# Complex Networks and Evolutionary Computation

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## Intro

- What is a network.
- Examples
- Why should we study them

## Historical notes

## What´s all this about

- Many complex relationships can be expressed by means of *networks*.
- Networks are composed of *nodes* (subjects or agents) and *edges* (directed relations) or *arcs* (undirected relations).

## Graphs

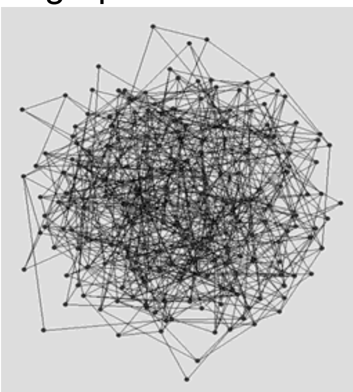
- Directed/undirected
- Geodesics
- Components/cliques
- Degree

## Bipartite graphs

- Mapping problem to bipartite graphs
- Some interesting algorithms
- Mapping 2-mode to 1-mode
- Solving the paper assignment problem

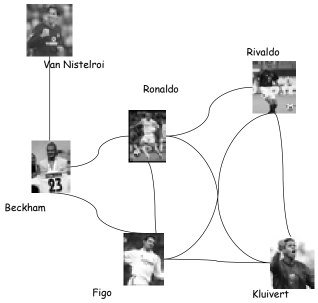
## Random graphs

- Generation
- How they look like
- Why they are not realistic



## Our first social network

- Co-team relations among soccer players.
- Van Nistelrooi, Beckham, Kluiwert, Ronaldo and Rivaldo.



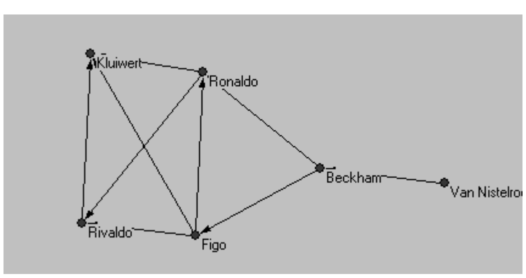
## In numerical terms

| Shared team with: | | | | | | |
|---|---|---|---|---|---|---|
| | | 1 | | | | |
| | | | 1 | 1 | | |
| | | | | 1 | 1 | 1 |
| | | | | | 1 | 1 |
| | | | | | | 1 |
| | | | | | | |

## Soccer backoffice conspiracy

- You want to find metastructures in the soccer players relationships.
- Identify core team players.
- Maximize goals score per euro spent.
- Be rich and famous.

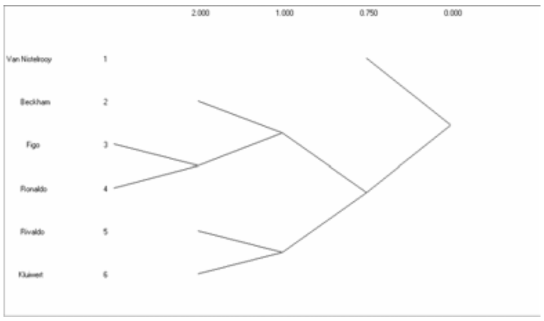
## Pajek shows



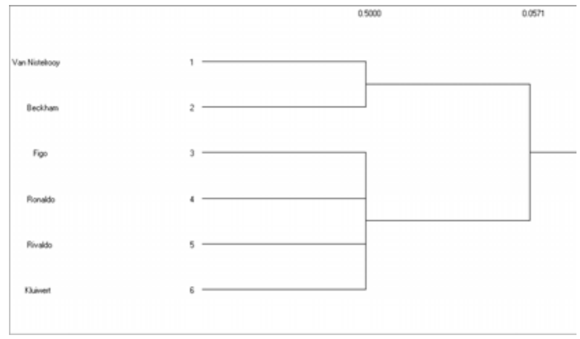

## It is obviously connected

- Choose *Network* ➔ *Cohesion* ➔ *Distance*
- *`Average distance (among reachable pairs) = 1.533`*
- *`Distance-based cohesion = 0.389`*
- It is also *reachable*.
- Several other things can be computed: geodesics (minimun distance between actors), maximum flow...

## But you must obey team discipline




## God creates them, and they gather



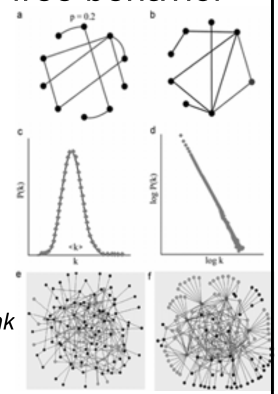

## But classes have always existed

```
Core/Periphery Class Memberships:

   1:  Figo Ronaldo Rivaldo
   2:  Van Nistelrooy Beckham Kluiwert

Blocked Adjacency Matrix

                    4 5 3   1 2 6
                    R R F   V B K
                   --------------
  4       Ronaldo |   1   |     1 |
  5       Rivaldo |       |     1 |
  3          Figo | 1 1   |     1 |
                   ---------------
  1 Van Nistelrooy |      |   1   |
  2       Beckham | 1   1 |       |
  6      Kluiwert |       |       |
                   ---------------
```

## Network links follow a power law

- Log-log links/rank plots show a straight line.
- Sometimes, link abundance plots too.
  - There are *1000 nodes with 1 links, 500 sites with 2 links...*
  - *Pareto 80/20 rule!*
  - *Rich get richer!*

## Which leads to scale-free behavior

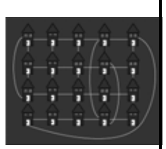


- There's no *preferred link size*
  - *Random networks link distribution, as proved by Erdös-Renyi, followed a Poisson distribution.*
- *Scale-free networks have no preferred scale.*
  - *Many incoming/outgoing link are unlikely, but possible.*

## Why do power laws arise?

- Preferential attachment (Barábasi)
  - Links are added preferentially to those with links.
- It's not always true
  - Sampling problems.
  - Link decay/aging.
  - Assortative/non-assortative networks.
- Other models: log-normal, stretched exponential

## It's a small world



- In small world networks, a few links are enough to connect any two components.
  - Path size scaling with size is logarithmic.
  - Doesn't work in random or regular networks.
- A few links are enough to convert a random network into a small world network.

## Complex networks

- clustering
- preferential attachment
- power law
- small world
- giant component

## Centrality measures

- Centrality measures indicate the relevance of a node (or link) within the network.
- Measures based on geodesics
  - Closeness
  - Betweenness
- Measures based on degree (or flow)
  - Bonacich power
  - Eigenvector centrality
  - …

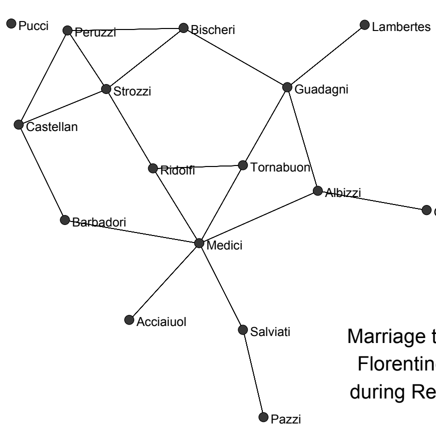
## Centrality measures

- Betweenness centrality measures how often a vertex appears on geodesics.
- High betweenness nodes may control information flow.

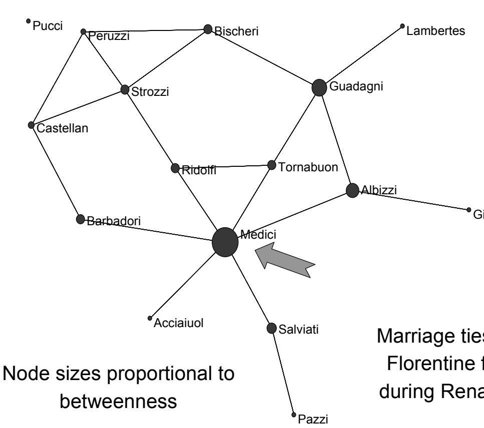
$$C_i^{BET} = \sum_{j<k} \frac{\boxed{\# g_{jik}}}{\boxed{\# g_{jk}}}$$

Number of geodesics from node *j* to node *k* that pass through node *i.*

Number of geodesics from node *j* to node *k*.




Marriage ties among Florentine families during Renaissance




Node sizes proportional to betweenness

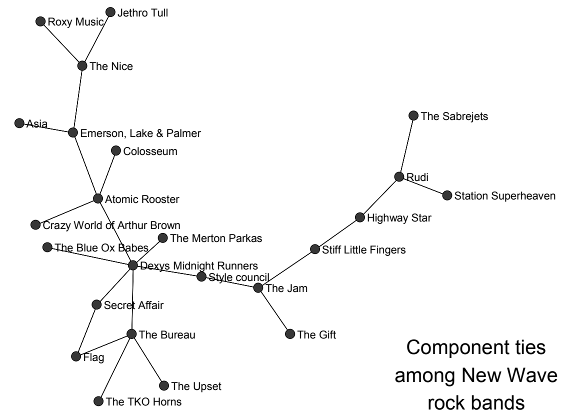
Marriage ties among Florentine families during Renaissance

## Centrality measures

- Closeness centrality measure how close a node is from the remaining nodes.
- High closeness nodes are the first to get new information (and the most efficient to spread it).

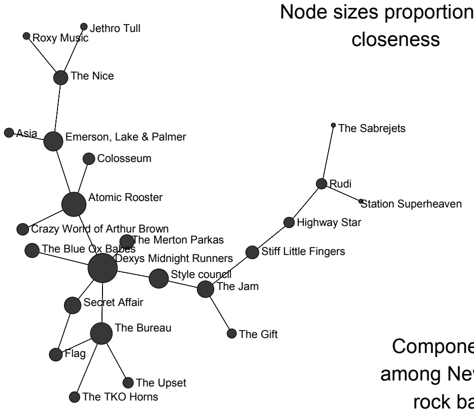
$$C_i^{CLO} = \frac{1}{\sum_j \boxed{d_{ij}}}$$

Length of the geodesic from node $i$ to node $k$.




Component ties among New Wave rock bands

Node sizes proportional to closeness




Component ties among New Wave rock bands

## Centrality measures

- Bonacich power measures the importance of a node's neighbors.
- High power nodes have the ability to influence the network directly or indirectly.

$$C_i^{POW} = \sum_j \boxed{A_{ij}}\left(\alpha + \boxed{\beta} C_j^{POW}\right)$$

Lower than the reciprocal of the largest eigenvalue

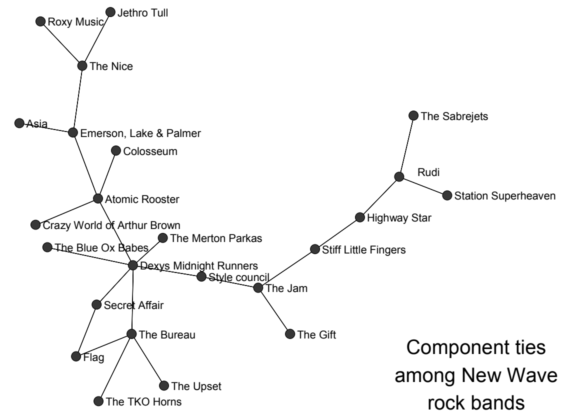
Adjacency matrix

## Centrality measures

- Eigenvector centrality is a weighted (inversely proportional to length) sum of the number of walks originating at a certain node.
- High power nodes have the ability to influence the network via multiple paths.

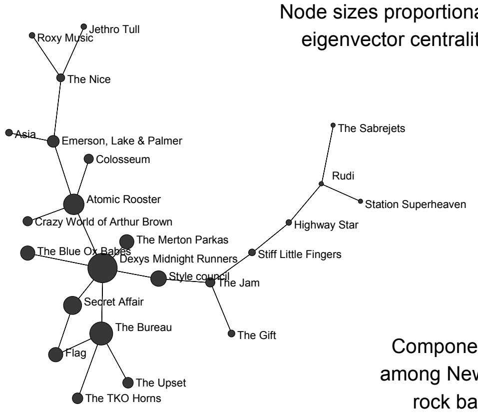
$$C_i^{EIG} = \boxed{\beta} \sum_j A_{ij} C_j^{EIG}$$

Reciprocal of the largest eigenvalue



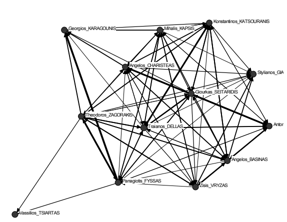

Component ties among New Wave rock bands

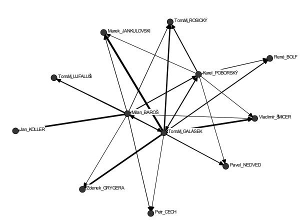
Node sizes proportional to eigenvector centrality




Component ties among New Wave rock bands

## Who would win? (Core/Periphery Concentration Ratio)



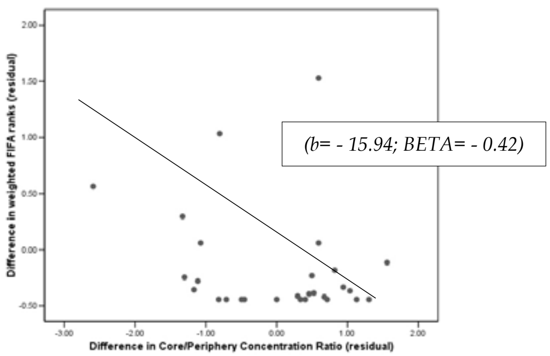


*Greece (vs Czech); CPCR = 0.816*

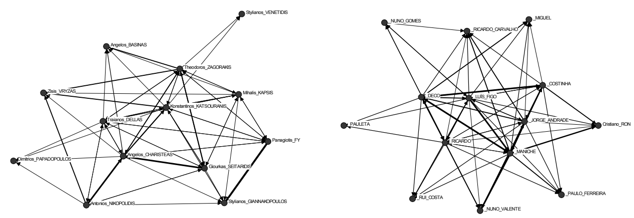
*Czech (vs Greece); CPCR = 1.000*

***Greece 1 : 0 Czech***

## C/P Concentration Ratio




## Who would win? (Out-degree Centralization)

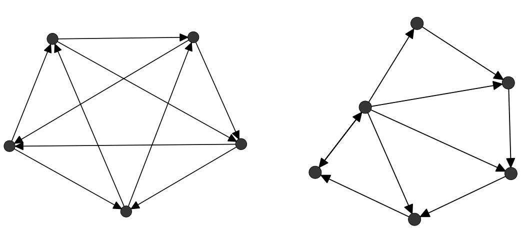


*Greece (vs Portugal); ODC = 111.81*

*Portugal (vs Greece); ODC = 262.72*

***Greece 1 : 0 Portugal***
*(Final Match)*

## Who would win? (Out-degree Centralization)



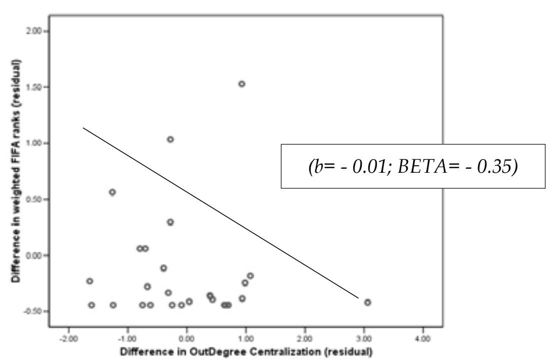
***ODC = 0.00***

**Each node has 2 out degree**

***ODC = 0.80***

**A node has 5 out degree, and all other nodes have 1 out degree**

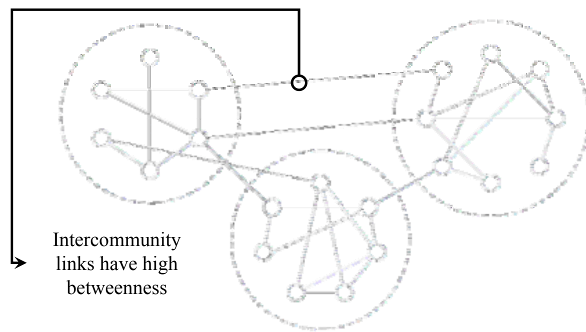
## Out Degree Centralization



## Network structure

- A graph partitioning problem
- Number of communities not known
- Two approaches
  - Agglomerative
  - Divisive

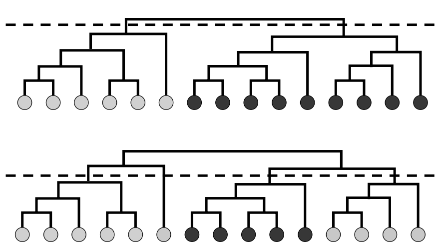
## Network structure




(example taken from [Newman, Girvan, 2004; cond-mat/0308217])

## Network structure

- Removal of high-betweenness edges results in a dendrogram.



## Network structure

- A modularity measure:

$$e_{ij} = \frac{\sum_{r \in C_i} \sum_{s \in C_j} A_{rs}}{\sum_r \sum_s A_{rs}}$$

fraction of edges between communities $i$ and $j$

$$a_i = \sum_j e_{ij}$$

fraction of edges connecting to community $i$

$$Q = \sum_i \left( e_{ii} - a_i^2 \right)$$

fraction of edges within a community

expected value

## Applications in EC - I

- Small world cellular EAs
  - Giacobini, Mike Preuss, and Tomassini
  - Effects of scale-free and small-world topologies on binary coded self-adaptive CEA.
  - Introduces a reproductive restriction, which *drasticall influences search.*

## Applications in ECII

- On the Importance of Information Speed in Structured Populations
  - Mike Preuss and Christian Lasarczyk
  - Changing reproduction strategy, small-world networks increase information flow
    - Which might not be good

## Applications in EC III

- Evolutionary reconstruction of networks
  - Mads Ipsen, Alexander S. Mikhailov
  - http://arxiv.org/abs/nlin/0111023
  - Tries to reconstruct several types of network (including small-world) from its laplacian spectra (set of eigenvalues).

## Applications in EC IV

- Small-world optimization algorithm for function optimization
  - Haifeng Du, Xiaodong Wu and Jian Zhuang
  - *Kenning* : connections is to small world as optimization algorithm is to…
    - Local and long-distance search operators make a small-world network (nodes: solutions; operators: links)
  - Competitive with GAs

## Applications in EC V

- Population structure and artificial evolution
  - Arthur Farley
  - Tests different graph structures with mating restriction
  - Small world networks have much the same properties as fully connected networks.

## Applications in EC VI

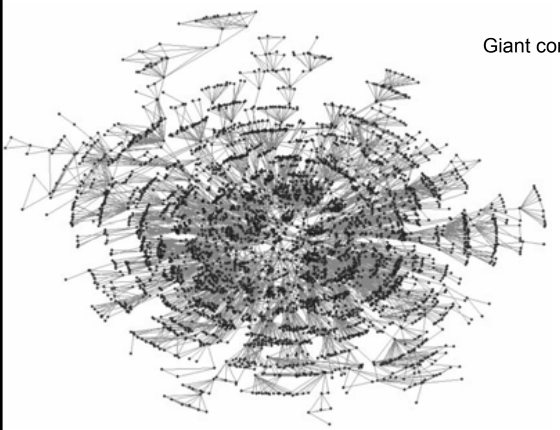
- On properties of genetic operators from a network analytical viewpoint
  - Hiroyuki Funaya and Kazushi Ikeda
  - Studies a GA as a complex network.
    - Populations as nodes, operators as edges.
  - Crossover creates long-distance connections: small world.
  - Worthwhile further investigation

## EC coauthorship network

- Scientific coauthorship is a key indicator of the social dynamics of our community.
- The structure of this complex network can provide some insight on the inner workings of EC as a science.
- Interesting questions:
  - How *typical* a research area is EC?
  - Is the area expanding or shrinking?
  - Are there sociometric stars? If so, who are they?

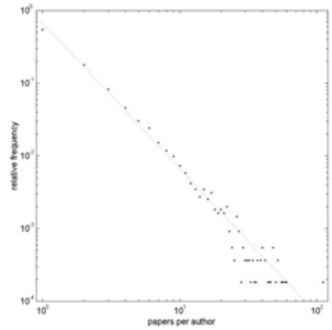
## EC coauthorship network

- Data taken from the DBLP.
  - 7,712 authors
  - 8,501 papers
- A giant component comprises 62.3% of the network (2nd largest component is 1.4%)
- Mean distance is 10.9
- Diameter is 21
- Clustering coefficient is 0.811



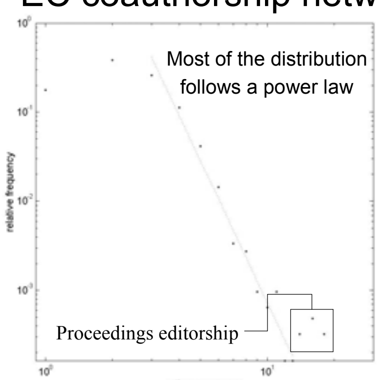
Giant component

## EC coauthorship network




EC follows Lotka's Law of Scientific Productivity

## EC coauthorship network

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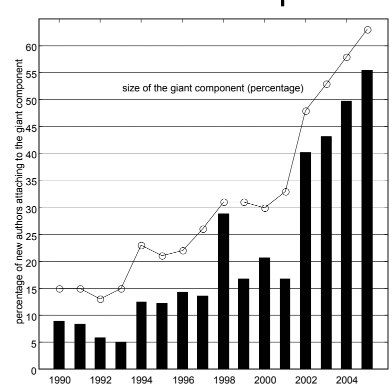

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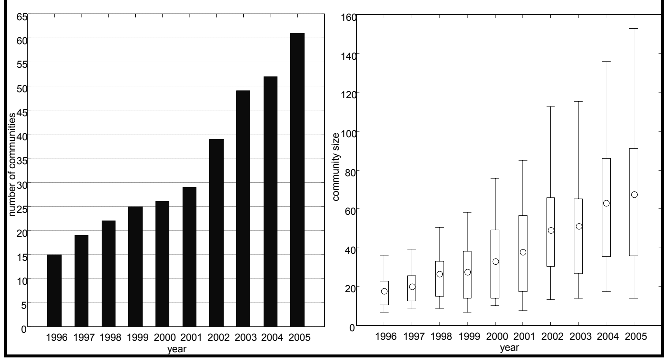
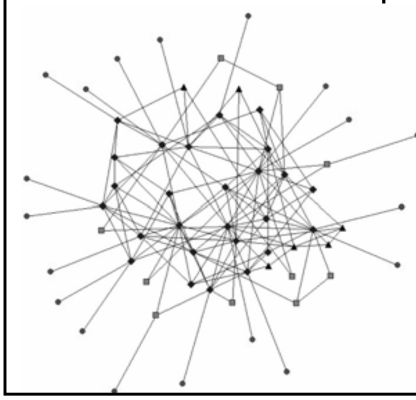
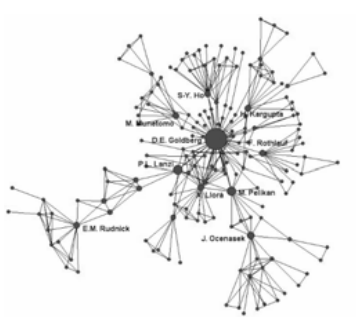



## EC coauthorship network



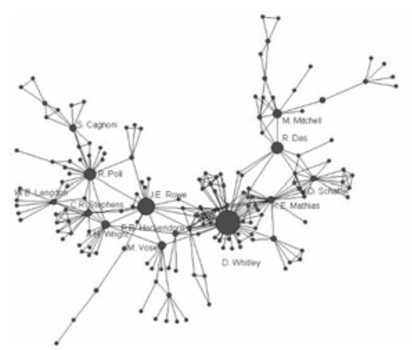
The community interaction graph shows a core-periphery structure

## EC coauthorship network




Largest community

## EC coauthorship network




Most central community

## EC coauthorship network

- Different centrality measures point to different authors:
  - Betweenness: Goldberg, Deb, Schoenauer, de Garis, …
  - Closeness: Deb, Michalewicz, Goldberg, Schoenauer, …
  - Power: Goldberg, Schoenauer, Deb, Keymeulen, …
  - Eigenvector: Keymeulen, Higuchi, Iwata, Kajitani, …
- Eigenvector centrality prone to hitchhiking.
- Pareto-dominance approach required.

## EC coauthorship network

- Resulting non-dominated fronts
  1. K. Deb, D.E. Goldberg
  2. Z. Michalewicz, M. Schoenauer
  3. T. Bäck, A.E. Eiben, H. de Garis, D. Keymeulen, B.Paechter, M. Tomassini, X. Yao
  4. D.B. Fogel, J.J. Merelo, T. Higuchi, K.A. De Jong, L.Kang, E. Lutton, R.E. Smith, L.D. Whitley
  5. H.A. Abbass, H.-G. Beyer, J. Branke, M. Dorigo, T.C. Fogarty,H. Iba, M. Keijzer, E.G. Talbi, M.D. Vose
- Connectedness (not scientific excellence) is measured.

## Conclusions

- Complex networks are cool
- And useful

## The End

¿Any questions?