Optimal Antenna Placement Using a New Multi-Objective CHC Algorithm

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ABSTRACT

Radio network design (RND) is a fundamental problem in cellular networks for telecommunications. In these networks, the terrain must be covered by a set of base stations (or antennae), each of which defines a covered area called cell. The problem may be reduced to figure out the optimal placement of antennae out of a list of candidate sites trying to satisfy two objectives: to maximize the area covered by the radio signal and to reduce the number of used antennae. Consequently, RND is a bi-objective optimization problem. Previous works have solved the problem by using single-objective techniques which combine the values of both objectives. The used techniques have allowed to find optimal solutions according to the defined objective, thus yielding a unique solution instead of the set of Pareto optimal solutions. In this paper, we solve the RND problem using a multi-objective version of the algorithm CHC, which is the metaheuristic having reported the best results when solving the single-objective formulation of RND. This new algorithm, called MOCHC, is compared against a binary-coded NSGA-II algorithm and also against the provided results in the literature. Our experiments indicate that MOCHC outperfoms NSGA-II and, more importantly, it is more efficient finding the optimal solutions than single-objectives techniques.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*; G.1.6 [**Numerical Analysis**]: Optimization—*Global optimization*

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General Terms

Algorithms, Experimentation, Performance

Keywords

Multi-objective optimization, radio network design, CHC

1. INTRODUCTION

Deciding base station (or antenna) locations is a basic task in the design and deployment of cellular networks as those used in mobile telecommunication systems. The placement of an antenna determines the area it covers (also called cell), therefore taking the decision of where to deploy all the antennae of a network influences the covered area, the coverage degree (the number of antennae covering a given region), and the number of required antennae. Consequently, an appropriate decision making affects directly the quality of the provided service and the system cost. The problem of finding optimal antenna location is known as *Radio Network Design* or RND.

The use of cellular networks is continuously increasing in the telecommunications sector: mobile telephony (with successive generations), wireless networks and, more recently, sensor networks. The number and complexity of the incoming networks are also growing, from a few tens of antennae in the first generations of mobile telephony networks to hundreds and thousands in most modern systems (e.g., sensor networks). As a consequence, it is necessary to make use of accurate and fast techniques to assist the design of these networks, allowing to cope with complex scenarios which would be unmanageable otherwise.

Among these techniques, metaheuristics [3] appear as popular tools able to provide satisfactory results to complex optimization problems in a reasonable amount of time. Thus, genetic algorithms [1, 4], simulated annealing [2], and differential evolution [12] have been applied to solve the RND problem. The best results have been reported using CHC (*Cross generational elitist selection, Heterogeneous recombination, and Cataclysmic mutation*) [2], a kind of genetic algorithm.

RND may be formulated as a bi-objective optimization problem, in which two contradictory goals have to be optimized: to maximize the area covered by the radio signal and to reduce the number of used antennae. However, previous works have solved the problem using a single-objective formulation by combining the values of both objectives; as a consequence, a unique solution is provided instead of the set of Pareto optimal solutions. This way, the decision maker is not provided with a number of solutions allowing to choose the most adequate one out of them. In this paper we deal with the bi-objective formulation of the RND and, instead of just using well-known multi-objective metaheuristics (e.g. NSGA-II [6] or SPEA2 [14]), we design a new multi-objective variant of CHC algorithm called MOCHC. The motivation driving us is to study whether MOCHC reports the best results when solving the RND in the multi-objective domain in the same way as CHC obtains the best results in the single-objective formulation of the problem. We are also interested in comparing both versions of CHC, to determine if reformulating the problem from single to bi-objective introduces any benefits. The contributions of our work can be summarized in the following:

- We propose a new multi-objective metaheuristic called MOCHC, and we apply it to the RND problem.
- To assess the performance of MOCHC in the multi-objective domain, we compare it against NSGA-II.
- We compare the results obtained by MOCHC with those previously published in the literature related to the single-objective formulation of RND.

The paper is structured as follows. In the next section, we detail the RND problem and its single-objective formulation. Section 3 describes the multi-objective formulation of the problem. The proposed multi-objective CHC algorithm is briefly described in Section 4. The results of the experiments are analyzed in Section 5. Finally, some conclusions and future lines of research are discussed in the last section.

2. THE CLASSIC RND PROBLEM

Let us assume that we intend to provide radio coverage to an urban scenario. The covered area is a key factor; ideally, we want to cover all the surface, but the available resources are limited and have a cost, so we would like to place as few antennae as possible. The telecommunication company carries out a preliminary study to get a set of suitable antenna locations: roofs of high buildings and places with good visibility (with no nearby obstacles) and far from restricted zones (e.g., hospitals or police stations). In our example, let us suppose that there are one hundred of available sites, which are distributed more or less uniformly on the area to cover. The RND problem considered in this work consists of selecting a subset of locations among the set of available locations to place antennae on them. The goal is twofold, since on the one hand we want to maximize the covered area and, on the other hand, the number of antennae should be minimized. Both objectives are clearly in conflict.

To manage the problem information, Calégari et *al.* [4] used a terrain discretization according to a square grid model to represent the area to cover. This model was used in latter works, and it is the one we use here. The grid used to model the terrain has a square shape and it contains 287×287 squares, which we call *target points*, each of them representing an atomic portion of the surface which is either absolutely covered or not. Figure 1 illustrates a simplified model of terrain discretization using a 10×7 grid. The considered problem is similar to the unicost set covering problem (USCP), which is known to be NP-hard.

The set of *available location sites* (ALS) to deploy the antennae is represented by means of a list of coordinates, indicating the squares of the grid containing the sites. A network design consists of a subset of coordinates of the ALS. Any valid subset is a *solution* to the RND problem. The solutions are coded using binary strings having a length equal to the cardinality of the ALS. Each bit in the string represents a site of the ALS. A '1' indicates the placement of an antenna in the corresponding site, while a '0' indicates an empty site.

The solution space of the problem is the set of possible values that the solution string can have, and its size, which depends on the number of available locations, is $2^{|ALS|}$. We define the size of an instance of the problem as the size of the ALS.

When an antenna is deployed in a site, it offers full coverage to a set of target points centered around the antenna (*cell*). In this work we consider the same model used in previous works: all the antennae have associated the same cell shape, independently of their location, and the shape is a square region of 41×41 target points. The shape of the cell was chosen in such a way that full coverage (100% of the area) could be achieved using 49 antennae arranged in a 7×7 grid ($7 \times 41 = 287$). See Figure 2 for the graphical representation of a partial coverage (left) and a full coverage (right) for the RND problem. This problem admits numerous extensions, like defining different types of antennae, finding their 3D orientation, or estimating signal propagation according to physical simulation of radio waves, what really transform the problem in a hard real world optimization problem [11, 13].

In the design of a cellular network, it is generally preferable to increment the covered area instead of using fewer antennae. In [4] a function measuring the quality of the solutions (*fitness function*) was defined (Equation (1)). This fitness function, which has to be maximized, combines both factors, coverage and number of antennae, into a unique value, so that single-objective techniques can be applied:

$$f(\vec{x}) = \frac{Coverage(\vec{x})^{\alpha}}{Number\ of\ antennae(\vec{x})} \tag{1}$$

where $Coverage$ is the ratio (in percentage) between the covered target points and the total number of target points (287×287 in our problem instance). In this formulation, parameter α allows to adjust the ratio of importance between coverage and number of antennae. In Calégari et al. [4] a value $\alpha = 2$ is suggested.

3. MULTI-OBJECTIVE FORMULATION

The RND problem previously described follows a single-objective formulation: there are two parameters to optimize, and they are combined in a unique fitness value. A multi-objective approach raises naturally by considering each objective as a separate goal. This way, we can define two functions to optimize, which are detailed in Equations (2) and (3). These two functions are to be minimized.

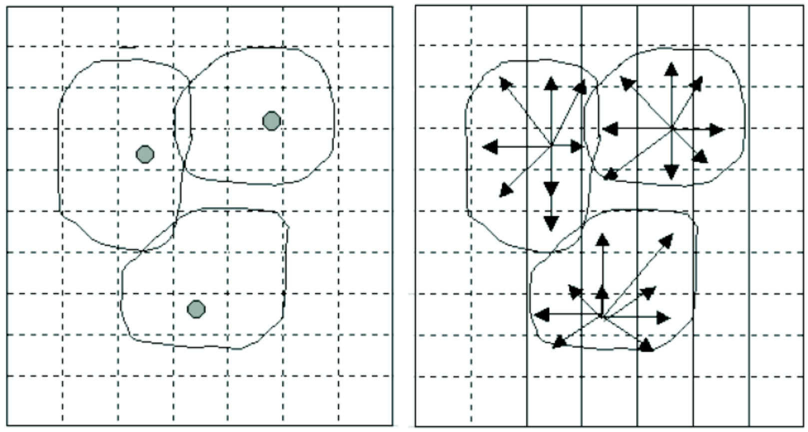

Figure 1: True antenna coverage (*left*). Discretized coverage using a grid model (*right*)

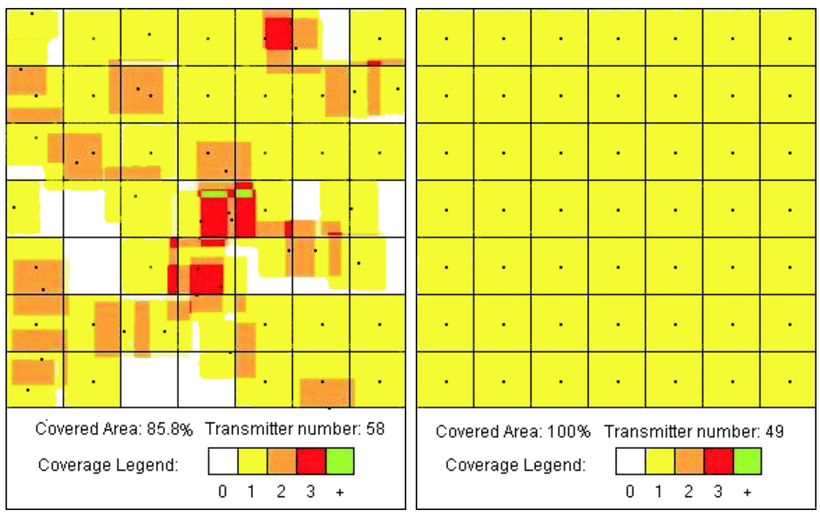

Figure 2: Graphical representation of a partial covered solution (*left*) and a full covered solution (*right*)

$$f_1(\vec{x}) = \mathit{Number\ of\ antennae}(\vec{x}) \quad (2)$$

$$f_2(\vec{x}) = 100 - \mathit{Coverage}(\vec{x}) \quad (3)$$

Switching from a single-objective to a multi-objective perspective has a strong influence in the search process. Given that we do not intend to find a unique optimal solution but the Pareto optimal set, the search diversification must be increased. This is interesting when employing metaheuristics, but it can be harmful if it produces an imbalance in the trade off diversification/intensification. The consequences can be serious if there exist subsets of solutions which are non-dominated but they are not very useful to the decision maker. In the case of the RND problem, solutions having few antennae are not interesting because the resulting coverage is poor, and they are implicitly penalized in the single-objective formulation (see Equation (1)) using a value of $\alpha > 1$, which makes the coverage to be the main objective instead of the number of antennae. In the proposed multi-objective formulation, we restrict the search including constraints indicating the range of desired values by defining two side constraints: a maximum of 60 antennae must be used (Equation (4)), and the solutions must achieve at least a 90% of coverage (Equation (5)). This way, we avoid wasting time exploring unpromising regions of the search space.

$$c_1(\vec{x}) = \begin{cases} f_1(\vec{x}) - 60 & (f_1(\vec{x}) > 60) \\ 0 & (f_1(\vec{x}) \leq 60) \end{cases} \quad (4)$$

$$c_2(\vec{x}) = \begin{cases} f_2(\vec{x}) - 10 & (f_2(\vec{x}) > 10) \\ 0 & (f_2(\vec{x}) \leq 10) \end{cases} \quad (5)$$

These constraints must be taken into account by the metaheuristic used to solve the problem. A foreseeable consequence of these constraints is that the feasible region of the search space is reduced, with a positive effect on the results.

The Pareto front of the RND problem studied in this work can be obtained analytically. Let us consider the following proposition:

PROPOSITION 1. *The Pareto front of the instances we use in this work is composed of points*

$$(x, y) = \left(n,\ 100 \cdot \left(1 - \frac{n}{49}\right)\right) \quad (6)$$

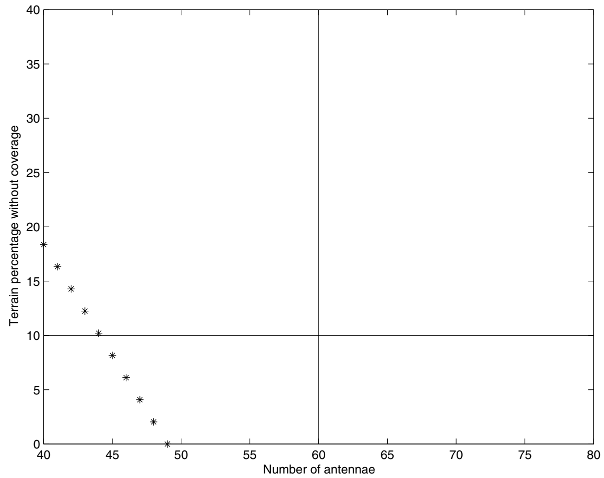

Figure 3: Pareto front of the considered RND instance problem

where $n \in [0, 49]$ is the number of antennae (first objective) and the second component is the percentage of uncovered terrain (second objective).

Proof. For any given number n of antennae, the maximum coverage that can be achieved ideally by placing those antennae is n times the coverage of a single antenna. This happens if all the cells associated to these antennae are completely included inside the terrain area, and no overlap exists between any two of those cells. In our instances of the problem, the coverage of a single antenna consist of 41×41 target points, i.e., $1/49$ (or 2.04%) of the total terrain area (287×287 points). Complete coverage can be achieved optimally using 49 antennae by placing them in the 49 predefined locations (included in the ALS) such that all the coverage regions are included inside the terrain, and no overlap is produced. By definition, if we place antennae in any subset of locations selected from those 49, all the coverage regions will still be included in the terrain and no overlap will be produced. Therefore, the Pareto front contains the solutions having (n antennae, $100 \cdot n/49$ coverage). Or, if we express it in terms of *uncoverage* (terrain without coverage) rather than coverage, we get Equation (6). □

The resulting Pareto front is shown in Figure 3. The point consisting of 49 antennae and 0% of terrain without coverage is the optimal solution produced by the single-objective algorithms, and it achieves a fitness value of 204.082 using Equation 1. The figure also includes the lines representing the side constraints (coverage of 90%, a maximum of 60 antennae).

4. MOCHC: A MULTI-OBJECTIVE CHC ALGORITHM

The algorithm CHC was proposed by Eshelman in 1991 [8]. It is an evolutionary algorithm which has not been widely used in the literature, although it has reported very good results [2, 5]. CHC works with a population of solutions, and follows a typical iterative behavior, producing in every step new solutions which are incorporated into the population replacing existing ones. The pseudocode of CHC is shown in Algorithm 1.

Algorithm 1 CHC

$t \leftarrow 0$
Initialize(Pa, $convergence\_count$, k) // Pa: population
while not ending_condition(t, Pa) **do**
  $Parents \leftarrow$ Selection_parents(Pa, $convergence\_count$)
  $Offspring \leftarrow$ HUX($Parents$)
  Evaluate($Offspring$)
  $Pn \leftarrow$ Elitist_selection($Offspring$, Pa) // Pn: new pop.
  if not modified(Pa,Pn) **then**
    $convergence\_count \leftarrow convergence\_count - 1$
    if $convergence\_count \leq -k$ **then**
      $Pn \leftarrow$ Restart(Pa)
      Initialize($convergence\_count$)
    end if
  end if
  $t \leftarrow t + 1$
  $Pa \leftarrow Pn$
end while

4.1 Classic CHC Algorithm

CHC was designed to work with binary-coded solutions. The algorithm works with a population of individuals (Pa in Algorithm 1). In every step, a new set of solutions (Pn) is produced by selecting pairs of solutions from the population (the parents) and recombining them. This selection is made in such a way that individuals which are too similar cannot mate each other.

CHC can be viewed as a kind of genetic algorithm which does not apply mutation to produce new solutions, but only a recombination mechanism called HUX. This procedure copies first the common information of both parents into both offspring, then copies half of the diverging information from each parent to each of the offspring, so that the Hamming distance among offspring and parents is the maximum. This is done in order to preserve the maximum amount of diversity in the population, as no new diversity is introduced during the iteration since there is no mutation operator. The next population is selected according to an elitist criterion, based on picking the best individuals among the old population and the new set of solutions Pn.

The absence of mutation and the elitist selection criterion make the population to converge. To delay this process, CHC applies an incest prevention mechanism: parent selection is carried out choosing individuals randomly, but the recombination is only performed if the parents are not very similar, i.e., if the Hamming distance between them is greater than a given threshold value ($convergence\_count$ in Algorithm 1). As the execution of the algorithm progresses, the population becomes more homogeneous and the number of solutions fulfilling the incest condition augments; as a consequence, the incest threshold has to be progressively decreased. Whenever an iteration has finished and the population remains unchanged, the $convergence\_count$ is decreased in one unit.

When the incest threshold gets to 0 (the minimum distance to combine two solutions is 0), after k iterations with no new individuals in the population it is assumed that the population has converged and the algorithm is stalled. A mechanism is then used to generate new diversity in the population: a *restart*. When restarting, the best solutions remain unchanged, and the rest are significantly (*cataclysmically*) modified using a bit-flip mutation with very high probability (in [8] a probability of 35% is suggested).

Table 1: Parameter settings of MOCHC

Population size	100
Crossover	*HUX*
Cataclysmic mutation	*Bit flip, Pm = 35%*
Preserved population	5%
Initial convergence count	25% *of the problem instance size*
Convergence value k	1
Parent selection	*Random with incest threshold*
New generation selection	*Elitist selection*
Ordering criterion	*Ranking and crowding distance*

Table 2: Parameter settings of NSGA-II

Population size	100
Mutation	*Bit flip, Pm* = $1/L$ (L = string length)
Crossover	*SPX, Pc* = 0.95
Parent selection	*Binary tournament*
New generation selection	*Elitist selection*
Ordering criterion	*Ranking and crowding distance*

4.2 Multi-Objective CHC

The multi-objective version of the CHC algorithm that we propose, called MOCHC, is based on the algorithm previously described. The most important modification concerns the elitist selection mechanism; instead of ordering the solutions according to a single scalar value, in MOCHC the solutions are ordered using a *ranking* and a *crowding distance* estimator similar to those used in NSGA-II [6]. Thus, the non-dominated solutions in the population are selected and removed from it, constituting the subset of rank 1. This process is iteratively repeated on the remaining individuals to obtain the subsets of rank 2, 3, and so on; the stopping condition is that the sum of the individuals in the obtained subsets is equal or greater than the population size. In the second case, the crowding distance estimator is applied to the solutions in the last subset to choose among them those having the biggest distance values.

When the population is stalled the restarting mechanism is applied by using a high disruptive mutation to all the solutions except to the best ones. In the single-objective CHC the solutions having best fitness were selected; in the multi-objective version this would mean to preserve the non-dominated solutions. Our approach to the restart process is to keep a percentage of the population which is selected after ordering it by ranking and crowding distance. The number of preserved solutions is a parameter of the algorithm; after performing a number of preliminary experiments, we choose a value of 5%. We include in Table 1 the parameter settings of MOCHC in this work.

To cope with the constraints presented in our formulation of the RND problem, we apply the same mechanism used in NSGA-II: in this algorithm, when two solutions are to be compared, the one having less overall constraint violation is preferred; otherwise, a Pareto dominance test is applied to choose the best solution.

5. EXPERIMENTS

The RND instances solved in this work are the same used previously in [1] and [2]. All of them are RND instances using antennae providing a square coverage, as it was described in Section 2. The smaller instance has 149 available locations, and additional random transmitter locations are added to get four new instances with 199, 249, 299, and 349 locations. In all the instances the optimum solution is the same as for the canonical 149 problem, and the random transmitters are added to deceive the solvers.

For each experiment, 50 independent runs have been executed to ensure statistical confidence. We include the mean and standard deviation in the tables (Tables 3 and 4), and the best result has a grey color background. Since we are dealing with stochastic algorithms, the following statistical analysis has been performed in all this work [7]. Firstly, a Kolmogorov-Smirnov test is performed in order to check whether the values of the results follow a normal (gaussian) distribution or not. If so, an ANOVA test is done, otherwise we perform a Kruskal-Wallis test. We always consider in this work a confidence level of 95% (p-value under 0.05) in the statistical tests, which means that the differences in the results cannot have occurred by chance with a probability of 95%. Successful tests are marked with "+" symbols in the last column in the tables; conversely, "−" means that no statistical confidence was found (p-value > 0.05).

The stopping condition is to find the optimal solution described in [1] (100% coverage with 49 antennae) or to perform one million function evaluations; this way, we can make a comparison among the results obtained by MOCHC and those presented in [1] and [2].

All the instances include the coordinates of the 49 locations allowing a coverage of 100% with the minimum number of antennae. As it was previously commented, these locations provide the optimal solution found in previous works, and they allow us to decide that the problem has been solved. The fitness function defined in Equation (1) applied to these solutions produces a value of 204.082.

To determine how competitive the proposed multi-objective CHC algorithm is, we have made two types of studies. On the one hand, we compare MOCHC against NSGA-II [6], a state-of-the-art multi-objective optimization algorithm. We have used a binary-coded NSGA-II with the parameter settings shown in Table 2. The mutation and crossover operators are, respectively, bit flip and SPX (single point crossover). The mutation probability Pm is $1/L$, where L is the chromosome length (the length of the ALS in the case of RND), and the crossover probability Pc is 0.95. On the other hand, we compare the results obtained by MOCHC against those obtained by single-objective metaheuristics in [1]: classic CHC, simulated annealing (SA), and dssGA8 (an eight island distributed genetic algorithm).

5.1 Metrics

To compare MOCHC and NSGA-II we use the hypervolume metric. This metric calculates the volume (in the objective space) covered by members of a non-dominated set of solutions Q (the region enclosed into the discontinuous line in Fig. 4, $Q = \{A, B, C\}$) for problems where all objectives are to be minimized [15].

Mathematically, for each solution $i \in Q$, a hypercube v_i is constructed with a reference point W and the solution i as the diagonal corners of the hypercube. The reference point can simply be found by constructing a vector of worst objective function values. Thereafter, a union of all hypercubes is found and its hypervolume (HV) is calculated:

$$HV = volume\left(\bigcup_{i=1}^{|Q|} v_i\right). \quad (7)$$

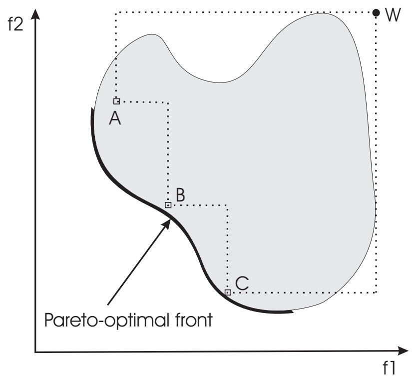

Figure 4: The hypervolume enclosed by the non-dominated solutions.

Algorithms with larger values of HV are desirable. Since this metric is not free from arbitrary scaling of objectives, we have evaluated the metric by using normalized objective function values.

The second metric we use is the computational effort (CE), which will allow us to compare MOCHC (and NSGA-II) against single-objective metaheuristics. This metric is defined as the number of solutions that have to be evaluated to solve the problem (to find the optimal solution). Lower values of CE are desirable: when comparing different techniques, the one having the lowest CE value is the most efficient one. Although were are dealing with two different types of techniques, single and multi-objective, we can compare them using CE because all of them use the same stopping condition (to find the optimal configuration defined in [1]).

5.2 Results

We analyze first the results obtained with the CE metric by all the algorithms, which are included in Table 3. At a first glance, it can be observed that the multi-objective algorithms are more efficient than the single-objective ones. MOCHC achieves a CE reduction compared against the classic CHC between 40% (149-size instance) and 59% (349-size instance). If we compare MOCHC against SA, the improvements are between 75% and 80%. If we take into account NSGA-II, this algorithm performs worse than the classic CHC in the 149-size instance (24% worse), but in the rest of the instances the improvements oscillate between 5% (199-size instance) and 24% (349-size instance). The comparison against SA is clearly favorable (improvements between 57% and 69%). In general, it can be observed that the larger the instance size (and, consequently, the problem complexity) the better the results obtained by the multi-objective metaheuristics.

We proceed now to analyze the two multi-objective metaheuristics. Considering the CE metric, MOCHC achieves better results than NSGA-II with statistical confidence: its CE values are better than the ones of NSGA-II in a range between 43% and 52%. The HV metric (see Table 4) indicates also that the non-dominated solution sets obtained by MOCHC produce statistically better Pareto fronts than NSGA-II.

In Figure 5 we include an execution trace of the two multi-objective metaheuristics. In both cases, we represent the

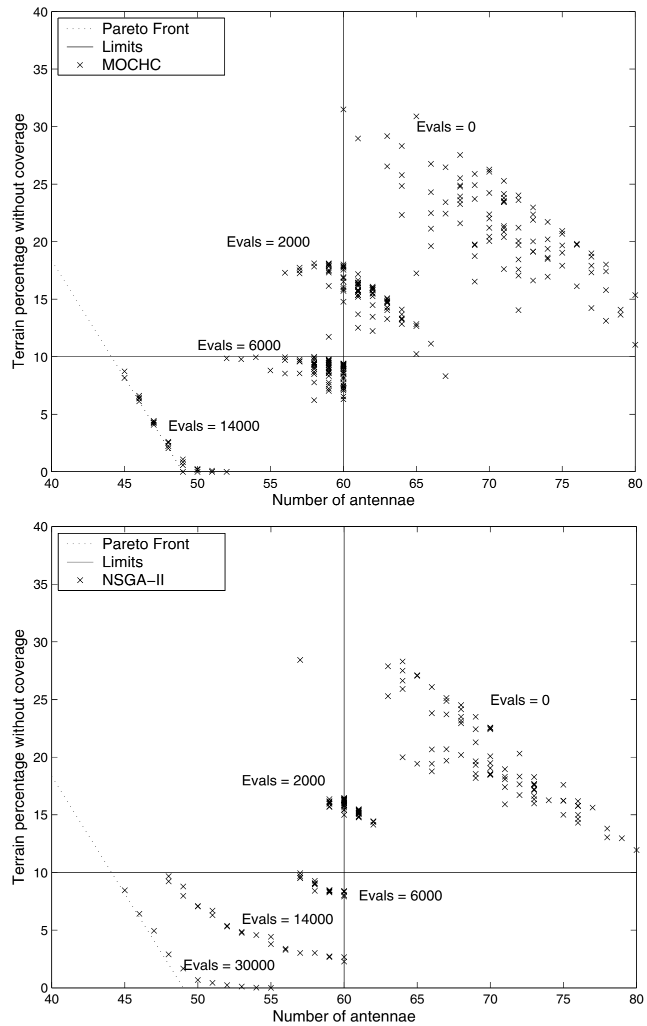

Figure 5: *Top:* Execution tracking of MOCHC. *Bottom:* Execution tracking of NSGA-II

Table 3: Computation effort (CE) metric values (number of evaluations)

Instance size	Single-objective techniques			Multi-objective techniques		
	SA [2]	CHC [2]	dssGA8 [1]	MOCHC	NSGA-II	
149	$8,676e{+}4$ $_{5,12e+4}$	$3,032e{+}4$ $_{2,84e+4}$	$7,859e{+}5$	$1,814e{+}4$ $_{6,50e+3}$	$3,745e{+}4$ $_{8,17e+3}$	+
199	$1,970e{+}5$ $_{8,54e+4}$	$7,862e{+}4$ $_{5,95e+3}$	$1,467e{+}6$	$3,998e{+}4$ $_{1,07e+5}$	$7,479e{+}4$ $_{1,55e+4}$	+
249	$3,341e{+}5$ $_{1,13e+5}$	$1,486e{+}5$ $_{9,67e+4}$	$2,481e{+}6$	$7,723e{+}4$ $_{2,29e+4}$	$1,418e{+}5$ $_{5,95e+4}$	+
299	$6,380e{+}5$ $_{1,80e+5}$	$2,289e{+}5$ $_{1,85e+5}$	$2,998e{+}6$	$1,136e{+}5$ $_{3,24e+4}$	$1,987e{+}5$ $_{4,46e+4}$	+
349	$8,108e{+}5$ $_{2,75e+5}$	$3,802e{+}5$ $_{2,03e+5}$	$4,710e{+}6$	$1,574e{+}5$ $_{4,68e+4}$	$2,871e{+}5$ $_{8,15e+4}$	+

Table 4: Hypervolume (HV) metric values

Instance Size	MOCHC	NSGA-II	
149	$4,672e{-}1$ $_{1,90e-2}$	$4,605e{-}1$ $_{2,16e-2}$	+
199	$4,726e{-}1$ $_{1,00e-2}$	$4,669e{-}1$ $_{2,11e-2}$	+
249	$4,699e{-}1$ $_{1,54e-2}$	$4,701e{-}1$ $_{1,17e-2}$	−
299	$4,730e{-}1$ $_{1,00e-2}$	$4,726e{-}1$ $_{6,20e-3}$	+
349	$4,731e{-}1$ $_{8,40e-3}$	$4,714e{-}1$ $_{9,60e-3}$	+

populations in the objective space in different phases of the execution of the algorithms, corresponding to the initial population and the solutions obtained after 2000, 6000, 14000, and 30000 (only NSGA-II) evaluations.

We can observe that the behavior of the two algorithms is similar: first, the populations concentrate inside the non-penalized search region and then, once inside it, they expand to explore the search space, converging progressively to the Pareto front. As it can be seen in Figure 5, MOCHC converges faster than NSGA-II, approaching the Pareto front after computing about 14000 function evaluations, while NSGA-II requires around 30000. We also observe that, when the non-penalized region has been reached, MOCHC maintains a more diversified population while NSGA-II tends to concentrate all the population in a front of solutions.

5.3 Discussion

The CHC algorithm was the best technique to solve the single-objective RND problem in [2], where other four metaheuristic algorithms were studied (a simulated annealing, a steady-state genetic algorithm, a generational genetic algorithm, and a distributed steady-state genetic algorithm). The results presented in the previous section have shown that the multi-objective version of CHC is even better, requiring less computational effort than the classic CHC algorithm to find the same optimal solution in all the considered problem instances.

This better performance of MOCHC can be explained according to several facts. First, the inclusion of two side constraints in the multi-objective formulation certainly restricts the search space, which works in favor of the multi-objective algorithms. Second, we have to consider the concept of *multi-objectivization*, introduced by Knowles, Watson, and Corne in [10]. The idea is that defining a multi-objective formulation of a single-objective optimization problem can implicitly reinforce diversification, so the search for the optimal solution is less likely of becoming trapped in a local minimum. Multi-objectivization was studied in [10] in the context of hill-climbing solvers; our experiences in this paper with CHC indicate that this behavior may possibly be extended to other metaheuristics. Finally, we have to consider that we implemented MOCHC from scratch, instead of taking as starting point the CHC implementation used in [2], so there may be some differences in the behavior of the two algorithms.

Deeping in the concept of multi-objectivization, we have to consider that the RND problem is multi-objective in nature, so there is no need of finding *artificial* or *helper* objectives [9]. Furthermore, in [10] it was stated that the relation expressed in Equation (8) should hold:

$$\forall \vec{x}^{opt} \in \vec{X}^{opt}, \exists \vec{x}^{*} \in \vec{X}^{*} / \vec{x}^{*} = \vec{x}^{opt} \tag{8}$$

where $\vec{x}^{opt}$ is an optimal solution to the single-objective problem, $\vec{X}^{opt}$ is the set of such solutions, and $\vec{x}^{*}$ and $\vec{X}^{*}$ have the same meaning considering the multi-objective formulation of the problem. This expression implies that the global optimum of the single-objective problem is one of the solutions of the Pareto optimal set in the multi-objective problem. This condition holds in the case of RND; in fact, the stopping condition of the multi-objective algorithms is to reach a solution fulfilling Equation (8).

6. CONCLUSIONS AND FUTURE WORK

In this paper we solve the RND problem using a multi-objective formulation of the problem. Our main contribution is MOCHC, a multi-objective version of CHC, a kind of genetic algorithm. We have evaluated MOCHC against NSGA-II, a state-of-the-art algorithm for multi-objective optimization, and the obtained results have been compared with those reported in the literature using single-objective metaheuristics.

The experiments carried out reveal that the multi-objective formulation of the RND is particularly adequate, because the existing results have been improved. The profit of using this approach is twofold: first, the optimal solutions are obtained using a lower number of function evaluations; second, instead of a single solution, the Pareto optimal set is obtained, thus allowing the decision maker to choose the best coverage/cost tradeoff solution.

MOCHC has proven to be more efficient than NSGA-II, requiring about a 50% less computation effort to get the Pareto front. Furthermore, the accuracy of MOCHC is also better than that of NSGA-II in four out of the five instances solved (with statistical confidence).

Some lines of future work include solving more complex formulations of the RND, considering different types of antennae, as well as comparing MOCHC against other state-of-the-arts metaheuristics to study whether the feature of fast convergence shown in this work also holds when solving other multi-objective optimization problems different from RND.

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