# Fractional Order Dynamical Phenomena in a GA

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**Abstract.** This work addresses the fractional-order dynamics during the evolution of a *GA*, which generates a robot manipulator trajectory. In order to investigate the phenomena involved in the *GA* population evolution, the crossover is exposed to excitation perturbations and the corresponding fitness variations are evaluated. The input/output signals are studied revealing a fractional-order dynamic evolution, characteristic of a long-term system memory.

## 1 The GA Trajectory Planning Scheme

This section presents a *GA* that calculates the trajectory of a two-link manipulator that is required to move between two points. The path is encoded directly, using real codification, as strings in the joint space to be used by the *GA* as: $[\Delta t, (q_{11}, q_{21}), \ldots, (q_{1j}, q_{2j}), \ldots, (q_{1m}, q_{2m})]$. The $i$th joint variable for a robot intermediate $j$th position is $q_{ij}$, at time $j\Delta t$. The fitness function $f$ adopted for evaluating the trajectories is defined as:

$$f = \beta_1 f_\tau + \beta_2 \sum_{j=2}^{m} \sum_{i=1}^{2} \dot{q}_{ij}^2 + \beta_3 \sum_{j=2}^{m-1} \sum_{i=1}^{2} \ddot{q}_{ij}^2 + \beta_4 \sum_{j=2}^{m} \dot{p}_j^2 + \beta_5 \sum_{j=2}^{m-1} \ddot{p}_j^2 \qquad (1)$$

The $f_\tau$ index represents the excessive torque that is demanded for the joints motors, $p_j$ is the $j$ cartesian arm position. This simple experiment consists on moving a robotic arm between two points. In the *GA* are adopted $p_c = 0.8$, $p_m = 0.05$, a 200 population size, a string size of $m = 7$ and a 3-tournament selection. The robot parameters are $l_i = 1$m, $m_i = 1$kg and $\tau_{i,max} = \{16, 5\}$Nm ($i = 1, 2$). Figure 1ab show the simulation results. The trajectory presents a smooth behavior, both in the space and time evolution and the required joint torques do not exceed the imposed limitations.

## 2 Fractional-Order Dynamics

The *GA* system is stimulated by perturbing the crossover probability $pc$ through a white noise signal, with a small amplitude (1%) during a time period $T_{exc}$, and

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the corresponding modification of the population fitness is evaluated. Therefore, the variation of the crossover probability and the resulting fitness modification of the *GA* population, during the evolution, can be viewed as the system input and output signals versus time, respectively.

The transfer function $H_n(j\omega)$, between the input and output signals, and the fractional order analytical approximation $G_n(j\omega)$ are depicted in figure 1c.

The numerical data of the transfer functions are approximated by analytical expressions with gain $k \in \Re$, one zero and one pole $(a,b) \in \Re$ of fractional orders $(\alpha,\beta) \in \Re$, respectively, given by $G_n(s) = k[\left(\frac{s}{a}\right)^{\alpha} + 1]/[\left(\frac{s}{b}\right)^{\beta} + 1]$ for the $n$th fitness percentile $P_n$. For evaluating the influence of the excitation period $T_{exc}$ several simulations are developed. The relation between the transfer function parameters $\{k, \alpha, \beta\}$ and $(T_{exc}, P_n)$ are showed in figure 1def.

## 3 Conclusions

Fractional-order models capture phenomena and properties that classical integer-order simply neglect. For the case under study the signal evolution have similarities to those revealed by chaotic systems. This conclusion confirms the requirement for mathematical tools well adapted to the phenomena under investigation. In this line of thought, this article is a step towards the signal and system analysis based on the theory of Fractional Calculus.

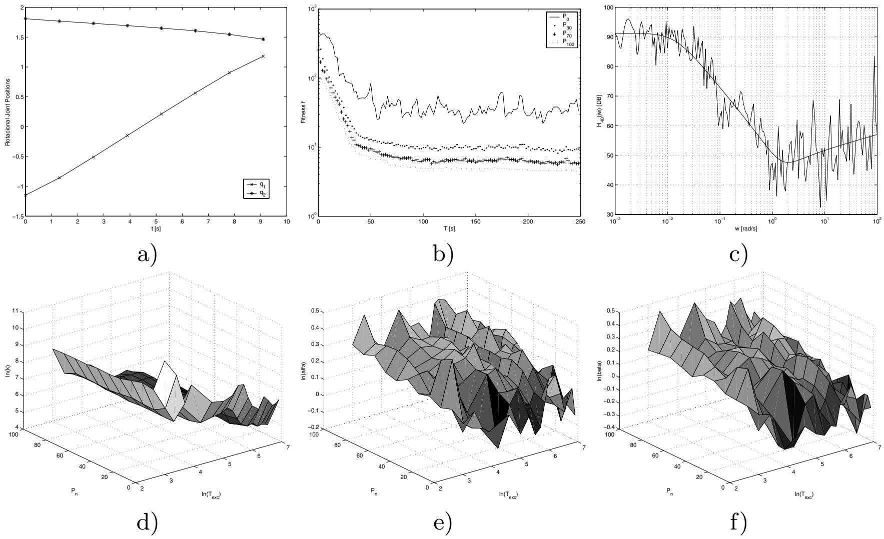


**Fig. 1.** a) Robot joint positions *vs.* time. b) Percentiles of the population fitness *vs.* $T$. c) $H_{40}(j\omega) = F\{\delta P_{40}(T)\}/F\{\delta p_c(T)\}$ and $G_{40}(j\omega)$ for the percentile $n = 40\%$. d) Estimated gain $\ln(k)$ *vs.* $(T_{exc}, P_n)$. e) Estimated zero fractional-order $\ln(\alpha)$ *vs.* $(T_{exc},\ P_n)$. f) Estimated pole fractional-order $\ln(\beta)$ *vs.* $(T_{exc},\ P_n)$.