

The Chebyshev expansion based passive model for distributed interconnect networks

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Abstract

A new Chebyshev expansion based model for distributed interconnect networks is presented in this paper. Unlike the moment methods, this new model is optimal and it does not require the knowledge of expansion points. An automatic order selection scheme is also included in the new model. By using the integrated congruence transform, we guarantee the passivity of the new model for distributed interconnect networks. Because of the orthogonality of Chebyshev polynomials, the Modified Gram-Schmidt algorithm can be simplified. In the experimental examples, the new model is found to be accurate and efficient.

1 Introduction

In recent years, with the rapid increase of signal frequency and decrease of feature sizes of high speed electronic circuits, interconnect has become a dominating factor in determining circuit performance and reliability in deep submicron designs. At high frequencies, people usually use distributed model in order to do accurate interconnect timing analysis and optimization. Previously, moment matching techniques[1][2][3] are used to generate passive reduced order models for interconnects. Because the moment matching approaches lack optimality property, they require multipoint and/or high order for practical circuit simulation, especially when the models need to capture skin effect. Moreover, for most practical circuits, it has difficulties in predicting expansion points. Researchers resort to Balanced Truncation Method[4]. Unfortunately, the optimality of this method is not proved and whether we can guarantee the passivity of the original system is unknown.

In this paper, we explore the method of using the Chebyshev expansion to generate an optimal passive reduced order model. This new method does not require the using of expansion points and it includes an automatic order selection scheme. Our algorithm consists of two main steps. In the first step, each line's voltage and current in the frequency domain are modeled by a set of finite order Chebyshev polynomials with respect to spatial variable or frequency variable. In the second step, an L^2 Hilbert space theory based integrated congruence transform is applied to the network to form its reduced order model. The passivity is preserved and the Chebyshev expansion coefficients of the input admittance matrix are matched. Because of the orthogonality of Chebyshev polynomials, we can simplify the Modified Gram-Schmidt Algorithm.

Chebyshev expansion has three useful properties[5] over mo-

ment matching methods: 1) It converges exponentially; 2) It has the orthogonal property; 3) It has the optimal property; It satisfies the discrete least-squares criterion. This optimal property has been frequently used in filter design where the Chebyshev expansion is applied as "the equal-ripple function"[6] which provides the minimum absolute deviation from the ideal-filter curve in passband.

In this paper, we use Chebyshev expansion in the modeling of the transmission lines which support TEM waves at all frequencies. Under this circumstance, the voltage and current along the transmission lines satisfy Telegraph equations. In the s domain, the Telegraph equations are written as:

$$\frac{dV(z, s)}{dz} = -(R + sL)I(z, s) \quad (1)$$

$$\frac{dI(z, s)}{dz} = -(G + sC)V(z, s) \quad (2)$$

where $V(z, s)$ and $I(z, s)$ are the voltage and current vectors along the transmission lines. z is the spatial variable and s is the frequency variable. R, L, G and C are per unit length parameters of the lines. After normalizing the lines, we have $z \in [0, 1]$. When we are dealing with the input admittance of a transmission line system, it is assumed that two vectors of voltage sources $V_{s1}(s)$ and $V_{s2}(s)$ are applied to the two ends of the line system. So the boundary conditions of the line equations are

$$\begin{bmatrix} V(0, s) \\ V(1, s) \end{bmatrix} = V_s(s) = \begin{bmatrix} V_{s1}(s) \\ V_{s2}(s) \end{bmatrix}$$

Transmission lines belong to infinite dimension system due to the fact that the voltage and current along the lines are continuous functions of spatial variable z and frequency variable s . In the following two sections, we derive the Chebyshev expansion of the voltage and the current with respect to variable z and s respectively.

2 Chebyshev expansion with respect to frequency variable

2.1 Integrated congruence transform

Let $s = i2\pi f$ where $i = \sqrt{-1}$ and f is the frequency of the system. Suppose $s \in i2\pi * [-f_{max}, f_{max}]$ where f_{max} is the maximum frequency of interest. The variables of Chebyshev expansions are defined in the range $[-1, 1]$ (we can also use $-i$ to i), so let $\bar{s} = \frac{s}{s_{max}}$ where $s_{max} = i2\pi * f_{max}$. By using \bar{s} instead of s , we have $\bar{s} \in [-1, 1]$. The voltage and current functions with respect to \bar{s} are $\bar{V}(z, \bar{s})$ and $\bar{I}(z, \bar{s})$. Sticking

to the use of notation V and I in the presentation as our voltage and current functions, we obtain

$$\frac{dV(z, \bar{s})}{dz} = -(\bar{R} + \bar{s}\bar{L})I(z, \bar{s}) \quad (3)$$

$$\frac{dI(z, \bar{s})}{dz} = -(\bar{G} + \bar{s}\bar{C})V(z, \bar{s}) \quad (4)$$

where $\bar{R} = R$, $\bar{L} = s_{max}L$, $\bar{G} = G$ and $\bar{C} = s_{max}C$. The boundary conditions are

$$\begin{bmatrix} V(0, \bar{s}) \\ V(1, \bar{s}) \end{bmatrix} = V_s(\bar{s}) = \begin{bmatrix} V_{s1}(\bar{s}) \\ V_{s2}(\bar{s}) \end{bmatrix} \quad (5)$$

Now rewrite Eq(3) and Eq(4) in the following form:

$$(\bar{s}M + N + T \frac{d}{dz})X(z, \bar{s}) = 0 \quad (6)$$

where

$$X(z, \bar{s}) = \begin{bmatrix} I(z, \bar{s}) \\ V(z, \bar{s}) \end{bmatrix} \quad (7)$$

$M = \begin{bmatrix} \bar{L} & \\ & \bar{C} \end{bmatrix}$, $N = \begin{bmatrix} \bar{R} & \\ & \bar{G} \end{bmatrix}$ and $T = \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$. In the case of an RLGC coupled line system with m lines, $I(z, \bar{s})$ and $V(z, \bar{s})$ are m -dimensional vectors. R , L , G and C are $m \times m$ dimensional matrices. $X(z, \bar{s})$ is a $2m$ dimensional vector, and M , N and T are $2m \times 2m$ matrices, where the element 1 in T is an $m \times m$ identity matrix. The boundary conditions remain in the form of Eq(5), where $V_{s1}(\bar{s})$ and $V_{s2}(\bar{s})$ are m -dimension vectors. The element $V_i(z, \bar{s})$ of $V(z, \bar{s})$ and $I_i(z, \bar{s})$ of $I(z, \bar{s})$ are expanded by the Chebyshev expansion with respect to frequency variable \bar{s} . Let $N = \max_{i=1, \dots, m}(N_{Vi}, N_{Ii})$ where N_{Vi} and N_{Ii} are the orders of the Chebyshev expansion of $V_i(z, \bar{s})$ and $I_i(z, \bar{s})$ respectively and N can reach ∞ . So, we have $V_i(z, \bar{s}) = \sum_{j=0}^{N-1} c_{Vi,j} T_j(\bar{s})$ where $T_j(\bar{s}) = \cos(j \arccos(\bar{s}))$. However, Chebyshev expansion is not incremental which means for different expansion orders, we have different Chebyshev coefficients and we can not use lower order Chebyshev coefficients. In order to overcome this problem, Clenshaw-Curtis Quadrature[7] is used instead of the general Chebyshev expansion formular. By the definition of Chebyshev expansion's Clenshaw-Curtis Quadrature form,

$$c_{Vi,0} = \frac{1}{N} \sum_{k=1}^{N-1} V_i(z, \bar{s}_k) \quad (8)$$

for $j \neq 0$,

$$c_{Vi,j} = \frac{V_i(z, 1) + (-1)^j V_i(z, -1)}{N} + \frac{2}{N} \sum_{k=1}^{N-1} V_i(z, \bar{s}_k) T_j(\bar{s}_k) \quad (9)$$

where $\bar{s}_k = \cos(\frac{k\pi}{N})$ with $k = 0, \dots, N$ are $N+1$ extreme points of $T_N(\bar{s})$. Since $c_{Vi,j}$ where $j = 0, \dots, N-1$ are functions of spatial variable z in Eq(8) and Eq(9), we denote them as $c_{Vi,j}(z)$. So, we have

$$V(z, \bar{s}) = \begin{bmatrix} \sum_{j=0}^{N-1} c_{V0,j}(z) T_j(\bar{s}) \\ \sum_{j=0}^{N-1} c_{V1,j}(z) T_j(\bar{s}) \\ \dots \\ \sum_{j=0}^{N-1} c_{Vm,j}(z) T_j(\bar{s}) \end{bmatrix} \quad (10)$$

similarly,

$$I(z, \bar{s}) = \begin{bmatrix} \sum_{j=0}^{N-1} c_{I0,j}(z) T_j(\bar{s}) \\ \sum_{j=0}^{N-1} c_{I1,j}(z) T_j(\bar{s}) \\ \dots \\ \sum_{j=0}^{N-1} c_{Im,j}(z) T_j(\bar{s}) \end{bmatrix} \quad (11)$$

We rewrite Eq(7) as:

$$X(z, \bar{s}) = \begin{bmatrix} I(z, \bar{s}) \\ V(z, \bar{s}) \end{bmatrix} = C_x(z) \bar{T}(\bar{s})$$

where $\bar{T}(\bar{s}) = [T_0(\bar{s}) \ T_1(\bar{s}) \ \dots \ T_{N-1}(\bar{s})]^T$,

$$C_x(z) = \begin{bmatrix} C_{I0,0}(z) & C_{I0,1}(z) & \dots & C_{I0,N-1}(z) \\ C_{I1,0}(z) & C_{I1,1}(z) & \dots & C_{I1,N-1}(z) \\ \dots & \dots & \dots & \dots \\ C_{Im,0}(z) & C_{Im,1}(z) & \dots & C_{Im,N-1}(z) \\ C_{V0,0}(z) & C_{V0,1}(z) & \dots & C_{V0,N-1}(z) \\ C_{V1,0}(z) & C_{V1,1}(z) & \dots & C_{V1,N-1}(z) \\ \dots & \dots & \dots & \dots \\ C_{Vm,0}(z) & C_{Vm,1}(z) & \dots & C_{Vm,N-1}(z) \end{bmatrix}$$

Let $U(z)$ be the orthonormal matrix and matrix $C_x(z)$'s column vectors are in the span of the column vectors of $U(z)$. Then there exists a coefficient matrix \hat{C} , $C_x(z) = U(z)\hat{C}$. Let $\hat{x}(\bar{s}) = \hat{C}\bar{T}(\bar{s})$, we have

$$X(z, \bar{s}) = U(z)\hat{x}(\bar{s}) \quad (12)$$

If we split $U(z)$ as

$$U(z) = \begin{bmatrix} u_i(z) \\ u_v(z) \end{bmatrix}$$

and substitute Eq(12) into Eq(6). Premultiplying a $U^T(z)$ on the both side of Eq(6) and integrating with z from 0 to 1 as in [1], we can express the input admittance matrix as

$$\hat{Y}(\bar{s}) = \hat{b}^T (\bar{s}\hat{M} + \hat{N})^{-1} \hat{b} \quad (13)$$

where $\hat{M} = \int_0^1 (u_i^T(z)\bar{L}u_i(z) + u_v^T(z)\bar{C}u_v(z))dz$, $\hat{N} = \hat{N}_1 + \hat{N}_2$ with $\hat{N}_1 = \int_0^1 (u_i^T(z)\bar{R}u_i(z) + u_v^T(z)\bar{G}u_v(z))dz$ and $\hat{N}_2 = \int_0^1 (u_v^T(z)\frac{du_i(z)}{dz} - \frac{du_v(z)}{dz}u_v(z))dz$ and $\hat{b} = \begin{bmatrix} u_i(0) \\ -u_{ci}(1) \end{bmatrix}$. $U(z)$ is called integrated congruence transform matrix. The above procedure to obtain the \hat{M} and \hat{N} is called integrated congruence transform. Note here \hat{b}, \hat{M} and \hat{N} are all generated based on $U(z)$ which is different from the moment based method. In the moment based method, all these matrices depend on $U_M(z)$ where $U_M(z)$ is the integrated congruence transform matrix based on the moment method. For cases when we know the resonant frequency of the system and we might need more expansion points around the resonant frequency, segment Chebyshev expansion can be applied. In these cases, the frequency range is segmented into several subranges. In each subrange, Chebyshev expansion is used. Then the congruence transform matrix is formed based on the multi-range Chebyshev expansion, which is similar to the multi-point moment matching method.

2.2 Passivity and Chebyshev term preservation

We can prove that the admittance matrix generated by the Chebyshev expansion based integrated congruence transform in Eq(13) is passive and the proof follows that for the moment based integrated congruence transform[1] and is omitted.

Theorem 1[8] The admittance matrix (Eq(13)) is passive. If we derive the Chebyshev expansion based integrated transform from Eq(6), we have the following theorem.

Theorem 2[8] Let $C_x(z) \equiv C(N, z) = \left\{ \begin{bmatrix} c_{I0}(z) \\ c_{V0}(z) \end{bmatrix}, \begin{bmatrix} c_{I1}(z) \\ c_{V1}(z) \end{bmatrix}, \dots, \begin{bmatrix} c_{IN-1}(z) \\ c_{VN-1}(z) \end{bmatrix} \right\}$. $C(N, z)$ is a set of Chebyshev expansion coefficient matrices of $X(z, \bar{s})$ from order 0 – $(N - 1)$.

Let $U(z)$ be the orthonormal matrix based on Chebyshev expansion method. So, $U(z) = [U_1(z), U_2(z), \dots, U_k]$ and $C(N, z) \in \text{colspan}(U(z))$, then $Y(\bar{s}_j) = \hat{Y}(\bar{s}_j)$ and $C_{Y_k} = C_{\hat{Y}_k}$ where

$$C_{Y_0} = \frac{1}{N} \sum_{j=1}^{N-1} Y(\bar{s}_j)$$

$$C_{\hat{Y}_0} = \frac{1}{N} \sum_{j=1}^{N-1} \hat{Y}_c(\bar{s}_j)$$

$$C_{Y_k} = \frac{Y(1) + (-1)^k Y(-1)}{N} + \frac{2}{N} \sum_{j=1}^{N-1} Y(\bar{s}_j) T_k(\bar{s}_j)$$

and

$$C_{\hat{Y}_k} = \frac{\hat{Y}(1) + (-1)^k \hat{Y}(-1)}{N} + \frac{2}{N} \sum_{j=1}^{N-1} \hat{Y}_c(\bar{s}_j) T_k(\bar{s}_j)$$

with $k = 1, \dots, N$.

Thus, this new integrated congruence transform preserves the coefficients of the Chebyshev expansion. The admittance matrix in Eq(13) can be estimated if $U(z)$ can be constructed from $C(N, z)$. By Chebyshev expansion, the j th element of $C(N, z)$ can be represented as

$$C_j(z) = \frac{\begin{bmatrix} I(z, 1) \\ V(z, 1) \end{bmatrix} + (-1)^j \begin{bmatrix} I(z, -1) \\ V(z, -1) \end{bmatrix}}{N} + \frac{2}{N} \sum_{k=1}^{N-1} \begin{bmatrix} I(z, \bar{s}_k) \\ V(z, \bar{s}_k) \end{bmatrix} T_j(\bar{s}_k)$$

2.3 Chebyshev coefficient matrix derivation

Since

$$\frac{d \begin{bmatrix} I(z, \bar{s}_k) \\ V(z, \bar{s}_k) \end{bmatrix}}{dz} = - \begin{bmatrix} 0 & \bar{G} + \bar{s}_k \bar{C} \\ \bar{R} + \bar{s}_k \bar{L} & 0 \end{bmatrix} \begin{bmatrix} I(z, \bar{s}_k) \\ V(z, \bar{s}_k) \end{bmatrix} \quad (14)$$

By using diagonalization, we have

$$\bar{A} = \begin{bmatrix} 0 & \bar{G} + \bar{s}_k \bar{C} \\ \bar{R} + \bar{s}_k \bar{L} & 0 \end{bmatrix} = P^{-1} \text{diag}(\lambda) P \quad (15)$$

where $\text{diag}(\lambda)$ is a diagonal matrix with eigenvalues on the diagonal and P is the eigenvector matrix. Let σ represent the diagonal matrix with positive eigenvalues, so

$$\text{diag}(\lambda) = \begin{bmatrix} \sigma & \\ & -\sigma \end{bmatrix}$$

The solutions for Eq(14) are

$$V(z, \bar{s}_k) = \bar{P}_{21} e^{\sigma z} K_1(\bar{s}_k) + \bar{P}_{22} e^{-\sigma z} K_2(\bar{s}_k)$$

$$I(z, \bar{s}_k) = \bar{P}_{11} e^{\sigma z} K_1(\bar{s}_k) - \bar{P}_{12} e^{-\sigma z} K_2(\bar{s}_k)$$

where

$$P^{-1} = \begin{bmatrix} \bar{P}_{11} & -\bar{P}_{12} \\ \bar{P}_{21} & \bar{P}_{22} \end{bmatrix}$$

By Theorem 2, with known $Y(\bar{s}_k)$ and with input voltage sources as identity matrix, we have a linear system with $N \times 2m$ equations. After solving this linear system, we can obtain $K_1(\bar{s}_k)$ and $K_2(\bar{s}_k)$. Then by the Modified Gram-Schmidt algorithm we can generate $U(z)$.

In terms of accuracy, we can see from the experimental results in Section 5 that both of the Chebyshev expansion w.r.t frequency variable \bar{s} method and the moment expansion method can achieve accurate results. The efficiency of these two methods depends on the dimension of the integrated congruence transform. Let $U_M(z)$ be the integrated congruence transform for the moment expansion method. If we are dealing with m coupled lines, the row numbers of $U_c(z)$ and $U_M(z)$ are $2 \times m$. The column numbers of $U(z)$ and $U_M(z)$ are all determined by the number of expansion terms w.r.t frequency variable \bar{s} . However, Chebyshev expansion has the optimal property and it converges exponentially. After Modified Gram-Schmidt algorithm, we have less number of columns in $U(z)$ than in $U_M(z)$. This is verified by the examples in Section 5.

3 The Chebyshev expansion with respect to spatial variable

In this section, we focus on the Chebyshev expansion with respect to spatial variable. Practically, this method may be only efficient for low order cases, but theoretically, from this point we can extend integrated congruence transform idea to other orthogonal polynomials.

3.1 Integrated congruence transform

When deriving the Chebyshev expansion with respect to spatial variable, we have to restrict the spatial variable to $[-1, 1]$. If

$$z = \frac{(\bar{z} + 1)}{2} \quad (16)$$

we have $\bar{z} \in [-1, 1]$. If we substitute Eq(16) into Eq(1) and Eq(2) and stick to the use of V, I in the presentation, we have

$$\frac{dV(\bar{z}, s)}{dz} = -\frac{1}{2}(R + sL)I(\bar{z}, s) \quad (17)$$

$$\frac{dI(\bar{z}, s)}{dz} = -\frac{1}{2}(G + sC)V(\bar{z}, s) \quad (18)$$

Then $V(\bar{z}, s)$ and $I(\bar{z}, s)$ are expanded by Chebyshev expansion with respect to spatial variable \bar{z} . Following the

same procedure as in the last section, we have $X(\bar{z}, s) =$

$$\begin{bmatrix} I(\bar{z}, s) \\ V(\bar{z}, s) \end{bmatrix} = \bar{T}(\bar{z})C_x(s) \text{ where}$$

$$\bar{T}(\bar{z}) = \begin{bmatrix} T_0(\bar{z}) & T_1(\bar{z}) & \cdots & T_{N'-1}(\bar{z}) & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & T_0(\bar{z}) & \cdots & T_{N'-1}(\bar{z}) \end{bmatrix}$$

and

$$C_x(s) = \begin{bmatrix} C_{I0,0}(s) & C_{I1,0}(s) & \cdots & C_{Im,0}(s) \\ C_{I0,1}(s) & C_{I1,1}(s) & \cdots & C_{Im,1}(s) \\ \cdots & \cdots & \cdots & \cdots \\ C_{I0,N'-1}(s) & C_{I1,N'-1}(s) & \cdots & C_{Im,N'-1}(s) \\ C_{V0,0}(s) & C_{V1,0}(s) & \cdots & C_{Vm,0}(s) \\ C_{V0,1}(s) & C_{V1,1}(s) & \cdots & C_{Vm,1}(s) \\ \cdots & \cdots & \cdots & \cdots \\ C_{V0,N'-1}(s) & C_{V1,N'-1}(s) & \cdots & C_{Vm,N'-1}(s) \end{bmatrix}$$

where $N' = \max_{i=1,\dots,m}(N'_{V_i}, N'_{I_i})$ and N' can reach ∞ . N'_{V_i} and N'_{I_i} are the numbers of Chebyshev expansion terms of the voltage and current elements in $V(\bar{z}, s)$ and $I(\bar{z}, s)$ respectively. Suppose $U_T(\bar{z})$ is the orthonormal matrix and $\bar{T}(\bar{z})$'s column vectors are in the span of the column vectors of $U_T(\bar{z})$. Then it is obvious that there exists a coefficient matrix \bar{C} , $\bar{T}(\bar{z}) = U_T(\bar{z})\bar{C}$. Assume $U_T(\bar{z}) = \begin{bmatrix} u_{Ti}(\bar{z}) \\ u_{Tv}(\bar{z}) \end{bmatrix}$.

If we premultiply a $U_T^T(\bar{z})$ on both sides of Eq(6) and integrate with \bar{z} from -1 to 1 , we can derive the input admittance matrix as $\hat{Y}_T(s) = \hat{b}_T^T (s\hat{M}_T + \hat{N}_T)^{-1} \hat{b}_T$ where $\hat{M}_T = \frac{1}{2}(\int_{-1}^1 (u_{Ti}^T(\bar{z})Lu_{Ti}(\bar{z}) + u_{Tv}^T(\bar{z})Cu_{Tv}(\bar{z}))d\bar{z})$, $\hat{N}_T = \hat{N}_{T1} + \hat{N}_{T2}$ with $\hat{N}_{T1} = \frac{1}{2}(\int_{-1}^1 (u_{Ti}^T(\bar{z})Ru_{Ti}(\bar{z}) + u_{Tv}^T(\bar{z})Gu_{Tv}(\bar{z}))d\bar{z})$ and $\hat{N}_{T2} = \int_{-1}^1 (u_{Tv}^T(\bar{z})\frac{du_{Ti}(\bar{z})}{d\bar{z}} - \frac{du_{Tv}(\bar{z})}{d\bar{z}}u_{Ti}(\bar{z}))d\bar{z}$ and $\hat{b}_T = \begin{bmatrix} u_{Ti}(-1) \\ -u_{Ti}(1) \end{bmatrix}$. Here \hat{b}_T, \hat{M}_T and \hat{N}_T are all generated based on $U_T(\bar{z})$.

3.2 Relationship with integrated congruence transform based on moment

Theorem 3[8] Let $U_M(\bar{z})$ be the integrated congruence transform based on the moment method and $U_T(\bar{z})$ be the orthonormal matrix whose column vectors span the same space as the column vectors of $\bar{T}(\bar{z})$. There exists a coefficient matrix C such that $U_M(\bar{z}) = U_T(\bar{z})C$. Both $U_M(\bar{z})$ and $U_T(\bar{z})$ are nonsingular and orthonormal matrices for the same functional space w.r.t. the spatial variable \bar{z} . Based on the orthogonality property of Chebyshev polynomials, we can give the algorithm for the formation of congruence transform matrix:

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Simplified Modified Gram-Schmidt Algorithm
{ Input: Chebyshev polynomials T and order N
Output: Transformation matrix U.
function U=SMGM(T, N)
{ Split T into Teven and Todd;
Ueven = Modified - GramSchmidt(Teven, int(N/2));
Uodd = Modified - GramSchmidt(Todd, int(N/2));
U = construct(Ueven, Uodd, N);
return(U);
}

```

Because the even order Chebyshev polynomials are orthogonal to odd order Chebyshev polynomials, we save half of the running time on the Modified Gram-Schmidt Algorithm. Moreover, in this case, the integrated congruence

transform is independent of the line parameters and input currents or voltages. We can construct the integrated congruence transform before analyzing the circuits. Let $U_M(z)$ be the integrated congruence transform for moment expansion method. For m coupled lines, the row numbers of $U_T(z)$ and $U_M(z)$ are $2 \times m$. From Theorem 3, it can be proved that the column number of $U_T(z)$ is determined by the terms of the polynomial we use to approximate $e^{-\lambda_{max}z}$ where λ_{max} is the eigenvalue of matrix A in Eq(15) with maximum real absolute value.

Integrated congruence transform is not only applicable to moment matching and Chebyshev expansion methods. It can also be extended to other orthogonal polynomial expansions. We have

$$X(z, s) = \begin{bmatrix} I(z, s) \\ V(z, s) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^M I_i(z)g_i(s) \\ \sum_{j=1}^N V_j(z)f_j(s) \end{bmatrix}$$

If g_i and f_j represent other orthogonal polynomial expansions and M and N can reach ∞ . Denote $Q(z) = [\begin{smallmatrix} g_1(z) \\ \vdots \\ g_M(z) \end{smallmatrix} \quad 0 \quad \cdots \quad 0 \quad \begin{smallmatrix} f_1(z) \\ \vdots \\ f_N(z) \end{smallmatrix}]$, then $X(z, s) = Q(z)\hat{x}(s)$

where $\hat{x}(s) = [I_1(s)^T \quad I_M(s)^T \quad V_1(s)^T \cdots V_N(s)^T]^T$. Thus, instead of one continuous function for voltage and current along each line respectively, we use a group of continuous functions to approximate voltage and current along the line. Then by finding the orthonormal matrix $U(z)$ so that $U(z)C = Q(z)$ where C is a constant matrix, we obtain $X(z, s) = U(z)\hat{x}(s)$ just as in Eq(12). Then all the results for the Chebyshev expansion w.r.t. spatial variable method are applicable here.

4 Order selection for Chebyshev expansion method

In this section, we discuss the order selection technique which is called relative error method. This technique can be applied to both Chebyshev expansion methods. It requires the users to offer the threshold value $Err_{threshold}$. Let $Y_n(\bar{s})$ denote the admittance matrix estimation by using order n expansion, and $Y_{n+1}(\bar{s})$ denote the order $n+1$ expansion case. We define the relative error as $err = \frac{\|Y_{n+1}(\bar{s}) - Y_n(\bar{s})\|}{\|Y_n(\bar{s})\|}$. If $err \leq Err_{threshold}$, the order of expansion is set as n . Otherwise, we have to go on using higher order expansions to estimate $Y(s)$ until $err \leq Err_{threshold}$ is satisfied. For norm, we can use 2-norm or other norm definitions, i.e. H_∞ definition: $\|Y(\bar{s})\|_\infty = \max_z \lambda(Y(\bar{s}))$, because different norms have inequality relationships. We implemented both Chebyshev expansion methods. The Chebyshev expansion w.r.t. \bar{z} method usually produces integrated congruence transforms column numbers larger than the Chebyshev expansion w.r.t. \bar{s} method. Though we save running time on the construction of the integrated congruence transform by using the Chebyshev expansion w.r.t. \bar{z} method, it takes longer to estimate the input admittance matrix than the Chebyshev expansion w.r.t. \bar{s} method where dense matrix multiplication is involved. Based on our experience, after the order estimation, if the Chebyshev expansion w.r.t. \bar{z} method has an order over 20, it is wise to choose the Chebyshev expansion w.r.t. \bar{s} method in terms of efficiency.

5 Examples

We have successfully tested a number of examples. Some of them are presented here.

Example 1.

This circuit consists of a single line with parameters $R = 0.01\Omega/cm$, $L = 2.5nH/cm$, $C = 1pH/cm$, $d = 1cm$, a load resistor and a source resistor of 50Ω , which match the characteristic impedance of the line at high frequencies. The voltage source is a pulse. Fig.1 shows the output voltage waveform V_3 obtained by the SPICE simulation and our Chebyshev expansion w.r.t. \bar{s} method. The waveform is nearly exact. Three expansion points are needed for the moment matching model. From Table 1, we can observe that a 6-column integrated congruence transform is generated by the moment based method, whereas the Chebyshev expansion w.r.t. \bar{s} method only has 1 column. This example shows that the Chebyshev expansion w.r.t. \bar{s} further reduces the size of integrated congruence transform. Note in this case, the only column in Chebyshev expansion w.r.t. \bar{s} is not equivalent to the 0th moment at $s = 0$. It should be an average of $T_0(\bar{s})$ coefficients at different \bar{s}_i points. If we need to capture the exact DC point response, interpolation can be used in Chebyshev expansion so that at $s = 0$ it has the exact value instead of the average value.

Example 2.

This circuit consists of 4 coupled lines with neighbor coupling. The waveforms of V_5 are shown in Fig.2, where the solid, dashed and dotted lines correspond to the result of SPICE simulation, the time domain simulation with our model and the result of the moment method. The circled waveforms in Fig.2 are magnified and presented in Fig.3. On a DEC5000/125 machine, SPICE spends 785s to finish the simulation, the multipoint moment method needs 51s and our new model only need 38s.

Example 3.

This is an example borrowed from [9]. The circuit has two coupled line systems, each of which consists of three coupled lines. The time-domain response of V_{out} are shown in Fig.4. The solid line represents the exact solution where the coupled lines are modeled by their exact multipoint characteristic model. The dashed line corresponds to our model. For moment method, a moment matching set $MS = \{(0Hz, 5), (1.5GHz, 2), (3GHz, 2), (4GHz, 2), (5GHz, 2)\}$ for each line system is used. And the moment based integrated congruence transform has 13 columns. The solid and dashed lines are indistinguishable. Compared with the model used in [9], not only that our model is guaranteed passive and theirs not, but also that our model order is much lower (a 40-th moment matching model at zero frequency is used for each coupled line system in [9]). This also shows the advantage of Chebyshev expansion based method over a single point based moment matching method.

Example 4.

This is a clock net consisting of 73 lossy transmission lines, 2895 resistors and 2777 capacitors driven by a cascade of two inverters, as shown in Fig.5. The waveforms at PIN117 are shown in Fig.6, where the solid, dashed and dotted lines correspond to the result of SPICE simulation and the time domain simulation with our model. For each line, moments are matched at 0 frequency with order 4 and at a high frequency with order 0 is used. Taking into consideration of the split of complex moments, the integrated congruence transform has 7 columns whereas Chebyshev expansion based integrated congruence transform has 4 columns. It takes SPICE 180.3min, the multipoint moment matching method 8.5 min and the new model implemented in SWEC 6.3 min.

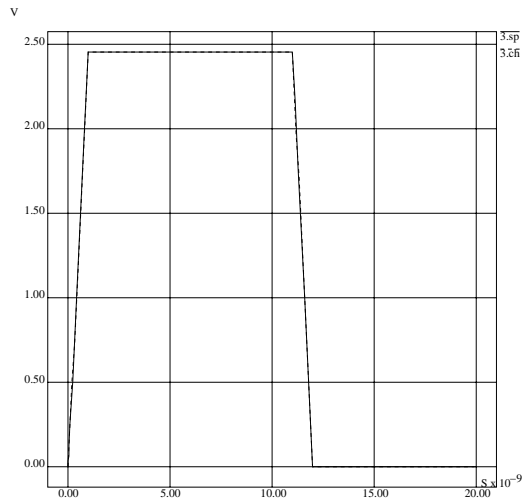


Figure 1: Waveform of example 1

6 Conclusions

We have presented a new algorithm for optimal passive model order reduction with Chebyshev expansion for distributed interconnect networks. Without given the expansion points, this new method generates a better passive model than the multipoint moment based method in terms of efficiency and order. Experiments show that this new model works well. In the near future, we aim to apply this new method to frequency-dependent interconnect modeling problem.

7 Reference

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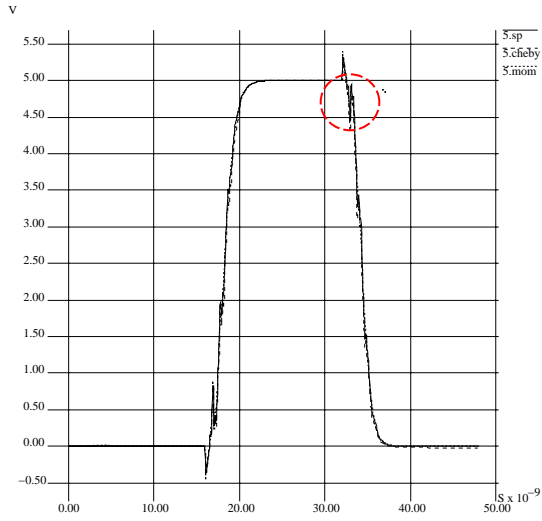


Figure 2: Waveform of example 2

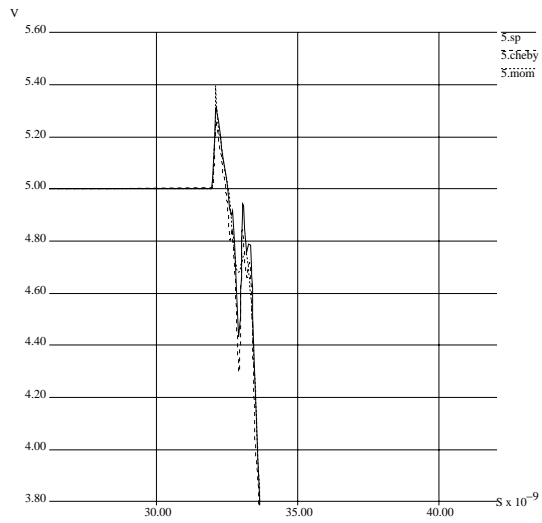


Figure 3: Magnified part of Figure 2

Table 1: Column numbers of the integrated congruence transform

Examples	mom	cheby w.r.t. \bar{s}	cheby w.r.t. \bar{z}
ex1	6	1	10
ex2	8	3	22
ex3	13	5	25
ex4	7	4	30

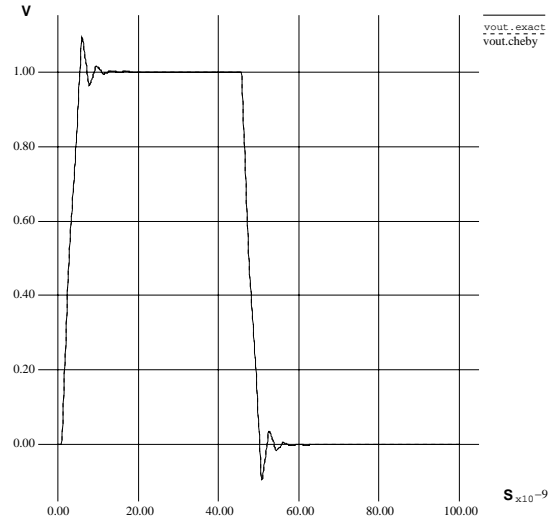


Figure 4: Waveform of example 3

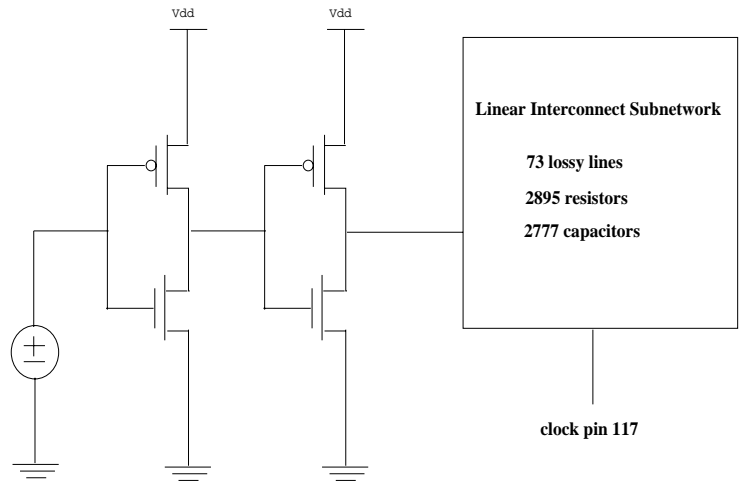


Figure 5: Circuit diagram of example 4

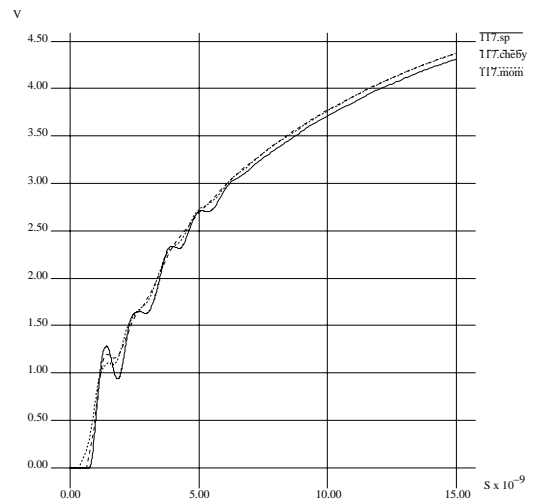


Figure 6: Waveform of example 4