A New Multi-Objective Evolutionary Algorithm for Solving High Complex Multi-Objective Problems

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ABSTRACT
In this paper, a new multi-objective evolutionary algorithm for solving high complex multi-objective problems is presented based on the rule of energy minimizing and the law of entropy increasing of particle systems in phase space, Through the experiments it proves that this algorithm can quickly obtains the Pareto solutions with high precision and uniform distribution. And the results of the experiments show that this algorithm can avoid the premature phenomenon of problems better than the traditional evolutionary algorithm because it can drive all the individuals to participate in the evolving operation in each generation.

Categories and Subject Descriptors
F.2 [Analysis of Algorithms and Problem Complexity]: Design and Analysis of Algorithm.

General Terms
Algorithm, Design, Theory.

Keywords
Evolutionary Algorithm, Multi-Objective Optimization Problem, Free Energy, Entropy, Pareto Front.

1. INTRODUCTION
The classical method[1][2] of solving multi-objective optimization problems is, first, to transfer the multi-objective problems into a single-objective problem by introducing some parameters before searching, then, by adjusting these parameters, to solve for different optimal solutions of the single-objective problem, and finally to find the optimal Pareto set of the multi-objective problems. But traditional evolutionary algorithms easily fall into the local solution areas and lead to the creation of the premature phenomenon of optimization problems. In this paper a new multi-objective evolutionary algorithm for solving high complex multi-objective optimization problems precisely and efficiently is presented based on the rule of energy minimizing and the law of entropy increasing of particle systems in phase space. Because of these properties, the algorithm ensures that all the particles have a chance to take part in the cross-over and mutation in all the iteration time, and solves for the global optimal solutions of the problem in the end.

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2. ENERGY AND ENTROPY OF PARTICLE
According to transportation theory[3], we assume that a closed system is composed of two open subsystems that may exchange energy and particles so that the entropy of the system increases, i.e. $S = S_1 + S_2$, where S_1 and S_2 denote the entropies of the first and second systems, respectively. We assume that the relationship between the micro-state number of the micro-canonical ensemble and the entropy function is $S = f(\Delta\Omega)$, and the two subsystems are independent of each other. As a result, $S = k_B \ln \Delta\Omega$, where k_B is called the Boltzmann constant. According to the entropy equilibrium equation and Boltzman H-theorem, we know that the entropy function is a monotonically increasing function of time in a closed system.

Now, we introduce the Helmholtz free energy $F = E - TS$, where E, T, and S are the average internal energy, temperature, and entropy, respectively. If the system is under a non-equilibrium state and is not influenced by other outside constraints, its Helmholtz free energy will decrease and F will be the minimum, and finally the particle system reach the equilibrium state.

3. ALGORITHM DESIGN
We consider the following multi-objective optimization problem[4]:

$$\min \mathbf{y} = \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \Lambda, f_k(\mathbf{x}))$$
$$\mathbf{e}(\mathbf{x}) = (e_1(\mathbf{x}), e_2(\mathbf{x}), \Lambda, e_m(\mathbf{x})) \le 0, \mathbf{x} = (x_1, x_2, \Lambda, x_n) \in \mathbf{X}$$
$$\mathbf{X} = \{(x_1, x_2, \Lambda, x_n) \mid l_i \le x_i \le u_i, i = 1,2, \Lambda, n\}, \mathbf{l} = (l_1, l_2, \Lambda, l_n)$$
$$\mathbf{u} = (u_1, u_2, \Lambda, u_n), \mathbf{y} = (y_1, y_2, \Lambda, y_k) \in \mathbf{Y}$$

where $\mathbf{x}$ is a decision vector, $\mathbf{y}$ an objective vector, $\mathbf{X}$ the decision vector space, Y the objective function vector space, $\mathbf{l}$ and $\mathbf{u}$ are the lower and upper bounds of the decision vector

space, and **e(x)** is the constrained function vector. We consider individuals of a population as particles in the phase space.

Algorithm Flow:

Step 1. Set $t=0$, and generate an initial population $P_0=\{\mathbf{x_1}(t),\mathbf{x_2}(t),\Lambda\ ,\mathbf{x_N}(t)\}$ at random.

Step 2. For particle $\mathbf{x}_i$, $i=1,2,\Lambda\ ,N$, select randomly a particle $\mathbf{x}_j$ in the population, $1\le j\le N\wedge j\ne i$, calculate the entropy: $S_i=\sum_{j=1}^{N}\left\|\mathbf{y_i}-\mathbf{y_j}\right\|_p$ and free energy: $p_i(t,\mathbf{f}(t))=\frac{1}{|\mathbf{f}(t)|}\left(\left|\sum_{j=1}^{k}(f_j(t,\mathbf{x}_i(t))-f_j(t-1,\mathbf{x}_i(t-1)))\right|\right)$, and then calculate the i^{th} particle's rank function value according to the following comparative regulations:

$if\quad((\mathbf{x}_i\,\pi\,\mathbf{x}_j)\vee(\mathbf{x}_i\,\pi{=}\mathbf{x}_j)\vee(\mathbf{x}_i\Leftrightarrow\mathbf{x}_j\wedge s_i>$
$s_j)\vee(\mathbf{x}_i\Leftrightarrow\mathbf{x}_j\wedge s_i=s_j\wedge p_i(t,\mathbf{f}(t))<=$
$p_j(t,\mathbf{f}(t)))\vee(\mathbf{x}_i\sim\mathbf{x}_j))$

then $\mathrm{Rank}(\mathbf{x}_i(t))=\mathrm{Rank}(\mathbf{x}_i(t))+1$;

else, $\mathrm{Rank}(\mathbf{x}_i(t))=\mathrm{Rank}(\mathbf{x}_i(t))$.

Step 3. Taking all the particles solved according to the rank function values sorted in the order from large to small as the evolving pool $\widetilde{P}_t=\{\widetilde{\mathbf{x}}_1(t),\widetilde{\mathbf{x}}_2(t),\Lambda\ ,\widetilde{\mathbf{x}}_N(t)\}$, select the biggest M particles $\widetilde{\mathbf{x}}_1(t),\widetilde{\mathbf{x}}_2(t),\Lambda\ ,\widetilde{\mathbf{x}}_M(t)$ on the forefront of the rank function values from the evolving pool, use Tao Guo' algorithm to do the crossover and mutation and generate l new particles.

Step 4. Add the newly generated l particles into the population P_t and get P_t' .

Step 5. Set $t=t+1$ and $N=N+l$ if the stopping criterion is satisfied, i.e., if the iteration time $t>T$ or the algorithm free energy $P(t,\mathbf{f}(t))=\frac{1}{|P_t|}\sum_{i=1}^{N}p_i(t,\mathbf{f}(t))<\varepsilon$, output P_t as the optimal Pareto set, and the calculated objective vector set corresponding to P_t is the optimal Pareto solutions. Otherwise, return to Step 3.

4. NUMERICAL EXPERIMENTS

In the experiments, four typical benchmark functions (ZDT1~ZDT4)[5] are tested for solving high complex multi-objective optimization problems by using this new algorithm in order to prove the computing performance of the algorithms. The results are as follows,

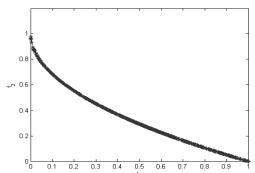

Figure 1: ZDT1

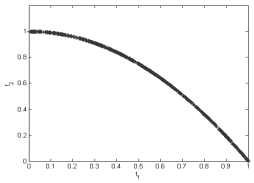

Figure 2: ZDT2

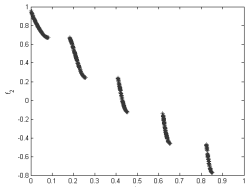

Figure 3: ZDT3

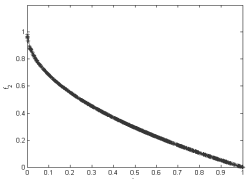

Figure 4: ZDT4

From above we can see that, by using this new algorithm we can not only solve for convex optimal Pareto front (Fig.1) and concave optimal Pareto front (Fig.2), but also can solve for discrete optimal Pareto front (Fig. 3). The convergent speed is more quickly and the performance is better than traditional multi-objective evolutionary algorithms.

5. CONCLUSIONS

Through above theoretical and experimental analysis, we conclude that the new multi-objective evolutionary algorithm we constructed has obviously improved the performance of the traditional EAs. The reason is that this new algorithm has combined with the rule of free energy minimization and the law of entropy increasing of particle system in the phase space, and then drives all the particles to participate in the crossover and mutation all the time so as to obtain the Pareto solutions quickly and precisely and avoid the premature phenomenon.

6. REFERENCES

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