OTTER: Optimal Termination of Transmission lines Excluding Radiation

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Abstract – The design automation of high-speed digital system interconnects is a challenging problem which requires controlling reflections from discontinuities and noise due to crosstalk. On-chip interconnect design automation is well defined since the objective is to minimize R and C to minimize delay. In contrast, for boards and packaging interconnect, the design objective is much more difficult to specify in terms of a metric.

This paper presents a generalized approach for RLC interconnect design automation. A new metric is defined that specifies the optimal design as a function of input signal rise-time, loading conditions on the line, parasitic resistance in the circuit and discontinuities in the interconnect. The approach is based on the recognition of the relation between moments of the responses and critical damping of the circuit. The metric is evaluated without any time-domain simulations to obtain the optimal termination. No simplifying assumptions are required for combinations of lumped and distributed lossy lines, with non-ideal terminations.

I. Introduction and Background

The design of system-level interconnects remains a challenging problem for high-speed digital systems. Without proper terminations, reflections from discontinuities and induced voltages due to crosstalk can adversely influence the delay and the signal integrity. It is often the case that an additional lumped resistance is introduced either in series with the driver (series termination) or in parallel with the load (parallel termination). The lumped resistance value is generally selected to be equal to the characteristic impedance of the transmission line.

Experienced designers recognize, however, that these termination schemes are "optimal" only for a limited class of interconnect circuits, and are inappropriate when any of the following conditions are true: (1)The unloaded line driver has a significant rise/fall time; (2)The circuit has significant capacitive loading; (3)There is significant parasitic resistance in the circuit (e.g. lossy line); (4)There are discontinuities or a number of transmission lines with different parameters.

When any or all of these conditions are true, the "optimal" value of a termination resistor will be less than the characteristic impedance value. Terminating the line with the characteristic impedance can overdamp the response and increase the circuit delay. Notice, however, that all of the conditions stated above are rather subjective. That is, the designer will have to determine what "significant" means due to the absence of any appropriate metrics. For this reason, designers usually select a termination resistance value through trial and error, requiring a detailed time-domain simulation at each iteration.

Previous work on a quantified approach to optimal termination has utilized an interconnect model to generate a simplified pole-zero description of the circuit behavior [3,17,18]. A single-lump RLC model is used for the interconnect path in order to generate a closed-form expression for the two-pole voltage response at the driving point, and the optimal series resistance is defined as the value which produces a critically damped circuit based on this two-pole model. Since the 2-poles are not the *exact* dominant poles of the system, the termination resistance value produced, in general, cannot be truly optimal. In [5] a distributed interconnect model is used for self-damped lossy transmission lines under the restriction that the highest frequency component transmitted by the line does not exceed the quarter-wavelength limit. However, the main limitation of this technique, as well as the single-RLC lump approaches, is their restriction to pin-to-pin nets and the inability to consider driver rise-time effects or loading effects on the interconnect.

In this paper, a completely new metric has been defined for optimal termination of transmission lines that uses the coefficients of the Taylor series expansion of the "exact" time domain response, as a symbolic function of the resistances in the circuit, to determine the conditions for critical damping. The optimal termination choice is shown to be a trade-off between the rise time of the signal and the acceptable amount of peak overshoot, and is a direct function of the loading conditions, driving signal characteristics and the resistivity of the line. The foundation for this approach lies in the relation between the Taylor series coefficients and the moments, which represent the time-weighted integrals of the exact time-domain responses. No time-domain analysis is required and there are no restrictions on the type of interconnect circuit. Moreover, the symbolic resistances can be either distributed or lumped circuit parameters. Self-termination of a transmission line has been considered in [15]. Examples of the optimal termination metric and the overall methodology are shown for series and ac terminations.

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II. Optimal Termination

Off-chip interconnect and packaging are generally modeled as combinations of distributed and lumped RLC elements. Since these interconnect circuits are characterized by a time-of-flight delay and ringing, defining and optimizing the delay is much more complicated than for the on-chip RC propagation case. For example, for an under-damped waveform the delay is determined by the settling time and the logic thresholds for the signal line. Increasing the resistance in the circuit will decrease the settling time, and hence the delay. However, excessive resistance will cause the line to be overdamped, and the time at which the waveform crosses some threshold, say 90% of the final value, will occur at a later point.

The most general and desirable condition for optimal termination is to decrease the time at which the signal first crosses some logic threshold and control the maximum overshoot and undershoot a signal can experience. Achieving this condition requires a positive value of termination resistance which is guaranteed to be less than or equal to the characteristic impedance of a given line.

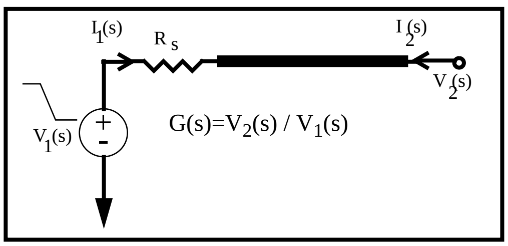

FIGURE 1: Open circuited lossless line with series termination at source end. l=0.2nH/mm, c=0.15pF/mm, *length*=25mm, Z_0=36.51, T_f=0.14ns.

From transmission line theory it is well known that a pure LC line, as shown in Fig.1, is perfectly terminated with a series impedance given by Z_0, the *characteristic impedance* of the line. With $R_s = Z_0$ there is maximum power transfer and no overshoot. With $R_s = 0$ and a parallel termination of $R_L = Z_0$, there is no reflection, and the line is equivalent to an infinitely long line insofar as the source is concerned [16].

A. The Moment Representation

Moment-matching techniques such as *A*symptotic *W*aveform *E*valuation (*AWE*) have been widely applied for efficient waveform estimation of large RLC lumped and distributed circuits [12,14]. Moments of a time-domain waveform, *V(t)*, are classically defined via the Laplace domain representation of the waveform, as follows:

$$V(s) = \int_0^{\infty} e^{-st} v(t)\, dt = m_0 + m_1 s + m_2 s^2 + \ldots \tag{1}$$

where m_k are the Maclaurin series coefficients of *V(s)*. Thus, the *k*-th moment, m_k is:

$$m_k = \frac{(-1)^k}{k!} \int_0^{\infty} t^k v(t)\, dt \tag{2}$$

The zero-th moment, m_0, is the time domain integral of the waveform from $t = 0$ to $t = \infty$. Similarly, the *k*-th moment, m_k, is the t^k-weighted time-domain integral of the waveform *v(t)*. Our definition of *optimal termination* is in terms of these moments.

Consider the lossless line in Fig.1, open-circuited at the end, and with R_s as the series termination. The transfer function *G(s)* is:

$$G(s) = \frac{V_2(s)}{V_1(s)} = R_s \frac{\sinh(\Gamma d)}{Z_0} + \cosh(\Gamma d) \tag{3}$$

where $\Gamma = s\sqrt{lc}$ is the *propagation constant* and $Z_0 = \sqrt{l/c}$ is the *characteristic impedance* of the transmission line. l and c are the inductance and capacitance of the transmission line per unit length respectively, and d is the length of the line. Expanding about $s=0$,:

$$G(s) = m_0 + m_1 s + m_2 s^2 + \ldots + m_k s^k + \ldots \tag{4}$$

Each moment m_k in (4), is a polynomial function of the series termination resistance and can be evaluated *symbolically* in terms of R_s:

$$m_k = a_p R_s^p + a_{p-1} R_s^{p-1} + \ldots + a_0, \; p \in N \tag{5}$$

For a positive value of R_s, as $R_s \to \infty$, $m_k \to \pm\infty$ (i.e. m_k grows in magnitude without bound) which is the asymptotic limit of the overdamped case. As R_s is reduced from infinity, therefore, the first zero-crossing of the moment expression m_k in (5), in terms of R_s, is the point at which the t^k-weighted time domain response *V(t)* has the integral from $t = 0$ *to* ∞ exactly equal to zero. As the order of the moment being considered is increased (larger value of k), this first zero-crossing of the moment m_k, as a function of R_s, signifies the point at which the circuit changes from overdamped to critically damped.

The moments of the output response $V_2(s)$ in Fig.1 for a step input are shown as a symbolic function of the series termination resistance in Fig.2. The first, second, fourth, sixth and eighth moments (m_1, m_2, m_4, m_6 and m_8) are plotted as a function of R_s. The moments magnitudes are not plotted on the same scale, but it is evident that the greatest zero-crossing of m_k, as k is increased, converges to a value of R_s equal to the characteristic impedance of the transmission line.

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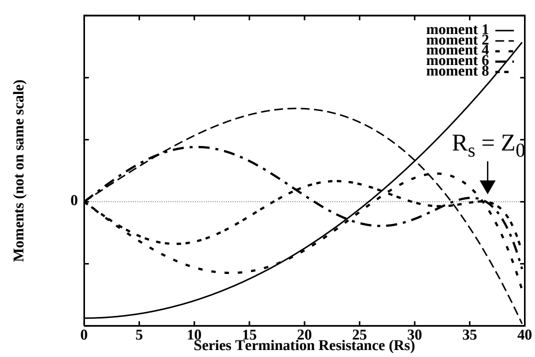

FIGURE 2: Moments m_1, m_2, m_4, m_6 and m_8 of the voltage response V_2 as a function of the termination resistance, R_s. Z_0=36.5 Ω

This *symbolic* approach to optimal termination via the moment representation provides a metric that is powerful and offers important advantages in terms of generality and efficiency. Considering loading effects on the termination requirements for a transmission line, for example, a low-loss transmission line with a capacitive load has the transfer function:

$$G(s) = sC_L(R_s \cosh(\Gamma d) + Z_0 \sinh(\Gamma d)) + \frac{R_s}{Z_0}\sinh(\Gamma d) + \cosh(\Gamma d) \tag{6}$$

where, $\Gamma = \sqrt{(r+sl)(sc)}$ and $Z_0 = \sqrt{(r+sl)/(sc)}$. A low-to-high input transition with non-zero rise-time, t_r, is given by:

$$U(s) = \frac{1-e^{-st_r}}{s^2 t_r} \tag{7}$$

This input can be convolved with the impulse response of the transmission line to obtain moments for the output response, *Y(s)*:

$$\begin{aligned} Y(s) &= G(s) \cdot U(s) \\ &= \left(m_0 + m_1 s + m_2 s^2 + \ldots\right)\left(\frac{1-e^{-st_r}}{s^2 t_r}\right) \end{aligned} \tag{8}$$

As explained above, the expression for *moment-3* of the output response is of the form:

$$m_3 = R_s^4 f_4 + R_s^3 f_3 + R_s^2 f_2 + R_s f_1 + R_s \tag{9}$$

where $f_i \equiv f(R, L, C, t_r, C_L)$ is a function of the load C_L and rise-time of the driving signal t_r.

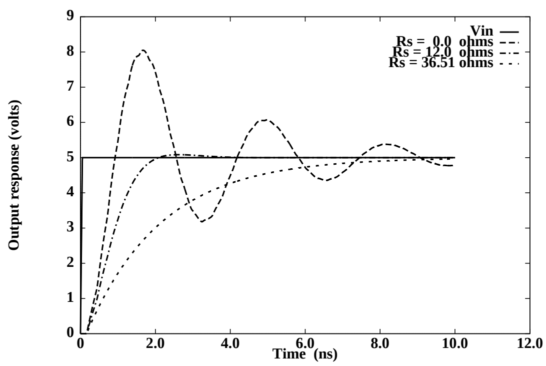

FIGURE 3: Output response of low-loss line (Z_0=36.51 Ω) with capacitive load (C_L) and optimal series termination resistance (R_s) obtained using m_3 via symbolic analysis. C_L= 50.0pF.

For a rise-time of *0.05ns* and a capacitive load of *50pF* on the circuit in Fig.1, the maximum root of m_3 as a function of R_s yields a value of 12Ω. Time-domain waveforms shown in Fig.3 contrast this termination value against a no-termination condition and termination equal to the characteristic impedance Z_0. Notice the underdamped response for $R_s=0$ and the overdamped signal for $R_s=Z_0$.

The above discussion shows that we can determine the point at which the circuit becomes critically damped without performing a time-domain analysis. Further, since m_1 is the t^1-weighted integral of the voltage waveform, m_2 is the t^2-weighted integral, and so on, considering optimal termination to be the highest zero-crossing of the *k*-th moment plotted against the series resistance ensures that a lower order moment allows more signal overshoot/undershoot at "optimal termination" than a higher moment. We will show next how to approximate the overshoot from the moments in order to facilitate fast delay/overshoot trade-off choices during optimization.

B. Laplace Domain Representation

The distributed model of a transmission line represents the system as an infinite pole system. It can be argued that, in general, the condition for optimal termination for a step-response translates in the Laplace domain to "What makes the first complex pole pair (pole-pair closest to the real-axis) of the infinite-order transmission line system become real ? [15]". For the LC line example in Fig.1, the movement of the first pole pair as a function of the termination value R_s is shown in Fig.4. When $R_s = 0$, the poles of the lossless transmission line system lie on the $j\omega$-axis. As R_s is increased, the poles move away from the $j\omega$-axis, and at $R_s = Z_0$, the first pole pair becomes a repeated real pole, after which the poles move in opposite directions on the real axis.

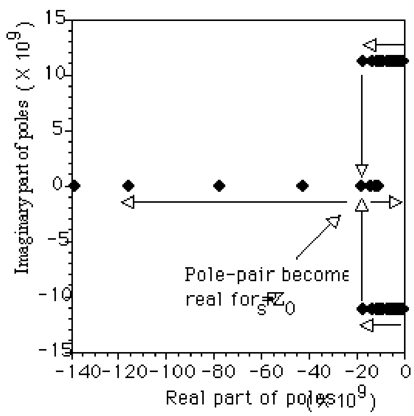

FIGURE 4: Movement of first pole-pair as a function of the termination resistance R_s. Pole-pair becomes real for $R_s = Z_0$.

C. Convergence of First Pole Pair

Moments, obtained as coefficients of a Maclaurin series expansion of an RLC distributed interconnect, can be used to *theoretically* show "convergence" of a single pole-pair model to the exact first pole-pair of a system as higher and higher moments of the system are used to derive the two-pole model. For a *q-th order* system defined in the form of poles and residues as:

$$G(s) = \left(\sum_{i=1}^{q} \frac{k_i}{s+p_i}\right), \tag{10}$$

the n-th moment, m_n, is defined as:

$$m_n = -\left(\frac{k_1}{p_1^n} + \frac{k_2}{p_2^n} + \ldots\ldots\ldots + \frac{k_q}{p_q^n}\right) \tag{11}$$

Since the complex poles of the system exist as complex conjugate pairs, the poles can be ordered as:

$$0 < |p_1| = |p_2| < |p_3| = |p_4| < |p_5| = |p_6| \ldots\ldots \tag{12}$$

In polar form, we rewrite (11) as:

$$\begin{aligned} k_i &= r_i e^{j\theta_i} \qquad \bar{k}_i = r_i e^{-j\theta_i} \\ p_i &= \gamma_i e^{j\rho_i} \qquad \bar{p}_i = \gamma_i e^{-j\rho_i} \end{aligned} \tag{13}$$

$$m_n = -\frac{r_1 e^{j\theta_1}}{\left(\gamma_1 e^{j\rho_1}\right)^n}\left(1 + \frac{e^{-j2\theta_1}}{\left(e^{-j2\rho_1}\right)^n} + \left(\frac{\gamma_1}{\gamma_2}\right)^n \frac{r_2 e^{j(\theta_2-\theta_1)}}{r_1\left(e^{j(\rho_2-\rho_1)}\right)^n} + \ldots\right) \tag{14}$$

Since $\gamma_1/\gamma_2 < 1$ (from (12)) and $r_2/r_1 < 1$ [13], all terms beyond the first two in (14) become arbitrarily small as $n \to \infty$. Thus, considering moments m_n, m_{n+1}, m_{n+2} and m_{n+3}, by the Caley-Hamilton theorem [1], the first two poles of the system can be exactly determined, and a two-pole model converges to system's first pole-pair.

As higher order moments are considered for determining the optimal termination for a transmission line system, the time-domain output-response of the interconnect is increasingly dominated by the first pole pair. Thus, the greatest zero-crossing of a high-order moment, say m_k, corresponds to a response due to the first-pole pair that contains minimal or no ringing. This corresponds to a real pole-pair [15], and hence follows our argument for optimal termination in Section II B. The convergence of the highest moment root for a loss-less line to Z_0 (as the moment-order m_k is increased) and the movement of the first pole-pair corroborates this argument.

D. Approximating Signal Overshoot/Undershoot

The Laplace domain treatment of a distributed model of a transmission line in Section B led to a criterion for optimal termination based on the first "exact" pole-pair of the infinite order system. For a loss-less line it can be shown using Fourier analysis that between the undamped and the critically damped conditions, the first pole-pair and its corresponding residues provide bounds on the signal amplitude. When the line is unterminated, the output response oscillates between V and 2V, as shown in Fig.5, where V is the amplitude of the step-input. For this output, the Fourier series is:

$$v(t) = V + \frac{4V}{\pi}\left(-cos\left(\frac{\pi}{2\sqrt{lcd^2}}\right)t + \frac{1}{3}cos\left(\frac{3\pi}{2\sqrt{lcd^2}}\right)t - \frac{1}{5}cos\left(\frac{5\pi}{2\sqrt{lcd^2}}\right)t + \dots\right) \quad (15)$$

Since $4/\pi > 1$, it is clear that the amplitude of the first harmonic, which corresponds to the first pole-pair of the lossless transmission line system, bounds its output response.

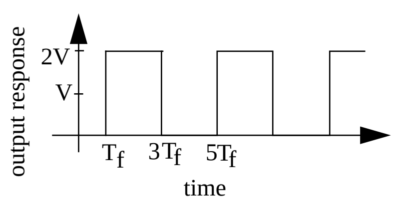

FIGURE 5: Output response of an unterminated, open-circuited lossless line. T_f = time of flight.

The exact first pole-pair for an RLC system can be obtained using moment shifting [1] which effectively involves exciting the system with a low frequency input, $1/s^p$, if moments m_p, m_{p+1}, m_{p+2}, m_{p+3} are used to converge to the actual pole-pair. From linear system theory, it is well known that low frequency signals "excite" the low frequency poles of a system more than the poles high in the *s*-plane. Thus the residues corresponding to the first pole pair increase in magnitude as the low frequency content of the input signal increases. For an input excitation given by $1/s^p$, as $p \to \infty$, the first pole-pair is said to have converged to the exact pole-pair, and the time-domain response due to this pole-pair bounds that due to the second order *AWE* approximation.

For optimal termination obtained using *moment-3*, the first pole-pair can be obtained by solving the quadratic equation:

$$a_2 s^2 + a_1 s + 1 = 0 \quad (16)$$

where,

$$a_2 = -\frac{m_4}{m_2}, \quad a_1 = -\frac{m_5}{m_4}, \quad m_3 = 0 \quad (17)$$

The exact first pole-pair for an RLC transmission line response provides a pessimistic approximation for the signal overshoot/undershoot as compared with a second order moment matching approximation (*AWE* [14]), as shown in Fig.6. However, a "tight" bound requires consideration of the second pole pair when the first pole-pair approaches the real-axis at optimal termination [15].

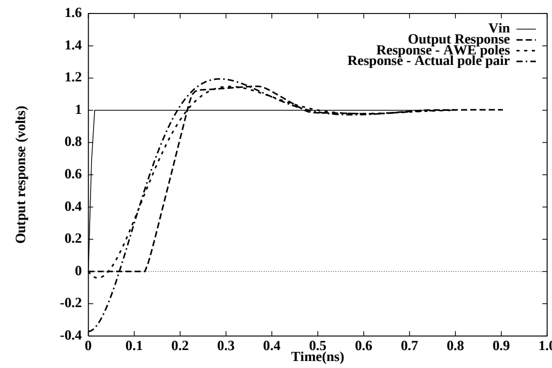

FIGURE 6: Time domain response of a low-loss transmission line with a capacitive load of 0.1pF. r=0.13Ω /mm, l=0.2nH/mm, c=0.15pF/mm. *length*=25mm, Z_0=36.51, T_f=0.14ns.

III. Variants Affecting Optimal Termination

A. Optimal Termination as Function of Capacitive Load

A capacitive load at the end of a transmission line due to the input capacitance of a receiver drastically affects the characteristics of the signal on the line. The charging and discharging of C_L has a time-constant of Z_0C_L for a lossless line. This time-constant should be small compared to the time-of-flight of the transmission line for the capacitive effects of the load to be negligible [2]. As shown in Fig.7, this relation can be easily analyzed with the metric from this paper.

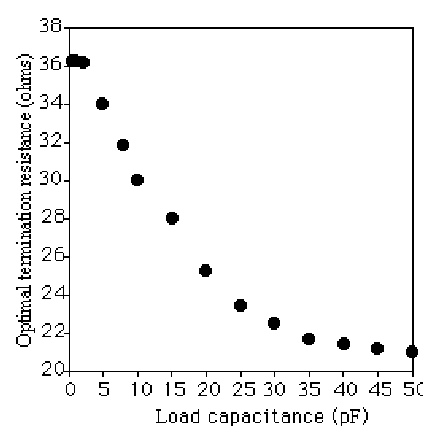

FIGURE 7: Optimal termination resistance (R_{opt}) as function of capacitive load (C_L). C_{line}=3.75pF. For C_L<C_{line}, R_{opt}=Z_0.

B. Optimal Termination as Function of Line-loss

Interconnections for multi-chip modules and thin-film wirings have significant resistance and cannot be approximated using a loss-less transmission line analysis. From empirical observation it is known that when $R_{line} < Z_0/2$ the transmission line behavior of the interconnect is dominant, and when $R_{line} > 5Z_0$, the line can be considered to be a distributed RC line[2]. For long lines and fast rising signals, inductive effects in interconnects are considerable, and appropriate termination for lossy lines with $R_{line} < 5Z_0$ can be obtained using the technique described in this paper.

The optimal termination value for a lossy transmission line has a monotonically decreasing dependence on the value of the line resistance, R_{line}, as shown in Fig.8. The moment representation results in a zero value of optimal termination for $R_{line} \approx \pi Z_0$, which can be shown to be the condition for self-termination for an unloaded lossy line [15]. Fig.9 shows the time-domain response for a lossy line with optimal termination obtained using m_3 versus a termination value equal to Z_0.

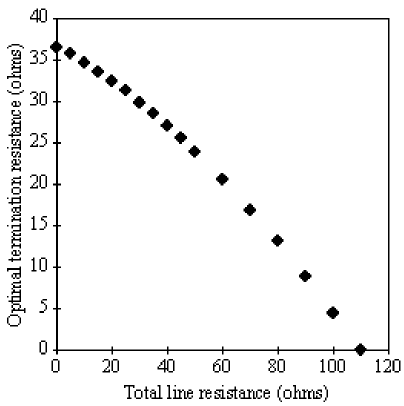

FIGURE 8: Optimal termination resistance, R_s, plotted as a function of the resistance of the transmission line, R_{line}. $Z_0 = 36.5\Omega$. Self termination condition for the line is $R_{line} = 109.5\Omega$.

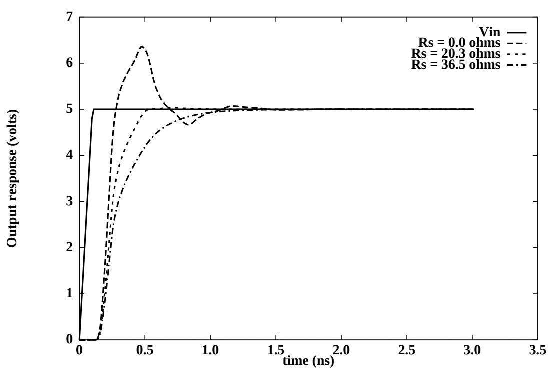

FIGURE 9: Time-domain response for lossy transmission line ($Z_0 = 36.51\Omega$, $R_{line} = 40\Omega$) with termination value $R_s = 0.0\Omega$, 20.3Ω (termination value from m_3) and 36.5Ω.

C. Optimal Termination as Function of Rise-time of Driver

Transmission line phenomenon becomes significant for fast-rising signals where the smallest wavelength component of the signal is comparable to the interconnect length. In terms of rise-time, t_r, this condition is empirically known to be equivalent to $t_r \approx 2.2T_f$, where T_f is the time-of-flight of the signal on the line [2].

Rise-time, t_r, of the input signal is determined by the rate at which the driver for the line is turned on. This depends on the driver characteristics and the ratio of the driver source resistance to the line impedance. Thus, owing to practical considerations, t_r is always > 0, and can be modeled as a saturated ramp.

As shown in Fig.10, the optimal terminal resistance, R_{opt}, for a transmission line reduces with increase in the rise-time, t_r, of the driving signal. The sensitivity of R_{opt} to variations in t_r is very high for values of $t_r \approx 2T_f$ which is also the condition when transmission line effects cease to be dominant, and the line can be considered to be a lumped circuit. Moment domain symbolic analysis gives an optimal termination value of zero for $t_r > 2T_f$ since the RC effects begin to dominate, and minimization of RC delay follows the usual strategy for delay reduction as in on-chip interconnects [2].

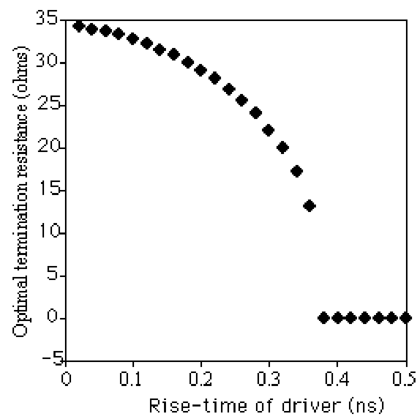

FIGURE 10: Optimal termination resistance, R_{opt}, as function of input signal rise-time, t_r. $T_f = 0.14$ns. R_{opt} obtained from moment analysis (using m_3) equals 0 for $t_r \approx 2T_f$.

The time-domain response for a lossy transmission line with a rise-time of 0.35ns, open-circuited at the end with a time-of-flight of 0.14ns, is shown in Fig.11. Considering the resistance of the line and rise-time, the moment analysis using *moment-3* yields a zero value of termination.

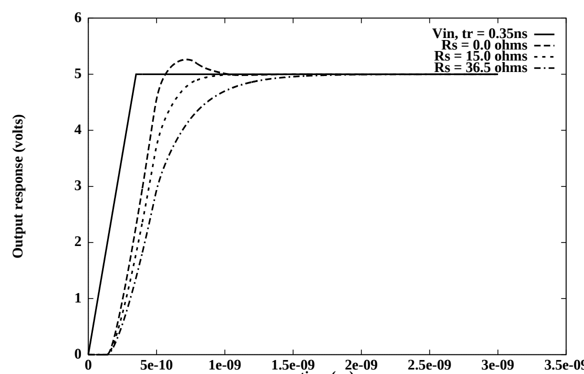

FIGURE 11: Output response of lossy line ($R_{line} = 60.0\Omega$, $Z_0 = 36.51\Omega$) for input signal rise-time, $t_r = 0.35$ns. $R_s = 0.0\Omega$ (optimal series termination resistance obtained from moment-3), $R_s = 15.0\Omega$ (from moment-6) and $R_s = 36.5\Omega$ (Z_0).

D. AC Termination

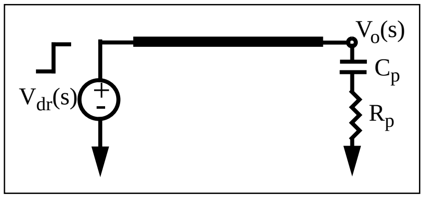

FIGURE 12: Low loss line terminated in parallel with R_pC_p combination. r=0.13Ω /mm, l=0.2nH/mm, c=0.15pF/mm, *length*=25mm.

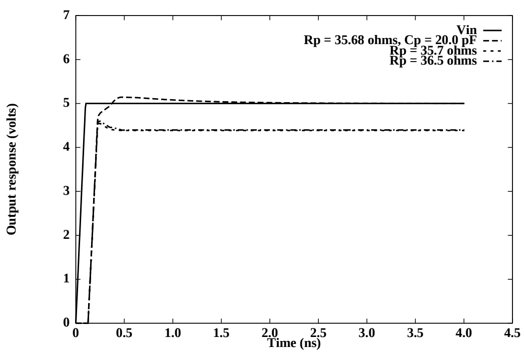

FIGURE 13: Output response of a low-loss line with parallel AC termination in contrast with parallel resistance termination. OTTER yields values of R_p=35.68Ω and C_p=20.0pF using the symbolic analysis approach described. R_{line}=5.0Ω .

Parallel termination of transmission lines results in unwanted dc power dissipation, and this can sometimes be eliminated using an RC parallel termination as shown in Fig.12. For a pure lossless line, the signal travels down the line unattenuated and in order that there are no reflections, the value of R_p should equal Z_0. C_p should be such that ringing at the output is minimal. Since the input signal in a lossy line gets attenuated along the line, obtaining the optimal values of R_p and C_p is a two-step process:

- Obtain the value of the *optimal parallel termination resistance* from moment analysis.
- Use the transfer function in (18) to obtain optimal value of the parallel capacitor, C_p, using moment analysis.

$$G(s) = \left(\frac{sC_p}{sR_pC_p + 1}\right) Z_0 \sinh(\Gamma d) + \cosh(\Gamma d) \qquad (18)$$

A result is shown plotted in Fig.13 for a low-loss transmission line with *AC* termination versus a parallel resistive termination with an optimal R.

IV. Conclusions

This paper presents a comprehensive and generalized approach for optimal termination of transmission lines using moment-based symbolic analysis. A new metric has been defined for terminating transmission lines based on the interpretation of moments of a signal as the time-weighted integral of the waveform in the time-domain. No time-domain simulations are required to obtain this optimal value of termination. Furthermore, the general framework of this methodology is shown to be conducive for incorporating rise-time effects of the input signal and loading conditions at the end of a line.

Towards the objective of providing a reliable metric that can efficiently evaluate the trade-off between the overshoot/undershoot of a signal and delay, future work will involve determining accurate bounds on the signal. Estimating the sensitivity of the signal to parameter changes owing to process variations is also an important consideration. Finally, calculating the moments as a symbolic function of various R's will be considered for more complex interconnect circuits.

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