

# Optimal Operating Conditions for Overhead Crane Maneuvering Using Multi-objective Evolutionary Algorithms

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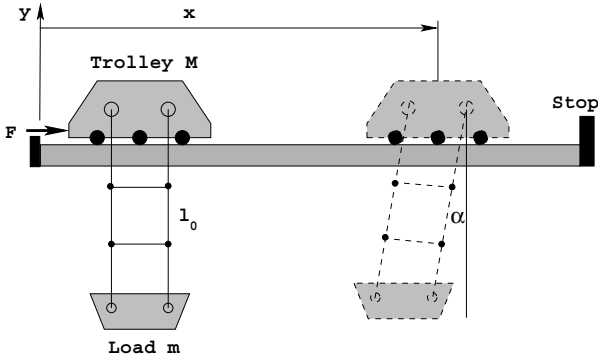
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**Abstract.** While operating a crane for maximum productivity, the time of operation and the required energy are two important conflicting factors faced by a crane operator. In such a case, trying to reach the destination too quickly demands a large energy supply, while a small powered motion requires longer time. In this paper, we consider such a problem for two different pairs of objectives and employ a multi-objective genetic algorithm for the task. Besides finding a set of trade-off optimized solutions (operating conditions), an analysis of these solutions reveals salient operating principles, which would be difficult to achieve by other means. The methodology demonstrated in this paper can be used for other similar engineering design and application problems.

**Keywords:** Crane maneuvering, Multi-objective GAs, Optimal trade-off, Dynamics of cranes.

## 1 Introduction

Overhead cranes are used in the industries, workshops, factories, docks, and other places to transport the heavy components from one place to another. In order to increase the productivity, individual such operation must be optimized to find what speed the overhead trolley must be moved so that the supplied energy to the crane and the overall operation time are minimum. The overall operation time has two components: (i) the trolley time which denotes the time needed for the trolley to move from the starting position to the destination point and (ii) the sway time which denotes the time needed by the hanging load to damped out its oscillation to a critical acceptable limit. Although not obvious, these two objectives have a conflicting effect. If an operator tries to reach the destination too fast (by spending too much energy) the trolley time will be saved, but the sudden stopping of the trolley will cause the remaining energy to be transferred to the hanging load, thereby starting a large-amplitude oscillation to the hanging



**Fig. 1.** A schematic of the overhead crane consisting of a trolley and a swaying load.

load. On the other hand, a careful and slow motion towards the destination point will, though not impart a large-amplitude oscillation to the hanging load, require a larger trolley time. Thus, it is important to know what and how to move the trolley right from the starting position so that certain goals are achieved.

Although there exist a number of classical multi-objective optimization techniques [8,1,6], multi-objective evolutionary algorithms (EMO) [3,9,2] have gained tremendous popularity in solving different kinds of engineering problems. In this paper, rather than finding one solution to the problem, we employ a multi-objective genetic algorithm — the elitist non-dominated sorting GA or NSGA-II [4] — to first find a set of trade-off optimal solutions for two conflicting objectives of operation. Thereafter, the obtained solutions are analyzed to reveal important operating principles for the task. By considering two different pairs of objectives, important information about the optimal crane operations are found. The methodology used in this study can be followed in handling similar other engineering problems.

## 2 Modeling the Dynamics of Crane Operation

Figure 1 shows a schematic model of the crane used in this study. In this simplified model, the cable connecting the trolley and the hanging load is considered of fixed length, however in practice such cables are varied in length while moving in order to lower or raise the load [5]. The fixed length assumption reduces one degree-of-freedom of the system, however, a similar study without this assumption can also be made. The system has two degrees-of-freedom: (i)  $x$  denoting the linear motion of the trolley along  $x$ -direction and (ii)  $\alpha$  denoting the angular motion of the hanging mass.

In the model, we assume that there is a time-varying force  $F(t)$  applied to the trolley in the direction of the destination (along positive  $x$ -direction). The trolley experiences a friction force  $f = \mu N$  ( $\mu$  being the coefficient of friction between the trolley and the guide and  $N$  is the normal force) opposite to its

motion. By considering the force balances in  $x$  and  $y$  directions of all forces acted on the trolley, we obtain the following two equations:

$$\begin{aligned} M\ddot{x} &= F + 2T \sin(\alpha) - \mu N, \\ N &= Mg + 2T \cos(\alpha), \end{aligned}$$

where,  $T$  is twice the tension in each cable,  $M$  is the mass (in kg) of the trolley, and  $\ddot{x}$  is the acceleration of the trolley in the  $x$ -direction. Performing a similar task for the hanging load (of mass  $m$  kg), we have the following two equations:

$$\begin{aligned} -2T \sin(\alpha) - c\dot{x}_1 &= m\ddot{x}_1, \\ 2T \cos(\alpha) - c\dot{y}_1 - mg &= m\ddot{y}_1, \end{aligned}$$

where,  $c$  is the coefficient of damping arising due to several factors on to the cable and the hanging mass. The variables  $x_1$  and  $y_1$  are the displacement of the hanging load in the  $x$  and  $y$  directions, respectively.

In addition, the following relationships between trolley and the hanging load motions can be written with variables along  $x$  and  $y$  directions:

$$\begin{aligned} x_1 &= x + l_o \sin(\alpha), & y_1 &= -l_o \cos(\alpha), \\ \dot{x}_1 &= \dot{x} + l_o \dot{\alpha} \cos(\alpha), & \dot{y}_1 &= l_o \dot{\alpha} \sin(\alpha), \\ \ddot{x}_1 &= \ddot{x} + l_o \ddot{\alpha} \cos(\alpha) - l_o \dot{\alpha}^2 \sin(\alpha). & \ddot{y}_1 &= l_o \ddot{\alpha} \sin(\alpha) + l_o \dot{\alpha}^2 \cos(\alpha). \end{aligned}$$

Here,  $l_o$  is the length of the cable,  $\dot{\alpha}$   $\ddot{\alpha}$  are the angular velocity and acceleration of the cable. By eliminating  $T$ ,  $x_1$  and  $y_1$  from the above expressions, we get the following two equations of motion of the trolley and the hanging mass:

$$\begin{aligned} \ddot{x} &= [F - c\dot{x} \sin^2(\alpha) + ml_o \sin(\alpha) \dot{\alpha}^2 + mg \sin(\alpha) \cos(\alpha) - f(ml_o \cos(\alpha) \dot{\alpha}^2 \\ &\quad - c\dot{x} \sin(\alpha) \cos(\alpha) - mg \sin^2(\alpha))] / (M + m \sin^2(\alpha) - fm \sin(\alpha) \cos(\alpha))(1) \\ \ddot{\alpha} &= -(\ddot{x} + r\dot{x} + g \tan(\alpha)) \frac{\cos(\alpha)}{l_o} - r\dot{\alpha}, \end{aligned} \tag{2}$$

where,  $r$  is the ratio of  $c$  to  $m$ . These two equations can be solved using a numerical integration technique and the variation of  $x$  and  $\alpha$  with time can be found. Here, we use an adaptive scheme for stable solutions to the above equations.

### 3 Energy and Time Minimizations

A little thought over the problem makes it clear that the two objectives (i) total energy supplied to the system and (ii) the total time for the block-load system to reach at the desired position and stabilize are the two conflicting objectives. The supplied energy will be minimum for the case of moving ever slowly towards the destination. But such a solution will require quite a long time to complete the task. On the other hand, reaching the destination with a large velocity and suddenly stopping at the destination would be a quick way to reach the destination, however some time needs to be spent for the sway of the

load to diminish. Although such a solution may not be the quickest overall time solution, there would exist a solution with a reasonable velocity which would minimize the overall time.

In this paper, we use the following parameters for the crane system:

$$\begin{aligned} M &= 20,000 \text{ kg}, & m &= 30,000 \text{ kg}, & \mu &= 0.1 \\ l_o &= 25 \text{ m}, & c &= 50 \text{ N-s/m}. \end{aligned}$$

Here, we use NSGA-II [4] for minimizing the above two objectives. Constraints are handled using the constraint-domination approach suggested elsewhere [3]. We use the following NSGA-II parameters: (i) population size = 150, (ii) maximum number of generations = 1,000, (iii) probability of crossover = 0.9, and (iv) probability of mutation = 0.01.

The decision variables in the crane operating problem are the magnitude and sequence of application of forces on the trolley till it reaches the destination. To keep matters simple, we have used a force  $F_0$  and a sequence of Boolean variables to denote the application of the force. A typical NSGA-II solution is as follows:

$$(F_0 \quad (1110010100))$$

Each time step is assumed to be of  $\Delta t = 4$  sec duration. Thus, in the above example, the trolley reaches the destination in  $(10 \times 4)$  or in 40 sec. The force  $F_0$  is always applied at the beginning of the first time step. Thus, in the binary string representing the sequence of operation, the first bit is always a 1. It is clear that with the above representation scheme, every solution may have a different size of the binary string. To avoid this problem of coding, we maintain a fixed length (of large size,  $\ell_{\max} = 150$ ) string and use the front part of the string. Thus, a NSGA-II solution will have a maximum trolley time of  $150 \times 4$  or 600 sec. The motion of the trolley-load is simulated with the pattern of application of force as dictated by a string and the corresponding  $F_0$  and as soon as the trolley reaches the destination, the string is not used further.

The force parameter is also treated as a binary string of length  $\ell_F = 10$  initialized in the range [100, 1670] N. Thus, the total string length of a NSGA-II solution is  $\ell_F + \ell_{\max} = 10 + 150$  or 160. The overall binary string is operated by a single-point crossover and a bit-wise mutation [7].

Three different implementations are adopted for the energy-time minimizations:

**Approach 1:** The magnitude of force is varied and the pattern of the application of force is kept periodic, such as on, off, on, off, etc (or (1010 ...)).

**Approach 2:** The pattern of application of force is varied as a series of Boolean variables (on or off) and the force is kept to a constant value ( $F_0$ ).

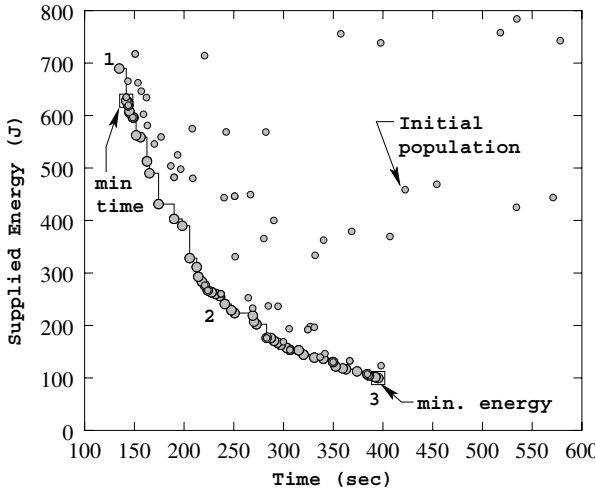
**Approach 3:** Both the pattern of application of force and the magnitude of force is varied.

In each case, the maximum string length for representing the pattern of force is kept to be 150. We discuss each of the above implementations and obtained results in the following subsections.

### 3.1 Approach 1: Variation of Magnitude of Force

Here, a fixed pattern of application of force is applied and the magnitude of the force is dictated by a GA solution. Thus, for each solution we keep track of the total work being done to move the trolley to the final destination and call it the energy to be supplied. The second objective is the total time required to reach the destination and to have the load sway to reduce to a permissible value. Here we call the system is stable if the angular sway is reduced to  $\alpha_c = 0.002$  rad.

Figure 2 shows the optimized non-dominated front, trading-off the two conflicting objectives. We observe that a small energy solution takes longer time



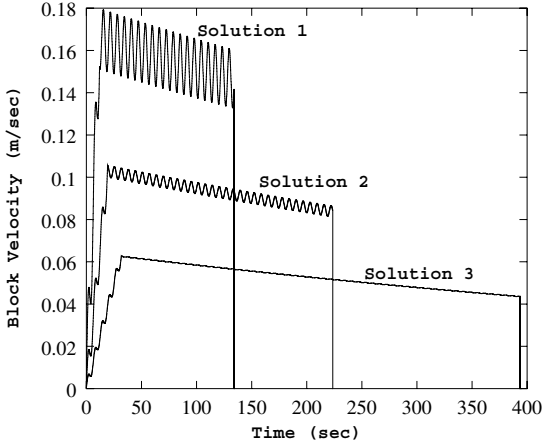
**Fig. 2.** Optimized trade-off solutions for Approach 1 are compared with the initial random population. Single-objective optimized solutions are also marked.

and a fast solution requires large energy. Although such a trade-off which was anticipated at the start of the study, the figure quantifies the terms and shows a number of such trade-off solutions. It then depends on the operator to choose a particular solution depending on the available time and energy at his/her disposal. The figure also shows the initial random population. This population is shown to get a clear idea of the extent of progress of NSGA-II in the objective space.

Figure 2 also shows two individual minima for the objectives. Since the individual minima comes on the the optimized front, it can concluded that the obtained NSGA-II solutions are the true non-dominated solutions or are very close to them.

To show the trade-off further, we choose three solutions from the optimized front (marked as 1, 2 and 3 in Figure 2) and show the time-variation of the

trolley's velocity with time in Figure 3. Since a periodic pattern is used for applying the force, a periodic variation in the velocity is observed. Solution 1 is the minimum-time solution and hence require a large energy. On the other



**Fig. 3.** The velocity variation of the trolley as it moves towards the destination for three different non-dominated solutions. The trade-off in their variations is clear.

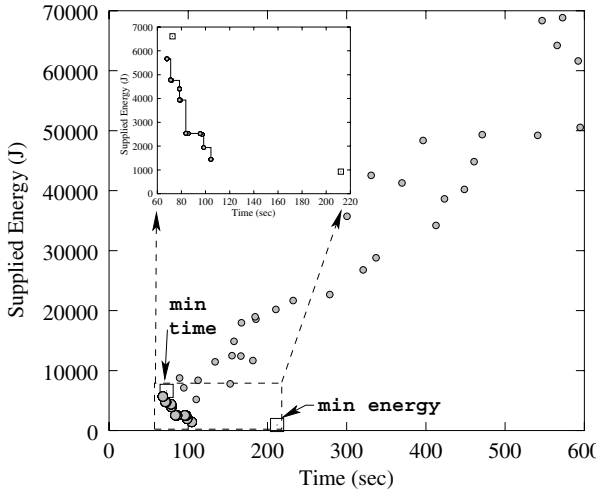
hand, solution 3 is the minimum energy solution. This solution suggests the smallest velocity of the trolley as it moves towards the destination, but requires the longest time to reach there.

### 3.2 Approach 2: Variation of Pattern of Application of Force

Here, we keep a constant force  $F_0 = 500$  N and vary the pattern of application of force. Such an implementation is quite practical as with a fixed energy source it can be assumed that the force applied to the trolley would be identical at different time steps and the user only needs to know with what sequence the force is to be applied.

Figure 4 shows the obtained optimized front for the same two objectives. The initial population is also marked. The inset figure shows the trade-off between supplied energy and the time more clearly. Individual minima are also shown in the plot. NSGA-II solutions are found to be non-dominated with these solutions.

Some interesting observations can be made when we investigate the force patterns, as shown in Table 1. With the increase in supplied energy the strings get smaller, meaning that the system reaches the destination quickly. In almost all solutions the force is continuously applied in the initial few time steps. Once the trolley has acquired the required energy to overcome the friction and other



**Fig. 4.** Optimized trade-off solutions for Approach 2 are compared with the initial random population. Single-objective optimized solutions are also marked.

dynamics, only occasionally the force is required to be applied. A general pattern for an optimal maneuvering seems to be to apply the force early on and let the frictional force reduce the motion of the trolley later on and till the trolley reaches its destination. Such a pattern is not hard-coded in NSGA-II. Such a property of the optimized solutions emerge as a desired mode of operation in an optimal manner. Such informations are useful to the operators and can be quite useful in real-world applications.

### 3.3 Approach 3: Variation of Force and Application Pattern

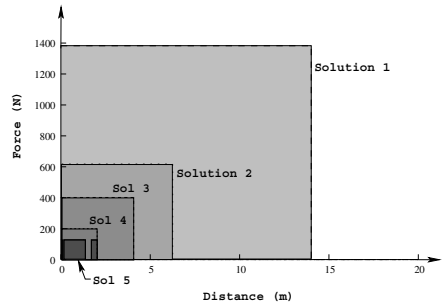
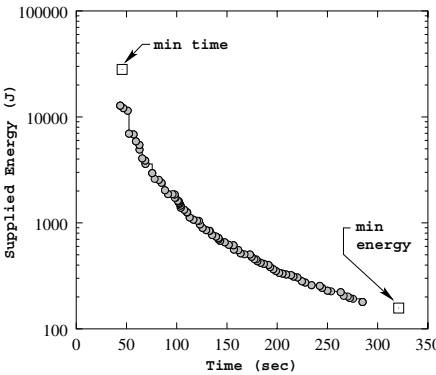
Next, we keep both the force and the application of force pattern as variables. Figure 5 shows the obtained non-dominated front. Once again, the individual minima (obtained using a single-objective GA with identical representation scheme and operators as in NSGA-II) are also shown on the plot. It can be observed that the NSGA-II frontier is non-dominated with these individual minima. Also, since the representation involves both  $F_0$  and force pattern, a wider non-dominated front is discovered compared to Figures 2 and 4.

In Table 2, we show the force and its application pattern for a few obtained trade-off solutions. It is clear that quicker solutions require more energy and a larger magnitude of force. Importantly, the force is required to be applied early on and then the trolley moves with its acquired energy till it reaches the destination. Smaller force solutions consume smaller energy and moves slowly to the destination. A similar pattern was also found in Approach 2.

To show the above pattern and force graphically, we have chosen five solutions from the entire range of the obtained solutions and the force variation is

**Table 1.** Trade-off solutions and corresponding patterns of application of force.

Time (sec)	Work (J)	Pattern
104.5	1440.446	111111000000000000000000
98.5	1938.642	1111110000001000000000
97.2	2487.745	10111100010001000000100
95.5	2520.041	111111000001000100000
85.8	2531.828	11111100011000000000
83.7	2539.399	11111101010000000000
79.0	3933.450	1111110000111001000
78.5	3937.728	1111110001111000000
78.4	4399.622	1111110001110001000
71.9	4759.727	11111110111000010
71.1	4779.473	111111111110001000
68.0	5670.620	11111111111001100



**Fig. 5.** Optimized trade-off solutions for Approach 3. Single-objective optimized solutions are also marked.

**Fig. 6.** Force versus distance moved by the trolley for five widely-distributed solutions on the obtained front.

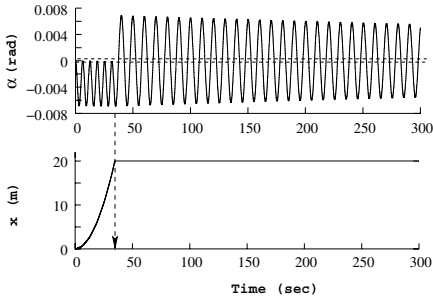
plotted with distance moved by the trolley in Figure 6. Since the area covered by such variations is related to the supplied energy, it is clear that the minimum time solution (solution 1 marked in Figure 6) requires the largest energy to complete the task, while solution 5 requires a small fraction of the energy needed in solution 1 to complete the task, but the time taken to reach the destination is 6.5 times more.

### 4 Trolley Time and Sway Time Minimizations

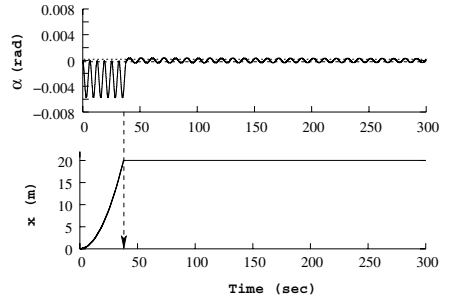
Now, we consider two components of the overall time of completion of the task: (i) minimization of the trolley time — time needed by the trolley to reach the



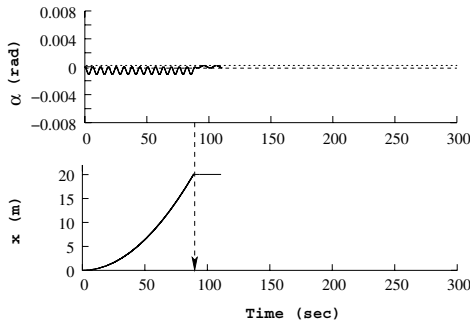




**Fig. 8.** Minimum trolley-time solution (sol. 1). Sway stabilizes after 4,248.1 sec.



**Fig. 9.** Intermediate solution (solution 2). Sway stabilizes after 870.6 sec.



**Fig. 10.** Minimum sway time solution (solution 3). Sway stabilizes after 0.3 sec.

**Table 3.** Objective values and corresponding solutions for a few of the non-dominated solutions.

Trolley Time (sec)	Sway Time (sec)	Force (N)	Pattern
34.6	4248.1	1673.460	(11111)
36.5	3800.2	1618.035	(11110)
37.8	1758.1	1413.269	(11111)
37.9	870.6	1404.032	(11111)
38.0	18.4	1402.492	(11111)
44.3	3.7	1031.451	(1111111)
88.9	0.3	260.117	(11100101011010011)

obtained trade-off front by NSGA-II, and two individual minima for the objectives obtained using a single-objective GA. A comparison with the individual minima indicates that the obtained NSGA-II solutions are non-dominated with the individual minima.

To get a better idea of the trade-off between these two objectives, we have shown the time-variation of the trolley and hanging load for three solutions – two extreme solutions and one intermediate solution on the obtained front in Figures 8 to 10. The bottom figure shows the variation of the trolley mass. It can be seen that when the trolley reaches 20 m destination mark, it is forced to stop. At that instant, the energy gets transferred to the hanging mass and it started to sway. Its motion decays due to the damping into the system. The critical angular displacements in positive and negative directions are marked with dotted lines in the figure.

The trade-off between the trolley time and the sway time is clear from these figures. Solution 1 requires smallest trolley time (34.6 sec), but gets into a larger amplitude oscillation which requires 4,248.1 sec to get damped to the critical  $\alpha_c$ . On other hand, solution 2 (intermediate one) takes slightly more trolley time (37.9 sec), but gets damped out to the limit within 870.6 sec. Solution 3 reaches the destination slowly requiring as large as 88.9 sec. But the slow arrival at the destination causes the hanging mass to get damped out to the limit almost immediately (in only 0.3 sec).

Table 3 shows the objective values and corresponding solutions for a few of the obtained trade-off solutions.

It can be observed that the applied force is inversely proportional to the elapsed trolley time, that is, for a quick (small time) arrival at the destination, more force (hence more energy) must be applied. Although most solutions require an early application of the force, as dictated by the pattern in the table, the smallest sway time requires a careful on-off application of the force till it reaches the destination.

If the time of completing the task is important, the summation of trolley and sway times can be optimized. For the solutions mentioned in the above table, the second-last solution seems to be the optimal solution. Although the above consideration of two-objective minimization would usually include this optimized solution, it also provides other useful information about the problem which would be useful to the operators or users.

## 5 Conclusions

In this paper, we attempted to find optimal operating conditions of an overhead crane in carrying a load over a distance. First, the task is optimized for two conflicting goals of design: the supplied energy and the task completion time. Using a multi-objective GA (NSGA-II), we have obtained a number of trade-off solutions. It has been observed that an operation requiring minimal time of completion demands for a large energy; on the other hand, an operation requiring minimal energy demands for a longer time of completion. In each case, an optimization of individual objectives has been performed to build confidence on the obtained non-dominated front.

Moreover, an investigation of the obtained trade-off solutions reveal the following important operating principles:

1. The bang-bang force model used in the study requires the forces to be applied early on so that the system acquire enough energy to complete the task. Although not obvious, such a strategy would enable the trolley to reach its destination with a minimal energy so that when stopped suddenly at the destination the hanging load does not sway much.
2. The applied force is inversely proportional to the time to reach the destination.

In another case study, the trolley time and the sway time are minimized using NSGA-II and a trade-off relationship between them is observed.

Although the application study considered here is a specific one related to the overhead crane operating conditions, the methodology used here can be used in other engineering design and applications. A consideration of more than one objective (with a conflict in them) in the optimization process is expected to produce a set of trade-off solutions. An investigation of such trade-off solutions should reveal important information about the problem, which may be difficult to obtain by any other means.

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