Genetic Algorithms and Mixed Integer Linear Programs for Optimal Strategies in a Student's "Sports" Activity

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ABSTRACT

This paper uses an entertaining student "sports" game to illustrate that GAs can be adapted to problems with uncertain properties and complexity. These problems can be solved easily through GAs within a few seconds. Contrary to this, using standard MILP techniques does not yield results in a reasonable time.

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General Terms: Algorithms, Performance

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1. INTRODUCTION

Metaheuristics like genetic algorithms GAs [4, 5], simulated annealing [6], tabu search [3], and other metaheuristics [2] have been applied to a wide range of problems. The key benefit of such metaheuristics is their ease of use and their good performance for a large number of different problems. In order to apply metaheuristics it is sufficient to define a fitness function that assigns fitness values to different solutions of a problem and to develop a representation search operators can be applied to. One of the large advantages of GAs is that they find good solutions if only limited structural knowledge is available.

Therefore, this paper uses the metaheuristic approach to optimize the non-trivial optimization problem of a "beer-run". In this setting, various and contradicting factors have to be considered, as the subsequent description of the problem shows.

2. OPTIMIZATION PROBLEM

In a "beer-run", a team of two has to cover a certain distance as fast as possible, while carrying a case of beer. There is the additional constraint that all beer has to be consumed by the two people before reaching the goal. Finding optimal strategies for the game is difficult as several counteracting effects occur. For example, due to consuming a beer the weight that has to be carried by the two persons decreases which leads to a speed increase. In contrast, the blood alcohol level (BAL) slowly increases over time which leads to a lower speed of the person drinking. Furthermore, there are tight restrictions for the objective function, since it is necessary to cover a given distance while consuming all the beer.

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The "beer-run" is a representative example for optimization problems where people (for example firefighters) have to move in hazardous and poisonous environments (for example polluted air or dangerous smoke) that affects their physical abilities. On the one hand inhaling polluted air is necessary to survive but on the other hand it reduces the speed (and physical abilities) of the humans. The "beer-run" can be seen as a formalized model for such situations and finding optimal strategies on when inhaling polluted air can be important.

2.1 Objective Function and Variables

The goal of the game is to travel a given distance d as fast as possible. There are two persons that have to carry a case of beer with n^b bottles. Each bottle contains beer of weight c^b. At each time point t the person j can decide to consume a bottle of beer, or not. $x_{t,j}$ is a binary variable. For $x_{t,j} = 1$, person j drinks a beer at time t. The maximum number of bottles that can be consumed by the two persons in the group are n^b. Drinking a bottle of beer has two effects. The weight c_t of the case decreases allowing the team to walk with a greater speed. $v_{t,j}$ indicates the speed of person j at time t. It is assumed that there is a maximum speed v_j^{max} for each person which is reached if no additional weight has to be carried. When carrying an additional weight, v_j^{max} is reduced by Δv_j^w for each additional kilogram of weight that has to be carried. As consuming a bottle of beer reduces the weight c_t of the case at time t, the speed $v_{t,j}$ of person j at time t increases. Furthermore, we assume that the two people carrying the case stay together and the speed of the team is equal to the speed of the slowest person. The second effect of consuming a bottle of beer is an increase of the blood alcohol level (BAL) which leads to a decrease of the speed of the person j that has consumed a bottle of beer. Δp_j denotes the increase of BAL (in parts per thousands) of the person j due to drinking one bottle of beer. The increase of the BAL leads to a decrease of the speed of the person. Δv_j^p denotes the decrease of the maximum speed v_j^{max} of person j if the blood alcohol level $p_{t,i}$ increases by one part per thousands (ppt).

For calculating the impact of consuming a beer on the BAL, we assume that the alcohol level increases linearly over one hour after the consumption [7] and then remains at this level during the whole walking period (until both persons reach the goal). The peak alcohol level is calculated according to the Widmark-Formula [7].

Consequently, a team of two "beer runners" has to decide when to have a drink (overall they have to drink n^b bottles) and which one the two team members drinks the bottle.

2.2 A Mixed Integer Linear Model

The problem of finding an optimal strategy can be formulated as an mixed integer linear problem. Due to space restrictions of this paper we refer to our technical report for further details [1]. To model the problem, we introduce an additional binary variable r_t which indicates if the two persons have reached the goal after time t ($r_t = 0$ means they reached the goal at time t). Therefore, the objective of the MILP is to minimize the sum of all r_t.

3. GENETIC ALGORITHM

For the problem at hand, we choose the following GA design. We use a simple standard GA [4] with population size N, fitness-proportional selection, standard one-point crossover, and bit-flipping mutation. We encode each solution using a genome that consists of a vector of n^b tuples. Each of the $i \in \{0, \ldots, n^b\}$ tuples consist of an integer number $t \in \{0, \ldots, t_{max}\}$ which indicates the time t a team member is having a drink and a binary variable $j \in \{0, 1\}$ which indicates which of the two team members is having the drink at that time. The one-point crossover operator is applied to two randomly chosen solutions, randomly selects a cutting point, and exchanges the sub-strings between both parental solutions. Mutation randomly changes t_i or j_i with some probability p_{mut}. The initial population of the GA is generated randomly assigning random values to the t_i and j_i. Only one person can consume a beer at time t invalid solutions can occur if $t_i = t_l$, where $i \neq l$. Therefore, we use an additional repair operator that ensures that only valid solutions can be created in the initial population and as result of the crossover and mutation operator.

For the GA, the calculation of the fitness is done according to the models described in [1]. Based on the models we implemented a discrete event simulator calculating the BAL $p_{t,j}$ and weight c_t of the case at every time t (the standard resolution was minutes). Based on these values the speeds $v_{t,j}$ are determined at each time point. As before, the BAL $p_{t,j}$ of person j depends on the previous BAL $p_{t-1,j}$ and Δp_j and the number of drinks in the previous hour. The weight of the case is calculated using the initial weight of the case and subtracting the weight of one beer for every drink consumed by one of the two team members. The speed depends on the speed decrease caused by the BAL and the remaining weight of the case.

4. RESULTS

We evaluate the performance of the GA and a MILP solver for the problem defined in Section 2. The maximal time allowed to reach the goal is $t_{max} = 180min$ ($t \in \{0, \ldots, 180\}$). For the experiments we use a resolution of one minute. Therefore, we have $2 \times 180 \times res$ decision variables $x_{t,j}$ for the MILP from Section 2.2 and the cardinality of the t_i for the GA is $t_{max} \times res$. We assume that males have a body weight of 80kg and females a body weight of 55kg. Furthermore, we assume a maximum running speed (with the empty case) of $v_j^{max} = 6km/h$. The speed decrease per kg additional weight is set to $\Delta v_j^w = 1/6 \frac{km}{hkg}$ and the speed decrease per BAL (in ppt) is set to $\Delta v_j^p = 1 \frac{km}{hppt}$. Each beer has a volume share of 5% alcohol.

Although we have not been able to solve the problem using CPLEX, we can use the MILP to verify that the solution that has been found by the GA is optimal. In the optimal solution found by the GA the team reaches the goal after 151 minutes. Therefore, we added the constraint $t_i = 0$ for $i > 152$ and CPLEX then solved the problem in a few minutes and confirms that the solution found by the GA is the optimal solution for the problem.

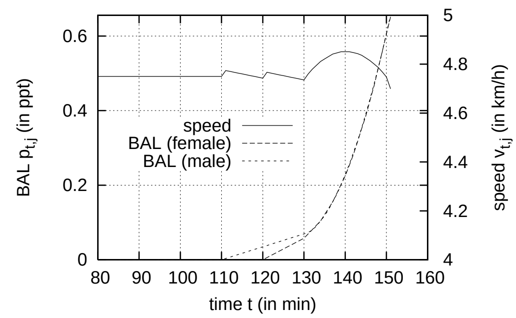

Figure 1: BAL and speed over time

Figure 1 shows the optimal strategy for a team of two different persons. We use $\Delta v_j^w = 1/6$ and assume a team that consists of a male with a body weight of $80kg$ and a female with a body weight of $55kg$. The optimal strategy allows the team to reach the goal after 153min. The plots reveal that the optimal strategy is to have the same BAL for both persons.

5. CONCLUSIONS

This paper applied two different optimization approaches, a MILP solver and a simple GA, to a specific problem denoted as “beer-run”. The problem was chosen as it has not been studied before and neither information about the properties of the problem nor its difficulty exists. The paper related the problem to similar problems from other domains and developed a MILP of the problem as well as a GA. The development of the different solution approaches (MILP versus GA) showed that it is possible to develop a GA that yields good results even if only little is known about the problem. Studying the performance of the two different approaches revealed that simple variants of the problem that can be solved to optimality by the GA in a few seconds can not be solved by the MILP solver with reasonable effort.

6. REFERENCES

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