Coevolutionary Convergence to Global Optima

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Abstract. We discuss a theory for a realistic, applicable scaled genetic algorithm (GA) which converges asymptoticly to global optima in a coevolutionary setting involving two species. It is shown for the first time that coevolutionary arms races yielding global optima can be implemented successfully in a procedure similar to simulated annealing.

Keywords: Coevolution; convergence of genetic algorithms; simulated annealing; genetic programming.

In [2], the need for a theoretical framework for coevolutionary algorithms *and* possible convergence theorems in regard to coevolutionary optimization ("arms races") was pointed out. Theoretical advance for coevolutionary GAs involving two types of creatures seems very limited thus far. [6] largely fills this void[1] in the case of a fixed division of the population among the two species involved even though there is certainly room for improvement. For a setting involving two types of creatures, [6] satisfies *all* goals advocated in [1, p. 270] in regard to finding a theoretical framework for scaled GAs similar to simulated annealing.

[4,5] contain recent substancial advances in theory of coevolutionary GAs for competing agents/creatures of a single type. In particular, the coevolutionary global optimization problem is solved under the condition that (a group of) agents exist that are strictly superior in every population they reside in.

Here and in [6], we continue to use the *well-established notation* of [3,4,5]. The setup considers two sets of creatures $\mathcal{C}^{(0)}$ and $\mathcal{C}^{(1)}$. Elements of $\mathcal{C}^{(0)}$ can, *e.g.*, be thought of as sorting programs while $\mathcal{C}^{(1)}$ can be thought of as unsorted tuples. The two types of creatures $\mathcal{C}^{(j)}$, $j \in \{0, 1\}$, involved in the setup of the coevolutionary GA are being encoded as finite-length strings over arbitrary-size alphabets $\mathcal{A}_j$. Creatures $c \in \mathcal{C}^{(0)}$, $d \in \mathcal{C}^{(1)}$ are evaluated by a duality $<c, d> \in \mathbb{R}$. In case of the above example, this expression may represent execution time of a sorting program c on an unsorted tuple d. Any population p is a tuple consisting of $s_0 \geq 4$ creatures of $\mathcal{C}^{(0)}$ followed by $s_1 \geq 4$ creatures of $\mathcal{C}^{(1)}$. This fixed division of the population is done here simply for practical purposes but is, in effect, in accordance with the evolutionary stable strategy in evolutionary game theory. In particular, the model in [6] does not refer to the multi-set model [7].

[1] Possibly, there exist significant theoretical results unknown to the author. Referee 753 claims in regard to [6]: "this elaborate mathematical framework that doesn't illuminate anything we don't already know" without giving further reference.

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The GA considered in [6] employs *very common* GA operators which are given by detailed, almost procedural definitions including explicit annealing schedules: multiple-spot mutation, practically any known crossover, and scaled proportional fitness selection. Thus, the GA considered in [6] is *standard* and by no means "out of the blue sky". Work by the authors of [1] and [4, Thms. 8.2–6] show that the annealing procedure considered in [6] is *absolutely necessary for convergence to global optima* and not "highly contrived". The mutation operator allows for a scalable compromise on the alphabet level between a neighborhood-based search and pure random change (the latter as in [4, Lemma 3.1]). The population-dependent fitness function is defined as follows: if $p = (c_1, \ldots, c_{s_0}, d_1, \ldots, d_{s_1})$, $\varphi_1{=}\pm 1$, then $f(d_\iota, p) = \exp(\varphi_1 \sum_{\sigma=1}^{s_0} <c_\sigma, d_\iota>)$. The fitness function is defined similarly for $c_1, \ldots, c_{s_0}$. The factors $\varphi_{0,1}{=}\pm 1$ are used to adjust whether the two types of creatures have the same or opposing goals. Referring to the above example, one would set $\varphi_0{=}{-1}$ and $\varphi_1{=}1$ since good sorting programs aim for a short execution time while 'difficult' unsorted tuples aim for a long execution time. The fitness function is then scaled with logarithmic growth in the exponent as in [4, Thm. 8.6] or [5, Thm. 3.4.1] with similar lower bounds for the factor $B{>}0$ determining the growth.

Under the assumption that a group of *globally strictly maximal* creatures exists that are evaluated superior in any population they reside in, an analogue of [4, Thm. 8.6] [5, Thm. 3.4.1] with similar restriction on population size is shown in [6]. In particular, the coevolutionary GA in [6] is strongly ergodic and converges to a probability distribution over uniform populations containing only globally strictly maximal creatures. [6] is available from this author. As indicated above, this author finds the concerns of referees unacceptable to a large degree.

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