Predicting Coupled Noise in RC Circuits By Matching 1, 2, and 3 Moments

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Abstract

This paper develops the noise-counterparts to familiar delay formulas like Elmore or PRIMO. By matching the first few moments of the network's transfer impedance, we obtain efficient and accurate predictions for maximum noise between two capacitively coupled RC networks. Unlike many crosstalk equations in the literature, the method applies to general topologies and models transition-time dependence as well. Efficient enough for large circuits, the moment-matching noise formulas developed here can serve as a key ingredient in CAD methodologies that ensure a layout is free of noise problems.

Introduction

Capacitively coupled noise can sabotage a deep sub-micron design, if not properly managed [1]. There is a clear need for efficient, accurate analysis of crosstalk, including its impact on timing[2]. On the one hand, some papers--[3][4][5][6]--propose formulas that predict or bound noise, but these papers usually postulate a simple topology, often coupled T networks. At the other extreme are papers that invoke the machinery of circuit simulation or general N-port reduction[7][8][9]. We seek a middle way—more general than the former, more efficient than the latter—that computes noise by matching the first few moments of the transfer function from aggressor to victim.

Consider the coupled RC networks in Figure 1. Our goal is to estimate *peak* noise—in closed form, without simulation--at each victim receiver, like R, due to a transition of the aggressor's driver d. We want to estimate how crosstalk varies from receiver to receiver depending on actual layout, and how it varies with the rise or fall time Δ of the aggressor's source.

An important application of peak noise is in calculating worst or best case delay in the presence of coupling. This is done by translating the timing threshold by an amount equal to the peak noise amplitude, as described in [7].

We solve the noise problem by analogy with familiar delay formulas like Elmore and, more recently, PRIMO and h-gamma, that are based on moment matching [10][11][12]. Whereas these *delay* formulas are based on the first several moments of the transfer function from driver to receiver on the *same* net, we develop analogous equations for *noise* based on the first several moments of the transfer function from the *aggressor* driver to a *victim* receiver. The resulting crosstalk formulas are quite general, tolerably accurate, and very efficient.

2. Coupled Circuit Equations

The nodal equations for a pair of coupled RC networks like those in Figure 1 can be written in block form as

$$\left\{\begin{bmatrix} C_{11} & C_{21}^T \\ C_{21} & C_{22} \end{bmatrix} s + \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix}\right\}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} e_d \\ 0 \end{bmatrix} J_d(s)$$

$$V_R(s) = \begin{bmatrix} 0 & e_R^T \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (2.1)$$

The matrix partitions correspond to the aggressor and victim nets. Blocks C_{11} and G_{11} are the capacitance and conductance matrices of the aggressor and $V_1(s)$ is the Laplace transform of net 1's nodal voltages. C_{22} G_{22} and $V_2(s)$ are the corresponding quantities for net 2 (the victim net). Normally, C_{11} and C_{22} are diagonal. Block C_{21} and C^T_{21} constitute the coupling between the nets. $J_d(s)$ is the Laplace transform of the current source at d, $V_R(s)$ the transform of the noise voltage at R. Unit vector e_d has 1 in the row corresponding to the nodal equations for node d where the driver is attached; unit vector e_R has a 1 in the row corresponding to the receiver's nodal voltage. The driver conductance g_d is included in G_{11}; the conductance of the quiescent victim driver, g_D, is included in G_{22}.

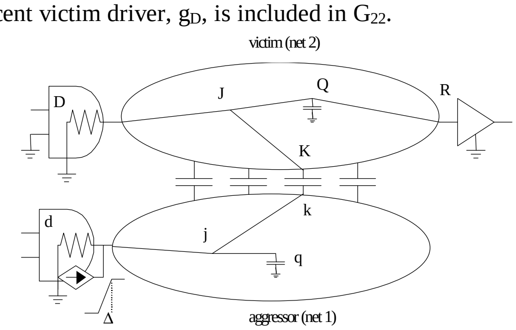

Figure 1 Two coupled RC Networks

3. Moment Calculation

We are interested in calculating the initial coefficients in the expansion of the transfer impedance

$$Z_{dR}(s) \equiv \frac{V_R(s)}{J_d(s)} = z_1 s + z_2 s^2 + z_3 s^3 + \ldots \qquad (3.1)$$

where z_0 is absent because there is no dc connection between the two circuits. We will first show how to calculate the z's, which are called moments, and then we will show how to estimate coupling noise using these moments.

To derive expressions for the moments, expand the nodal voltage vector in powers of s

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_1^{(0)} \\ V_2^{(0)} \end{bmatrix} + \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} s + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix} s^2 + \cdots \qquad (3.2)$$

The superscripts denote the term in the Taylor's expansion and the subscripts denote the block. Substituting (3.2) into (2.1) and equating powers of s gives us the recurrence relations

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$$\begin{bmatrix} V_1^{(n)} \\ V_2^{(n)} \end{bmatrix} = -\begin{bmatrix} G_{11}^{-1}(C_{11}V_1^{(n-1)} + C_{21}^T V_2^{(n-1)}) \\ G_{22}^{-1}(C_{21}V_1^{(n-1)} + C_{22}V_2^{(n-1)}) \end{bmatrix} \quad n > 0 \quad (3.3)$$

with initial conditions

$$\begin{bmatrix} V_1^{(0)} \\ V_2^{(0)} \end{bmatrix} = \begin{bmatrix} G_{11}^{-1} e_d \\ 0 \end{bmatrix} \quad (3.4)$$

Clearly (3.3) and (3.4) can be solved for as many terms as desired, provided G_{11} and G_{22} are nonsingular. We get the moments of (3.1) by taking the R'th components of $V_2^{(k)}$:

$$z_k = e_R^T V_2^{(k)} \quad (3.5)$$

The above equations are general, applying to any topologies for the coupled nets. If either or both of the nets are trees, however, the inversion of G_{11} (or G_{22}) simplifies considerably. For a tree, the conductance matrix can be factored in linear time without introducing fill. In fact, for trees it is possible to replace equations (3.3)-(3.5) by the calculation of a number of 'generalized' Elmore delays. See [13] for details.

We remark that while z_1 and z_3 are normally positive, z_2 is negative (this is due to the minus sign in (3.3) and the fact that the entries of C_{12} are all negative).

4. Moment Matching

Having explained how the first several moments of the transfer impedance can be calculated, we return to our primary task of considering how these moments can be used to predict coupling noise.

The general procedure for predicting noise or delay from a set of moments is this. First, judiciously select a family of functions $F = F^{(p_1,\ldots,p_m)}(s)$ with parameters $p_1,\ldots,p_m$. The **parameterized form** F is chosen so that the inverse transform $f(t) \equiv L^{-1}[F]$ has a shape similar to the expected impulse responses of actual circuits. Next, calculate values for the parameters so that the Maclaurin series of *F(s)* has the same initial coefficients as $Z_{Rd}(s)$; in other words, **match moments**. Finally, for a given source $J_d(s)$, take the noise or delay of the approximate waveform $\hat{v}_R(t) \equiv L^{-1}[F(s)J_d(s)]$ as an estimate for the noise or delay of the true waveform.

5. Admissible Forms

In what follows we require our parameterized forms to satisfy the following properties.

Definition. A trial function *f(t)* is **admissible** if:

$$\begin{aligned} &\text{(a)} \int_0^\infty f(t)dt = 1 \\ &\text{(b)} \int_0^\infty t f(t)dt = 1 \\ &\text{(c)}\ f(t) = 0, \quad t < 0 \\ &\text{(d)}\ f(t) \ge 0 \quad t \ge 0 \end{aligned} \quad (5.1)$$

Conditions (a) and (b) are normalization conditions, (c) expresses causality, and (d) captures the feature of RC circuits that the impulse response is non-negative [12].

Let $f_i \in S^a$ be an element from the class S^a of all admissible functions. Then

$$Z_i(s) \equiv z_1 s F_i(-\frac{z_2 s}{z_1}) = z_1 s + z_2 s^2 + \ldots. \quad (5.2)$$

where $F_i(s)$ is the Laplace transform of f_i. In other words, from any admissible function we can form an expression that matches the first two moments of $Z_{dR}(s)$. The quantity $t_{12} \equiv -z_2/z_1$ in (5.2), being positive with dimensions of time, can be though of as a **coupling time constant** from d to R.

We now develop coupled noise estimates based on matching 1, 2, or 3 moments.

6. One Moment Noise Estimate

The one moment noise estimate can be derived by an abstract argument that shows the result to be independent of the particular matching form used. Indeed, let f_i be *any* admissible form. For simplicity, assume that the driver is a Norton circuit with a saturating ramp current source:

$$J_R(s) = g_d V_{DD} \frac{1 - e^{-\Delta s}}{\Delta s^2} \quad (6.1)$$

Δ is the rise time of the ramp. The response of (5.2) to $J_R(s)$ can be shown to be

$$\begin{aligned} \frac{\hat{v}_R(t)}{g_d V_{DD}} &= L^{-1}\left[\frac{Z_i(s)}{\Delta s^2}(1 - e^{-\Delta s})\right] \\ &= \frac{z_1}{\Delta} \int_{(t-\Delta)/t_{12}}^{t/t_{12}} f_i(u)du \end{aligned} \quad (6.2)$$

In general, because of admissibility properties (a) and (b), we infer that

$$\hat{v}_R^{\max} \approx \frac{g_d V_{DD} z_1}{\Delta}, \text{ if } \Delta >> t_{12} \quad (6.3)$$

for *any* admissible f_i, since the *maximum* value of the integral in (6.2) for $\Delta >> t_{12}$ will always be approximately (but somewhat less than) 1. Expression (6.3) is a **one moment** estimate for max noise, since it uses only z_1; it is valid for driver transition times Δ that are large relative to the coupling time constant $t_{12} = -z_2/z_1$. A refined argument in [13] shows that the actual peak noise satisfies the bounds

$$\frac{g_d V_{DD} z_1}{\Delta} \ge v_R^{\max} \ge \frac{g_d V_{DD} z_1}{\Delta}\left(1 - \frac{t_{12}}{\Delta}\right) \quad (6.4)$$

Equation (6.3) is equivalent to the result in [14], but [14] does not develop the validity condition $\Delta >> t_{12}$.

7. Two Moment Noise Estimate

To match 2 moments, perhaps the simplest form is

$$F_1(s) = \frac{1}{(1+s)} \quad (7.1)$$

which is the Laplace transform of $f_1(t) = e^{-t}$. It is straightforward to show that $f_1(t)$ satisfies properties (5.1) and is therefore admissible.

Substituting e^{-u} for $f_i(u)$ in (6.2), we get for the noise due to ramp source (6.1)

$$v_R(t) = \begin{cases} \dfrac{g_d V_{DD} z_1}{\Delta}\left(1 - e^{-t/t_{12}}\right), & t < \Delta \\ \dfrac{g_d V_{DD} z_1}{\Delta}\left(e^{-(t-\Delta)/t_{12}} - e^{-t/t_{12}}\right), & t \geq \Delta \end{cases} \quad (7.2)$$

The peak noise occurs at $t = \Delta$:

$$v_R^{\max} = \frac{g_d V_{DD} z_1}{\Delta}\{1 - \exp(+z_1 \Delta / z_2)\} \quad (7.3)$$

This **two moment** formula for noise is derived in [13] by a different method that attempts to estimate the error or uncertainty in (7.3).

8. Three Moment Noise Estimate

To match three moments, we must use a form with one parameter. We consider a two pole and a gamma function.

Consider first the admissible two pole form:

$$F^{(x)}(s) \equiv \frac{1}{(1 + t_+ s)(1 + t_- s)}, \quad t_{\pm} = \frac{1 \pm x}{2} \quad (8.1)$$

where $|x| \leq 1$. We motivate (8.1) by suggesting that the factor with τ_+, say, captures in some way the average response from the driver to the coupling capacitors and τ_- captures the response from the coupling capacitors to the receiver.

The scaled function $Z^{(x)}(s) = z_1 s F^{(x)}(-z_2 s / z_1)$ matches the first three moments z_1, z_2, and z_3 provided we choose x so that

$$\frac{3 + x^2}{4} = \frac{z_3 z_1}{z_2^2} \quad (8.3)$$

Evidently, (8.1) can match z_1, z_2, and z_3 only if $z_3 z_1 / z_2^2$ is in the range [3/4, 1]. Experience shows that this restriction is rarely a problem in practice; see section 9 below.

The moment matching procedure is as follows. Assuming $z_3 z_1 / z_2^2$ is in the range [¾, 1], compute x from (8.3) and then compute the maximum value of

$$\hat{v}_R(t) = L^{-1}[Z^{(x)}(s) J_R(s)] \quad (8.4)$$

For a step input (i.e. $J_R(s) = g_d V_{DD} / s$), we get

$$\frac{(-z_2) v_R^{\max}}{\left(g_d V_{DD} z_1^2\right)} = \frac{1}{x}\left\{\left(\frac{1-x}{1+x}\right)^{\left(\frac{1-x}{2x}\right)} - \left(\frac{1-x}{1+x}\right)^{\left(\frac{1+x}{2x}\right)}\right\} \quad (8.5)$$

the quantity on the right varying from 2/e for x=0 to 1 for x=1. For finite ramp inputs, the maximum of (8.4) requires solving a transcendental equation numerically. Efficient methods of approximation are possible, but we omit details.

An alternative to (8.1) is the trial form,

$$F^{(y)}(s) = \frac{1}{(1 + s/y)^y} \quad (8.7)$$

whose inverse Laplace transform is the gamma function kernel

$$y^y t^{y-1} e^{-yt} / \Gamma(y) \quad (8.8)$$

This function has already been used in PRIMO and h-gamma for calculating delays; we want to see if it works also for crosstalk.

It is easy to show that (8.8) is an admissible function, and that $Z^{(y)}(s) = z_1 s F^{(y)}(-z_2 s / z_1)$ matches moments z_1, z_2, and z_3 provided

$$\frac{1}{2}\left(1 + \frac{1}{y}\right) = \frac{z_3 z_1}{z_2^2} \quad (8.9)$$

While equation (8.9) can be solved for y given any positive numbers z_1, $-z_2$, and z_3, in practice we restrict $z_3 z_1 / z_2^2$ to the range (0.5,1]: this keeps the step response finite at t=0.

The procedure for getting peak noise is the same as before. Assuming $z_3 z_1 / z_2^2$ is in the range (0.5,1], we compute y from (8.9), and then compute the maximum of $\hat{v}_R(t) = L^{-1}[Z^{(y)}(s) J_R(s)]$. For a step input,

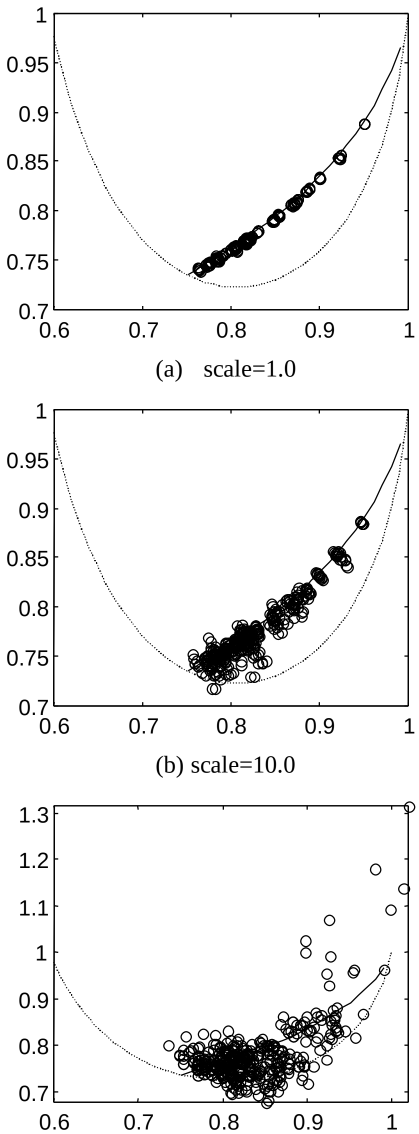
$$\frac{v_R^{\max}}{\left(\dfrac{g_d V_{DD} z_1^2}{(-z_2)}\right)} = \frac{y}{\Gamma(y)}\left(\frac{y-1}{e}\right)^{(y-1)} \quad y \geq 1 \quad (8.10)$$

For finite rise-times (i.e., $\Delta > 0$), (8.10) doesn't apply and we must solve a transcendental equation numerically to get $v_R^{\max}$.

9. Numerical Verification

In this section we test the adequacy of our noise estimates against a diverse set of 400 coupled RC networks. The set includes various net topologies, coupling lengths, positions of coupling along the two nets, like and opposed signal directions (*opposed* meaning that if two nets run horizontally and net 1's driver is at the *left* of its trace, then net 2's driver is at the opposite, *right* end of its trace). The set also includes an assortment of internal resistances for both aggressor and victim drivers. The traces are all on layer 4 using 0.18µm process technology. To show our theory under the *least* flattering circumstances, we use a *step* current source (Δ=0) for the aggressor's driver.

For even greater comprehensiveness, we evaluate the test set using three sizes (scalings) for the nets. First, we simulate the set with the nets having their original sizes (100-200µm); this scaling represents local interconnect. Next we increase the lengths of all traces by a factor of 10 while keeping everything else (driver characteristics, metal cross-sections, topology) the same. The resulting 1000-2000µm long nets are indicative of interconnect within a major functional block. Finally, we increase net lengths by a factor 100.0, resulting in nets 1-2cm long that represent global interconnect.



(a) scale=1.0

(b) scale=10.0

(c) scale=100.0

Figure 2. Simulated test set. Ordinate is $-v_R^{\max}/(g_d V_{DD} z_1^2/z_2)$; abscissa is $z_1 z_3/z_2^2$

Figure 2 plots *simulated* $v_R^{\max}$ versus $z_1 z_3/z_2^2$ for each scaling. All crosstalk values are normalized by $\left(-g_d V_{DD} z_1^2/z_2\right)$. For comparison, (8.5) and (8.10) are drawn as solid and dotted lines, respectively.

We make the following observations. The *one* moment formula (6.3), of course, is totally unserviceable for step inputs, it's prediction being infinite. Because the normalization factor is the limit of (7.3) as $\Delta \to 0$, the *two* moment formula (7.3) predicts 1.0 for all cases in Figure 2. With few exceptions, actual noise is 0.7 to 0.9 times the *two* moment prediction. Of the *three* moment formulas, (8.5) is quite accurate for small to medium sized circuits and is clearly superior to (8.10).

Bear in mind that Figure 2(c) is an exceedingly harsh example: in practice it is not likely (or at least not advisable) to have two global nets run in adjacent channels across the chip; nor will the driver in practice make a step transition. Despite these exacerbating conditions, (8.5) is still accurate to within about 30% of the true peak noise.

For the global circuits of Figure 2(c), the problem is not so much that (8.5) is deficient as that, in general, crosstalk for such circuits cannot be predicted accurately from *any* 3-moment formula. In this regime crosstalk just doesn't have a clean functional dependence on only z_1, z_2, and z_3.

10. Conclusion

The arguments and data in this paper, we feel, suggest that two and three moment formulas for crosstalk, like those for delay, can well serve as the CAD workhorse whenever efficiency is a premium and accuracy is not. Significantly more efficient than simulation, both two and three moment formulas apply to a wide range of net topologies, net sizes, driver strengths, and transition times. In our tests, neither formula erred by more than about 30%, with the 3-moment calculation being significantly more accurate than this except for the largest nets. By contrast, noise based on one moment, while fine for small nets, is restricted by the need for transition time to exceed the coupling time constant. The two and three moment formulas do not have this restriction.

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