for approximate function (2) because the ratio a_0/b_0 defines the static state which must be equal for both functions to provide existence of integral in numerator of (4). For the existence of integrals in (4) the conditions $m \le n, \tilde{m} \le \tilde{n}$ are necessary.

It is important to note that minimization of criterion (4) automatically avoids the problem of numerical instability of obtained approximation function. This can be explained as follows. The presence of unstable components in reduced model leads to unlimited growth of $\tilde{h}(t)$ and the criterion w respectively. Therefore the minimum of (4) could not be provided in this case.

For practical purposes two approximation problems are considered:

Problem A. Minimize the approximation error for the given orders of the numerator and denominator: $w \to min$ at $\tilde{m}, \tilde{n}$ fixed.

Problem B. Minimize the denominator order for the given approximation tolerance: $\tilde{n} \to min$ at $w < w_{max}$ and $\tilde{m} \le \tilde{n}$.

The formulation of the error in time domain is introduced for convenience of further derivation. Note that integral square errors (L_2 norms) in time and frequency domains are equal due to Parseval equality.

III. Principles of the Computational Method

Our aim is to construct an iterative Newton-like algorithm based on linearization of transfer function with respect to unknown coefficients of numerator and denominator at each step.

Let $\tilde{H}_{init}(s)$ be the initial approximation. Assuming the discrepancy between the initial and new approximation ($\Delta H = \tilde{H}(s) - \tilde{H}_{init}(s)$) to be sufficiently small it can be presented after linearization

$$\tilde{H} = \frac{\tilde{P}}{\tilde{Q}} = \frac{\tilde{P}_{init} + [\tilde{P} - \tilde{P}_{init}]}{\tilde{Q}_{init} + [\tilde{Q} - \tilde{Q}_{init}]} \approx \frac{\tilde{P}_{init}}{\tilde{Q}_{init}} + \frac{\tilde{P}}{\tilde{Q}_{init}} - \frac{\tilde{P}_{init}}{\tilde{Q}^2_{init}}\tilde{Q} \quad (5)$$

in the form:

$$\Delta H = \sum_{i=1}^{\tilde{m}} \tilde{a}_i \cdot \frac{s^i}{\tilde{Q}_{init}} - \sum_{i=1}^{\tilde{n}} \tilde{b}_i \cdot \frac{s^i \cdot \tilde{P}_{init}}{\tilde{Q}^2_{init}} - \sum_{i=1}^{\tilde{n}} \tilde{b}_i \cdot \frac{s^i \cdot \tilde{P}_{init}}{\tilde{Q}^2_{init}} - \left[H - \frac{\tilde{P}_{init}}{\tilde{Q}_{init}} - \frac{a_0}{\tilde{Q}_{init}} + \frac{b_0 \cdot \tilde{P}_{init}}{\tilde{Q}^2_{init}}\right] \quad (6)$$

Let the originals of Laplace functions in (6) be the following:

$$u(t) = L^{-1}\left(\frac{1}{s\tilde{Q}_{init}}\right) \qquad v(t) = L^{-1}\left(\frac{\tilde{P}_{init}}{s\tilde{Q}^2_{init}}\right)$$

$$e(t) = L^{-1}\left(H/s - \frac{\tilde{P}_{init}}{s\tilde{Q}_{init}} - \frac{a_0}{s\tilde{Q}_{init}} + \frac{b_0 \cdot \tilde{P}_{init}}{s\tilde{Q}^2_{init}}\right)$$

Functions $u(t)$, $v(t)$, $e(t)$ have the form similar to (3) and the corresponding parameters d_i, k_i can be easily determined after computation of roots of polynomials $Q(s)$ and $Q_{init}(s)$.

Taking into account that multiplication by s in Laplace domain corresponds to differentiation in time domain we obtain the following time domain representation of (6):

$$\Delta h = \sum_{i=1}^{\tilde{m}} \tilde{a}_i \cdot u^{(i)}(t) - \sum_{i=1}^{\tilde{n}} \tilde{b}_i \cdot v^{(i)}(t) - e(t) \qquad (7)$$

where superscript i means i-th derivative.

By this way the problem **A** is reduced to the classical least squares problem:

$$\left\| \sum_{i=1}^{k} c_i \cdot f_i(t) - e(t) \right\|^2 \to \min_{\{c_i\}} \qquad (8)$$

where $\{f\} = \begin{pmatrix} u \\ v \end{pmatrix}$ and $\{c\} = \begin{pmatrix} \tilde{a} \\ \tilde{b} \end{pmatrix}$.

This problem is solved by the conventional least squares algorithm :

1. Orthogonalization of the system of functions $f_i(t)(i=1...k)$ where f includes functions u and v from (7).

2. Decomposition of $e(t)$ in terms of the found orthogonal basis.

3. Determination of coefficients c_i by the solving of linear system with triangular matrix obtained in the orthogonalization process.

The orthogonalization process requires the computation of scalar product of functions f_i and *e:*

$$\langle f_i(t) f_j(t) \rangle = \int_0^{\infty} f_i(t) f_j(t) dt$$

$$\langle f_i(t) e(t) \rangle = \int_0^{\infty} f_i(t) e(t) dt$$

Note that computation of scalar products does not need the numerical integration because integrals of products of functions of type (3) can be easily presented in the algebraic form.

Solving (8) with fixed value k gives the solution of the problem **A.** To solve the problem **B**, in other words to provide the automatic choice of TF order it is necessary to minimize the number of functions k for the specified error:

$$k \to \min_{\{c_i\}} \quad \text{at} \quad \left\| \sum_{i=1}^{k} c_i \cdot f_i(t) - e(t) \right\| < tol \qquad (9)$$

In this case the orthogonalization process is stopped when current error reaches the allowable value.

This algorithm can be applied to solve problem **B** if the sequence of coefficients and functions in (9) is defined as:

$$\begin{aligned} c_1 &= \tilde{b}_1 & f_1(t) &= \frac{d}{dt} v(t) \\ c_2 &= \tilde{a}_1 & f_2(t) &= \frac{d}{dt} u(t) \\ &\dots & & \\ c_{2n-1} &= \tilde{b}_n & f_{2n-1}(t) &= v^{(n)}(t) \\ c_{2n} &= \tilde{a}_n & f_{2n}(t) &= u^{(n)}(t) \\ &\dots & & \end{aligned} \qquad (10)$$

After determining the coefficients c_i $(i=1,...,k_{min})$ the numerator and denominator coefficients $(\tilde{a}_i, \tilde{b}_i)$ can be obtained from (10). The orders of numerator and denominator are either equal ($\tilde{m} = \tilde{n} = k_{min}/2$ for even k_{min}) or differ by 1 ($\tilde{n} = (k_{min}+1)/2, \tilde{m} = \tilde{n} - 1$ for odd k_{min}).

The described algorithms for solving problems **A**, **B** can be called by one-step A-algorithm and one-step B-algorithm respectively. They do not provide exact solutions of initial problem (**A** or **B**) because of approximate equality in (5) which results from the linearization. Accurate solutions can be obtained by applying Newton-like iterative process for which after determination of coefficients $\tilde{a}_i, \tilde{b}_i$ the obtained TF can be considered as a new initial approximation ($\tilde{H}_{init}(s) = \tilde{H}(s)$) and the process is repeated until stopping criterion is reached. At the first iteration $\tilde{H}_{init}(s) = H(s)$.

IV. Experimental Results

The application of the developed algorithms for reducing of the order of rational transfer function to large set of test problems containing both active circuits and passive RC or RLC networks confirms their suitability for practical problems.

The presented algorithms were compared with widespread techniques to reduce linear models. Below some test examples are presented. For selected test examples the poles of transfer function can be real as well as complex-conjugate. The following properties of algorithm were observed:

-saving of numerical stability in model reducing of circuits with complex selective frequency characteristics (Test 1);

- capability of automatic computation of order of reduced models (Tests 2-5).

Test problem 1: Six-pole filter [17, fig.1]. This problem is the example of numerical stability lost for widespread AWE technique if the order of approximate polynomial increases from 4 to 5 [17]. The developed algorithm saves stability properties. It can be seen from fig. 1, where two obtained curves are presented: curve a1 corresponds to denominator order 4 and curve a2 corresponds to denominator order 5. The curve a2 demonstrates high frequency oscillations.

The examples (2 - 5) illustrate automatic choice of low order for allowable error. The results of reducing orders for test problems 2-5 are illustrated in Table 1. It contains the obtained orders of numerator and denominator $\tilde{m}$ and $\tilde{n}$ for the different values of the specified error tolerance and resulting errors of reduced models.

Test problem 2: RLC-circuit [18, fig.11]. This circuit is RLC-equivalent of transmission line given in [18]. The original transfer function (TF) order is 12. TF contains complex-conjugate poles.

For this test problem the specified error 10^{-7} leads to orders of 6/10, and the resulting approximation error $6.3 \cdot 10^{-9}$ is achieved (approximation a2). If error 0.1 is specified, then orders 3/3 are obtained and the resulting error equals to 0.07 (approximation a1). The corresponding time responses are given in fig. 2,a. This example is selective circuit and the distinction between two obtained approximations of magnitude-frequency characteristics can be seen from fig. 2,b.

Test problem 3: RLC-circuit [6, fig.1]. This circuit is also selective RLC-equivalent of transmission line. The original transfer function (TF) order is 10. TF contains complex-conjugate poles.

The approximated time and frequency responses that were obtained for specified error tolerances 0.1 (a1) and 10^{-7} (a2) are given in fig. 3,a and fig. 3,b respectively. Note that error tolerance 10^{-7} requires the denominator order 10. In this case the algorithm automatically reduces the numerator order only.

Test problem 4: RC-tree [16, fig.7]. The initial TF of this circuit is presented by relatively high order polynomials. The order of original transfer function (TF) is 17. In spite of this TF can be described by small group of dominant poles. This example illustrates also successful work of algorithm for networks with dominant TF poles.

Test problem 5: Opamp ua-741. This example is widespread circuit with active devices. It can be also described by small group of dominant poles and zeros. The order of the initial TF equals to 22.

V. SUMMARY

The special-purpose computational procedure based on optimal fitting approach has been developed to reduce the order of rational Laplace transfer function. It is directed to automatic generation of low-order macromodels. The fitting criterion was defined as normalized integral error of time-domain response.

The procedure provides solution of two types of approximation problems:

- determine transfer function of minimal error under specified order;

- determine transfer function of minimal order under specified error tolerance.

The procedure does not need the determination of the set of frequency points.

The algorithm implemented in the procedure is based on the Newton-like iterative process with functional least-squares method at each iteration step.

The experiments on test problems of different type confirmed the efficiency of the proposed approach. The standard stage of TF model reduction can be based on the developed approach.

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Table 1: RESULTS OF REDUCING

Example	result	specified error tolerance			
		10^{-1}	10^{-3}	10^{-5}	10^{-7}
Test 2: RLC cicuit m=4 n=12	$\tilde{m}$	3	2	4	6
	$\tilde{n}$	3	8	8	10
	error w	$7.\ 10^{-2}$	$3.1\ 10^{-4}$	$2.5\ 10^{-7}$	$6.3\ 10^{-9}$
Test 3: RLC cicuit m=6 n=10	$\tilde{m}$	0	1	2	4
	$\tilde{n}$.	1	7	8	10
	error w	$2.9\ 10^{-1}$	$4.4\ 10^{-5}$	$1.3\ 10^{-6}$	$2.0\ 10^{-8}$
Test 4: RC-tree m=0 n=17	$\tilde{m}$	1	3	3	4
	$\tilde{n}$.	2	3	5	6
	error w	$1.9\ 10^{-2}$	$9.3\ 10^{-4}$	$4.6\ 10^{-6}$	$4.4\ 10^{-8}$
Test 5: ua741 m=0 n=22	$\tilde{m}$	0	1	2	3
	$\tilde{n}$	1	3	3	3
	error w	$8.9\ 10^{-2}$	$4.9\ 10^{-4}$	$6.7\ 10^{-6}$	$3.5\ 10^{-8}$

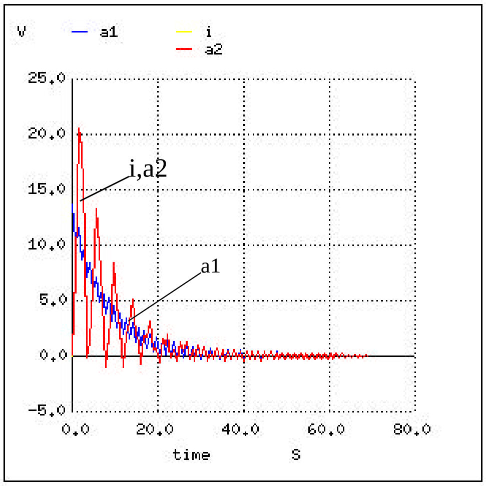
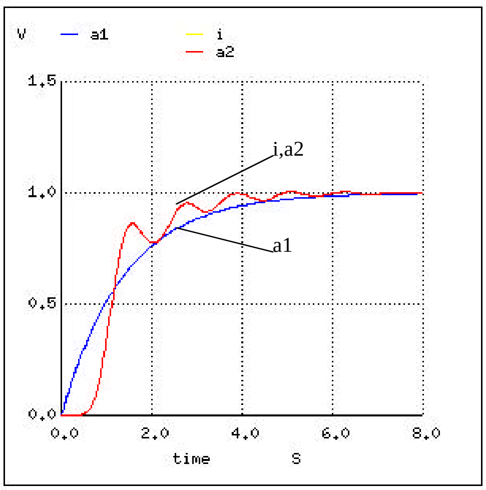
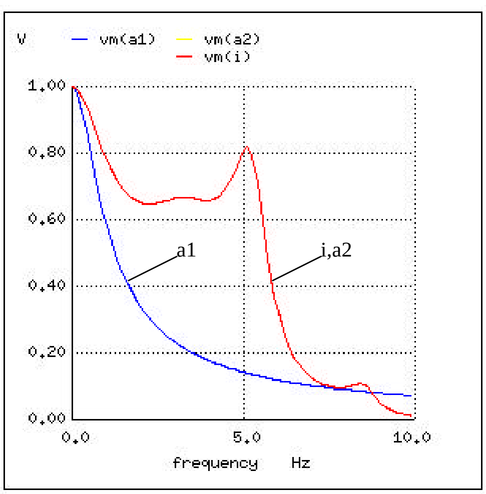


Fig.1 Test problem 1: Time-domain responses for original TF(I) and obtained approximations (a1, a2)

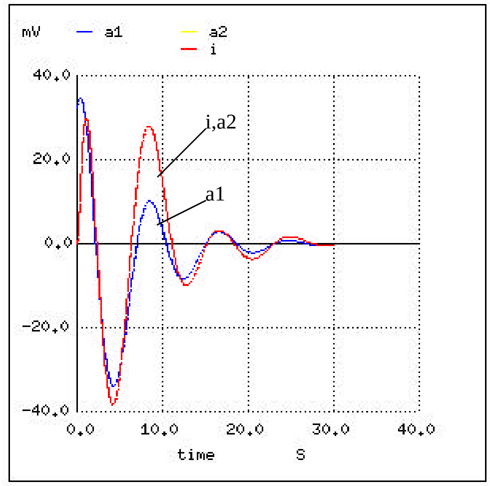


a)

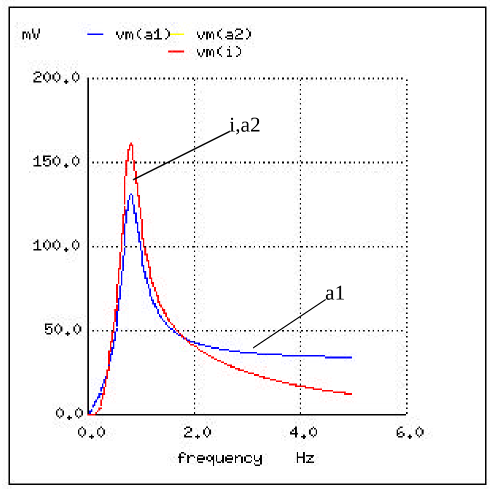


b)

Fig. 2 Test problem 2: Time-domain (a) and frequency-domain (b) responses for original TF(I) and obtained approximations (a1,a2)



a)



b)

Fig. 3 Test problem 3: Time-domain (a) and frequency-domain (b) and responses for original TF(I) and obtained approximations (a1, a2)