



Functional and Reliability Modelling of Swarm Robotic Systems

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This talk



- Can we mathematically model the functional and reliability properties of swarm robotic systems?
- Case studies
 - Functional Modelling of adaptive swarm foraging: *a macroscopic probabilistic approach*
 - Reliability and Scalability Modelling for swarm taxis: *the k-out-of-N model*

Case study: Foraging robots

Roomba, iRobot

Slugbot (BRL)



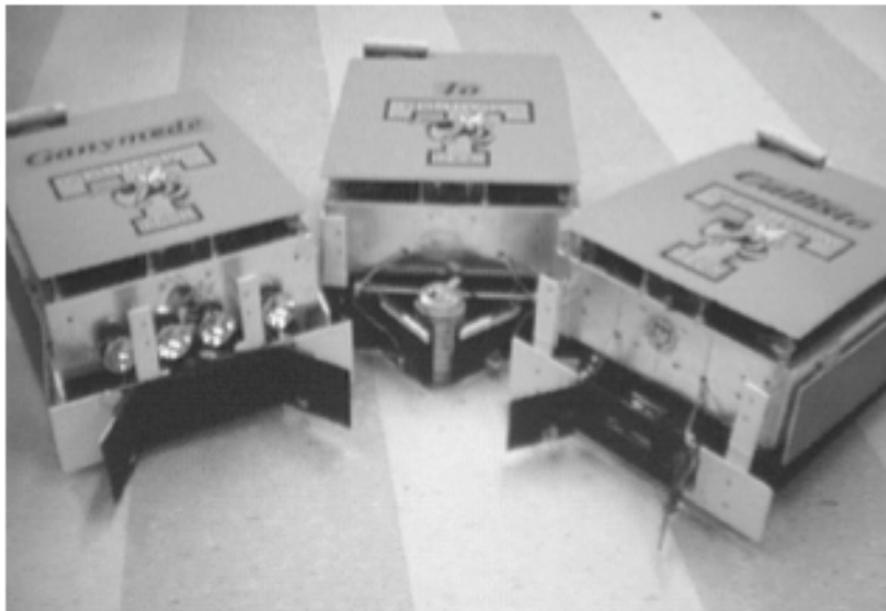
Zoë, Wettergreen et al, 2005



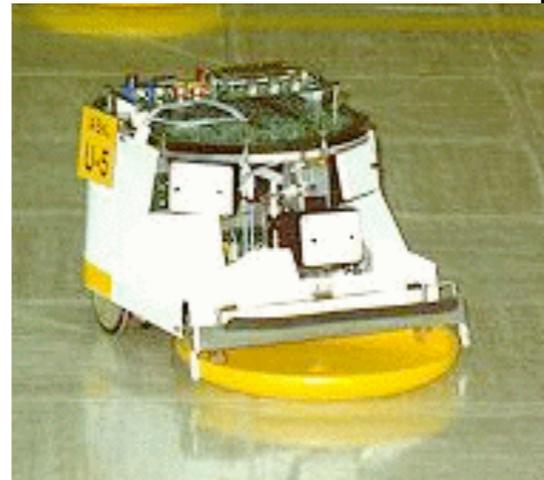
Demeter, Pilarski et al, 1999

Multi-Robot Foraging

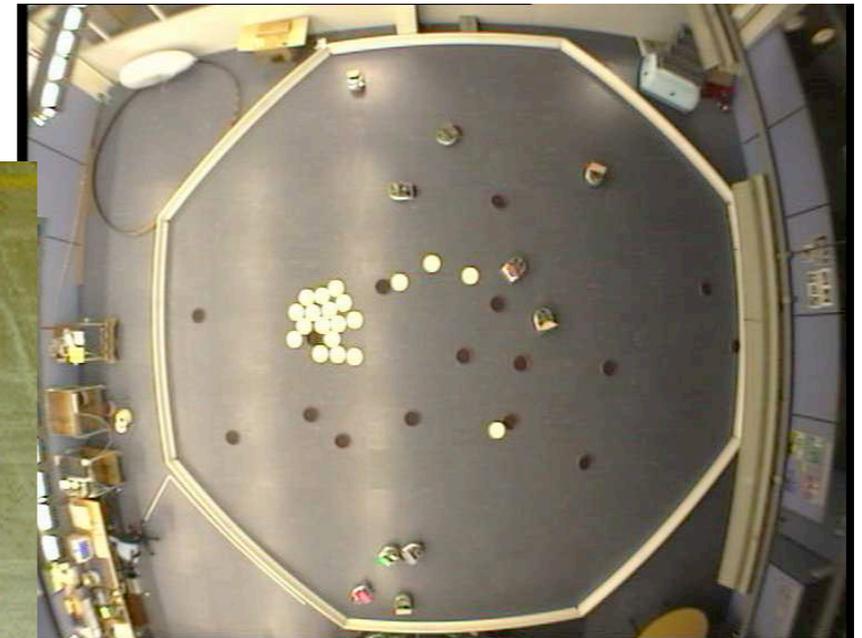
Soda can collecting



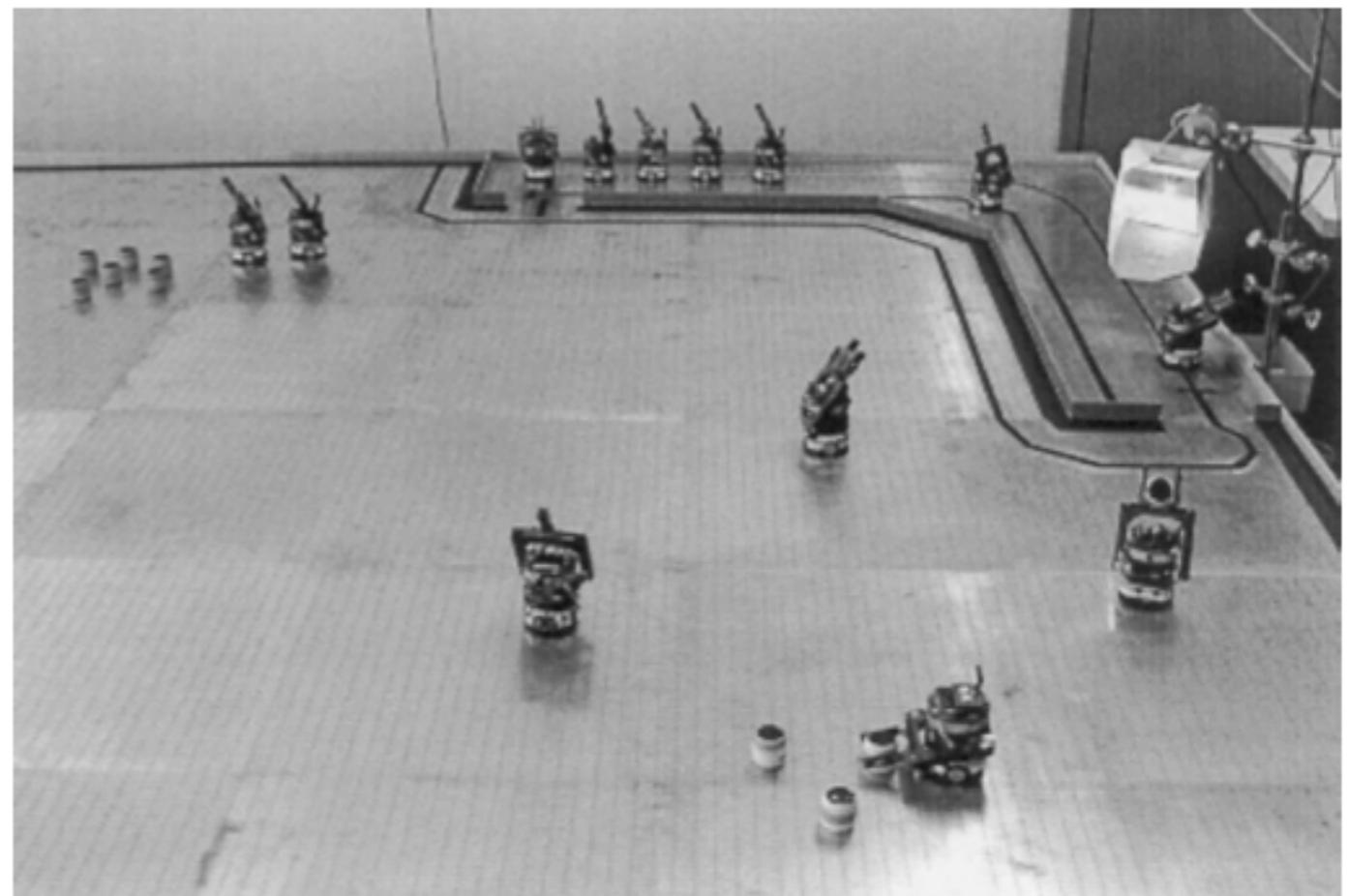
Balch et al. Io, Ganymede and Callisto: A multiagent robot trash-collecting team. *AI Magazine*, 16(2):39–53, 1995.



Melhuish et al.



Puck clustering

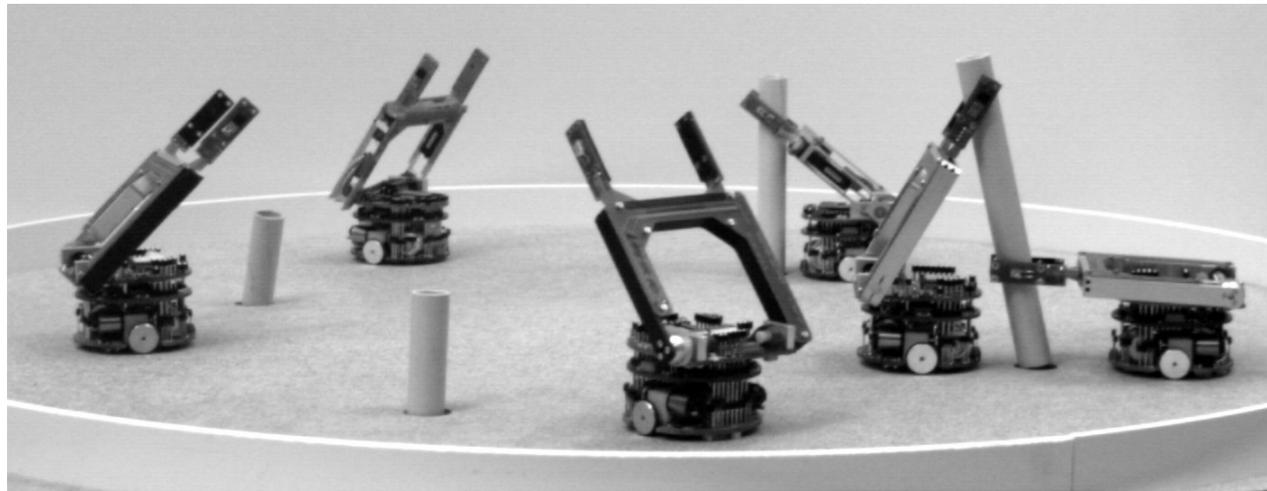


Multi-robot foraging

M. Krieger and J.-B. Billeter. The call of duty: Self-organised task allocation in a population of up to twelve mobile robots. *Jour. of Robotics & Autonomous Systems*, 30:65–84, 2000.

Multi-Robot Foraging 2

Collective manipulation

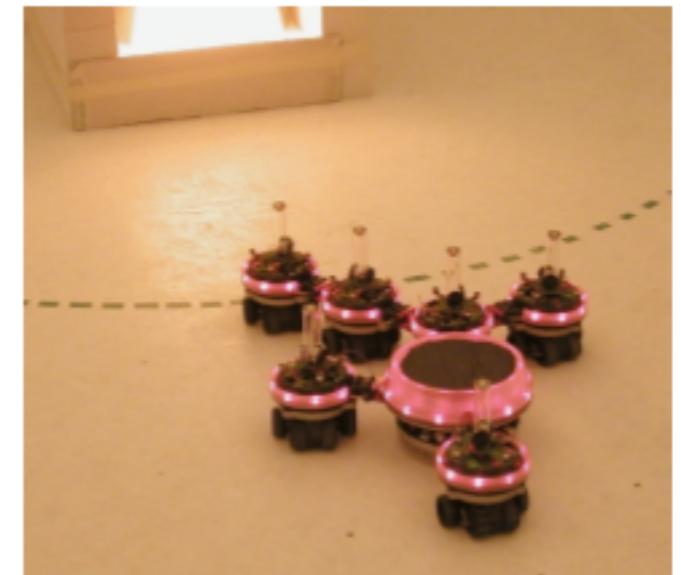
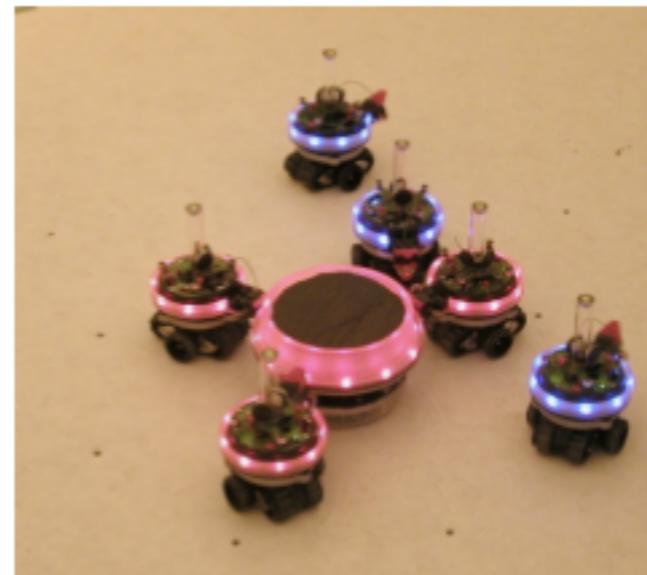
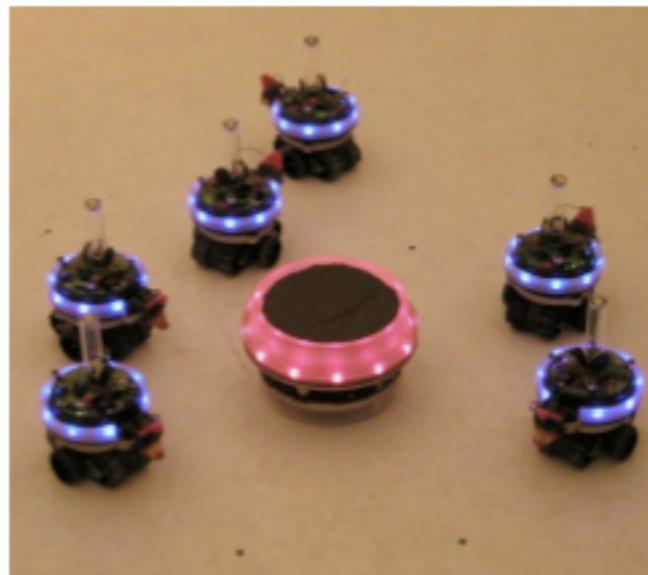


A. J. Ijspeert, A. Martinoli, A. Billard, and L. M. Gambardella. Collaboration through the exploitation of local interactions in autonomous collective robotics: The stick pulling experiment. *Autonomous Robots*, 11(2):149–171, 2001.



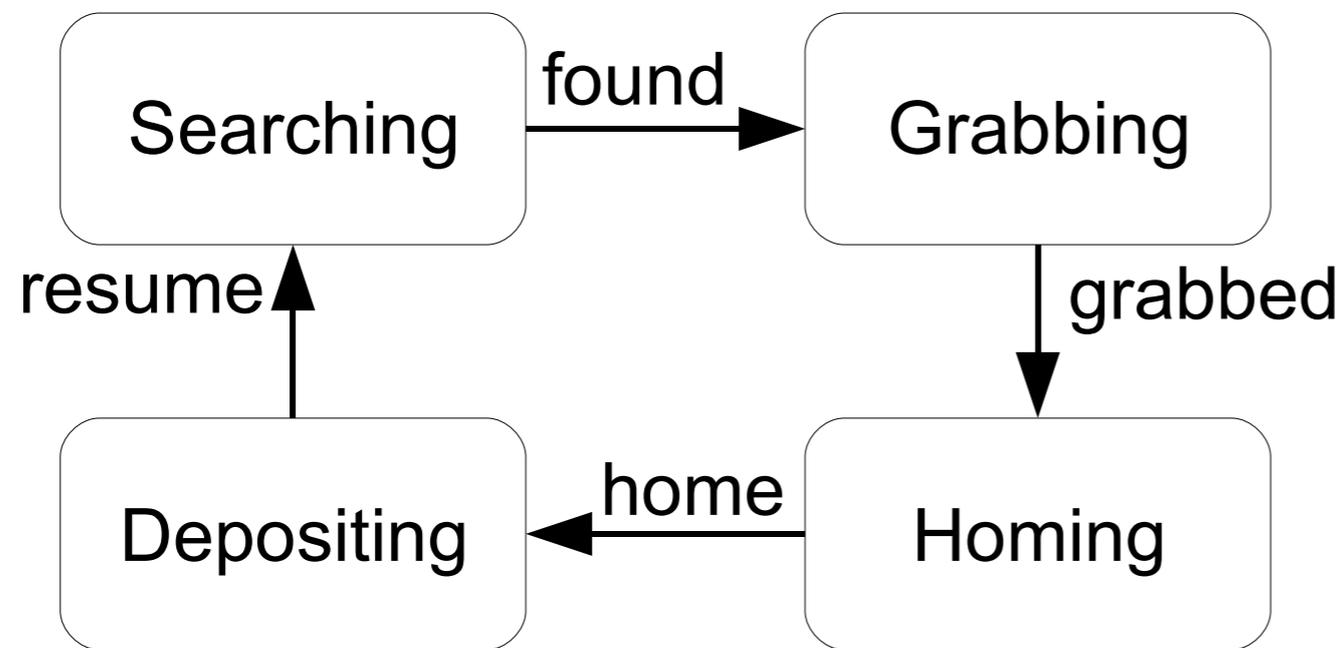
Search and Rescue, Prof Andreas Birk, Jacobs Uni, Bremen

Collective transport

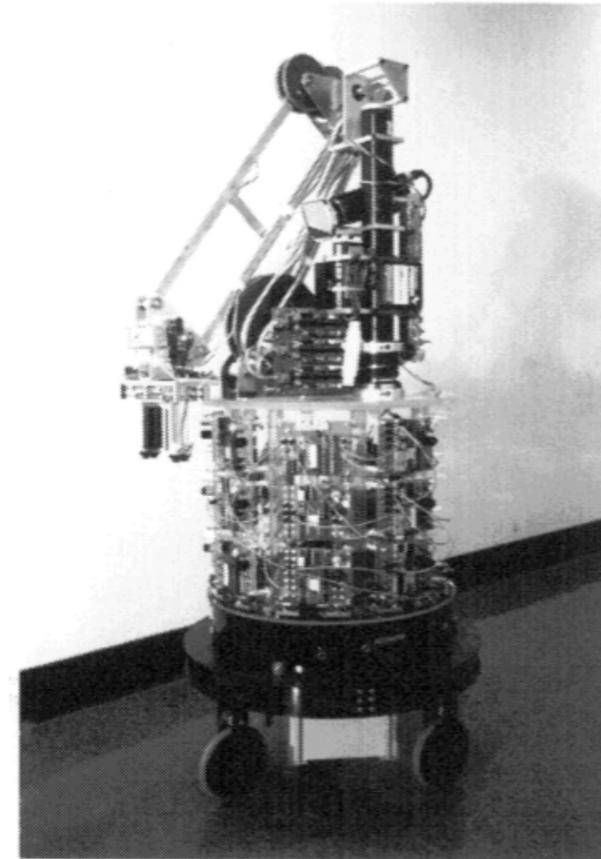


M. Dorigo, E. Tuci, T. Groß, V. Trianni, T.H. Labella, S. Nouyan, and C. Ampatzis. The SWARM-BOT project. In Erol Sahin and William Spears, editors, *Swarm Robotics Workshop: State-of-the-art Survey*, number 3342 in *Lecture Notes in Computer Science*, pages 31–44, Berlin Heidelberg, 2005. Springer-Verlag

Finite State Machine for basic foraging



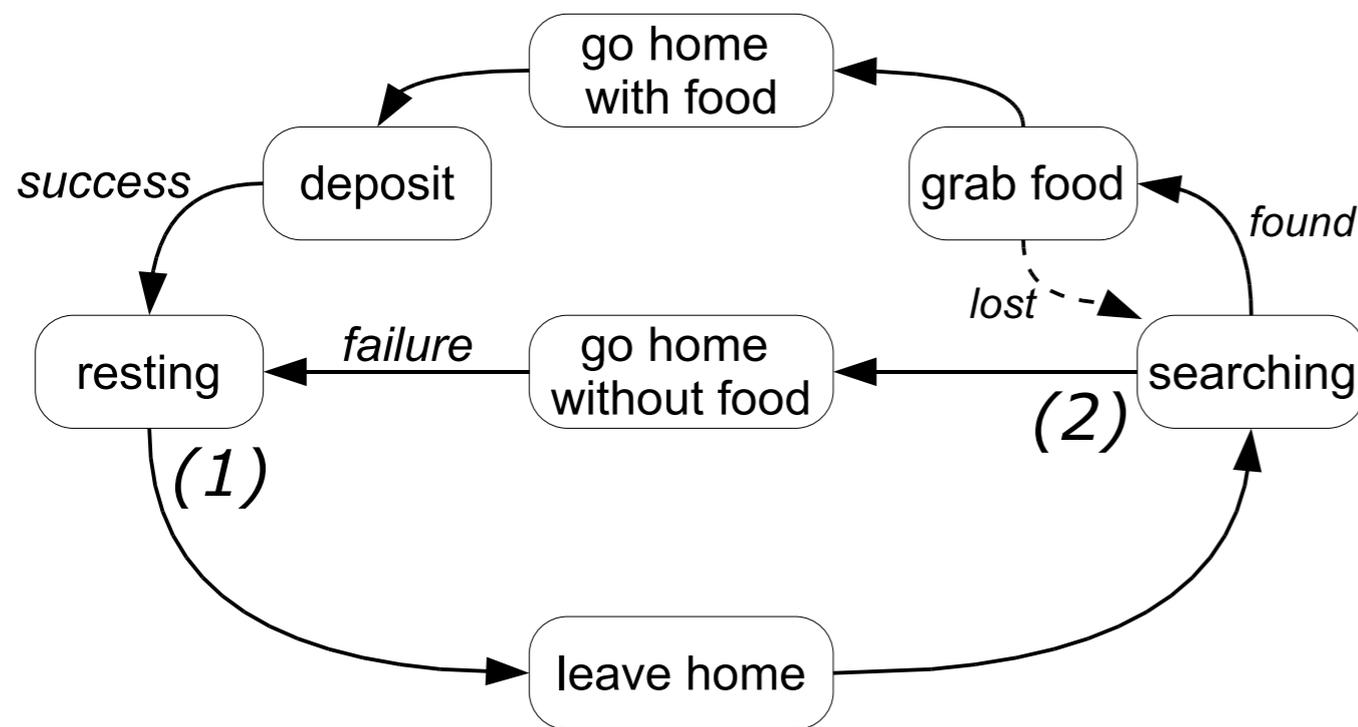
Four basic states provide an abstract model for single or multi robot foraging



Herbert

J. H. Connell. Minimalist Mobile Robotics: A colony-style architecture for an artificial creature. Morgan Kaufmann, 1990.

Generalised FSM for foraging with division of labour



- Robots leave the nest (1) when some threshold condition is met
 - e.g. resting time is up or net swarm energy drops below a certain value
- Robots abandon search (2) when
 - e.g. searching time is up or robot energy falls below a certain value
- We seek an algorithm in which robots can locally adjust their thresholds so that the overall ratio of resters to foragers adapts to the amount of food in the environment

Note: 'food' is a metaphor for any objects to be collected

Energy foraging

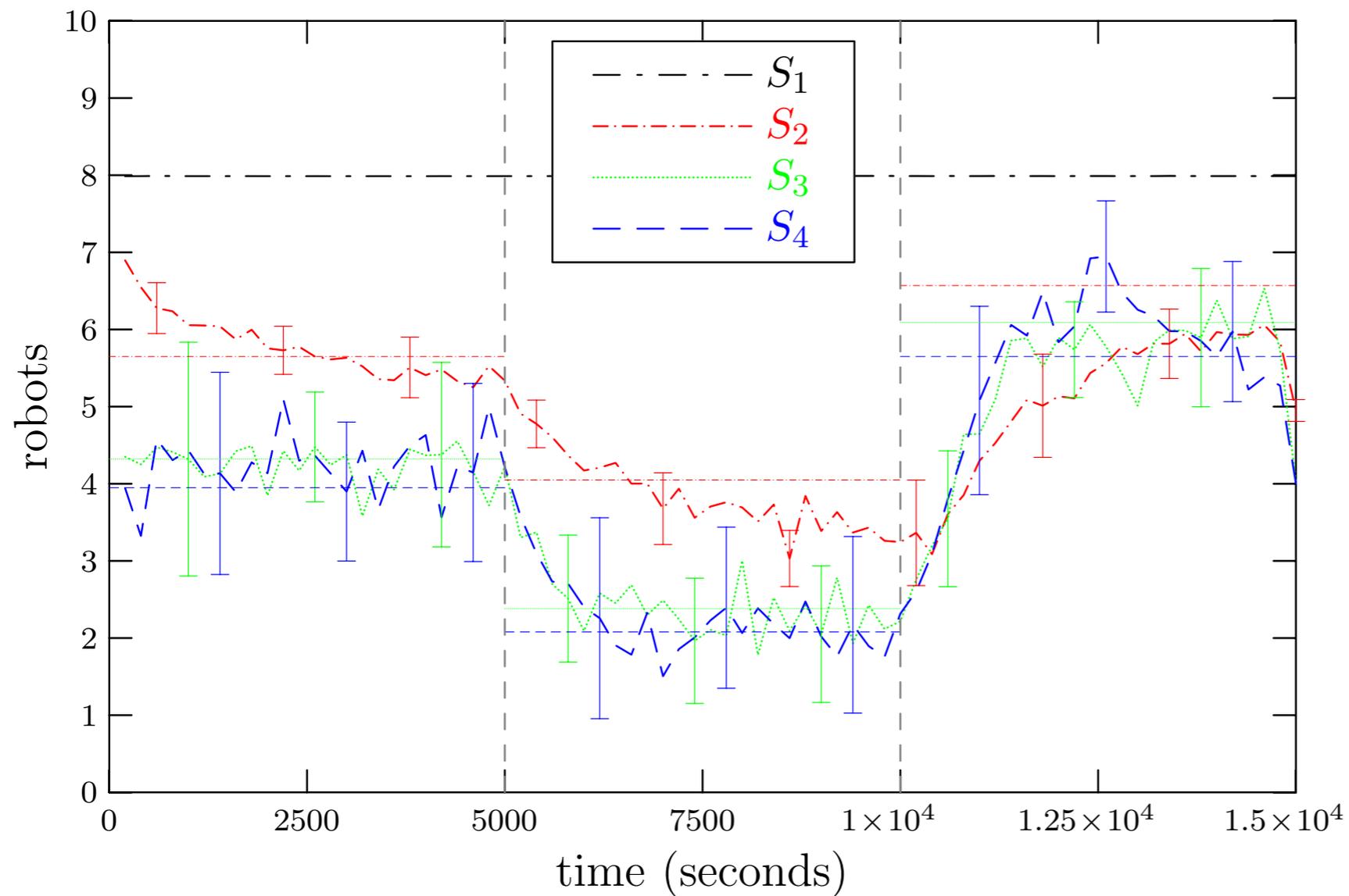
- Consider the special case of multi-robot foraging in which robots are foraging for their own energy. For an individual robot foraging costs energy, whereas resting conserves energy.
 - Each robot consumes energy at A units per second while searching or retrieving and B units per second while resting, where $A > B$
 - Each discrete food item collected by a robot provides C units of energy to the swarm
 - The average food item retrieval time, is a function of the number of foraging robots x , and the density of food items in the environment, ρ , thus $t = f(x, \rho)$

Strategies for cooperation

- Each robot has a search time threshold T_s and a rest time threshold T_r
 - Internal cues. If a robot successfully finds food it will reduce its T_r ; conversely if the robot fails to find food it will increase its T_r
 - Environment cues. If a robot collides with another robot while searching, it will reduce its T_s and increase its T_r times
 - Social cues. When a robot returns to the nest it will communicate its food retrieval success or failure to the other robots in the nest. A successful retrieval will cause the other robots in the nest to increase their T_s and reduce their T_r times. Conversely failure will cause the other robots in the nest to reduce their T_s and increase their T_r times

	internal cues	social cues	environment cues
S_1 (baseline)	×	×	×
S_2	✓	×	×
S_3	✓	✓	×
S_4	✓	✓	✓

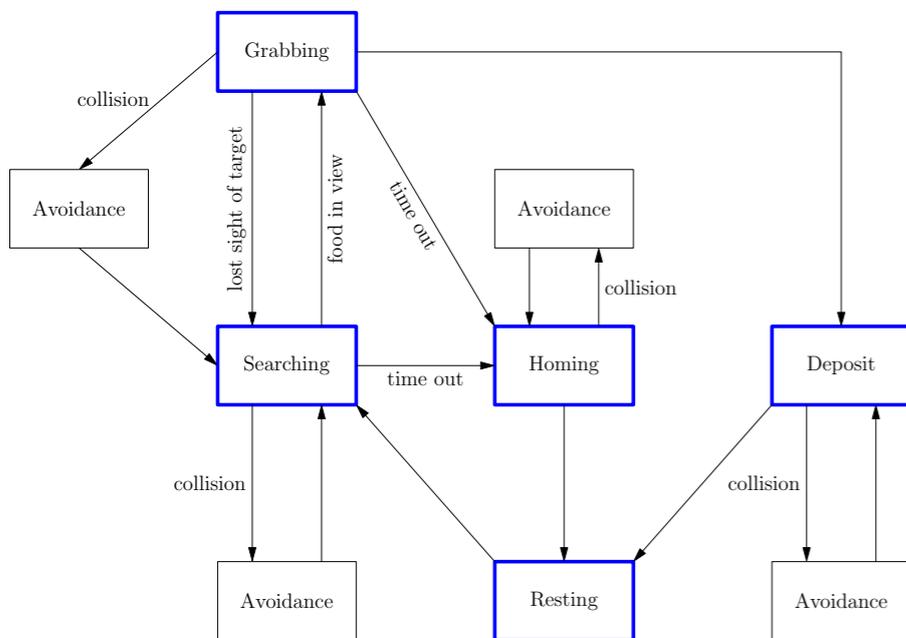
Adaptive foraging with changing food density



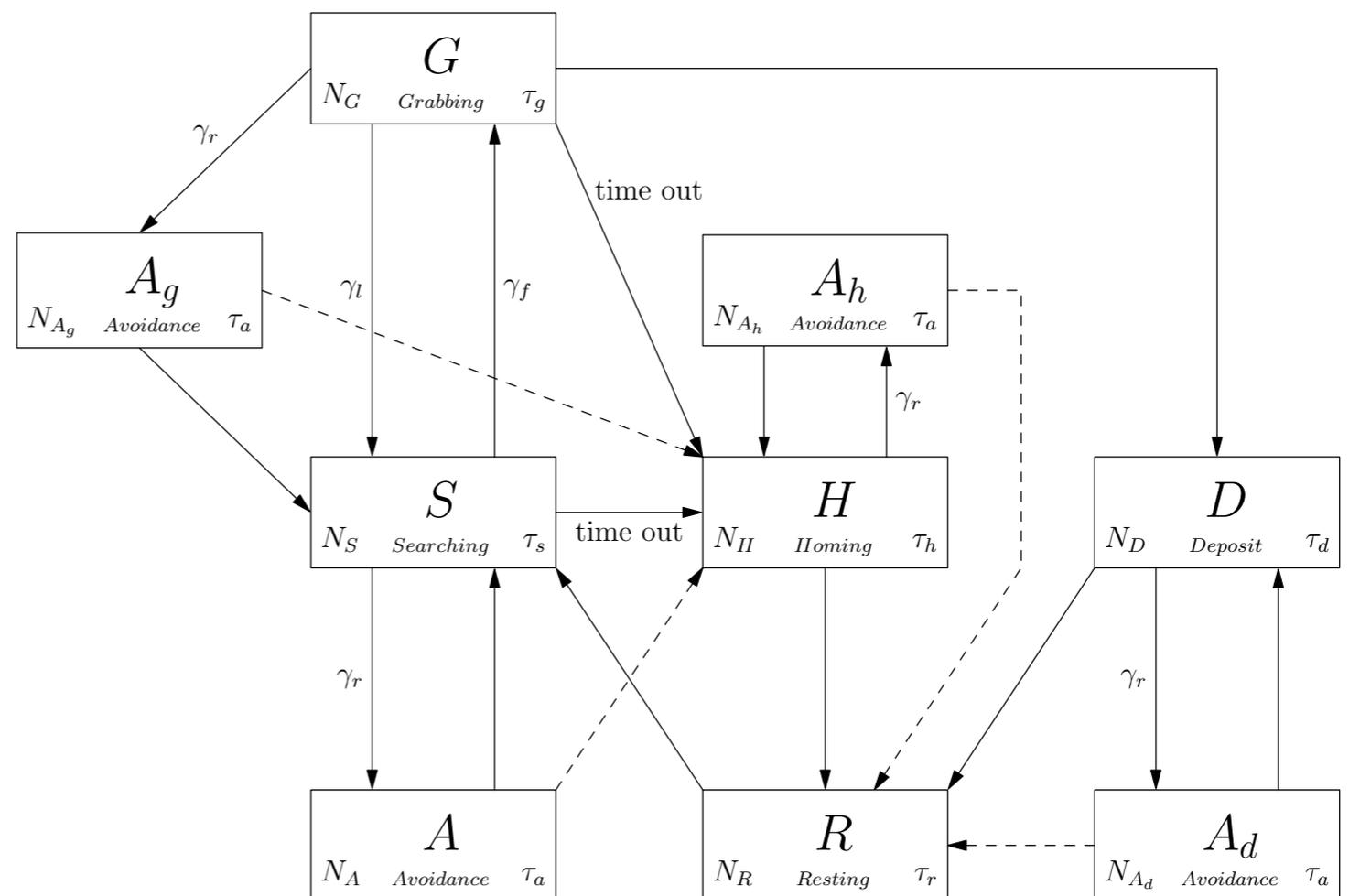
Number of foraging robots x in a foraging swarm of $N = 8$ robots. S_1 is the baseline (no cooperation strategy); S_2 , S_3 and S_4 are the three different cooperation strategies. Food density changes from 0.03 (medium) to 0.015 (poor) at $t = 5000$, then from 0.015 (poor) to 0.045 (rich) at $t = 10000$. Each plot is the average of 10 runs.

Developing the PFSM

Finite State Machine



Probabilistic Finite State Machine (PFSM)



PFSM parameters:

N number of robots in state .

\mathcal{T} time in state .

γ_f probability of finding food

γ_l probability of losing it

γ_r probability of collision

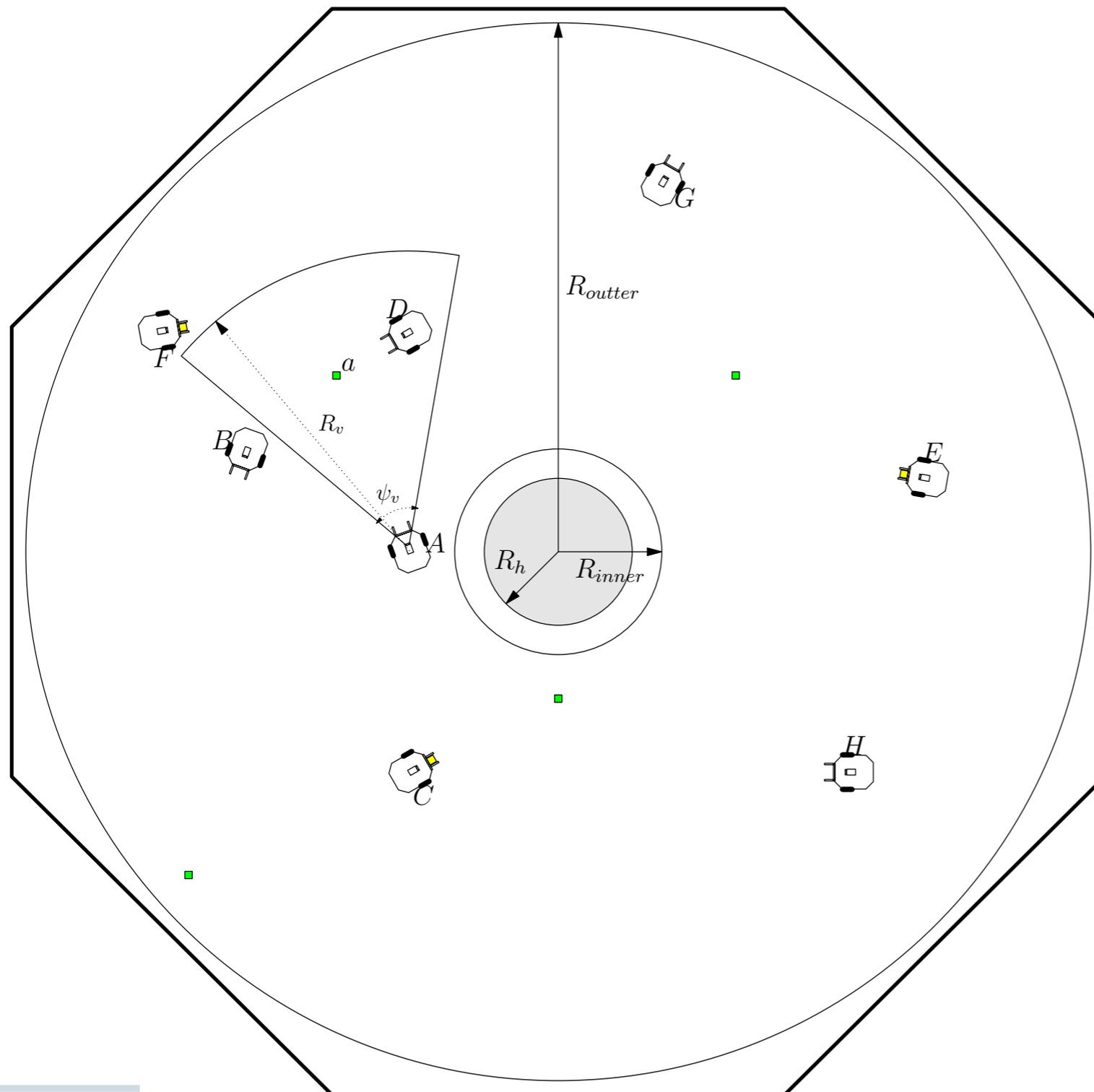
Difference equations

- For the PFSM we next develop a set of difference equations, e.g.

$$\begin{aligned} N_S(k+1) = & N_S(k) + \gamma_l(k)N_G(k) + \Delta_R(k - T_r) + [\Delta_A(k - T_a) - \Omega_A(k - T_a)] \\ & + [\Delta_{A_g}(k - T_a) - \Omega_{A_g}(k - T_a)] - \gamma_r(k)N_S(k) - \gamma_f M(k)N_S(k) \\ & - \Gamma_S(k+1) \end{aligned}$$

This appears complex because of multiple sampling rates and different priorities of behaviours

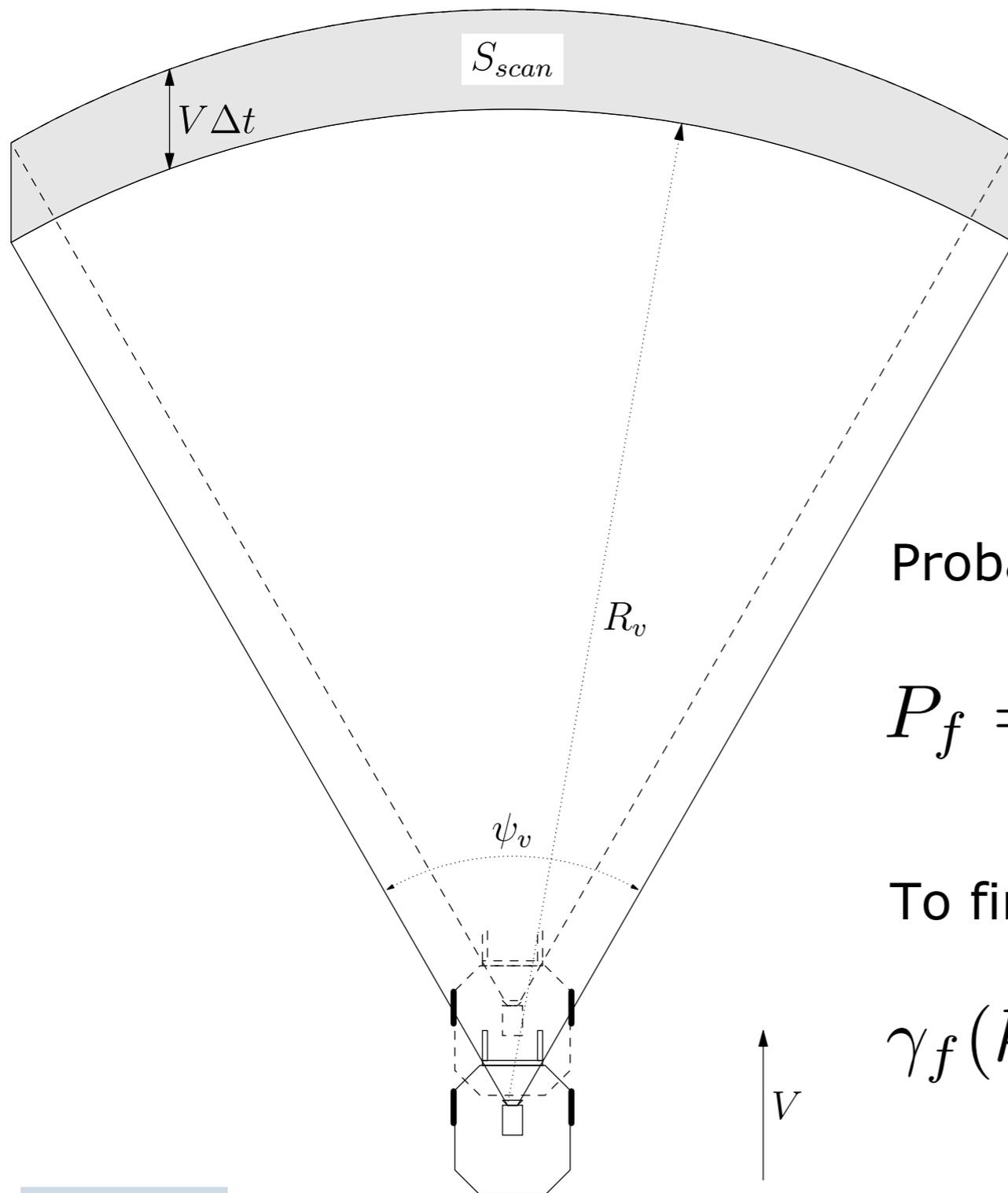
Geometrical estimation of state transition probabilities



- Three simplifying assumptions:

- food items are uniformly distributed
- robots have an equal probability of occupying any position in the arena
- the relative heading between any two robots varies uniformly in the range 0 to 360

probability of finding a food item: γ_f



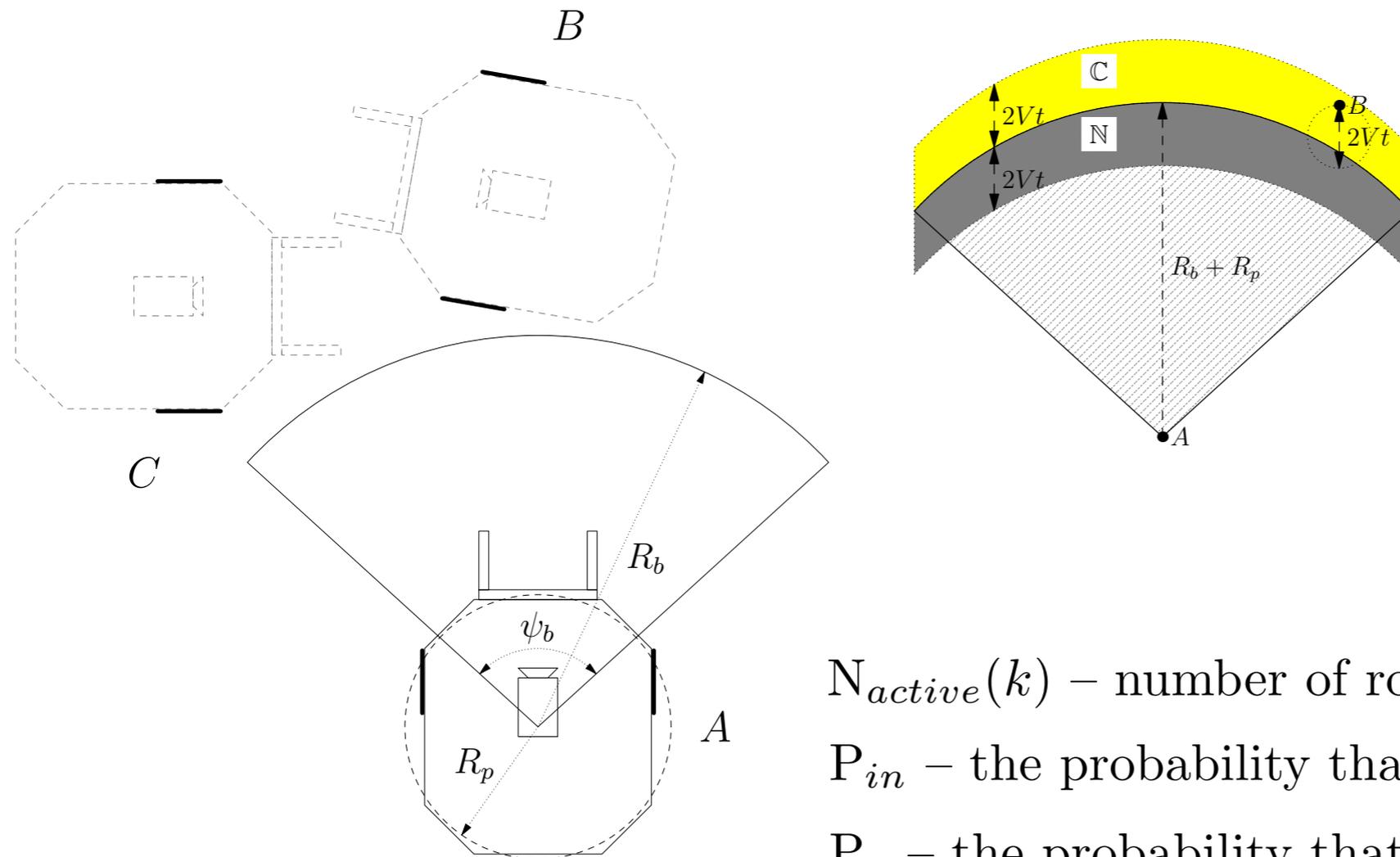
Probability to find 1 food item:

$$P_f = \frac{S_{scan}}{S_f} = \frac{\psi_v R_v V \Delta t}{\pi(R_{outer}^2 - R_{inner}^2)}$$

To find at least 1 of $M(k)$ food items:

$$\gamma_f(k) = 1 - (1 - P_f)^{M(k)} \approx P_f M(k)$$

collision probability: γ_r



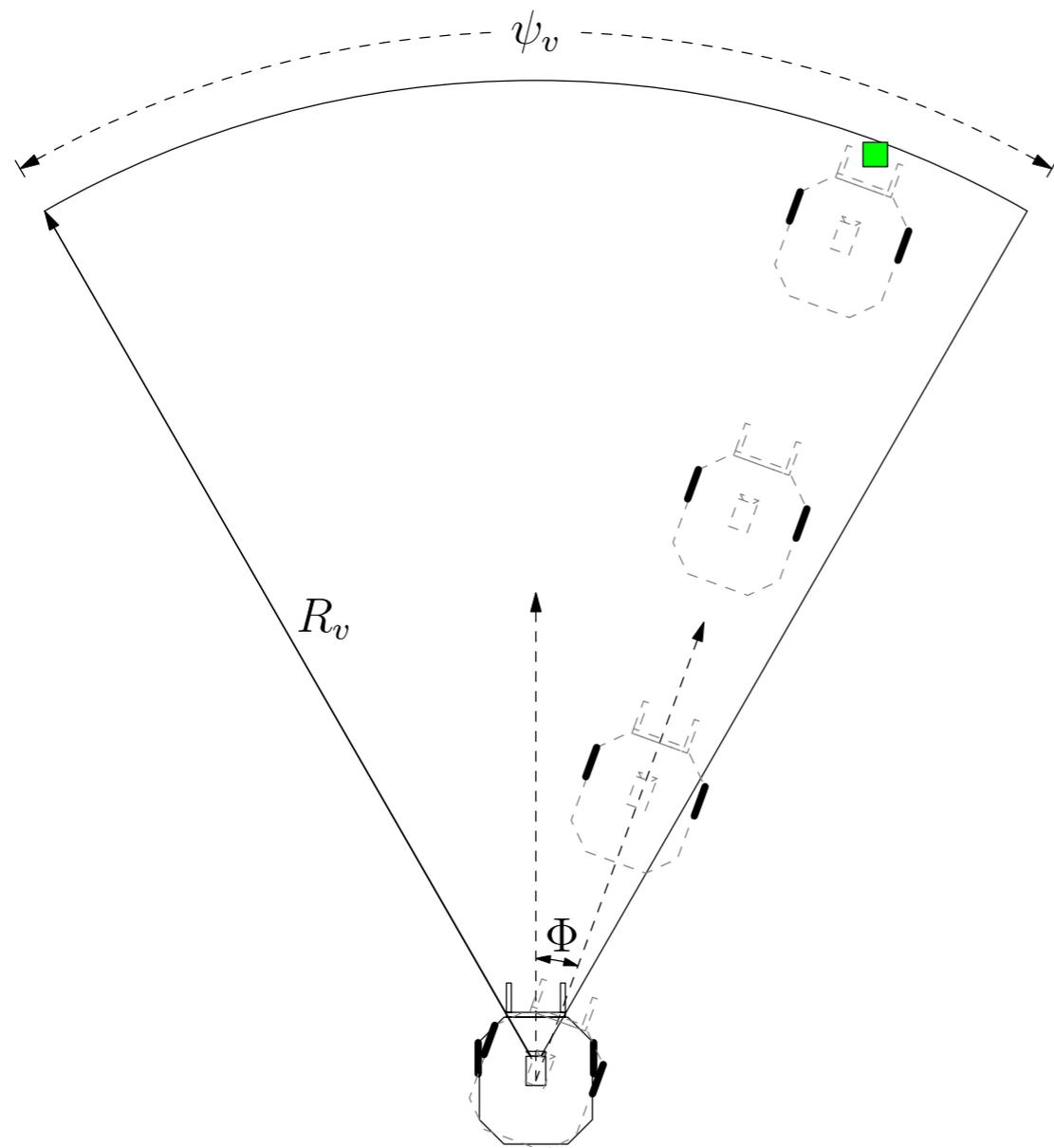
$N_{active}(k)$ – number of robots outside the nest

P_{in} – the probability that a robot is in area \mathbb{C}

P_a – the probability that a robot in area \mathbb{C} will move to \mathbb{N}

$$\gamma_r(k+1) = 1 - (1 - P_{in}P_a)^{N_{active}(k)-1}$$

Estimation of time parameter τ_g



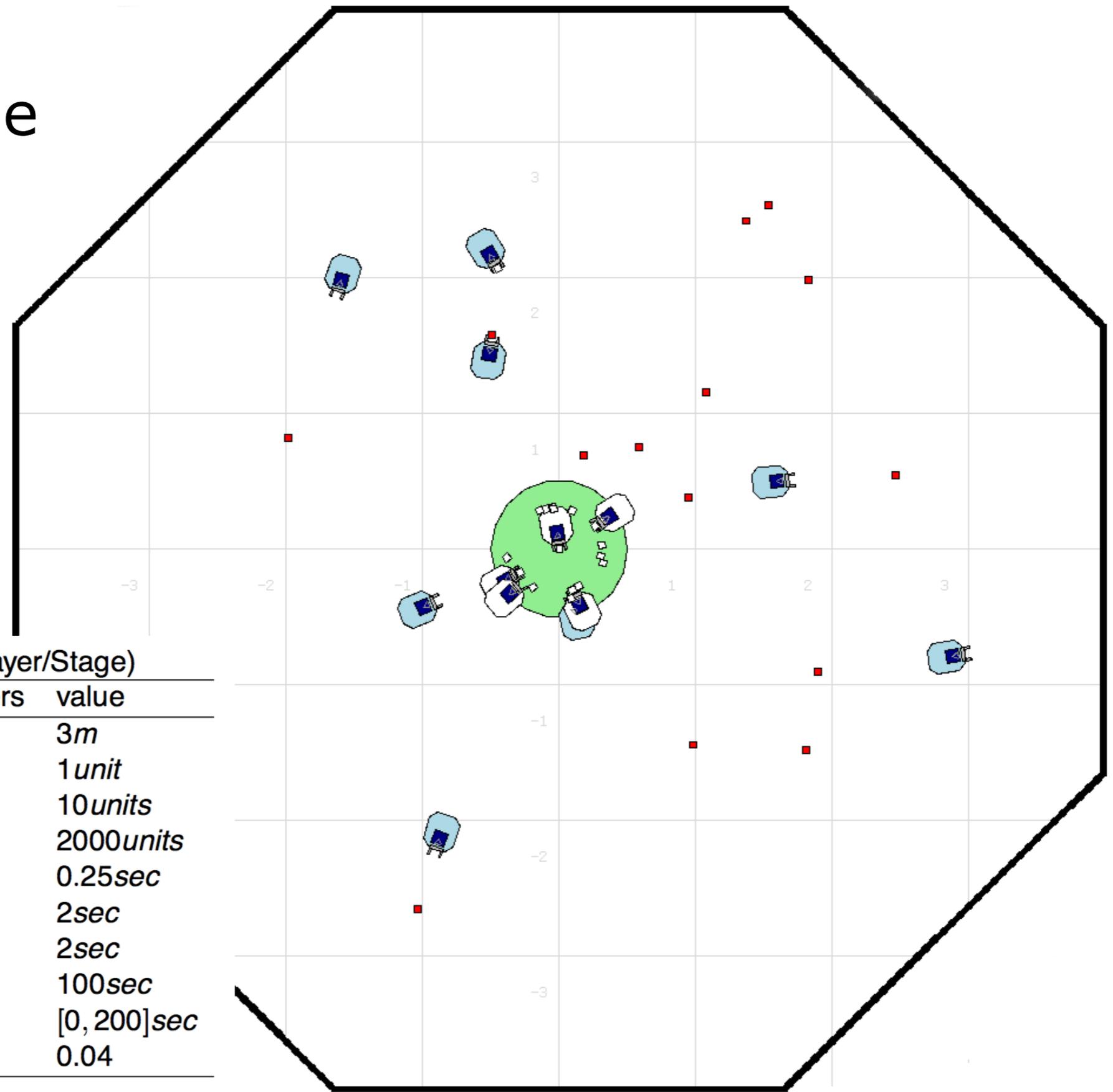
When a food item is in view the robot needs to

1. turn to face the food
2. move forward until close enough to grab it
3. grab and lift it

Average grabbing time:

$$\tau_g = \frac{\psi_v}{2\omega_1} + \frac{R_v}{V} + t_l$$

Validation of the model

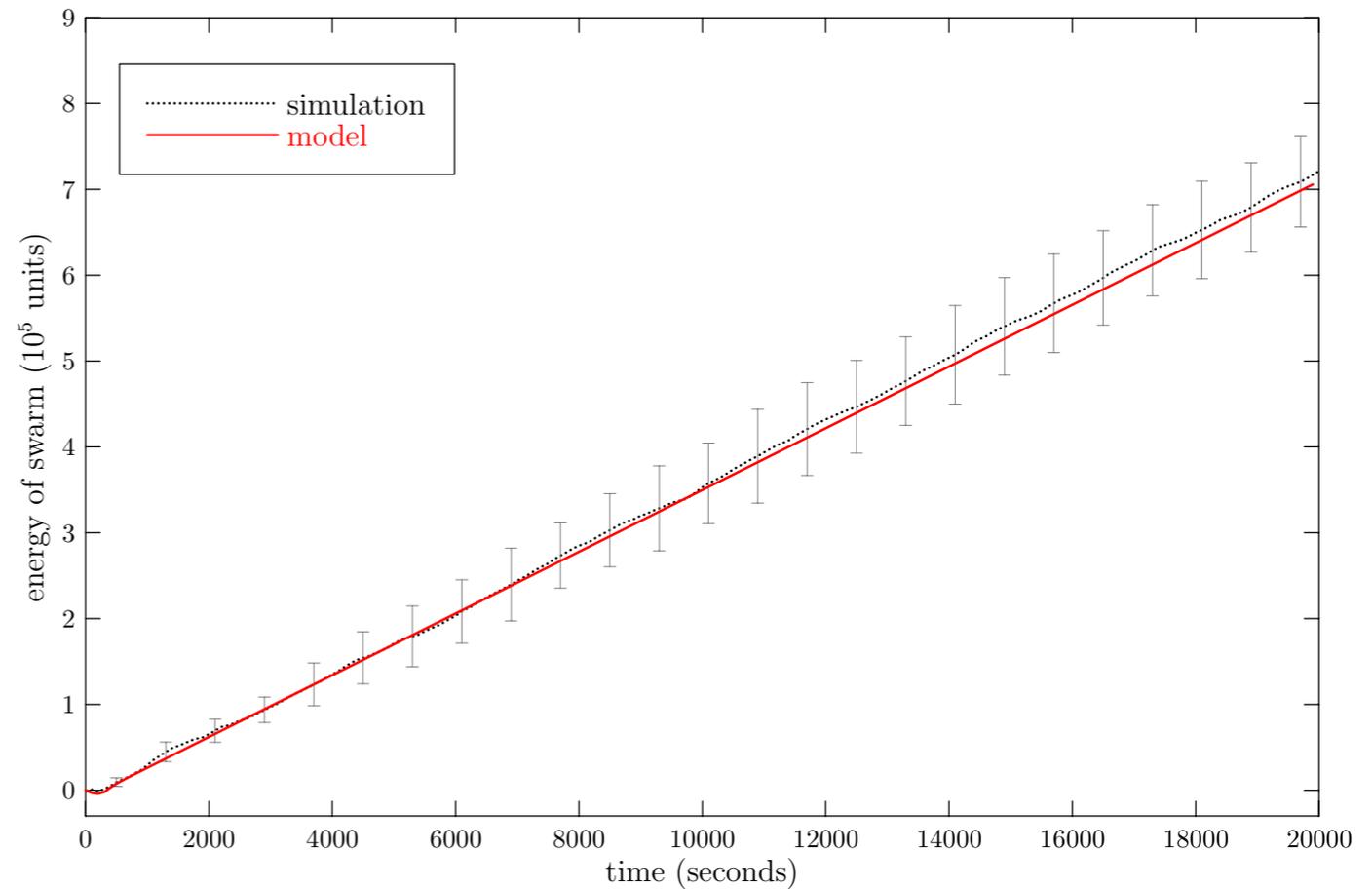
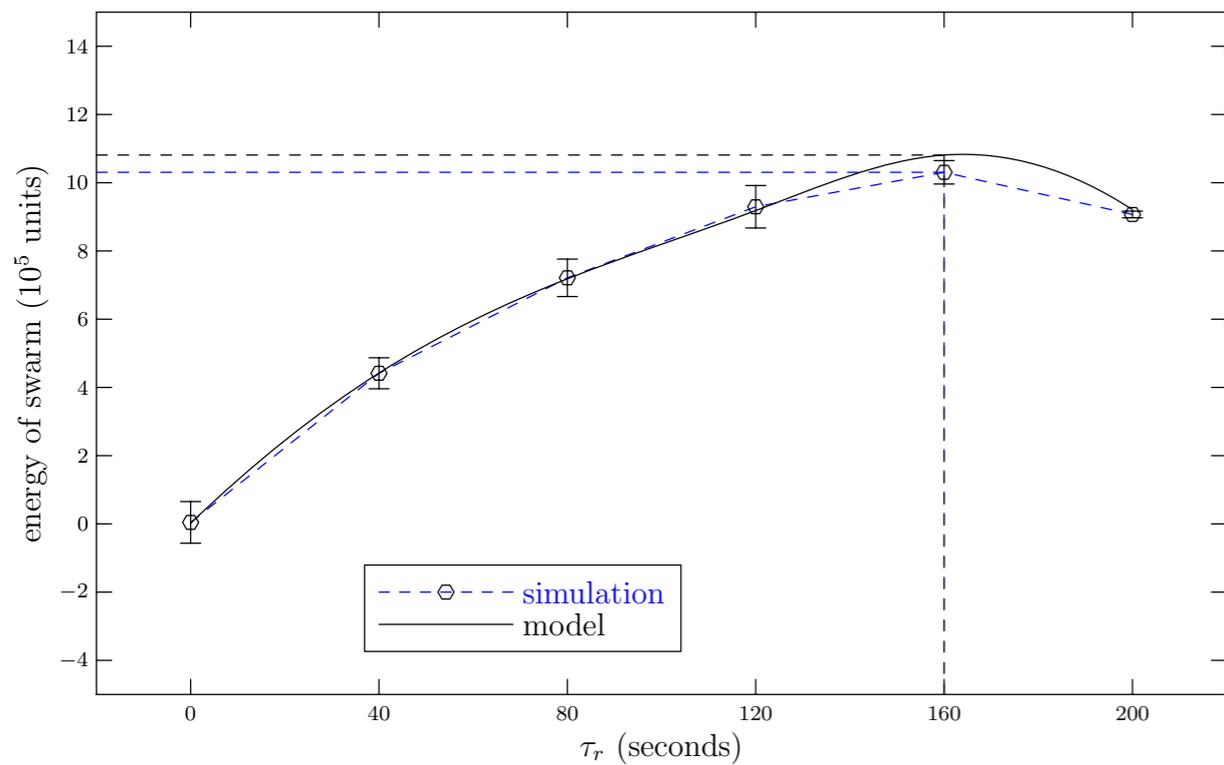


parameters for simulation (Player/Stage)

parameters	value	parameters	value
V	$0.15m/s$	R_{outer}	$3m$
w_1	$15^\circ/s$	E_r	$1unit$
w_2	$15^\circ/s$	αE_r	$10units$
ψ_v	60°	E_c	$2000units$
ψ_b	95°	Δt	$0.25sec$
R_v	$2m$	t_l	$2sec$
R_b	$0.4m$	τ_a	$2sec$
R_p	$0.13m$	τ_s	$100sec$
R_h	$0.5m$	τ_r	$[0, 200]sec$
R_{inner}	$0.7m$	ρ_{new}	0.04

validation of the model (2)

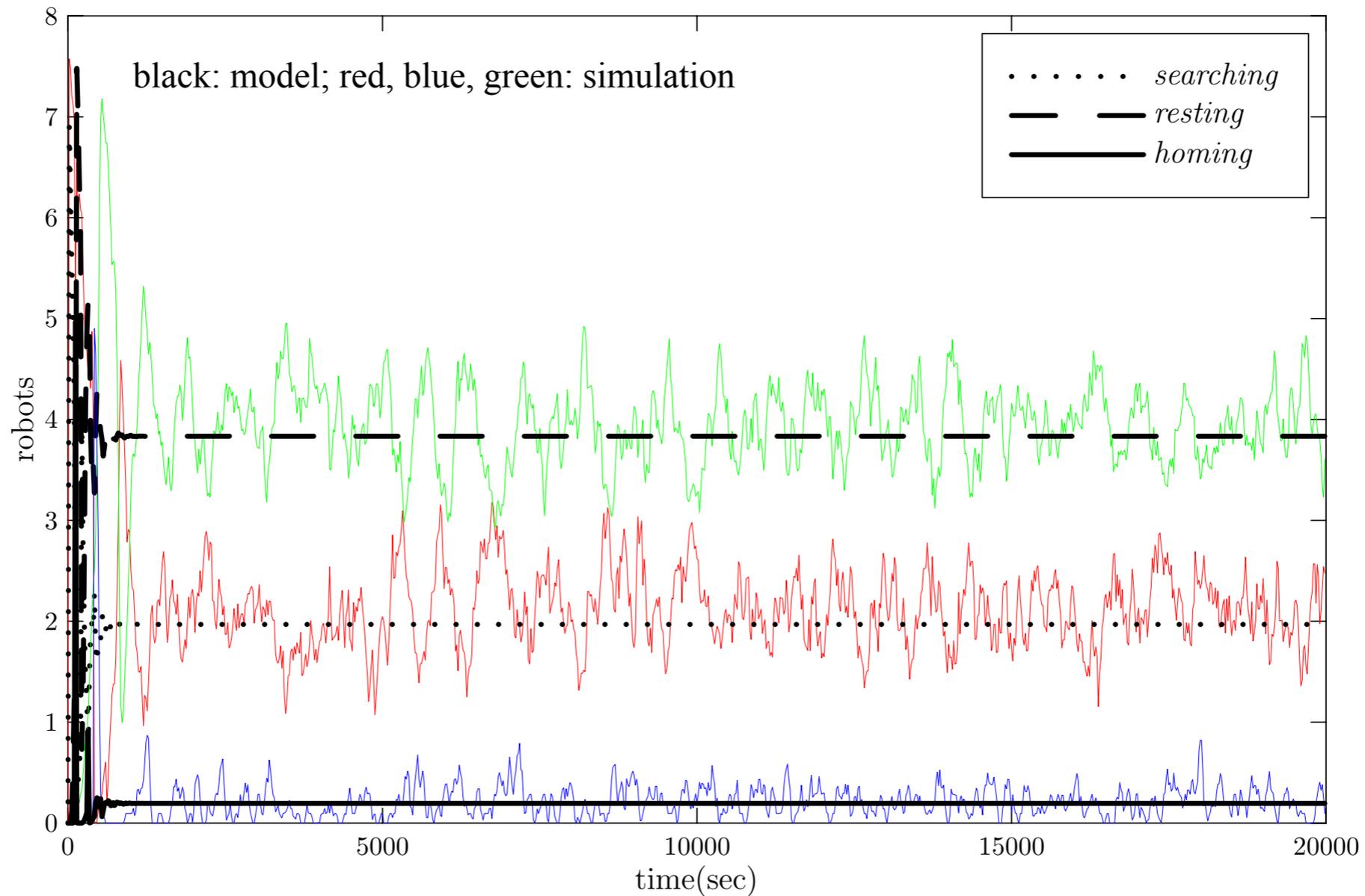
Net swarm energy, (left) varying resting time threshold τ_r , (right) for $\tau_r = 80$ s



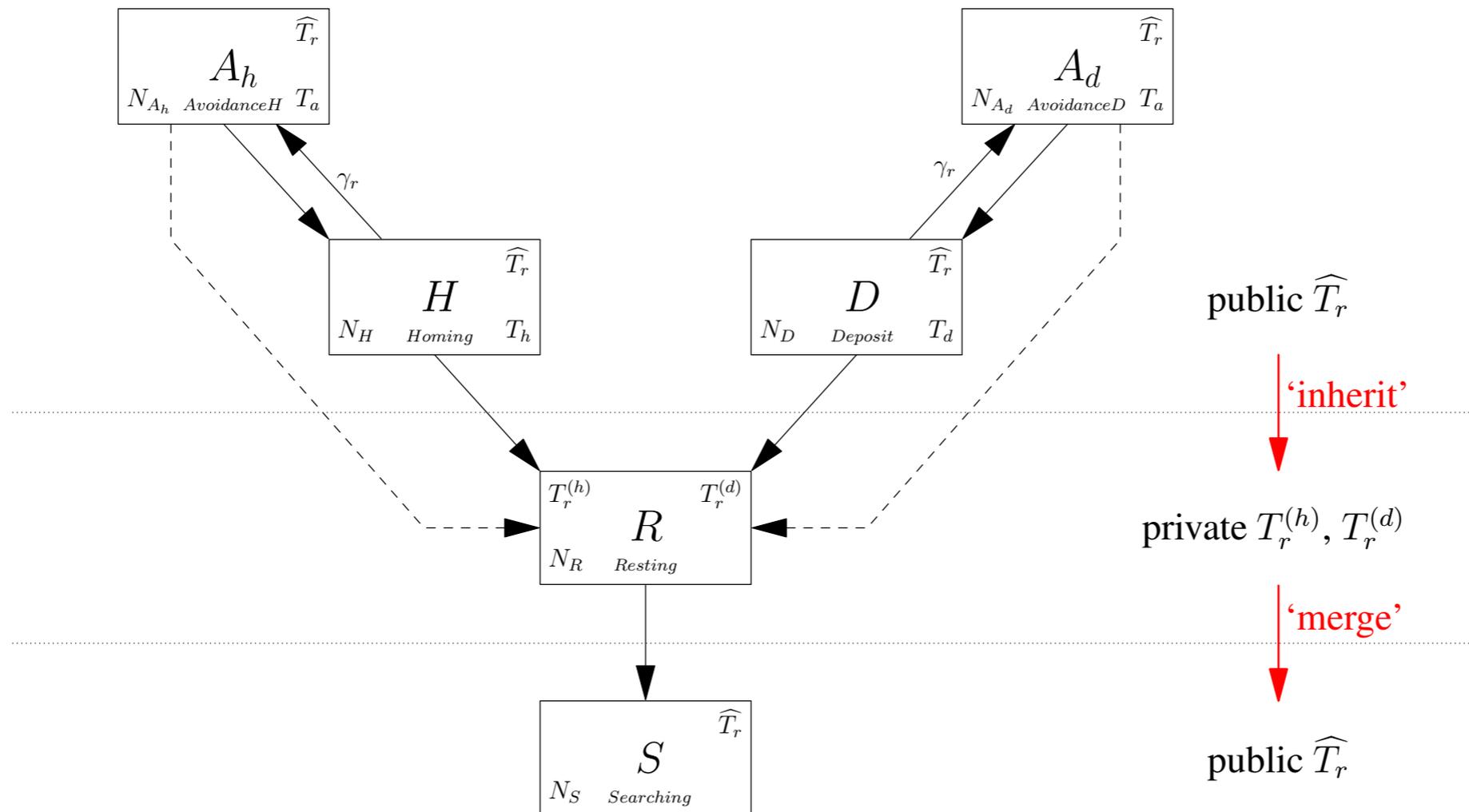
$$E(k+1) = E(k) + E_c \Delta_D(k - T_d) - E_r N_R(k) - \alpha E_r (N_0 - N_R(k))$$

validation of the model (3)

Average number of robots in states *searching*, *resting* and *homing* for $\mathcal{T}_r = 80s$



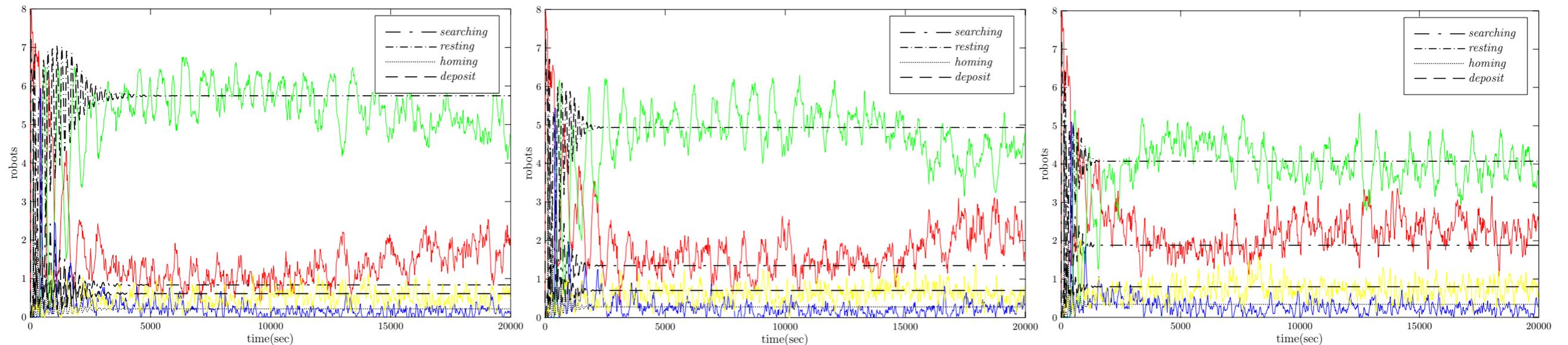
Modelling adaptive foraging



We introduce the concept of short time lived sub-PFSMs, with 'private' parameters

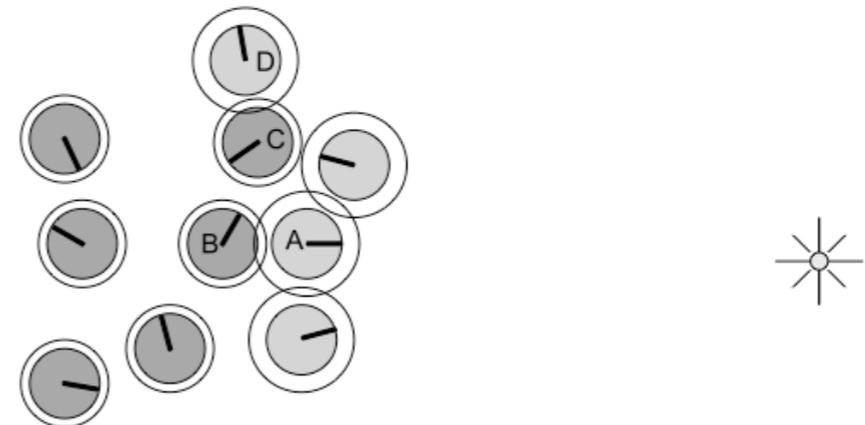
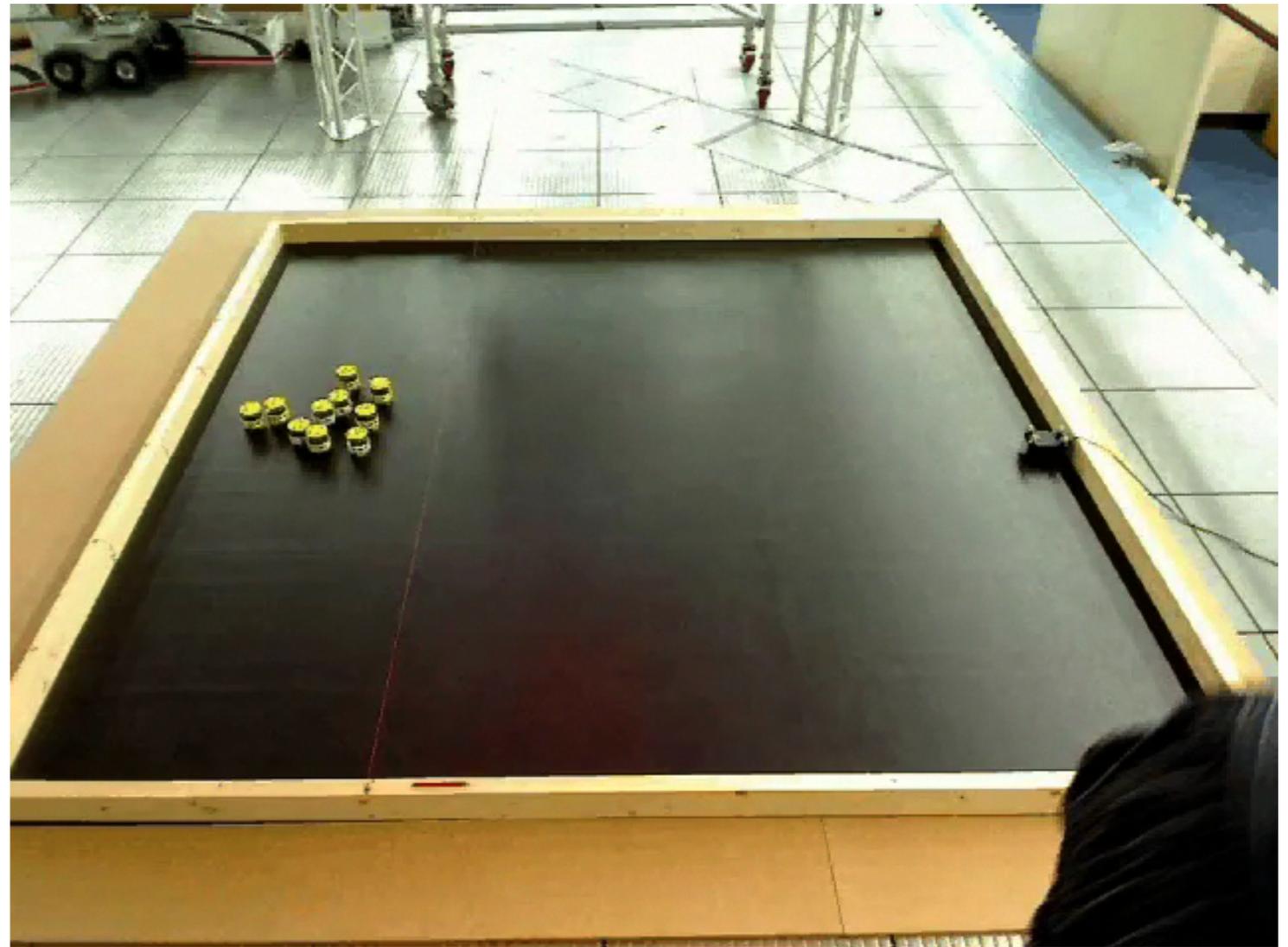
Model of adaptive foraging: validation of the model

Variable food density: 0.45, 0.4, 0.35



Reliability Modelling: emergent swarm taxis

- How does it work...
- Robots have simple aggregation:
 - short range: obstacle avoidance (repulsion)
 - longer range: maintain number of connected neighbours (attraction)
- Each robot also has a simple beacon sensor
 - symmetry breaking mechanism: *illuminated* robots have a slightly larger avoid radius than *occluded* robots



Failure modes and effects analysis

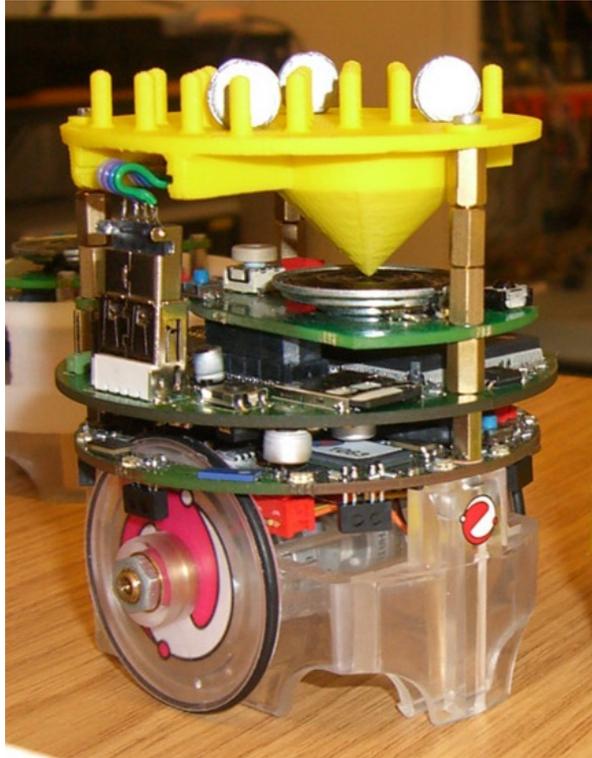
<i>Hazard</i>	<i>Description</i>
H1	Motor failure
H2	Communications failure
H3	Avoidance Sensor failure
H4	Beacon Sensor failure
H5	All Systems failed

	<i>Failure Effect</i>
E1	Motor failure impedes the translational motion of the swarm (anchors the swarm)
E2	Lost robot(s) loose in the environment
E3	Robot collisions with obstacles or target

Linking hazards and effects to swarm behaviours

<i>Swarm Behaviour</i>	H1	H2	H3	H4	H5
Aggregation	-	e2	-	-	-
Ad-hoc Network	-	e2	-	-	-
Object Taxis	E1	e2	-	-	-
Obstacle Avoidance	E1	e2	e3	-	-
Object Containment	E1	e2	e3	-	-

Introduce worst-case partially failed robots

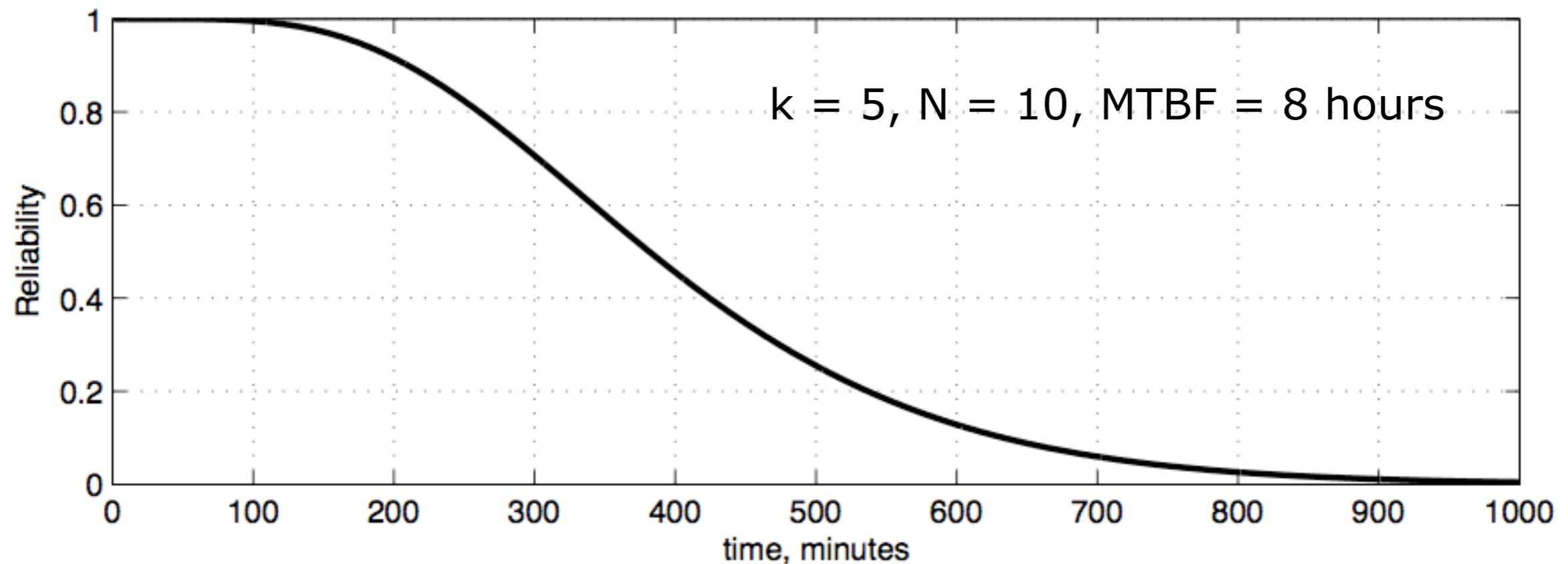


2 robots
failure mode H1

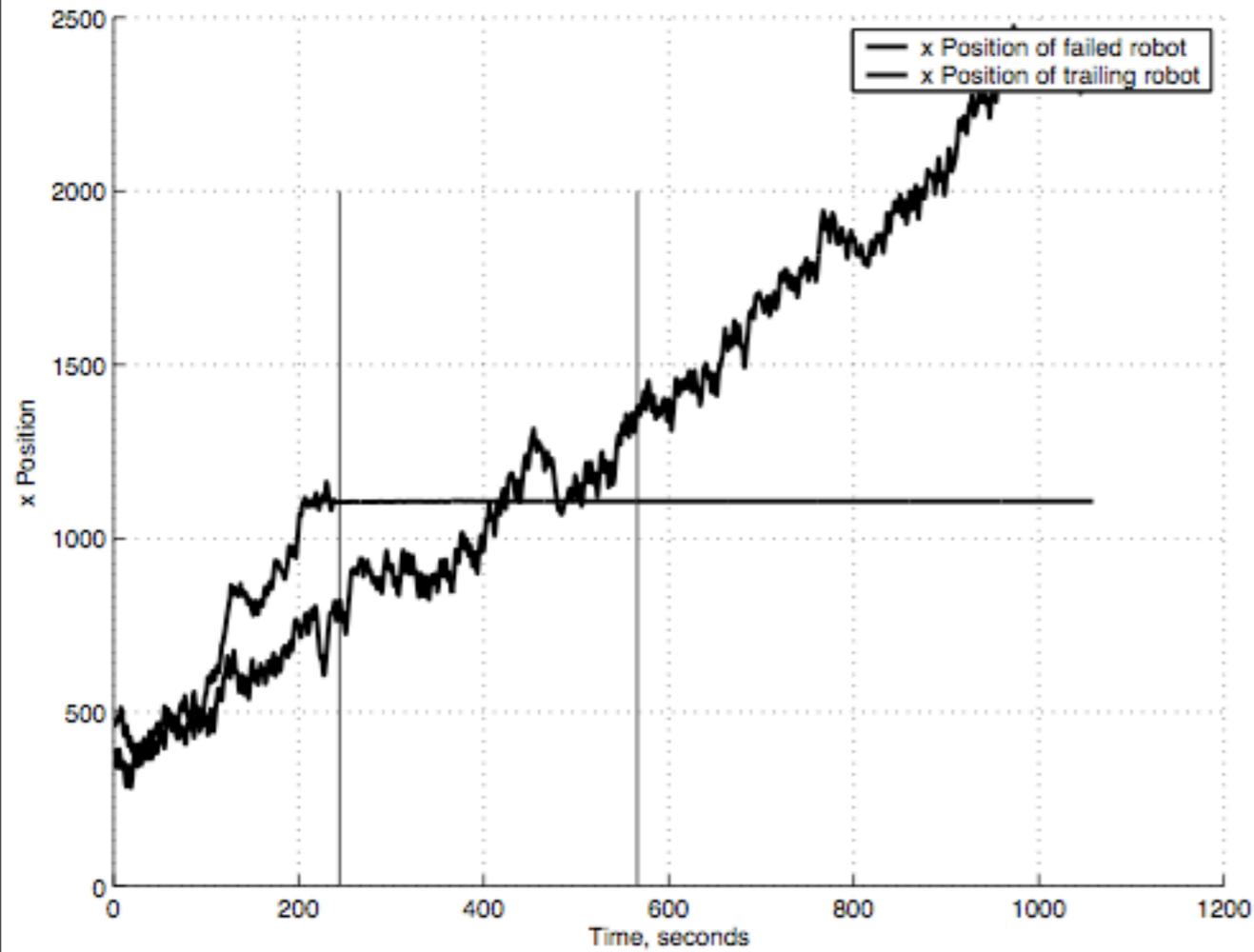
The k-out-of-N reliability model

The probability that at least k out of N robots are working at time t:

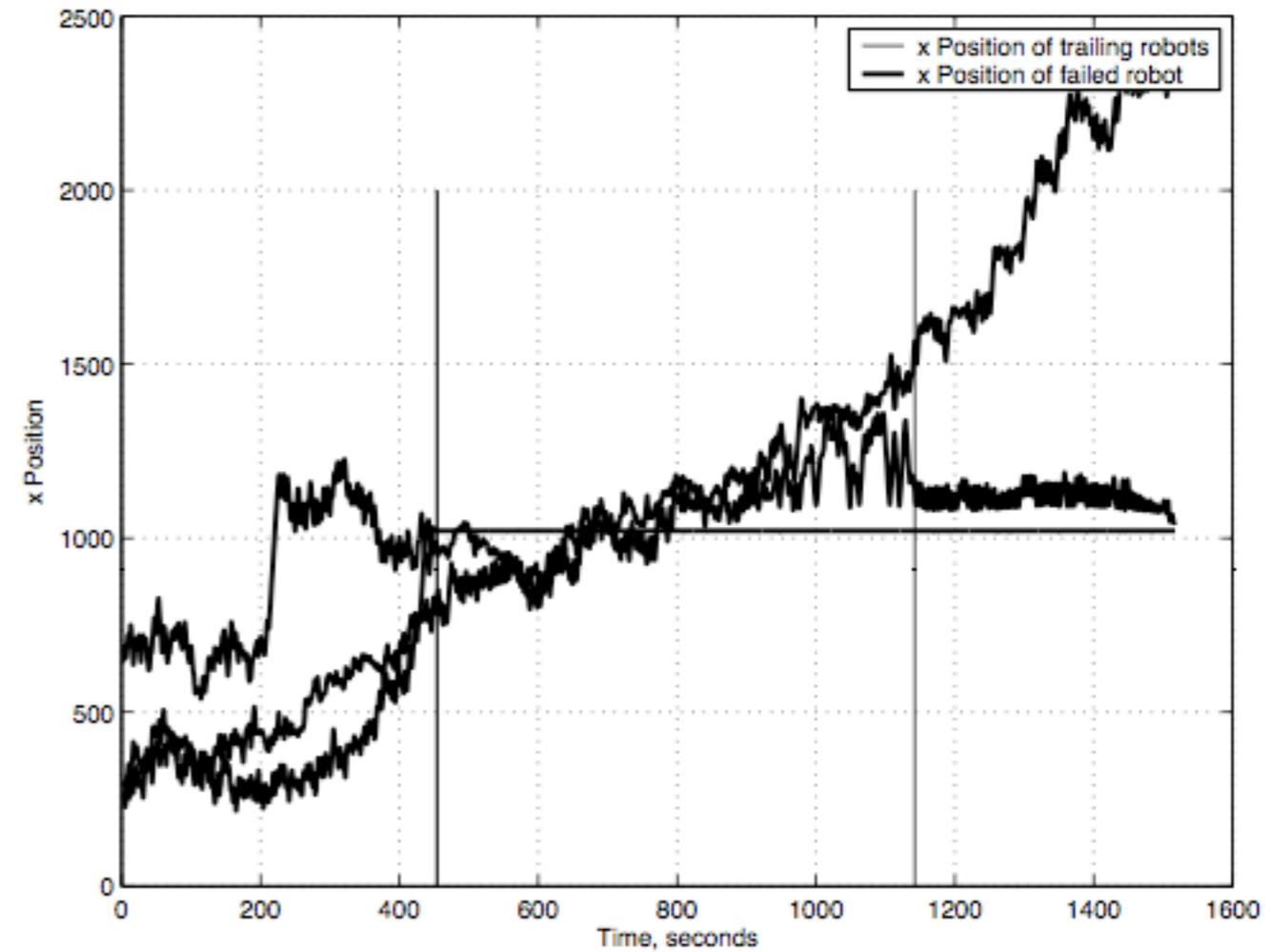
$$P(k, N, t) = \sum_{i=k}^N \binom{N}{i} (e^{-t\lambda})^i (1 - e^{-t\lambda})^{N-i} \quad \lambda = \frac{1}{MTBF}$$



Swarm self-repair



Single robot complete failure H5



Single robot partial failure H1

Estimate k for partial failure H1

- Conservatively $k = 0.9N$
 - in other words, we believe the swarm can tolerate 10% of H1 failures at any one time (i.e. within swarm self-repair time)

Estimate swarm self-repair time

Since a robot can fail anywhere in the swarm the average distance the swarm needs to move to escape the failed robot is half the diameter of the swarm, i.e. $t = d/2v$, $d = \text{swarm diameter}$, $v = \text{swarm velocity}$

We know

$$v(N) = CN^{-\frac{1}{2}} \quad \text{and} \quad d(N) = D\sqrt{N}$$

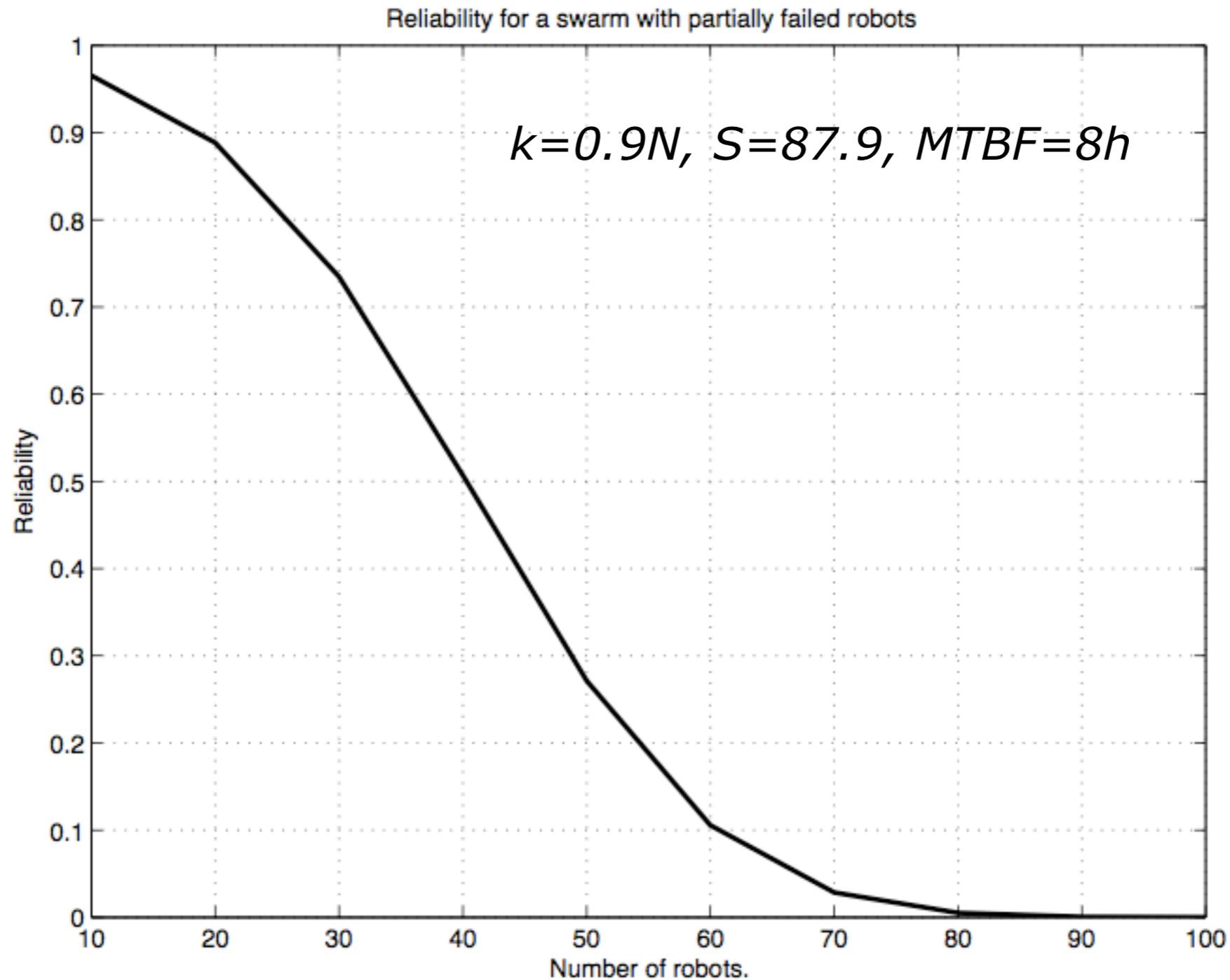
Thus

$$t(N) = \frac{D}{2C}N$$

Therefore swarm self repair time t is linear with N .

With $N=10$ and 1 partially failed robot mean swarm self repair time was measure as 870s, thus the constant $S = D/2C = 87.9$

Reliability as a function of swarm size



Discussion

- The frequent assumption, that swarm systems are automatically scalable and robust, is seriously incorrect
- This result strongly suggests that scaling systems (which rely on emergence or self-organising mechanisms) requires more sophisticated internal mechanisms for dealing with worst-case failures:
 - an *immune system*

Thank you

- Acknowledgements

- Prof Alcherio Martinoli, EPFL; Dr Julien Nembrini, UWE -> EPFL; Dr Wenguo Liu, BIT, Beijing - UWE Bristol; Dr Jan Dyre Bjercknes, UWE Bristol

- Papers

- Winfield AFT, Liu W and Bjercknes JD, 'Modelling and Reliability of Collective Systems', Symbiotic Multi-Robot Organisms, eds P Levi and S Kernbach, in press.
- Liu W, Winfield AFT, Sa J. 'A macroscopic probabilistic model of adaptive foraging in swarm robotics systems'. In Proc. 6th Vienna International Conference on Mathematical Modelling (Mathmod 2009), Special Session on Modelling the Swarm, Vienna, February 11 – 13, 2009.
- Winfield AFT, Liu W, Nembrini J and Martinoli A, 'Modelling a Wireless Connected Swarm of Mobile Robots', Swarm Intelligence, 2 (2-4), 241-266, 2008.
- Bjercknes JD, Winfield AFT and Melhuish C, 'An Analysis of Emergent Taxis in a Wireless Connected Swarm of Mobile Robots', Proc. IEEE Swarm Intelligence Symposium (SIS 2007), Hawaii, April 2007.
- Winfield AFT and Nembrini J, 'Safety in Numbers: Fault Tolerance in Robot Swarms', Int. J. Modelling Identification and Control, 1 (1), 30-37, 2006.
- Winfield AFT, Harper CJ and Nembrini J, 'Towards Dependable Swarms and a New Discipline of Swarm Engineering', in SAB'04 Swarm Robotics workshop, eds. Sahin E and Spears W, Springer-Verlag, LNCS 3342, pp 126-142, 2005.