A Fixed-Length Subset Genetic Algorithm for the p-Median Problem

Andrew Lim[1,2] and Zhou Xu[2]

[1] Dept Industrial Engineering and Engineering Management, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong
[2] Dept of Computer Science, National University of Singapore, 3 Science Drive 2, Singapore 117543
{iealim@ust.hk,xuzhou@comp.nus.edu.sg}

Abstract. In this paper, we review some classical recombination operations and devise new heuristic recombinations for the fixed-length subset. Our experimental results on the classical p-median problem indicate that our method is superior and very close to the optimal solution.

1 Fixed-Length Subset Recombinations

We study the Fixed Length Subset Genetic Algorithm (*FLS-GA*), whose candidate solutions are represented by the fixed-length subset (*FLS*), which can be defined as any subset with a fixed size for a given set. In FLS-GA, we adopt a *subset encoding* [CHWS97], which uses a list of elements to represent the candidate FLS. [Rad93] studies two *pure recombinations* for FLS, which are Random Respectful Recombination (*RRR*) and Random Assorting Recombination (*RAR*). We extend them to *heuristic recombinations*.

1. Construct candidate set S', and inherited pattern s_0, from FLSs A and B;
2. Choose sub-optimal FLS from S' using the heuristic procedure $H(S', s_0)$. Let the result of $H(S', s_0)$ be the child of recombinations;

For FLS-GA, we could use RRR and RAR respectively to construct candidate set S', and inherited pattern s_0, leading the following two heuristic recombinations.

- Heuristic RRR (*H-RRR*)
  The inherited pattern $s_0 = A \cap B$ and the candidate set $S' = A \cup B - A \cap B$. Thus the size of the inherited pattern $|s_0|$ is equal to $|A \cap B|$, while the size of the candidate set $|S'|$ is $|A \cup B| - |A \cap B|$;
- Heuristic RAR (*H-RAR*)
  Each element of the inherited pattern s_0, is chosen from s_0 with probability q. And each element of the candidate set S', is chosen from $A \cap B - s_0$ with probability p_0, or from $A \cup B - A \cap B$ with probability p_1, or from $S - A \cup B$ with probability p_2.

Moreover, to balance the diversity and pattern, an *adaptive heuristic recombinations* can be designed as follows.

E. Cantú-Paz et al. (Eds.): GECCO 2003, LNCS 2724, pp. 1596–1597, 2003.

– Threshold H-RAR (*T-H-RAR*)
  Different heuristic recombinations are used in different situations of diversity.
  1. If the diversity of the current population is less than a threshold H, we adopt H-RAR with a smaller q and bigger p_2 to increase the diversity;
  2. If the diversity of the current population is larger than a threshold H, we adopt H-RAR with a bigger q and smaller p_2 to decrease the diversity;

2 FLS-GA Application to P-Median Problem (PMP)

PMP [ODE03] can be formulated as a FLS optimization problem.

– Instance:
  1. A set $S = \{v_1, .., v_m\}$ with m vertices;
  2. A positive number $p \leq m$;
  3. A distance matrix $D_{m \times m}$, where D_{ij} represents the distance between vertex v_i and vertex v_j;
  4. A function $F(s) = \sum_{\forall v_i \in S} (\min_{\forall v_j \in s} D_{ij})$, $\forall s \subseteq S$;
– Constraint: $|s| = m$
– Output: A fixed length subset s^*, where $s^* \subseteq S$ and $|s^*| = m$.
– Objective: To minimize the value of $F(s^*)$;

Two heuristic procedures are adoptted for when we applying FLS-GA to PMP.

– Decreasing Heuristics (*DH*)
  For a given candidate set S' and an inherited pattern s_0, firstly add all elements of S to the result FLS. Then select the rest $(p - |s_0|)$ elements from $S' - s_0$ one by one. In each round, the element, which decreases the objective function most, is added to the result FLS.
– Increasing Heuristics (*IH*) [ODE03]
  For a given candidate set S' and an inherited pattern s_0, delete $(|S'| - p)$ elements from S' one by one. In each round, the element, which is excluded from s_0 and increases the objective function the least, is deleted from S'.

The experimental results have shown that the FLS-GAs with heuristic recombinations over-perform the GA with pure genetic operations, and that the hybrid heuristic genetic recombinations exhibit the best results over all. For most problems, FLS-GAs with T-H-RAR can obtain solutions that are very close to the optimal solution (the gap $< 0.1\%$).

References

[CHWS97] Kelly D. Crawford, Cory J. Hoelting, Roger L. Wainwright, and Dale A. Schoenefeld. A study of fixed-length subset recombination. In Richard K. Belew and Michael D. Vose, editors, *Foundations of Genetic Algorithms 4*, pages 365–378. Morgan Kaufmann, San Francisco, CA, 1997.

[ODE03] Alp O., Z. Drezner, and E. Erkut. An efficient genetic algorithm for the p-median problem. *Annals of Operations Research*, 2003. forthcoming.

[Rad93] Nicholas J. Radcliffe. Genetic set recombination. In *Foundations of Genetic Algorithms 2*, San Mateo, CA, 1993. Morgan Kaufmann Publishers.