Simulation of the Effects of Timing Jitter in Track-and-Hold and Sample-and-Hold Circuits

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ABSTRACT

In this paper, we analyze the effect of jitter in track and hold circuits. The output spectrum is obtained in terms of the system function of the track and hold. It is a fairly general model in which the effect of input as well as clock jitter can be included. The clock can have an arbitrary duty cycle, so that the circuit could also approximate a sample and hold. Using this model, it is possible to simulate the effects of jitter in a track and hold using a standard circuit simulator. Three cases are analyzed - long term jitter, correlated jitter with exponential autocorrelation and white noise jitter. These results are verified using Monte Carlo simulations.

Categories and Subject Descriptors: B.7.2 Design Aids: Simulation

General Terms: Design

Keywords: Jitter, Sampling circuits

1. INTRODUCTION

One of the limiting factors in the performance of analog to digital (A/D) converters is the jitter in the sampling clock. It is possible that the clock has both amplitude noise as well as phase jitter (or equivalently timing errors). However, it is well established that phase errors in the clock result in a larger deterioration in performance.

The presence of timing errors in the clock means that we effectively have a time-distorted signal, giving rise to errors in the sampled voltage and hence a reduced SNR at the output of the A/D converter. The methods that have been proposed to predict the performance of the converter in the presence of jitter essentially study the properties of the following random process:

$$g(t) = f(t + x(t)) \tag{1}$$

where $x(t)$ is the random process characterizing the timing jitter. One possibility is to assume that the jitter is much smaller than the mean time period of the clock. The above equation can then be linearized. This converts the timing error to an additive voltage error, which gives rise to an increased noise floor. This approach has been used in [1, 2]. The linear approximation fails to give reasonable results when the jitter is large. This is especially true when the clock has long term jitter [3, 4]. Nonlinear discrete time models for jitter analysis have been proposed in [5, 6, 7]. In these models, timing errors are converted to errors in the value of the impulse sampled signal. The resultant samples are assumed to be perfectly periodic and a conventional discrete time analysis is done to obtain the power spectral density. In [6, 7], the focus is on long term jitter. In [5], the effects of correlated jitter is also analyzed

A typical Nyquist rate A/D converter contains a track and hold circuit followed by conversion of the held value to a digital signal. In order to evaluate the A/D converter, we need to know the performance limits of the track and hold circuit. None of the above models can adequately describe the spectrum at the output of a track and hold circuit. Linearized models are not an option if there is long term jitter and the discrete-time models are not a correct representation of the output spectrum of the track and hold circuit. In this paper, we derive an expression for the output spectrum of the track and hold circuit with jitter, in terms of its system function. It is a fairly general model in which the effect of input as well as clock jitter can be included. The clock can have an arbitrary duty cycle, so that the circuit could also approximate a sample and hold. Using this model, it is possible to simulate the effects of jitter in a track and hold using a standard circuit simulator. We compare the spectrum at the output of the track and hold with the spectrum obtained by doing a discrete time analysis of the held value. Three cases are analyzed - long term jitter, correlated jitter with exponential autocorrelation and white noise jitter. These results are verified using Monte Carlo simulations.

2. ANALYSIS OF THE TRACK AND HOLD WITH JITTER

2.1 System Function Representation

Consider a track and hold that has clock jitter. Assume that the input to the system is a jitter-free tone at ω_o. Let the timing jitter of the clock be characterized by the random process $x(t)$. This implies that the output of the system should also exhibit some degree of timing uncertainty. In the case of driven systems (as opposed to free running systems such as a open loop oscillator), there is at most a constant

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phase delay between the input and the output. Therefore, the timing uncertainty of the clock will get directly reflected at the output. However, the zero-crossings of the input will not get affected by the clock jitter. Therefore, the output of the track and hold can be written as:

$$\begin{aligned} y(t;\omega_o) &= e^{j\omega_o t} H(j\omega_o, t + x(t)) \\ &= e^{j\omega_o t} \sum_{n=-\infty}^{\infty} H_n(\omega_o) e^{jn\omega_s (t+x(t))} \end{aligned} \quad (2)$$

where $H(j\omega_o, t)$ is the system function of the track and hold and $H_n(\omega_o)$ are the harmonics of the system function at ω_o. The autocorrelation of the output signal, $R_y(t, t-\tau;\omega_o) = E\{y(t;\omega_o)y(t-\tau;\omega_o)\}$, can be written as:

$$\begin{aligned} R_y(t, t-\tau;\omega_o) = \sum_n \sum_m H_n(\omega_o) H_m(\omega_o)^* e^{j(\omega_o + n\omega_s)\tau} \\ \times \; E\{e^{j\omega_s(nx(t) - mx(t-\tau))}\} e^{j(n-m)\omega_s t} \end{aligned} \quad (3)$$

If $x(t)$ is a wide sense stationary process with Gaussian statistics, $R_y(t, t-\tau;\omega_o)$ can be written as:

$$\begin{aligned} R_y(t, t-\tau;\omega_o) = \sum_n \sum_m H_n(\omega_o) H_m(\omega_o)^* e^{j(n-m)\omega_s t} \\ \times \; e^{j(\omega_o + n\omega_s)\tau} e^{-\omega_s^2 (n^2+m^2)\frac{\sigma_x^2}{2}} e^{\omega_s^2 nm R_x(\tau)} \end{aligned} \quad (4)$$

$R_y(t, t-\tau;\omega_o)$ is clearly cyclostationary in spite of the jitter. The stationary part of the autocorrelation can be written as:

$$\begin{aligned} R_{ys}(\tau;\omega_o) &= e^{j\omega_o \tau} \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{jn\omega_s \tau} e^{-n^2\omega_s^2(\sigma_x^2 - R_x(\tau))} \\ &= e^{j\omega_o \tau} \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{jn\omega_s \tau} e^{-n^2\omega_s^2\sigma_x^2} \\ &\qquad \times \sum_{k=0}^{\infty} \frac{(\omega_s^2 n^2 R_x(\tau))^k}{k!} \end{aligned} \quad (5)$$

If we have an exponentially decaying autocorrelation ($R_x(\tau) = \sigma_x^2 e^{-a|\tau|}$), $R_{ys}(\tau;\omega_o)$ can be written as:

$$\begin{aligned} R_{ys}(\tau;\omega_o) = e^{j\omega_o \tau} \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{jn\omega_s \tau} e^{-n^2\omega_s^2\sigma_x^2} \\ \times \sum_{k=0}^{\infty} \frac{(\omega_s^2 n^2 \sigma_x^2)^k e^{-ka|\tau|}}{k!} \end{aligned} \quad (6)$$

The power spectral density at the output can thus be written as:

$$\begin{aligned} S_{ys}(\Omega;\omega_o) = \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{-n^2\omega_s^2\sigma_x^2} \delta(\Omega - \omega_o - n\omega_s) \\ + \sum_{n=-\infty}^{\infty} \sum_{k=1}^{\infty} \frac{|H_n(\omega_o)|^2 e^{-n^2\omega_s^2\sigma_x^2} (\omega_s^2 n^2 \sigma_x^2)^k}{k!} \\ \times \frac{2ka}{(ka)^2 + (\Omega - \omega_o - n\omega_s)^2} \end{aligned} \quad (7)$$

The output power spectral density continues to be delta functions at various harmonics with reduced power in all harmonics other than the zeroth harmonic (which corresponds to ω_o). In addition, the correlation in the jitter results in an infinite series of Lorentzians about all harmonics, other than $n = 0$. This increases the noise floor. It is also interesting to note that the signal power at ω_o depends very slightly on the jitter. At ω_o, the contribution of the second term is negligible. It is also clear that the noise level will reach a maximum for some value of the correlation frequency 'a'. For values of 'a' much smaller than the clock frequency, the noise floor will be low, since the Lorentzian function can be approximately written as

$$L \approx \frac{ka}{(\Omega - \omega_o - n\omega_s)^2}$$

For very large values of 'a', once again the noise floor is low since the Lorentzian function can now be approximated to $1/ka$. It reaches a maximum when $a = (\Omega - \omega_o - n\omega_s)/k$. Since typically only the first few harmonics are significant, the noise levels are close to the maximum when the value of 'a' is a little higher than the clock frequency. This behaviour is, of course, a consequence of the model used. Since the jitter power is bounded ($=\sigma_x^2$), the noise floor will have to eventually drop as the effective bandwidth becomes large.

In the presence of long term jitter, the output autocorrelation is asymptotically stationary [3] and can be written as:

$$R_y(\tau;\omega_o) = e^{j\omega_o \tau} \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{jn\omega_s \tau} e^{-\frac{1}{2}\omega_s^2 n^2 c|\tau|} \quad (8)$$

where $c = \sigma_x^2 f_s$ and σ_x is the period jitter in the clock. The power spectral density is thus:

$$\begin{aligned} S_y(\Omega;\omega_o) = \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 \frac{\omega_s^2 n^2 c}{\frac{1}{4}\omega_s^4 n^4 c^2 + (\Omega - \omega_o - n\omega_s)^2} \\ + |H_o(\omega_o)|^2 \delta(\Omega - \omega_o) \end{aligned} \quad (9)$$

This implies that the output spectrum is a delta function at the signal frequency. It is a Lorentzian function at all the harmonics. Even in the presence of long term jitter, the signal power is only marginally affected by the clock jitter.

If the jitter $x(t)$ is a white noise process $R_x(\tau) = 0$ for a non-zero value of τ. As a result, we have:

$$\begin{aligned} R_{ys}(\tau;\omega_o) &= e^{j\omega_o \tau} \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2 e^{jn\omega_s \tau} e^{-n^2\omega_s^2\sigma_x^2}, \; \tau \neq 0 \\ &= \sum_{n=-\infty}^{\infty} |H_n(\omega_o)|^2, \qquad \tau = 0 \end{aligned} \quad (10)$$

Clearly, there is a discontinuity at $\tau = 0$. It is not possible to find an expression for the spectral density. This is because continuous-time white noise is basically an unphysical process. However, it is possible to find the power spectral density, if we consider a sampled signal and use discrete time analysis [5, 6]. This is discussed in section 3.

Any periodic input signal can be expanded as a Fourier series. The same analysis can be used for each of the harmonics. $S_y(\Omega)$ will then be a superposition of the power spectral density obtained for each of the harmonics. Also, it can be easily seen that the any input jitter can be taken into account by adding appropriate terms to the input tone.

It is interesting to note that in all cases, the signal power (output spectrum at ω_o) is relatively unaffected by the jitter. This is the consequence of our initial observation that the clock jitter does not affect the zero-crossings of the input. This is a useful result which can be used to distinguish between input jitter and clock jitter in experiments with track and hold circuit. In section 5 we verify this using Monte Carlo simulations, but some additional insight can be obtained by looking at jitter in a track and held signal. This is done in the following section.

2.2 Track and Hold - signal representation

Assuming that the duty cycle of the clock is nominally 0.5, the track and held signal corresponding to $f(t) = e^{j\omega_o t}$ can be written as:

$$\begin{aligned} g(t) = &\sum_n e^{j\omega_o t}\big(u(t-nT_s-T_s/2-x_{n/2})-u(t-nT_s-x_n)\big) \\ &+\sum_n e^{j\omega_o(nT_s+T_s/2)}\big(u(t-(n+1)T_s-x_{n+1}) \\ &\qquad - u(t-nT_s-T_s/2-x_{n/2})\big) \end{aligned} \tag{11}$$

In the above equation, x_n, $x_{n/2}$ and x_{n+1} are the are samples of the random process $x(t)$ at nT_s, $nT_s + \frac{T_s}{2}$ and $(n+1)T_s$. $u(t)$ is the unit step function. After a rather involved, but straightforward analysis, the stationary power spectral density can be written as:

$$S_g(f) = S_{g1}(f) + S_{g2}(f) \tag{12}$$

where

$$\begin{aligned} S_{g1}(f) = &\sum_{n,m} e^{j(\omega-\omega_o)T_a} E\Bigg\{(T_s/2+V)e^{j(\omega-\omega_o)Y} \\ &\times \Big(\frac{\sin((\omega-\omega_o)(T_s/2+V)/2)}{((\omega-\omega_o)(T_s/2+V)/2)}\Big)^2\Bigg\} \end{aligned} \tag{13}$$

and

$$\begin{aligned} S_{g2}(f) = &\sum_{n,m} e^{j(\omega-\omega_o)T_a} \\ &\times E\Bigg\{(T_s/2+V)e^{j(\omega-\omega_o)Y}\Big(\frac{\sin(\omega(T_s/2+V)/2)}{(\omega(T_s/2+V)/2)}\Big)^2\Bigg\} \end{aligned} \tag{14}$$

In the above equations,

$$\begin{aligned} T_a &= (n-m)T_s \\ Y &= x_n - x_m \\ V &= x_m - x_{m/2} \end{aligned}$$

In the presence of long term jitter, the value of Y could become quite large. However, V is representative of the period jitter and its standard deviation is of the order of σ_x. For practical jitter values, this is usually a fraction of the clock period. Now, V is Gaussian with $E\{V\} = 0$ and its variance is independent of the running index m. Therefore at $\omega = \omega_o$, the spectral density can be written as:

$$S_g(f) = \left(\frac{1}{2} + \frac{1}{2}E\left\{\left(\frac{\sin(\omega_o(T_s/2+V)/2)}{(\omega_o(T_s/2+V)/2)}\right)^2\right\}\right)\delta(f-f_o) \tag{15}$$

It is clear that even in the presence of long term jitter, the output power spectral density of the track and hold at the input frequency is a delta function. The signal power at this frequency depends very mildly on the jitter. Even if σ_x is $0.5T_s$ (which is highly unlikely), the signal power changes by about 2.5%. For 1% clock jitter, the change in the signal power is 0.03%. This essentially means that the output signal power at ω_o is virtually independent of the jitter. Note that output power spectral density at ω_o due to the "tracked" part of the signal is unaffected by jitter. This is because irrespective of the clock jitter, the zero crossings of the input signal are preserved in the track and hold. As expected, the "held" part of the signal is slightly affected by jitter.

3. COMPARISON WITH THE SPECTRUM OF SAMPLED SIGNALS

In an A/D converter, the "held" value is converted to a digital number and the output spectrum is obtained by finding the discrete Fourier transform (DFT) of these samples. In order to study the effect of clock jitter on this spectrum, we can find the Fourier transform of the discrete time auto-correlation of these "held" values. This of course, excludes the effect of quantization. This analysis is similar to that done in [6], but we also include the effect of correlated jitter. The assumption here is that the signal is not significantly attenuated by the track and hold, which is a reasonable assumption. It will be seen that this spectrum differs in many respects from the output spectrum of the track and hold.

Assume that we have N samples of the periodic signal $f(t) = e^{j\omega_o t}$. Since there is jitter in the sampling, there is an error in the value of each sample. However, when finding the spectral density, we assume that the samples are perfectly periodic i.e. there is no timing error. Thus the timing error in the sampling clock is converted to a voltage error in the input signal and then analyzed. This is essentially what we do, when we find the DFT of the output of an A/D converter. If there is jitter in the sampling, the n^{th} sample, g_n, can be written as:

$$g_n = e^{j\omega_o(nT_s+x_n)} \tag{16}$$

x_n is the random process representing the timing jitter and T_s is the time period of the sampling clock. The discrete time auto-correlation can thus be written as:

$$E\{g_n g_m^*\} = e^{j\omega_o(n-m)T_s} E\{e^{j\omega_o(x_n-x_m)}\} \tag{17}$$

If $x(t)$ is a Gaussian white noise process, x_n and x_m are uncorrelated for all n,m. Using the expression for the characteristic function of the Gaussian processes, we get

$$\begin{aligned} R_g(n-m) &= e^{j\omega_o(n-m)T_s}e^{-\omega_o^2\sigma_x^2}, \quad n \neq m \\ &= 1, \qquad\qquad n = m \end{aligned} \tag{18}$$

Clearly, g is a wide sense stationary random process. The power spectral density of g(n) can be written as:

$$S_g(f) = e^{-\omega_o^2\sigma_x^2}\delta(f-f_o) + \frac{(1-e^{-\omega_o^2\sigma_x^2})}{f_s} \tag{19}$$

If there is long term jitter, then $x(t)$ is a Weiner process. In this case, the autocorrelation can be written as:

$$R_g(n-m) = R_g(k) = e^{j\omega_o kT_s}e^{-|k|\frac{\omega_o^2\sigma_x^2}{2}} \tag{20}$$

The corresponding power spectral density is given by:

$$S_g(f) = \frac{1}{f_s} \frac{1-A^2}{1+A^2-2A\cos(\frac{2\pi(f_o-f)}{f_o})} \tag{21}$$

where

$$A = e^{-\frac{\omega_o^2\sigma_x^2}{2}} \tag{22}$$

The power in a frequency bin Δf around f_n can be written as:

$$P(f_n) = \frac{1}{\pi}\left[\tan^{-1}\left\{\frac{1+A}{1-A}\tan\left(\frac{\pi(f_o-f_n+0.5\Delta f)}{f_s}\right)\right\}\right] - \frac{1}{\pi}\left[\tan^{-1}\left\{\frac{1+A}{1-A}\tan\left(\frac{\pi(f_o-f_n-0.5\Delta f)}{f_s}\right)\right\}\right] \tag{23}$$

As expected, at $f_n = f_o$ the signal power tends to 1 as A tends to 1, that is, if there is no jitter. As the jitter becomes larger, the signal power decreases and noise level increases. Unlike the case of white noise jitter, the spectrum is approximately a Lorentzian - for frequencies close to the signal frequencies, the power decreases as $1/(f-f_o)^2$. This is different from the spectrum at the output of the track and hold, which is a delta function at the signal frequency.

The case of correlated jitter with exponential autocorrelation is now considered. We assume that the discrete time autocorrelation is a sampled version of the continuous autocorrelation, i.e.

$$R_{xd}(k) = \sigma_x^2 e^{-a|k|T_s} \tag{24}$$

Assuming $x(t)$ has Gaussian statistics, we can write:

$$R_g(k) = e^{j\omega_o kT_s} e^{-\omega_o^2\sigma_x^2} \sum_{l=0}^{\infty} \frac{(\omega_o^2\sigma_x^2)^l}{l!} e^{-la|k|T_s} \tag{25}$$

The corresponding power spectral density is given by:

$$S_g(f) = \frac{e^{-\omega_o^2\sigma_x^2}}{f_s} \sum_{l=0}^{\infty} \frac{(\omega_o^2\sigma_x^2)^l}{l!} \frac{1-B^2}{1+B^2-2B\cos(\frac{2\pi(f_o-f)}{f_o})} \tag{26}$$

where

$$B = e^{-laT_s}$$

The power in a frequency bin Δf about f_n can be written as:

$$P(f_n) = \frac{e^{-\omega_o^2\sigma_x^2}}{\pi} \sum_{l=0}^{\infty} \frac{(\omega_o^2\sigma_x^2)^l}{l!} \left[\tan^{-1}\left\{\frac{1+B}{1-B}\tan\left(\frac{\pi(f_o-f_n+0.5\Delta f)}{f_s}\right)\right\} - \tan^{-1}\left\{\frac{1+B}{1-B}\tan\left(\frac{\pi(f_o-f_n-0.5\Delta f)}{f_s}\right)\right\}\right] \tag{27}$$

It can easily be seen that if a is large, i.e. the samples are relatively uncorrelated, the value of B tends to zero for $l \neq 0$. In this case, $S_g(f)$ tends towards white noise limit given by equation (19). If a is small, i.e. the samples are highly correlated, the value of B tends to one and consequently $P(f_n)$ at $f_n = f_o$ tends to one. $P(f_n)$ is relatively small for all other frequencies and decreases with an increase in correlation. This behaviour is quite different from the continuous time case, where the noise level reaches a maximum for some value of a and then begins to decrease as a increases.

4. COMPUTATION

The track and hold circuit is modeled using a switch and capacitor. In order verify the theory, the results obtained using the analytical expressions were compared with Monte Carlo simulations. The following procedure was adopted for the simulations. The differential equation representing the track and hold circuit is integrated for a fixed number of clock cycles after steady state is attained. The integration was done using the trapezoidal rule along with time step control based on the local truncation error. Jitter was simulated as follows. In the case of long term jitter, x_i is effectively given by

$$x_i = \sum_{j=1}^{i} y_j \tag{28}$$

where y_j are samples of a zero mean Gaussian random process and are uncorrelated with each other. These samples can easily be obtained using a standard normal random number generator. In the case of exponentially correlated jitter, samples of x_i were obtained using the method described in [8].

A fixed length of the steady state waveform is stored in a table. In order to find the output power spectrum, we need to first find the DFT of this waveform. To do this, the steady state waveform is re-sampled at regular intervals. Linear interpolation was found to be sufficient for this purpose. The power spectrum, $P(f_n)$, of $v(t)$ at the frequency bin centered at f_n is then given by [9]

$$P(f_n) = \frac{|V(f_n)|^2}{N^2} \tag{29}$$

In the above equation, N is the total number of points used in the DFT analysis and $V(f_n)$ is the n^{th} point of the DFT of $v(t)$. This is assuming a rectangular window is used. If any other window is used, an appropriate scale factor has to be included [9]. The average of the power spectrum of 2-5 samples of the steady state waveform was used for the comparisons.

In order to compute the power spectrum using equations (7) and (9), we require the harmonics of the system function. To get these harmonics, we require the steady state output waveform over one clock cycle, which can be obtained by solving for the track and hold circuit using a perfectly periodic clock. Once the harmonics are computed, the power spectrum is obtained by integrating equations (7),(9) over a frequency range Δf centered at f_n. This power spectrum can then be directly compared with the results of Monte Carlo simulations.

5. RESULTS

We have compared the results obtained using the analytical expressions against Monte Carlo simulations. For these comparisons, it was assumed that the clock frequency is 1MHz and the signal frequency is 100kHz. The capacitance value used was 1pF and the ON resistance of the switch is assumed to be $10k\Omega$. In all cases, a rectangular window was used. Windowing effects are clearly visible in most of the plots, especially in and around "delta functions".

Figures 1(a) and (b) show a comparison of the results obtained for long term jitter. The period jitter was assumed to be 1% and 10% of the clock period in the two cases respectively. It is clearly seen that the harmonics due to the clock have a Lorentzian structure. Whereas, at the frequency corresponding to the input frequency, the spectrum is a delta function as predicted by the theory. Moreover, the output signal power at this frequency is relatively unaffected by the jitter levels. As expected however, the power at the clock harmonics reduces with increasing jitter. This is unlike the spectrum of the held values, where it is distinctly a Lorentzian function at the input frequency and the signal power reduces with increasing jitter as shown in Figure 2. Figure 3 shows a comparison of the spectrum of the "held" values with the results of the discrete time analysis. As expected, the spectrum now becomes a Lorentzian function, since the zero-crossing information of the input is no longer accurate. The analytical results are seen to match well with the Monte Carlo simulations. Figure 4 shows the effect of long term input jitter. It is assumed that the clock is perfectly periodic. As expected, at the signal frequency a Lorentzian spectrum is obtained, whereas the harmonics due to the clock are relatively unaffected by the jitter. Therefore, at the output of the track and hold, it is possible to distinguish between input and clock jitter.

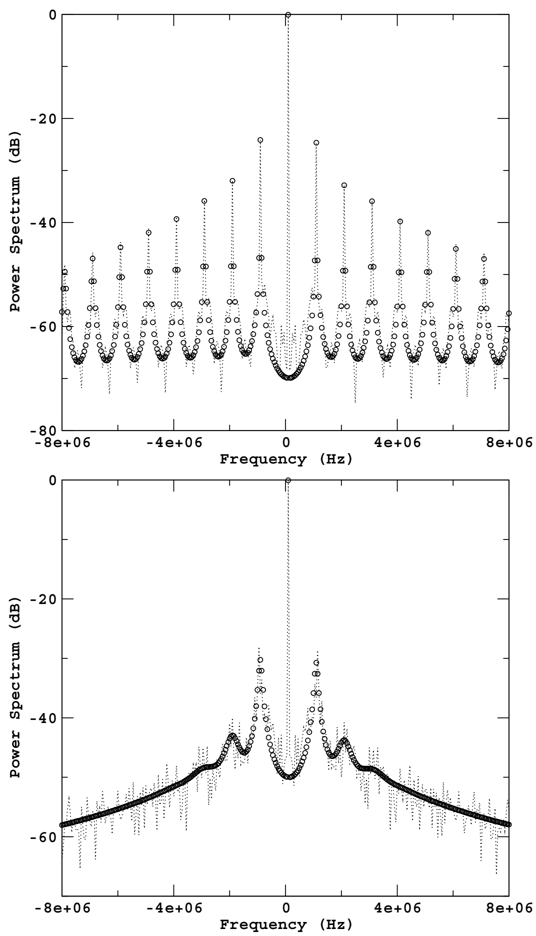

Figure 1: Spectrum at the output of the track and hold for 1% and 10% clock jitter. The dotted line is the average of two Monte Carlo simulations and circles represent values computed using the analytical expressions.

Figures 5(a) and 5(b) show the spectrum at the output of the track and hold if the clock jitter is correlated. The correlation time is $10^{-3}s$ and $10^{-7}s$ in the two cases. The jitter level is assumed to be 1% of the clock period. The noise level is clearly lower for longer correlation times. The crosses in 5(b) are the result of Monte Carlo simulation for uncorrelated samples. This noise level is seen to be about the same as that for correlation times of $10^{-7}s$. If the correlation time is decreased further, the noise level obtained using Monte Carlo simulations will not change. However, as explained previously, the noise level using the analytical expressions will start to drop. Since Monte Carlo simulations are basically discrete time simulations, their behaviour mimics the behaviour of spectrum obtained using discrete time analysis.

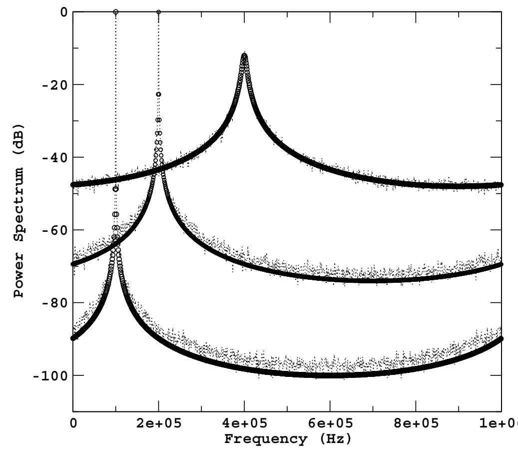

Figure 2: Comparison of the power spectrum of an impulse sampled signal including long term jitter with Monte Carlo simulations. The jitter levels are considered are .1%, 1% and 10%.

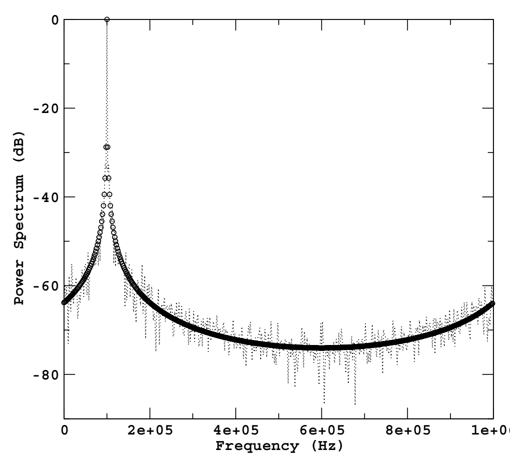

Figure 3: Comparison of the spectrum of the "held" values with spectrum obtained using discrete time analysis. The clock jitter is 1%.

6. CONCLUSIONS

In this paper, we have proposed a technique for simulating the effects of jitter in a track and hold circuit. We have

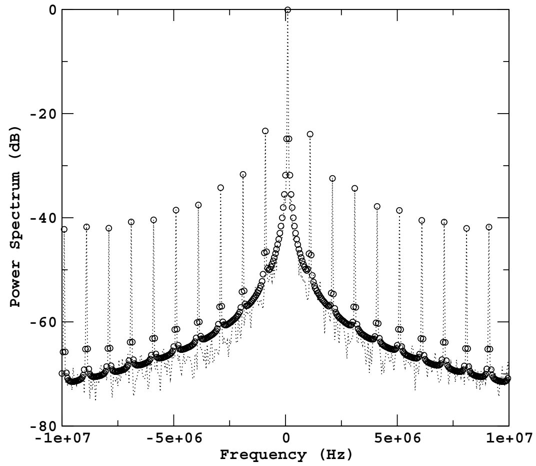

Figure 4: Effect of long term input jitter. The dotted line is the result of Monte Carlo simulations and the circles represent values computed using the analytical expressions

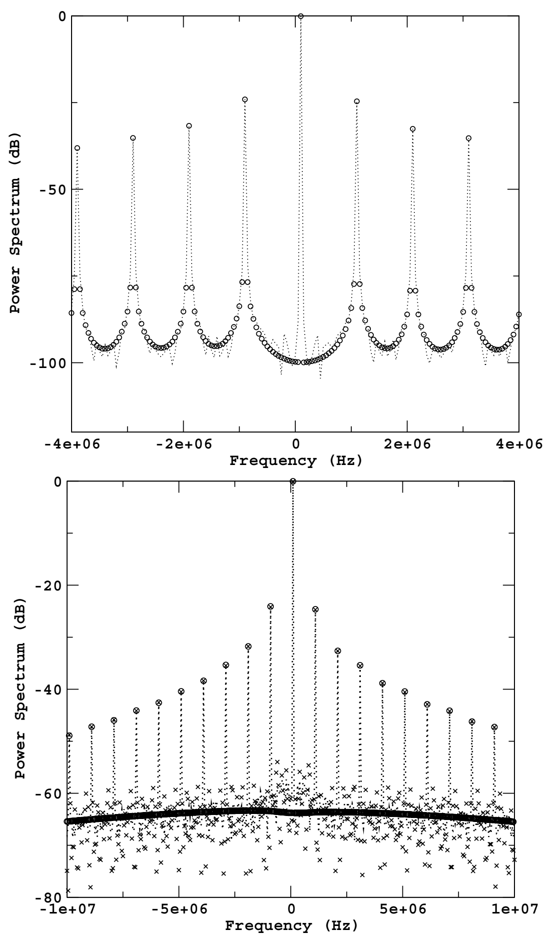

Figure 5: The effect of having correlated clock jitter. The correlation times are $10^{-3}s$ and $10^{-7}s$ respectively. The dotted lines are the results of Monte Carlo simulations. The crosses in (b) are due to Monte Carlo simulations with uncorrelated samples.

derived an expression for the output spectrum of the track and hold with jitter in terms of its system function. This makes it possible to simulate the effects of jitter using a standard circuit simulator. The model is quite general and it is possible to include the effects of both input as well as clock jitter. However, it does not take into account second order effects such as the effect of timing errors in one state variable on the others. This is usually not significant in a track and hold circuit, but it may have to be accounted for if the method is to be extended to a general switched capacitor circuit.

The main conclusions from the jitter analysis of the track and hold are the following. The output spectrum of the track and hold at the signal frequency continues to be a delta function in the presence of clock jitter. The signal power is also relatively unaffected by jitter, even in the presence of long term clock jitter. When the jitter is correlated, the noise level drops with increasing correlation times. At the output of the track and hold, it is possible to distinguish between the effect of input jitter and clock jitter quite easily.

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