Properties of the Bersini Experiment on Self-Assertion

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ABSTRACT
The approach of H. Bersini to shape-spaces and in particular his definition of affinity are analysed. It is shown that the definition of the affinity function in Bersini style implies a special form of an affinity region, namely a rhombus. However, variants of the function can be defined with rectangular or square but rotated affinity regions. In all cases, the affinity function has the form of a pyramid over the affinity region. The definition of the affinity function can be modified in such a way that it describes a lopsided pyramid. Experimental results with a reimplementation of Bersini's simulation procedure show that the form of the affinity region has a strong influence on the form of the recognition/tolerance separation of the shape-space.

Categories and Subject Descriptors
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General Terms
Algorithms, Experimentation

Keywords
Shape-space, affinity function, affinity region.

1. INTRODUCTION

The mostly used definition of shape-spaces is the one introduced by Perelson and Oster in [6]. According to this definition, the interaction between elements of the immune system (cells, antibodies, or molecules) and antigens is determined by properties of shape. Actually, this approach is an abstraction from the real immune system, where the interaction is essentially based on electrical forces due to the charge distribution on the surface of the molecules. The next step of abstraction, then, is the representation of the shape properties by a string of parameters of certain types of values like binary, integer, real, or symbolic.

A basic notion in the Perelson/Oster shape-space is that of complementarity, which means that an immune element and an antigen must have complementary shapes in order to exert affinity on each other. Different types of affinity have been defined, depending on the type of the shape-space as a vector, but all of them are based on some distance measure like Euclidian distance or Hamming distance.

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In [1], Bersini introduced an alternative definition of a shape-space which on first glance departs considerably from the Perelson/Oster definition. On second glance, it turns out that it is a shape-space based on $\Re^2$ with complementary affinity. However, Bersini uses a special definition of affinity which makes his shape-space particularly interesting. This definition incorporates complementarity as mirror image positions in the space together with a fixed affinity region where the immune elements (antibodies) are attracted with graded force. Bersini's approach has been adopted and modified in several ways by Hart and Ross [5] and Hart [4] who demonstrate the properties of this kind of shape-space by some simulation experiments.

Bersini's paper is based on the work he has done in the group of F. Varela for several years and which he has published in a number of papers like in [2] or [3] among others. The main objective of Bersini's work is to support the self-assertion view of the immune system as it was developed and propagated in Varela's group by simulation experiments. However, the system can also be used to simulate the self-recognition view. Nevertheless, Bersini's basic definition of a shape-space and the affinity in this space is highly interesting and worth to be considered in more detail than obviously he himself did. This is the aim of this paper.

In the next section we give a detailed analysis of Bersini's affinity definition which reveals the consequences of that definition, in particular the form of the affinity function and the affinity region. In section 3, some variants of the definition are introduced with different affinity regions. Section 4 gives a detailed description of an asymmetric version of the affinity function, motivated by the work of Hart and Ross. This definition deviates on first glance from Bersini's approach but is still in the lines of his definition as will be shown. Finally, in section 5 we present some experimental results of a simulation with the various affinity functions and give an interpretation of these results.

2. BERSINI'S SHAPE-SPACE AND AFFINITY FUNCTION

The shape-space is defined as a two-dimensional space. One point in this space is marked as the center of the space. An immune cell can be identified by its position in the space. Nothing is said about the size of the space, in fact, it does not play any role in the concept. The notion of complementarity comes in by the definition of the affinity. The idea is to define a region of affinity that is arranged around the point which is in the spatially symmetrical position with respect to the center of the space. This is illustrated in figure 1 (according to [1]).

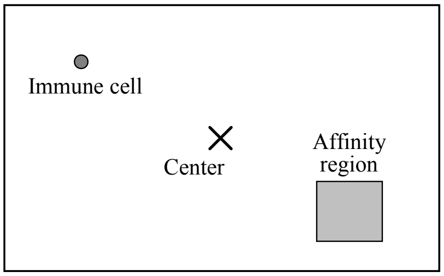

Figure 1. The shape-space according to Bersini

The affinity region is the zone where the immune cell exerts an affinity on antigens or other cells. The affinity decreases from the center of that region to the borders. Bersini assumes that the affinity region is a square of a certain size L.[1] The position of the shape-space center is (c_1, c_2), the immune cell i is at position (i_1, i_2), and an arbitrary element is at position (x_1, x_2). Each immune cell is provided with a concentration $C_i(t)$ at time t, initially it is $C_i(0)$. Based on these notations, the affinity exerted by the immune cell i is defined by a two-dimensional function

$$aff_i(x_1, x_2) = C_i(t) \cdot \left(L - \left(|2c_1 - i_1 - x_1| + |2c_2 - i_2 - x_2|\right)/2\right) \tag{1}$$

The factor ½ in definition (1) is not important; it influences the size of the affinity region which is in principle determined by the value of L, so that it can be incorporated in L. It will be omitted in the remainder of this paper.

It is important to consider the properties of the function $aff_i(x_1, x_2)$ in detail. First, it can adopt negative values, but a negative affinity does not make sense, therefore it should be restricted to regions where it is positive or at least zero. Second, it has a maximum at position $(2c_1 - i_1, 2c_2 - i_2)$, i.e. exactly at the point symmetrical to the position of the immune cell, because this is the only point where $(|2c_1 - i_1 - x_1| + |2c_2 - i_2 - x_2|) = 0$. The maximum value is $C_i(t) \cdot L$. Third, it is zero if either $C_i(t) = 0$, which means that the concentration of i has dropped to 0, or if $L = |2c_1 - i_1 - x_1| + |2c_2 - i_2 - x_2|$. The last equation includes four different cases, depending on whether the two absolute value terms on the right hand side are greater or less than zero. Assume both are greater than zero. Then we get the linear equation

$$x_1 + x_2 = (2c_1 - i_1) + (2c_2 - i_2) - L \tag{2}$$

This equation describes a straight line with gradient –1. In a similar way the other three cases produce the following lines

$$x_1 - x_2 = (2c_1 - i_1) - (2c_2 - i_2) - L \tag{3}$$
$$x_1 - x_2 = (2c_1 - i_1) - (2c_2 - i_2) + L \tag{4}$$
$$x_1 + x_2 = (2c_1 - i_1) + (2c_2 - i_2) + L \tag{5}$$

These four lines enclose just the affinity region. The center of this area is the point $(2c_1 - i_1, 2c_2 - i_2)$, i.e. the point where the affinity function has its maximum. Figure 2 shows the form of the affinity region for the parameter values $c_1 = 5$, $c_2 = 3$, $i_1 = 2$, $i_2 = 4$, and $L = 1$. The affinity region is a rhombus with length of the side $\sqrt{2}$ and therefore size 2. It may be surprising that the size of the affinity region is not equal to L (= 1), but this is a consequence of the definition of the affinity function.

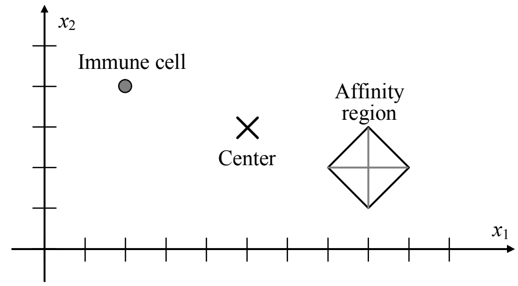

Figure 2. The shape-space according to Bersini's affinity function

The intersection points of the four straight lines that form the rhombus can be easily computed. For instance, the intersection of the lines of equations (2) and (3) and of lines number (2) and (4) respectively are $(2c_1 - i_1 - L, 2c_2 - i_2)$ and $(2c_1 - i_1, 2c_2 - i_2 - L)$ respectively. These two points together with the center of the affinity region form an isosceles right-angled triangle with legs of length L and therefore with a hypotenuse (which is the side of the rhombus) of length $L \cdot \sqrt{2}$. Therefore the size of the affinity region is $2L^2$.

Inside the rhombus and only there, the affinity function has positive values, decreasing linearly from the center to the sides, thus the function has the form of a pyramid. Each of the lines enclosing the affinity region divides the two-dimensional space in two halves. Consider e.g. line number (2). For the points of the space for which holds $x_1 + x_2 > (2c_1 - i_1) + (2c_2 - i_2) - L$ and which are located right above the line, the value of the affinity function is greater than zero because here we have the first case considered above where

$$aff_i(x_1, x_2) > 0 \Leftrightarrow C_i(t) > 0 \wedge L > (2c_1 - i_2 - x_1) + (2c_2 - i_2 - x_2) \tag{6}$$

The last part of this condition slightly transformed is

$$L > (2c_1 - i_2) + (2c_2 - i_2) - (x_1 + x_2) \tag{7}$$
$$x_1 + x_2 > (2c_1 - i_2) + (2c_2 - i_2) - L \tag{8}$$

which is exactly the property of the points in question. Similar considerations can be made for the other lines.

3. SOME VARIANTS OF BERSINI'S AFFINITY FUNCTION

As was shown in the previous section, Bersini's affinity function has the nice property that it not only describes the course of that function but also the form and location of the affinity region by dividing the two-dimensional space by a rhombus. In this section we will present three variants of Bersini's affinity function with different forms of the affinity region. The first one defines the affinity region as a rectangle in axes parallel position and the second one as a square in an arbitrarily rotated position. The third variant is an affinity function with a circular affinity region. The affinity function has the form of a cone. The first form of the affinity function is the following:

$$aff_i(x_1, x_2) = C_i(t) \cdot \left(L - \left(\left|(2c_1 - i_1 - x_1) - s(2c_2 - i_2 - x_2)\right| + \ldots\right)\right) \tag{9}$$

[1] Hart and Ross use a slightly different definition. They assume a space of restricted size and define the symmetry of positions with respect to the size of the space.

s is used as a "scaling parameter" to differentiate the lengths of the sides. The range of s is $0 < s$. Since the value of the expression that is subtracted from L is greater or equal to 0, the maximum value of the affinity function is achieved if

$$\begin{aligned}&\left|(2c_1 - i_1 - x_1) - s(2c_2 - i_2 - x_2)\right| + \\ &\quad \left|(2c_1 - i_1 - x_1) + s(2c_2 - i_2 - x_2)\right| = 0\end{aligned} \tag{10}$$

From (10) it follows that the center of the affinity region is the point with maximum affinity, like in section 2. As before, $aff_i(x_1, x_2)$ is zero if $C_i(t) = 0$ or

$$L = \begin{pmatrix}\left|(2c_1 - i_1 - x_1) - s(2c_2 - i_2 - x_2)\right| + \\ \left|(2c_1 - i_1 - x_1) + s(2c_2 - i_2 - x_2)\right|\end{pmatrix} \tag{11}$$

Equation (11) results in four different cases like in section 2 and it is easy to see that from these cases the following four equations for the borderline of the affinity region can be derived:

$$x_1 = 2c_1 - i_1 - L/2 \qquad x_1 = 2c_1 - i_1 + L/2 \tag{12}$$

$$x_2 = 2c_2 - i_2 + L/2s \qquad x_2 = 2c_2 - i_2 - L/2s \tag{13}$$

Obviously, this is a rectangle with side lengths L and L/s respectively in axes parallel position. Its size is L^2/s. The second variant of Bersini's definition is the following:

$$aff_i(x_1, x_2) = C_i(t) \cdot \left(L - \begin{pmatrix}\left|(2c_1 - i_1 - x_1) - r(2c_2 - i_2 - x_2)\right| \\ + \left|r(2c_1 - i_1 - x_1) + (2c_2 - i_2 - x_2)\right|\end{pmatrix}\right) \tag{14}$$

Here the range of r can be chosen as $-1 \le r \le 1$. This definition is similar to that of equation (9) but more symmetrical insofar as both coordinates of the region are modified. As before, the center of the region is the point of maximum affinity. As above, the regions with $aff_i(x_1, x_2) = 0$ are the border lines of the affinity region (except for the case $C_i(t) = 0$). They are defined by the following four equations:

$$(r+1)x_1 - (r-1)x_2 = (r+1)(2c_1 - i_1) - (r-1)(2c_2 - i_2) - L \tag{15}$$

$$(r-1)x_1 + (r+1)x_2 = (r-1)(2c_1 - i_1) + (r+1)(2c_2 - i_2) + L \tag{16}$$

$$(r-1)x_1 + (r+1)x_2 = (r-1)(2c_1 - i_1) + (r+1)(2c_2 - i_2) - L \tag{17}$$

$$(r+1)x_1 - (r-1)x_2 = (r+1)(2c_1 - i_1) - (r-1)(2c_2 - i_2) + L \tag{18}$$

Again, lines (15) and (18) are parallels and also lines (16) and (17). The gradient of line (15) is $(r+1)/(r-1)$ and that of line (16) is $-(r-1)/(r+1)$, so these two lines are orthogonal, correspondingly for the others. Thus, the affinity region is in fact a rectangle. The corners of the rectangle are

$$\left(2c_1 - i_1 - \frac{L}{r^2+1}, 2c_2 - i_2 + \frac{rL}{r^2+1}\right) \tag{19}$$

$$\left(2c_1 - i_1 - \frac{rL}{r^2+1}, 2c_2 - i_2 - \frac{L}{r^2+1}\right) \tag{20}$$

$$\left(2c_1 - i_1 + \frac{rL}{r^2+1}, 2c_2 - i_2 + \frac{L}{r^2+1}\right) \tag{21}$$

$$\left(2c_1 - i_1 + \frac{L}{r^2+1}, 2c_2 - i_2 - \frac{rL}{r^2+1}\right) \tag{22}$$

All sides have the same length, namely $L\sqrt{2}/\sqrt{r^2+1}$, so the rectangle is in fact a square of size $2L^2/(r^2 + 1)$ rotated by some angle against the axes. The length of the sides and the size of the square depend on r. The maximum of the size is achieved for $r = 0$ and the minimum for $r = 1$. Consider these two special cases. For $r = 1$ we get from (14) the definition of equation (9) with $s = 1$, i.e. the square is parallel to the axes. For $r = 0$ we get Bersini's original definition (equation (1) except for the factor ½), i.e. the square is a rhombus.

The third variant of the affinity function defines an affinity region with circular form and thus the function has the form of a cone. The definition of the function is simply that of a circle with center $(2c_1 - i_1, 2c_2 - i_2)$ and with L as the radius:

$$\begin{aligned}&aff_i(x_1, x_2) = \\ &\quad C_i(t) \cdot \left(L - \left|\sqrt{(x_1 - (2c_1 - i_1))^2 + (x_2 - (2c_2 - i_2))^2}\right|\right)\end{aligned} \tag{23}$$

The affinity function has its maximum at the center of the circle since here the square root disappears. It is zero where $L = \left|\sqrt{(x_1 - (2c_1 - i_1))^2 + (x_2 - (2c_2 - i_2))^2}\right|$ which is equivalent to $L^2 = (x_1 - (2c_1 - i_1))^2 + (x_2 - (2c_2 - i_2))^2$ and this is exactly the borderline of the circle. For points outside the circle the affinity function has negative values since for those points $L < \left|\sqrt{(x_1 - (2c_1 - i_1))^2 + (x_2 - (2c_2 - i_2))^2}\right|$. The size of the circle is πL^2. Figure 3 shows this type of affinity region. The affinity function has the form of a cone.

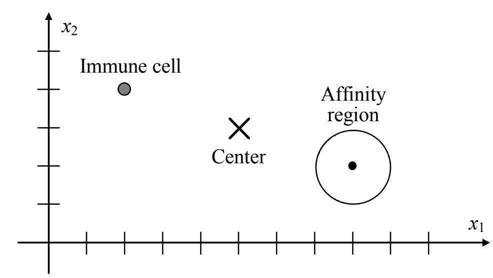

Figure 3. The shape-space with circular affinity region

4. AN ASYMMTERIC VARIANT OF BERSINI'S AFFINITY FUNCTION

This variant of the affinity function is the most general one. It is motivated by the work of Hart and Ross who describe some experiments with asymmetric affinity regions. We want to define such an asymmetric region in accordance with the type of affinity function that has been used throughout this paper. In all the variants of the affinity function described so far the point with maximum affinity is the center of the region and in this sense they are symmetric. Thus, an asymmetric region can be defined as one where the point of maximum affinity is not the center. However, any modification of the definition of the function itself only affects the form, the size, and the position of the affinity region but keeps its symmetry.

In order to define an affinity function with asymmetric affinity region we have to depart somehow from the type of functions discussed in sections 2 and 3, while still keeping the essential properties, namely linear gradients from the point of maximum affinity to the sides of the region and negative values outside the region. In addition, the affinity region should be a rectangle of arbitrary size, position, and rotation angle with respect to the axes. In contrast to Bersini's definition which is based on the center of the affinity region and implies a point symmetric region, the new definition is based on the affinity region which is chosen as an arbitrary rectangle. The rectangle is shaped by four straight lines which are parallel or orthogonal to each other in an appropriate way. Figure 4 illustrates these prerequisites.

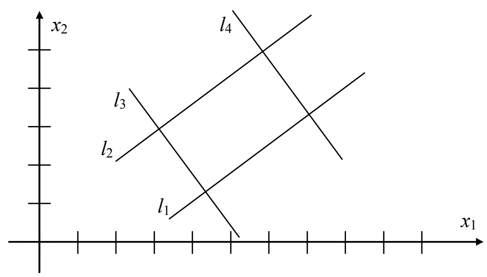

Figure 4. Four straight lines forming a rectangle

Because of the properties of being parallel or orthogonal, respectively, the lines can be defined by the following equations:

$$l_1 \quad ax_1 - x_2 = b_1 \tag{24}$$
$$l_2 \quad ax_1 - x_2 = b_1 + d_1 \tag{25}$$
$$l_3 \quad x_1 + ax_2 = b_2 \tag{26}$$
$$l_4 \quad x_1 + ax_2 = b_2 + d_2 \tag{27}$$

The vertices of the rectangle can be computed from these equations in the usual way yielding

$$c_1 = \left(\frac{b_2 + ab_1}{a^2+1}, \frac{ab_2 - b_1}{a^2+1}\right) \tag{28}$$
$$c_2 = \left(\frac{b_2 + ab_1 + d_2}{a^2+1}, \frac{ab_2 - b_1 + ad_2}{a^2+1}\right) \tag{29}$$
$$c_3 = \left(\frac{b_2 + ab_1 + ad_1}{a^2+1}, \frac{ab_2 - b_1 - d_1}{a^2+1}\right) \tag{30}$$
$$c_4 = \left(\frac{b_2 + ab_1 + d_2 + ad_1}{a^2+1}, \frac{ab_2 - b_1 + ad_2 - d_1}{a^2+1}\right) \tag{31}$$

The two sides of the rectangle have the lengths

$$\frac{d_1}{\sqrt{a^2+1}} \quad \text{and} \quad \frac{d_2}{\sqrt{a^2+1}} \tag{32}$$

respectively. The point of maximum affinity shall be located inside the rectangle at an arbitrary position, and the affinity function should have linear gradients from that point to the sides of the rectangle. Figure 5 illustrates this situation. t is the point of maximum affinity. The lines m_1, …, m_4 mark the edges of the (lopsided) pyramid that shall be formed by the desired affinity function. The point t can be computed like the point c_4 as the crossing point of a straight line parallel to l_1, shifted by some value e_1, and a line parallel to l_3, shifted by e_2:

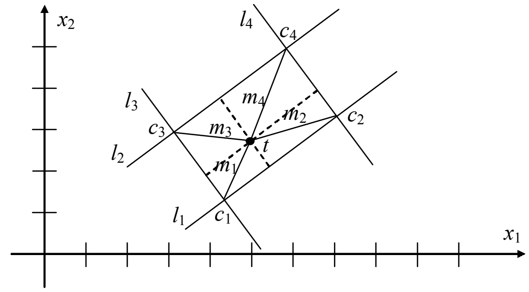

Figure 5. The form of the affinity function with asymmetric affinity region

$$(t_1, t_2) = \left(\frac{b_2 + ab_1 + e_2 + ae_1}{a^2+1}, \frac{ab_2 - b_1 + ae_2 - e_1}{a^2+1}\right) \tag{33}$$

The affinity function shall have the form of a pyramid and the sides of a pyramid have linear gradients. The height of the pyramid is denoted by h. h corresponds to the concentration of the element i in the definitions of section 2 and 3, i.e. $C_i(t)$. In order to compute the gradients, we need the distances between t and all sides of the affinity region as indicated in figure 5 by the dashed lines. These lines together with the sides of the rectangle form four small rectangles included in the affinity region, thus the distances can be computed in the same way as the lengths of the sides of the outer rectangle (cf. equation (32)). This gives the following values:

$$dist(t, l_1) = \frac{e_1}{\sqrt{a^2+1}} \qquad dist(t, l_2) = \frac{d_1 - e_1}{\sqrt{a^2+1}} \tag{34}$$
$$dist(t, l_3) = \frac{e_2}{\sqrt{a^2+1}} \qquad dist(t, l_4) = \frac{d_2 - e_2}{\sqrt{a^2+1}} \tag{35}$$

Now consider an arbitrary point $x = (x_1, x_2)$ inside the triangle formed by the lines l_1, m_1, and m_2 (this triangle will be denoted as Δ_1 in the following, correspondingly for the other three triangles) and assume its distance from l_1 is v_1. The value of the affinity function at x can then be computed as

$$aff_{i1}(x_1, x_2) = \frac{hv_1}{dist(t, l_1)} \tag{36}$$

However, a value like that of equation (36) can even be computed for points outside the triangle Δ_1, i.e. for arbitrary points in the affinity region and even outside as we will see later. In addition, the equation (36) can be generalized such that it holds in the same way for all sides l_j:

$$aff_{ij}(x_1, x_2) = \frac{hv_j}{dist(t, l_j)} \tag{37}$$

In order to get rid of the parameters e_j contained in the expressions $dist(t, l_j)$ and v_j the values v_j are computed similar to the expressions $dist(t, l_j)$ with a shifting value f_j (corresponding to e_j):

$$v_1 = \frac{f_1}{\sqrt{a^2+1}} \qquad v_2 = \frac{d_1 - f_1}{\sqrt{a^2+1}}$$
$$v_3 = \frac{f_2}{\sqrt{a^2+1}} \qquad v_4 = \frac{d_2 - f_2}{\sqrt{a^2+1}} \tag{38}$$

From equation (37) the values of e_1 and e_2 can be computed using the coordinates of t and correspondingly the new values f_1 and f_2 using the coordinates of x as follows:

$$e_1 = at_1 - t_2 - b_1 \qquad e_2 = t_1 + at_2 - b \tag{39}$$

$$f_1 = ax_1 - x_2 - b_1 \qquad f_2 = x_1 + ax_2 - b \tag{40}$$

Inserting the e-values in the $dist(t, l_j)$ expressions and the f-values in the v-expressions and altogether in the equations of type (37) we get the values of the affinity function for the points in the four triangles separately:

$$aff_{i1}(x_1, x_2) = h\,\frac{ax_1 - x_2 - b_1}{at_1 - t_2 - b_1} \tag{41}$$

$$aff_{i2}(x_1, x_2) = h\,\frac{d_1 - (ax_1 - x_2 - b_1)}{d_1 - (at_1 - t_2 - b_1)} \tag{42}$$

$$aff_{i3}(x_1, x_2) = h\,\frac{x_1 + ax_2 - b_2}{t_1 + at_2 - b_2} \tag{43}$$

$$aff_{i4}(x_1, x_2) = h\,\frac{d_2 - (x_1 + ax_2 - b_2)}{d_2 - (t_1 + at_2 - b_2)} \tag{44}$$

Each of the four values $aff_{ij}(x_1, x_2)$ can be computed for arbitrary points inside and outside the affinity region. It is easy to see that they can adopt negative values for points outside the affinity region, more precisely: $aff_{i1}(x_1, x_2)$ is negative for points below the line l_1, correspondingly for the other lines. Thus, $aff_{i1}(x_1, x_2)$ defines an inclined plane which is zero at line l_1, negative below and positive above the line, correspondingly four the other three values $aff_{ij}(x_1, x_2)$. This is indicated in figure 6 for $aff_{i1}(x_1, x_2)$ and $aff_{i3}(x_1, x_2)$.

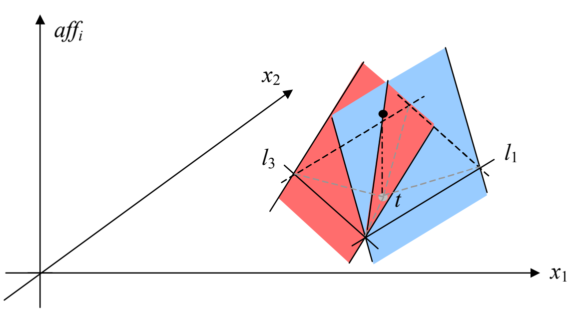

Figure 6. Inclined planes formed by the affinity values $aff_{i1}(x_1, x_2)$ and $aff_{i3}(x_1, x_2)$

Now we are ready for the definition of the affinity value for an arbitrary point in the two-dimensional space:

$$\begin{aligned} aff_i(x_1, x_2) &= \min_{j=1,\ldots,4} aff_{ij}(x_1, x_2) \\ &= h \cdot \min\left\{ \begin{matrix} \dfrac{ax_1 - x_2 - b_1}{at_1 - t_2 - b_1}, & \dfrac{d_1 - (ax_1 - x_2 - b_1)}{d_1 - (at_1 - t_2 - b_1)}, \\ \dfrac{x_1 + ax_2 - b_2}{t_1 + at_2 - b_2}, & \dfrac{d_2 - (x_1 + ax_2 - b_2)}{d_2 - (t_1 + at_2 - b_2)} \end{matrix} \right\} \end{aligned} \tag{45}$$

In order to show that this definition has the required properties we first must check its value at point t. From equations (41) – (44) it is easy to see that $aff_{ji}(t_1, t_2) = h$ for $i = 1, \ldots, 4$. Next, it must be shown that $aff_i(x_1, x_2) = aff_{i1}(x_1, x_2)$ for all points in the triangle Δ_1, correspondingly for the other three triangles. In order to do this, the equations of the lines m_1 and m_2 are required (cf. figure 5). m_1 has the general equation $x_2 = px_1 + q$. The parameters p and q can be determined by inserting the coordinates of the points c_1 (cf. equation (28)) and t since both points are located on m_1. This yields equation (46) for m_1 and correspondingly equation (47) for m_2.

$$m_1 \quad x_2 = \frac{ab_2 - b_1 - t_2(a^2+1)}{b_2 + ab_1 - t_1(a^2+1)}x_1 + \frac{t_2(b_2 + ab_1) - t_1(ab_2 - b_1)}{b_2 + ab_1 - t_1(a^2+1)} \tag{46}$$

$$m_2 \quad x_2 = \frac{b_2 + ab_1 + d_2 - t_2(a^2+1)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)}x_1 + \frac{t_2(ab_2 + b_1 - ad_2) - t_1(b_2 + ab_1 + d_2)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)} \tag{47}$$

Though these equations look a bit clumsy it is easy to show that e.g. t is on both lines. Now we claim that for the points inside Δ_1 the affinity value must satisfy the following condition:

$$\begin{aligned} & x_2 \geq ax_1 + b_1 \\ \wedge\ & x_2 \leq \frac{ab_2 - b_1 - t_2(a^2+1)}{b_2 + ab_1 - t_1(a^2+1)}x_1 + \frac{t_2(b_2 + ab_1) - t_1(ab_2 - b_1)}{b_2 + ab_1 - t_1(a^2+1)} \\ \wedge\ & x_2 \leq \frac{b_2 + ab_1 + d_2 - t_2(a^2+1)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)}x_1 + \frac{t_2(ab_2 + b_1 - ad_2) - t_1(b_2 + ab_1 + d_2)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)} \\ \Leftrightarrow\ & \min_{j=1,\ldots,4} aff_{ij}(x_1, x_2) = aff_{i1}(x_1, x_2) \end{aligned} \tag{48}$$

The three parts of this condition state that x must be located above l_1, below m_1, and below m_2. Actually, we do not have to show the first part of the condition since $aff_{i1}(x_1, x_2)$ is defined for points below l_1 as well, thus the condition can be weakened by skipping the first line. Assume $aff_{i1}(x_1, x_2) \leq aff_{i3}(x_1, x_2)$. This is equivalent to

$$\frac{ax_1 - x_2 - b_1}{at_1 - t_2 - b_1} \leq \frac{x_1 + ax_2 - b_2}{t_1 + at_2 - b_2} \tag{49}$$

A straightforward reformulation of inequation (49) results in the condition

$$x_2 \leq \frac{ab_2 - b_1 - t_2(a^2+1)}{b_2 + ab_1 - t_1(a^2+1)}x_1 + \frac{t_2(b_2 + ab_1) - t_1(ab_2 - b_1)}{b_2 + ab_1 - t_1(a^2+1)} \tag{50}$$

which is exactly the second part of condition (48). Note that the expressions of all steps of this reformulation are logically equivalent. Similarly, the assumption $aff_{i1}(x_1, x_2) \leq aff_{i4}(x_1, x_2)$ leads to the condition

$$x_2 \leq \frac{b_2 + ab_1 + d_2 - t_2(a^2+1)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)}x_1 + \frac{t_2(ab_2 + b_1 - ad_2) - t_1(b_2 + ab_1 + d_2)}{ab_2 + b_1 - ad_2 - t_1(a^2+1)} \tag{51}$$

and this is the third part of condition (48). Finally, to complete the proof it has to be shown that $aff_{i1}(x_1, x_2) \leq aff_{i2}(x_1, x_2)$. This assumption leads to the condition

$$ax_1 - x_2 - b_1 \leq at_1 - t_2 - b_1 \tag{52}$$

which states that the points on line l_1 are located below a line parallel to l_1 and through t which is trivially true. Condition (52) describes the fact that the two inclined planes defined by $aff_{i1}(x_1, x_2)$ and $aff_{i2}(x_1, x_2)$ intersect at that line and that for points below the line $aff_{i1}(x_1, x_2) \le aff_{i2}(x_1, x_2)$ holds and for points above it $aff_{i1}(x_1, x_2) \ge aff_{i2}(x_1, x_2)$. In total we have proved condition (48). For a complete proof the same must be done for the triangles Δ_2, Δ_3, and Δ_4. However, these cases are similar to the case of Δ_1 and are left to the reader.

Bersini's original definition of the affinity function can be considered as a special case of the function given by equation (45). We just have to insert for the parameters in (45) the special values that characterize Bersini's definition: $a = 1$, $d_1 = d_2 = 2L$, and $t = (2c_1 - i_1, 2c_2 - i_2)$. Lines l_1 and l_3 (cf. figure 5) both go through the point $(2c_1 - i_1, 2c_2 - i_2 - L)$, thus we get the equations

$$b_1 = (2c_1 - i_1) - (2c_2 - i_2 - L) \quad b_2 = (2c_1 - i_1) + (2c_2 - i_2 - L) \tag{54}$$

Inserting all these values in equation (45) yields

$$aff_i(x_1, x_2) = h \cdot \mathrm{min}\left\{ \frac{x_1 - x_2 - b_1}{t_1 - t_2 - b_1}, \frac{2L - (x_1 - x_2 - b_1)}{2L - (t_1 - t_2 - b_1)}, \frac{x_1 + x_2 - b_2}{t_1 + t_2 - b_2}, \frac{2L - (x_1 + x_2 - b_2)}{2L - (t_1 + t_2 - b_2)} \right\} \tag{55}$$

Since $t_1 - t_2 - b_1 = 2c_1 - i_1 - (2c_2 - i_2) - (2c_1 - i_1 - (2c_2 - i_2) - L) = L$ and $t_1 + t_2 - b_2 = 2c_1 - i_1 + 2c_2 - i_2 - (2c_1 - i_1 + 2c_2 - i_2 - L) = L$ equation (55) can be transformed to

$$\begin{aligned} aff_i(x_1, x_2) &= h/L \cdot \mathrm{min}\left\{ \begin{array}{l} x_2 - x_1 - (2c_2 - i_2) + 2c_1 - i_1 + L, \\ 2L - (x_2 - x_1 - (2c_2 - i_2) + 2c_1 - i_1 + L), \\ x_2 + x_1 - (2c_2 - i_2) - (2c_1 - i_1) + L, \\ 2L - (x_2 + x_1 - (2c_2 - i_2) - (2c_1 - i_1) + L) \end{array} \right\} \\ &= h/L \cdot \mathrm{min}\left\{ \begin{array}{l} (x_2 - x_1) + L - (2c_2 - i_2) + (2c_1 - i_1), \\ -(x_2 - x_1) + L + (2c_2 - i_2) - (2c_1 - i_1), \\ (x_2 + x_1) + L - (2c_2 - i_2) - (2c_1 - i_1), \\ -(x_2 + x_1) + L + (2c_2 - i_2) + (2c_1 - i_1) \end{array} \right\} \end{aligned} \tag{56}$$

Considering the regions where $aff_i(x_1, x_2) = 0$ we get from equation (56) the lines of equations (4), (3), (2), and (5) in this order. This shows that Bersini's affinity function is just a special case of that of equation (45).

5. SOME EXPERIMENTAL RESULTS

Bersini not only presented a new approach to the definition of affinity in his paper but also a simulation procedure which was aimed to support the self-assertion view. The procedure is based on the total affinity that is exerted by all immune elements on some element j. Let (x_1, x_2) be the position of that element. The total affinity on j is defined by

$$Aff_j = \sum_i aff_i(x_1, x_2) \tag{57}$$

The simulation procedure tries to keep the concentration of an immune element j between the two bounds *low* and *high*. It controls the concentration by the following steps

if $low < Aff_j < high$ then $C_j(t) = C_j(t) + 1$

else $C_j(t) = C_j(t) - 1$

if $C_j(t) = 0$ then *element j is eliminated*

A new element is added at each time step with some initial concentration $C_j(0)$. In contrast to antibodies, the concentration of antigens can only decrease according to the following rules:

if $low < Aff_j$ then $C_j(t) = C_j(t) - k*(Aff_j/low)$

if $C_j(t) = 0$ then *antigen j is eliminated*

k is a time rate. New elements are added to the system at random positions. We implemented the system in Matlab which is well suited for simulations and graphical output. Depending on the values of the parameters *low*, *high*, and k, the system typically evolves in such a way that a line develops consisting of immune elements with high concentration which divides the shape-space in two or more sections. Neighboring sections have the property that in one of them immune elements, in particular antigens, can survive, a so-called *tolerant zone*, while in the other the elements are removed, a so-called *reactive zone*. Bersini interpreted this result as a confirmation of the self-assertion view of the immune system. Figure 7 shows a typical result of this kind, achieved after 22.000 simulation steps.

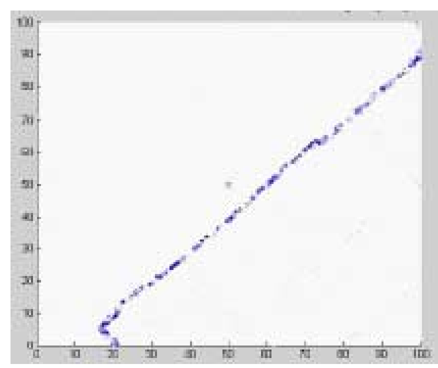

Figure 7. The result of a simulation run with the original affinity function

The dark (blue) points on the line indicate the antibodies with high concentrations. On the right hand side below the line is the tolerant zone. It still contains other immune elements but with low concentration compared to that of the elements on the line. We were interested in the behavior of the system when operating on the various affinity functions defined in the previous sections. We emphasized the course of the separating lines depending on the form and the orientation of the affinity region by straight (red) lines. Also, the respective affinity regions are added to the figure. Figure 8 shows an alternative case for the rhombus like region.

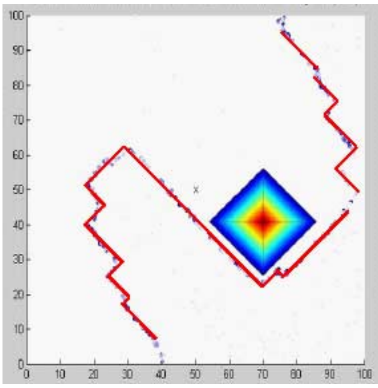

Figure 8. The result of a simulation run with the rhombus as affinity region

The affinity region can be rotated as defined by the function of equation (15) by an arbitrary angle. For an angle of 45° we get an axes parallel square. The result of the simulation is shown in figure 9. For the affinity function of equation (10) (axes parallel rectangle) a typical result of the simulation is that of figure 10.

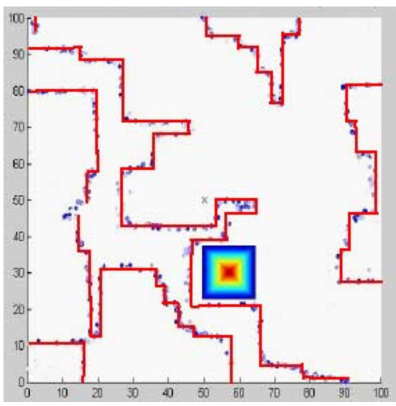

Figure 9. The result of a simulation run with the affinity region as a square

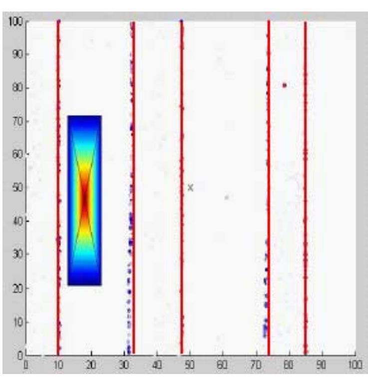

Figure 10. The result of a simulation run with the rectangle as affinity region

For an affinity function defined as a cone (cf. equation 24) (or a paraboloid), i.e. affinity functions with circular affinity region, the separating lines are no longer straight, rather they have the form of curves like that of figure 11.

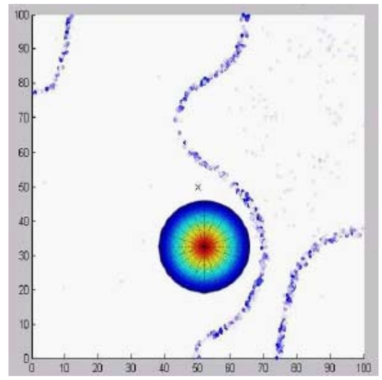

Figure 11. The result of a simulation run with a circular affinity region

From figures 8 to 11 it is obvious that the form of the separating lines strongly depends on the form of the affinity region. This is confirmed by the results of the simulation runs with the affinity functions of equations (45), i.e. the asymmetric functions forming lopsided pyramids. Figure 12 shows a result for an affinity function of this type. Also for these functions the separating lines obviously depend only on the form and the size of the affinity region.

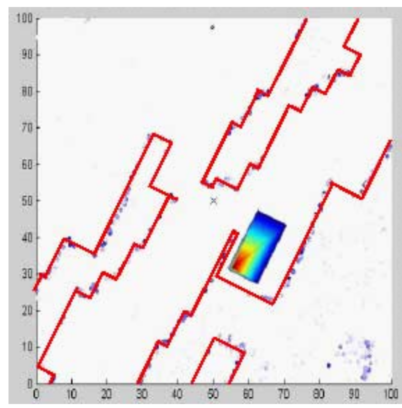

Figure 12. The result of a simulation run with a lopsided pyramid as affinity function

A closer look on the results of the simulation runs reveals that the tolerant zone and the reactive zone are closely related to each other. Actually, the reactive zones are point symmetrical to the corresponding tolerant zones and are even identical to the tolerant zones in their sizes except for a band of width L (the parameter that determines the size of the affinity region). The rest of the reactive zone can be easily seen as being identical to the corresponding part of the tolerant zone as figures 13 and 14 show. Bersini wondered "why a completely symmetrical simulation leads to unsymmetrical outcome" [1]. But actually, the outcome is symmetrical in the sense described here, and the fact that it is point symmetrical to the center of the space is a direct consequence of the complementarity based affinity function.

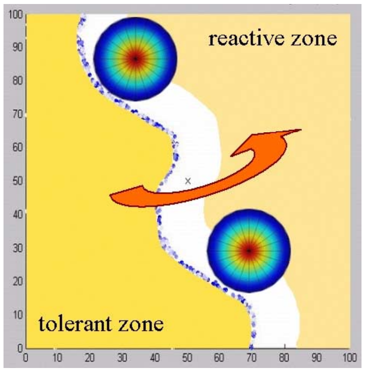

Figure 13. Correspondence between tolerant and reactive zones for a circular affinity region

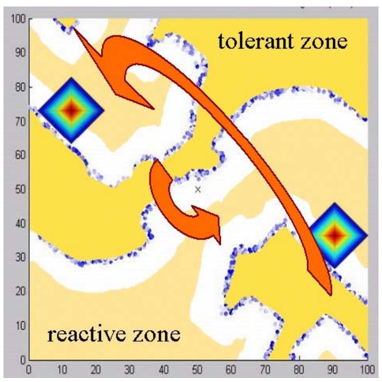

Figure 14. Correspondence between tolerant and reactive zones for a rhombus like affinity region

Bersini interpreted his experiment in the context of a simulation of the immune system with the aim to support the self-assertion view. But he also mentioned possible applications in AIS. We think that the results of the simulation runs can be considered as classifiers. The separating lines that are produced give a clear distinction between to classes of entities, those in the reactive zone and those in the tolerant zone. The separating lines can be considered as approximations to piecewise linear functions if the underlying affinity function uses a Manhattan metric.

Look for instance at figure 8 or 9. From the coordinates of the antibodies on the separating line the definition of the linear pieces can be easily derived. The composition of these pieces describes the separating line. By means of this line for each point in the space it can be determined to which of the two zones or classes it belongs. If for instance the shape-space is taken as a representation of a two-dimensional data set, the separation represents a classification of the set into two classes that was achieved by the simulation. Thus, the simulation can be considered as a training process.

This type of separation of the data space by a number of linear equations is similar to one that can be achieved by a decision tree procedure. Figure 9 is an example for an axes parallel separation, while figure 8 shows a non-axes parallel separation. The form of the separation depends on the form and the size of the affinity region, as has been shown in this paper, but also on the training examples, i.e. the antigens and antibodies. This was already described by Bersini who noticed that the results of his experiments strongly depended on the positions of the antigens that were introduced.

6. CONCLUSION

We have tried to reveal some of the properties of Bersini's experiment on self-assertion. In the center of this experiment is the affinity function. We have shown that and how the results of the simulation runs strongly depend on that function. The function implicitly defines a certain region of the shape-space, the affinity region. This region has to be taken into account because it is exactly that part of the shape-space where the affinity function has positive values.

Bersini used a Manhattan distance function (or metric) for his definition; therefore his shape-space is a Manhattan space. But this is not a necessary restriction, rather the affinity function can be generalized in different ways, for instance using Euclidian metric as we have done in the definition of equation (23). The function need not have a maximum at the center of that region as in Bersini's original function, it may have the maximum at an arbitrary point inside the affinity region.

We believe that Bersini's affinity function has further interesting properties which are worth an investigation. For instance, the two zones, at least the tolerant zone, may have an internal structure, i.e. subzones with varying concentrations of the antibodies. We assume that the concentrations of antigens and antibodies may have a typical temporal course which is different for those in the reactive zone and the tolerant zone and in particular for the antibodies in the separating lines. One could also wonder what would happen if an antigen with very high concentration is introduced into the shape-space, once the separation of the space has been established. It should have some influence and, according to the algorithm, could even lead to a reshaping of the separation. Finally, a more practical problem is: Can the Bersini experiment possibly be used as a data mining method? Which could be its advantages, how would it deal with standard problems in that field? For this purpose, however, it should be extended to higher dimensions.

7. REFERENCES

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