The Grand Challenge in Non-Classical Computation: From Its Qualities To Its Quantities: A Position Paper

R. E. Smith and Jon Caves
Faculty of Computing, Engineering,
and Mathematical Sciences
University of The West of England, Bristol
[robert.smith,jon.caves]@uwe.ac.uk

1 Position

Non-classical computation is often described by analogies and qualities, rather than quantities. For instance, biologically-inspired computation is often defended by appeals to the success of nature, with no more quantitative substance than "birds do it, bees do it." Inspiration from physics is often just as qualitative (it would seem that "spin glasses do it" too). Terms like "intelligence," "knowledge," "life," "emergence," and "chaos" are thrown around in computational circles as if they were concrete and decided, while they are better seen as persistently nebulous and moot, in both science and philosophy.

As another relevant point of interest, take the history of artificial intelligence (AI) (a field which has often been associated with non-classical computation). The repeated pattern of development is that a technology *is* AI, until it is well understood, when it becomes "*mere* engineering." This "moving yardstick" of AI begs the question as to whether non-classical computation is simply poorly understood (but classical) science operating in the realm of engineered computation. Perhaps poorly-understood computation is isomorphic to non-classical computation.

For instance, consider the qualities of "emergence" and "the edge of chaos" (where emergence is often thought to occur). In surveying the literature, there are several indications of "emergence at the edge of chaos", but little that can be called a situation-independent method for its quantitative identification. In light of the "moving yardstick" phenomena of AI, one might ask whether these concepts are also a socio-psychological phenomena of the moment. Perhaps emergence is simply the appearance of results that were not anticipated, due to a limitation in current understanding, and the edge of chaos is simply the place where current understanding fails or breaks down.

The position expressed in this paper is that more focus is needed on providing quantitative descriptions of the nebulous qualities of non-classical computation. While quality is the ultimate goal of any design activity, we should not rely on wishful mnemonics, colorful analogies, and appeals to prosaic qualities as the basis for our endeavors in this area.

Science has, in fact, always quantified qualities for which only insufficient models of underlying phenomena exist. This is well illustrated in fluid mechanics, where many aspects of flow (for instance, in turbulent or non-Newtonian regimes) remain less than fully understood. One technique common to fluids is the use of *dimensionless numbers* as primary descriptors of phenomena. Flows are most often described by a subset of 5 such numbers (Weber, Froude, Reynolds, Mach, and the coefficient of pressure); *even though it is clear that such simplified models do not capture a complete picture of the flow.* The limited, but careful, quantitative, situation-independent measures implied by dimensionless numbers are often adequate for scientific and engineering purposes. Judgments about these limited models are the art of this pursuit, but the art is backed by a careful methodology, and does not rely entirely on qualitative description.

It is *not* the suggestion of this position paper that we should always resort to micro-scale models of the complex systems often associated with non-classical computation, and disregard simpler models as "too qualitative". Indeed, "complete" models will often be as complex as the system under consideration, and will therefore yield no real insight. However, there are avenues for meaningful, quantitative modeling where we have too often resorted to pictures and prose alone.

The following sections attempt to illustrate this point with an experiment that uses a dimensionless number approach to quantify the edge of chaos. It draws on two previous quantitative efforts in this area, in an attempt to unify them into a single, situation-independent approach.

2 Illustration

2.1 Kaufman's Edge of Chaos

One of the most famous demonstrations of edge of chaos phenomena is that offered by Kauffman [3][4]. Kauffman examines the behaviour of *random Boolean networks* (RBN). In an RBN, each of the *N* nodes has as its inputs *K* randomly selected outputs of other nodes. Each node contains a random Boolean function of its inputs, which determines its output. RBNs operate as iterative dynamical systems. Any RBN (operating deterministically) settles into an attractor. Kauffman examines the statistical properties of the attractors of many randomly determined networks with various values of *N* and *K.* He first considers the length of such attractors, and discovers that above a certain value of *K* (about 3), the size of attractors expands exponentially with *N*. Below this value the size of attractors is roughly polynomial in *N*. Next, Kauffman considers the stability of these attractors, and shows that for the same intermediate values of *K* that are associate with a transition in attractor size is also a transition in attractor stability. In essence, at the edge of chaos, attractors are relatively large, yet also relatively stable. Beneath this edge of chaos value, attractors are simple, but exercise only a fractional part of the underlying system's phase space. Above the edge, much of the phase space is spanned by the attracting behaviour, but with small perturbations, this behaviour is essentially random. At the edge, a large part of the phase space is accessed by the system's steady state behaviour, in a way that is relatively robust in the face of perturbations.

2.2 Crutchfield's Edge of Chaos

Crutchfield (working with the Shalizi's [1][2][5][6]) takes a distinct approach to the edge of chaos. Rather than starting with an abstracted dynamical system model, Crutchfield infers models (which he calls *epsilon machines*) from the behaviour of extant, representative systems. An epsilon machine is essentially a stochastic automaton which is inferred to represent the statistical dynamics of a given system. An exhaustive method for inferring epsilon machines is provided by the CSSR algorithm [5][6]. Once an epsilon machine is inferred by CSSR, two important statistical properties can be examined. First, since the machine is stochastic, one can easily determine its *entropy*, *H*. Second, one can determine a metric of the machines size*,* called the statistical complexity *C*. Each of these can be measured in the convenient unit of bits. Crutchfield considers a symbolic form of the well-known *logistic equation* . This system has a single variable, *x*, and a single parameter, lambda, Its behaviour is of particular interest, since for various values of lambda, the system is a classic illustration of the period doubling route to chaos. A symbolic form of this system can be created by outputting 0 for values of $x < 0.5$, and 1 for values of $x > 0.5$.

For systems with lambda below the Feigenbaum number (where the system transitions to chaos) , the relationship between *C* and *H* is strictly increasing and linear. For higher values of lambda, complexity *C* decreases with a rougly linear bound. For the transition value, another interesting phenomena can be observed Cinsider the size (*L*) of the window used to infer the epsilon machine in the CSSR algorithm. The effort applied to inference varies exponentially with *L*. At the Feigenbaum number (at not elsewhere in the spectrum of lambda) more inference effort simply increases the complexity of the inferred machine. The exhaustive statistical inference process of CSSR does not converge.

Crutchfield's explorations indicate another form of the edge of chaos, with properties that are distinct, but not unrelated, to those of Kauffman. For systems beneath the edge, "machine complexity" (*C*) increases linearly with entropy. Such machines are relatively "simple". For values above the edge, machine complexity is bounded, and the complexity of the machine's behaviour is accounted for by mere randomness. Crutchfield points out that this mere randomness is yet another form of "simplicity." However, at the edge, entropy remains bounded, but inferred machine complexity seems unbounded.

2.3 Are These the Same Edges?

While there are vague conceptual similarities in these two descriptions of the edge of chaos, they are far from quantitatively unified. One must ask whether they describe the same *quantitative* phenomena.

In some sense the "attractor size" and "inferred machine size" concepts in these presentations are similar, as are the "stability to perturbation" and "entropy of inferred machine" concepts. This may provide an avenue for unification for the concepts.However, any effort to unify these two approaches is partially frustrated by the different ways that results are presented. In Kauffman, two graphs are presented that separately illustrate size and stability of system attractors versus system parameters. In Crutchfield, one graph represents size of a inferred stochastic machine versus entropy of that machine, with system parameters only included implicitly.

To explore this, we employ the following procedure. We generate RBNs, in a process similar to that employed by Kauffman. However, in an attempt to present the size and stability concepts together, we perturb the RBN system dynamics throughout the system's operation. Specifically, with some small probability, the output of any node will be complemented.

In such a system, measuring the attractor size is no longer straightforward. Instead, we infer an epsilon machine, using the CSSR algorithm, and consider its size and entropy.We wish to be as general as possible, and present information in a relatively system independent fashion. Therefore, we resort to a comparison of dimensionless numbers.

First, we will define the *dimensionless input entropy* as the entropy (in bits) of random perturbations to the system divided by a description length of the underlying system in bits. Note that the description length is the sum of the entropy and the bumber of bits needed to represented the deterministic aspects of the machine.

We similarly define the *dimensionless output entropy.* This will be the entropy of a machine *inferred* from the output behaviour, divided by the size of that inferred machine (entropy plus description length) in bits. For the CSSR inference process, this is simply *H/(H+C)*.

As an illustration of the use of these quantities, Figure 1 shows the results with various RBNs with *N*=8 and the CSSR inference effort *L*=16.

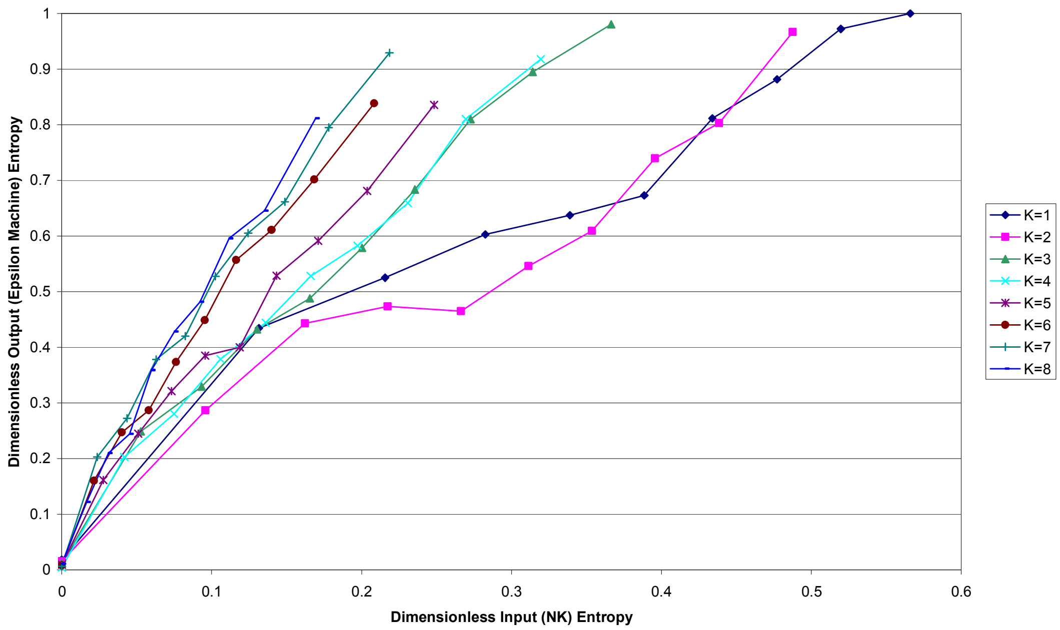

Figure 1: Dimensionless output entropy versus dimensionless input entropy for RBNs with *N*=8 and various values of *K*.

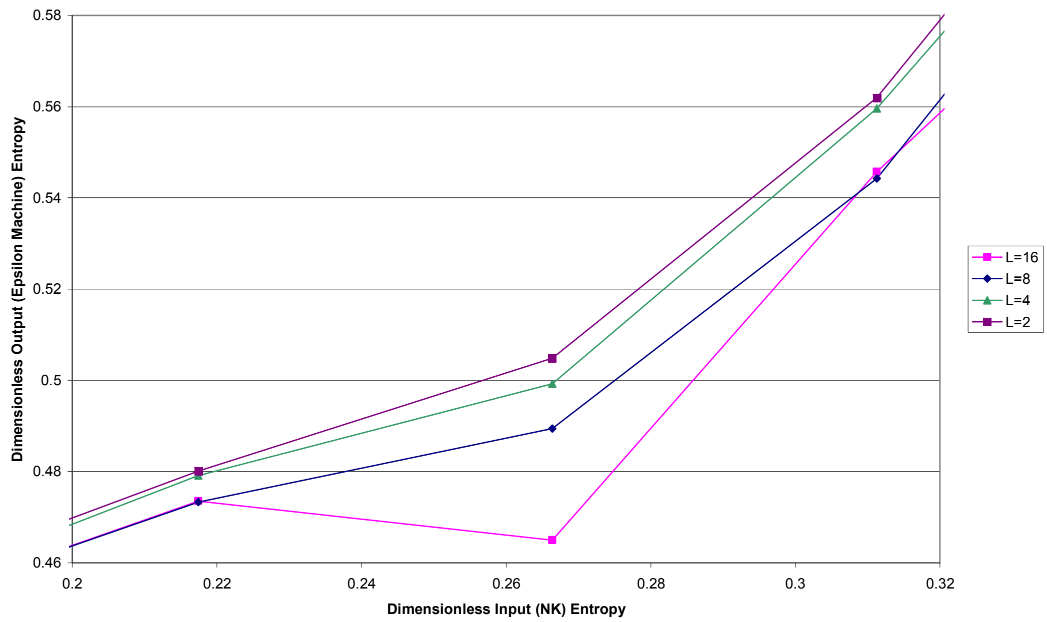

Figure 2: Detail of the *K*=2 line Figure 1, supplemented with lines representing various levels of inference effort (window size *L*) employed in the CSSR algorithm.

As expected, the dimensionless output entropy generally increases with dimensionless input entropy. Also as one might expect, the rate of increase generally increases with *K*. However, at *K*=2*,* there is an exception. For this value, the increase of output entropy as a function of input entropy is lower than any other *K* value, even *K*=1. There is a region where output randomness actually *declines* respect to input randomness (between dimensionless input entropy = 0.1 and 0.3). This means that despite the introduction of more randomness at the input, the randomness of the output, relative to the inferred machine size, does not increase.

Figure 2 further considers the *K*=2 case, and zooms in on the area where dimensionless output entropy seems to decline with increasing dimensionless input entropy. It shows that as we apply greater effort to inferring the dynamics of this machine, the effect is increased: dimensionless input entropy declines as a function of inference effort. This is because more and more deterministic machine structure emerges from the exhaustive CSSR process as inference effort is increased. This counter-intuitive effect does not occur for other *K* values in Figure 1. We suggest that this generalized phenomena is a quantitative indication of the edge of chaos.

3 Final Comments

Although the approach discussed above is promising, and unifies past concepts of the edge of chaos under one qualitative umbrella, it has limitations. In particular, it is a *variational* approach, requiring investigation of several system configurations to evaluate regions where increases in dimensionless input entropy decrease dimensionless output entropy.

However, the approach does suggest a set of easily obtainable quantities for experimenting with the edge of chaos phenomena. It should be possible to derive expressions for dimensionless entropy (randomness expressed in bits divided by machine description length) for a wide variety of systems, and examine them for edge of chaos phenomena. This sort of uniform, quantified treatment of such phenomena is important to the advancement of complex-systems-based, non-classical computation.

Acknowledgements

The authors gratefully acknowledge support provided for this work by the EPSRC and Airbus under the Industrial CASE studentship project entitled "CARD: Complexity in Aerospace Reliability and Design."

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