

# EASY – a System for Computer-Aided Examination of Analog Circuits

G. Dröge<sup>1</sup>

M. Thole<sup>2</sup>

E.-H. Horneber<sup>2</sup>

<sup>1</sup>SICAN Braunschweig GmbH  
Richard-Wagner-Straße 1  
D-38106 Braunschweig

<sup>2</sup>Institut für Netzwerktheorie  
und Schaltungstechnik  
Langer Kamp 19c  
D-38106 Braunschweig

## Abstract

The EASY analog design system includes a qualitative analysis tool for examination of the principal aptitude of a chosen circuit structure, as well as a symbolic analysis component. It allows the deduction of compact but sufficiently accurate design equations. These tools support the first steps of the design process and give insight in the behavior of the analog circuit.

## 1 Introduction

Apart from tools for simulation and sizing of circuits with fixed structures like opamps, CAD software that supports the design of general analog circuits is rarely available. Increasing attention is attracted on symbolic analysis of circuits which is tackled by a number of research groups [6, 8, 9].

The Experimental Analog Design System EASY aims at the support of the very first design steps, i. e. the selection and examination of possible circuit structures and the automatic deduction of design formulae. This is performed by the two main components shown in figure 1: the qualitative analysis component and the component for symbolic setup and approximation of design equations. It is based on the computer algebra program MACSYMA [7]

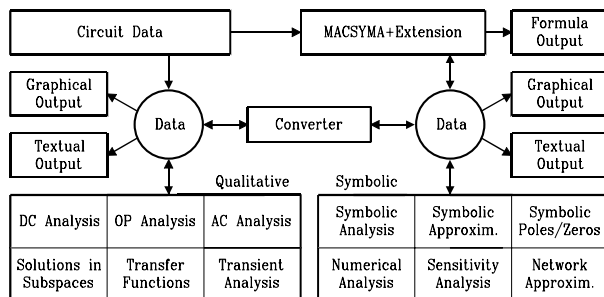


Figure 1: EASY System

that is used for equation manipulation, sensitivity calculation and symbolic pole/zero analysis. Evaluating the symbolic results for some estimated parameter values ('design points'), compact design formulae can be deduced. They allow to identify the significant design parameters and, therefore, giving insight in the circuit's behavior. The qualitative analysis supports the examination of non-standard circuit structures with no preassigned element values. This is done by introducing qualitative values to analyze circuit states and the circuit's behavior. Clearly, sizing of circuits of application of symbolic analysis only makes sense, if the principle design objective is covered by the circuit's structure. In the following, an overview on the tools and options of the EASY system is given, demonstrated on the basis of the example circuit of a two stage transistor amplifier [1] shown in figure 2 (a detailed discussion of the algorithms can be found in the references).

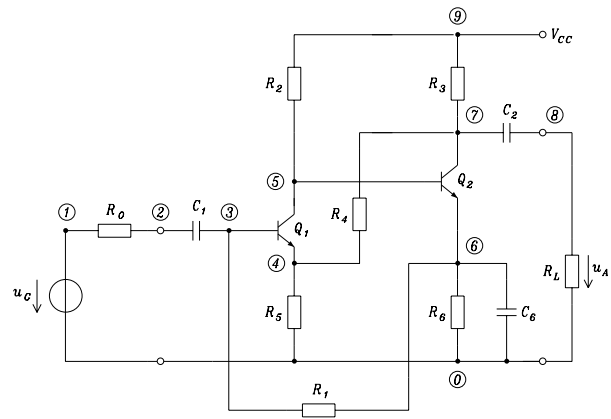


Figure 2: Two stage coupled amplifier

## 2 Qualitative Tools

At the beginning of a design process the designer has to verify which circuit structure is suitable for a given pur-

pose. The calculation of the possible circuit's behavior helps avoiding undesirable side-effects.

To support this task the qualitative tools offer different kinds of analyses which are similar to numerical simulation techniques regarding their functional purposes [10].

To analyze the circuit without preceding dimensioning the qualitative values *positive*, *zero* and *negative* (denoted by  $[+]$ ,  $[0]$ ,  $[-]$ ) are introduced. For example, the qualitative value of a resistor can be assigned *positive*. Corresponding the results can be interpreted, e. g. the qualitative value *negative* as a result of a small-signal analysis means *decreasing*.

## 2.1 DC Analysis & Operating Point Analysis

The main purpose of a qualitative DC analysis is to determine possible states of the nonlinear elements. In addition, all node voltages, branch voltages and branch currents as well as their correlations can be determined [11, 12].

A qualitative operating point analysis takes into account the initial conditions of the dynamic elements while a qualitative DC analysis computes the steady-state of the circuit.

The qualitative DC analysis applied on the example circuit (figure 2) shows that saturation is possible for transistor  $Q_2$ , however the cutset equation including node 1, 2 and 3

$$I(U_G) + I(R_1) + I(Q_{1BE}) + I(Q_{1BC}) = [0]$$

forbids saturation for transistor  $Q_1$ . The set of possible solutions contains nine elements caused by the branch quantities  $U(C_1)$  and  $I(Q_{2BC})$  which build a subspace of the solution space with nine elements. All other network variables can be determined unambiguously as shown in table 1 with  $Q_1$  and  $Q_2$  in forward-on state.

All node voltages are positive except  $V_8$  which of course is *zero*. Remarkable is the orientation of the branch currents of the resistors  $R_1$  and  $R_4$ . The current of  $R_1$  and  $R_4$  are *negative* regardless of the circuit's sizing or whether  $Q_2$  is saturated or not.

## 2.2 Subspaces of Solution

The complete relations between all network variables can be quite complex, however the interesting correlations of certain variables are relatively simple.

Therefore it is often helpful viewing only a subspace of the circuit's solution space by eliminating multiple occurrences of states in a subset of network variables. This tool can be used for all qualitative analyses.

## 2.3 Small-Signal Analysis

The qualitative small-signal analysis calculates the circuit response for a given input signal. It is assumed that the input signal is infinitely small. Hence nonlinear elements can not leave their domains which have to be calculated

Table 1: Variables with certain states

Variable	State	Variable	State	Variable	State
$I(V_{CC})$	$[-]$	$I(U_G)$	$[0]$	$I(C_1)$	$[0]$
$I(C_2)$	$[0]$	$I(C_6)$	$[0]$	$I(R_0)$	$[0]$
$I(R_1)$	$[-]$	$I(R_2)$	$[+]$	$I(R_3)$	$[+]$
$I(R_4)$	$[-]$	$I(R_5)$	$[+]$	$I(R_6)$	$[+]$
$I(R_L)$	$[0]$	$I(Q_{1BE})$	$[+]$	$I(Q_{1BC})$	$[0]$
$I(Q_{1CE})$	$[+]$	$I(Q_{2BE})$	$[+]$	$I(Q_{2BC})$	$[0]$
$I(Q_{2CE})$	$[+]$	$U(V_{CC})$	$[+]$	$U(U_G)$	$[+]$
$U(C_2)$	$[+]$	$U(C_6)$	$[+]$	$U(R_0)$	$[0]$
$U(R_1)$	$[-]$	$U(R_2)$	$[+]$	$U(R_3)$	$[+]$
$U(R_4)$	$[-]$	$U(R_5)$	$[+]$	$U(R_6)$	$[+]$
$U(R_L)$	$[0]$	$U(Q_{1BE})$	$[+]$	$U(Q_{1BC})$	$[-]$
$U(Q_{1CE})$	$[+]$	$U(Q_{2BE})$	$[+]$	$U(Q_{2CE})$	$[+]$
$V(9)$	$[+]$	$V(1)$	$[+]$	$V(2)$	$[+]$
$V(3)$	$[+]$	$V(7)$	$[+]$	$V(8)$	$[0]$
$V(6)$	$[+]$	$V(5)$	$[+]$	$V(4)$	$[+]$

by a qualitative steady-state analysis before. Changing the models of the dynamic elements a qualitative frequency response can be calculated.

Without an input signal the stability of a circuit can be proven. If the circuit is stable, the set of possible solutions contains only the trivial solution zero. Otherwise the network may be instable under certain conditions. This can be used to determine meta-stable states of oscillators or to evaluate amplifiers.

Figure 3 shows the results of a qualitative AC analysis

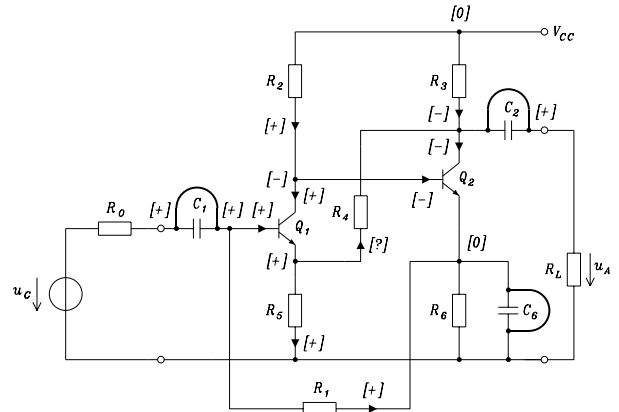


Figure 3: Results of a qualitative AC analysis

applied on the example circuit (figure 2) with both transistors in forward-on state. All capacitors are treated as short circuits. The set of possible solutions contains 27 elements caused by  $i(V_{CC})$ ,  $i(R_4)$ ,  $u(R_4)$  and  $i(C_6)$  which build a sub-

space of the solution space with 27 elements. Again, all other network variables have certain states.

With  $C_6$  modeled as an open loop, the node voltages  $v_5$  and  $v_6$  change to

$$v_5 = \begin{bmatrix} + \\ 0 \\ - \end{bmatrix} \quad \text{and} \quad v_6 = \begin{bmatrix} + \\ + \\ [ + \ 0 \ - ] \end{bmatrix},$$

which leads to a set of possible solutions for the subspace  $(v_5, v_6)$  with five elements. If  $v_5$  is *positive* or *zero*,  $v_6$  is always *positive*. If  $v_5$  is *negative* the node voltage  $v_6$  is uncertain.

The other node voltages are not affected. If  $Q_2$  is saturated and all capacitors are treated as short circuits, node voltage  $v_8$  is uncertain. This means that  $v_8$  might also be *zero* or *negative*.

Additional analyses show that the network remains stable even if dynamic effects of the bipolar transistors are considered.

## 2.4 Transient Analysis

The qualitative transient analysis is the counterpart of the numerical transient analysis. Its purpose is the calculation of time dependencies. Analog to the numerical simulation the result is a qualitative plot of network variables versus time represented by qualitative state vectors for each time step. These vectors contain the nominal value and the time derivatives of a network variable [10].

All possible trajectories as well as steady-states or oscillations can be evaluated.

# 3 Symbolic Tools

## 3.1 Symbolic and Numerical Analysis

Symbolic circuit analysis allows a mathematical formulation of dependencies between particular circuit quantities, such as voltages or currents. This can be done without fixed values for all circuit elements. Therefore, symbolic analyses may already be used before or during the sizing of the circuit. Thus the time-consuming first phase of the analog circuit design process is supported. This is the main area in which symbolic analysis differs from numerical analysis (for example realized within the circuit simulator *SPICE*), which is only useful for circuit verification after sizing is completed.

Symbolic analysis may generate various network functions in symbolic form, e. g. the input-to-output voltage or current gain, the input or output impedance or the common-mode rejection ratio. Numerical results can immediately be obtained by simply replacing the parameters by numerical values.

## 3.2 Sensitivity Analysis

To realize circuit specifications, it is necessary to know which elements have influence on corresponding circuit quantities. These elements can be determined by use of sensitivity analysis. Parameterized sensitivity expressions can be calculated easily by differentiating the solution expressions from the symbolic analysis.

Consider the two stage bipolar amplifier of figure 2. Figure 4 shows the pertinent sensitivities of the output elements (load resistance  $R_L$  and coupling capacitance  $C_2$ ) with respect to the output voltage  $u_A$ . It can be seen for which values of the frequency  $f$  and load resistance  $R_L$ , the output elements have no influence on  $u_A$ .

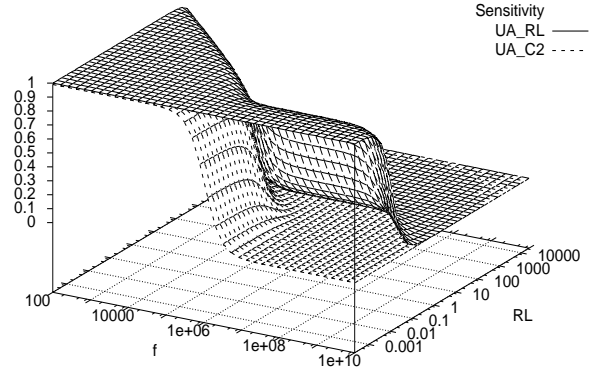


Figure 4: Sensitivity plot

## 3.3 Symbolic Approximation

Usually, the results from symbolic analysis, i. e. mathematical expressions, are so complex that they cannot be used until they are reduced to their essential information. This means, approximation methods are necessary to generate compact formulae [9]. These formulae establish the design equations given in literature for many known circuits.

The magnitude plot of the given example is shown in figure 5. For several design points (which only differ in the frequency) approximated transfer functions have been generated:

$$\begin{aligned} H_0(s) &= -\frac{C_1 C_2 R_5 R_6 R_L s^2}{C_6 R_5 R_6 s + R_6 + R_5}, \\ H_3(s) &= -\frac{C_1 C_2 g_{m2} R_L s^2 / g_{o2} g_{\pi 2}}{1 + (C_2 R_L + C_1 R_0) s + C_1 C_2 R_0 R_L s^2}, \\ H_6(s) &= -\frac{g_{m2}}{g_{o2} g_{\pi 2} R_0}, \quad H_{12}(s) = -\frac{1}{c_{\mu 2} R_0 s}. \end{aligned}$$

Each of them describes the behavior of the circuit in particular frequency intervals. The symbolic expressions are

very simple and enable good insight into the circuit's behavior. Moreover, they reflect very well parameter dependencies if the design point is varied. The main circuit elements are still part of the approximated expressions whereas the elements of little or no influence are effectively deleted.

### 3.4 Symbolic Pole-Zero Determination

Another important characteristic of linear circuits concerns the location of their poles and zeros. Again, expressions in symbolic form may be used for tuning specific circuit behavior. Their knowledge allows a wide range of application: e. g. pole-zero compensations, detection of instabilities, and determination of bandwidths and cutoff-frequencies.

In order to analyze the dynamic behavior of the example circuit, all transistors are replaced by a small signal high-frequency model. Numerical analyses show that the circuit has six zeros and seven poles. An exact symbolic pole and zero calculation is not possible. Therefore, special approximation methods [4], [5] have to be used. They guarantee the generation of very compact and also accurate symbolic expressions. The following poles and zeros have been calculated:

$$p_2 = -\frac{1}{2\pi C_2 R_L}, \quad p_3 = -\frac{1}{2\pi C_1 R_0},$$

which determine the lower cutoff-frequency and

$$p_4 = -\frac{g_{o2}g_{\pi 2}}{2\pi C_{\mu 2}g_{m2}}, \quad p_5 = -\frac{g_{o1}g_{\pi 1}}{2\pi C_{\mu 1}g_{m1}},$$

and

$$z_5 = -\frac{g_{o1}g_{\pi 1}}{2\pi C_{\mu 1}g_{m1}},$$

which fixes the high cutoff-frequency (see figure 5).

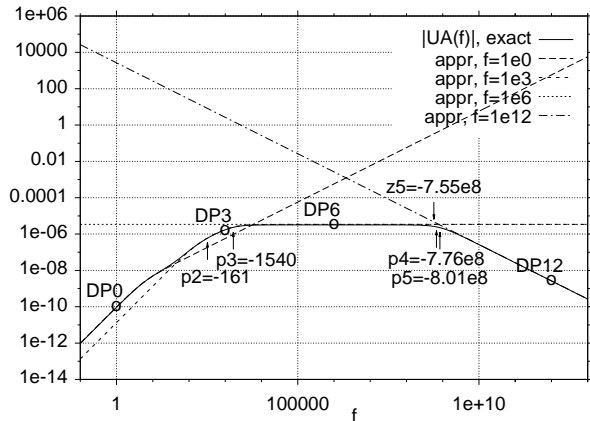


Figure 5: Magnitude plot

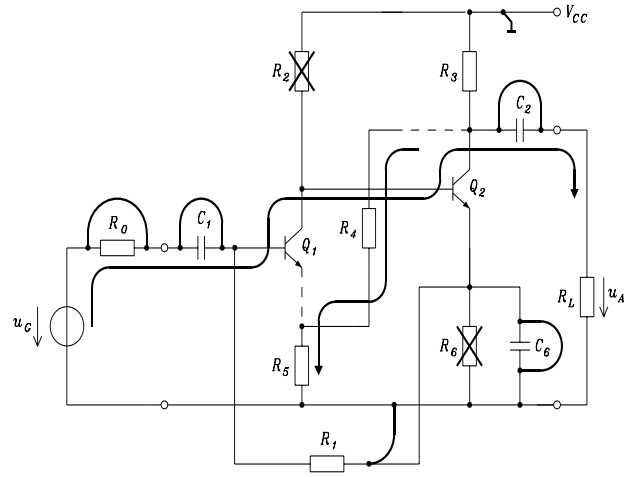


Figure 6: Approximated circuit with signal paths

### 3.5 Network Approximation and Signal Flow Analysis

Not all circuit characteristics can be expressed mathematically. For example, the signal flow is frequently used to explain the behavior of a circuit. For this problem network approximation methods and signal flow analyses [3] are necessary to locate different signal paths within a circuit.

The main idea of the method is the transference of approximations from equation level to network level [2]. By comparing the orders of magnitude of network quantities elements can be simplified to short circuits or open loops. This reduces the variety of possible signal paths. Figure 6 shows the approximated circuit. The broken lines indicate those partial branches where the current is neglectably low. If this occurs only on one side of an element, the signal flow is directed from the neglectable side to the other side where the current is still significant. Further signal paths can be obtained from the input, output and controlled sources. All these single signal paths together form one complete signal path passing the two transistor stages and one feedback path  $R_4$ .

## 4 Conclusion

The qualitative and the symbolic component of the EASY design system include a variety of analysis options which support insight in the behavior of a designed circuit. The resulting design equations can be used for sizing and optimization of the designed circuit even if non-standard circuit structures are considered.

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