Logical gates by glider-gun dynamics – the X-rule

Andrew Wuensche[1] and José Manuel Gómez Soto[2]

[1]Discrete Dynamics Lab, London, UK, `http://www.ddlab.org`
[2] Universidad Autónoma de Zacatecas, Unidad Académica de Matemáticas. Zacatecas, Zac. México
andy@ddlab.org, jmgomezuam@gmail.com

Introduction

This is a very brief overview of our recent paper (Gomez-Soto and Wuensche, 2015) about a new Life-like Cellular Automata (CA), the X-rule – a 2d, binary CA with a Moore neighborhood ▦ and a λ parameter analogous to the game-of-Life (Berlekamp et al., 1982, Chapter 25), but not based on birth/survival and not fully isotropic. Glider-guns based on periodic oscillations between stable barriers were constructed, and interactions combining multiple glider-guns and eaters/reflectors were arranged and synchronised precisely to create the logical gates NOT, AND, OR and NOR required for logic universality, and potentially universality in the Turing sense, though this will require further work.

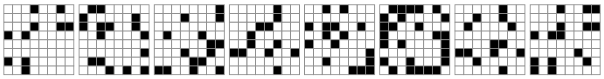

Figure 1: The rule-table of the X-rule – 512 neighborhood outputs are shown in descending order of their values, from left to right, then in successive rows from the top.

Glider-guns

The X-rule's glider-guns are analogous to Gosper's in the game-of-Life(Berlekamp et al., 1982, Chapter 25), but different in that they are constructed from a kit of parts, gliders and reflectors, that can be put together in many combinations to produce periodic oscillators based on bouncing/reflecting behaviour – pairs of gliders bouncing against each other and trapped between reflectors from which other glider types are ejected at periodic intervals.

This was achieved by introducing specific non-isotropic outputs within an isotropic precursor rule. Increasing the gap between reflectors increases the glider-gun period and reduces glider ejection frequency. These and other emergent structures enable flexible and versatile computational dynamics. Snapshots of the two basic glider-guns, GGa and GGb, are shown in figures 2 and 3, and figure 6 and 7 show the compound glider-gun GGc. So far GGa and GGc have provided the components for logic circuits.

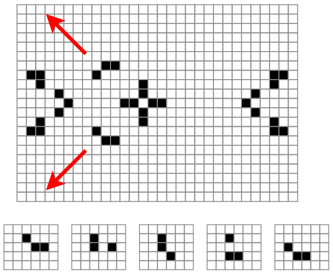

Figure 2: Basic glider-gun GGa shooting gliders Ga SW and NW, speed=c/4. The SE and NE directions requires a compound glider-gun (figure 6). *Inset*: the 4 phases of Ga SW.

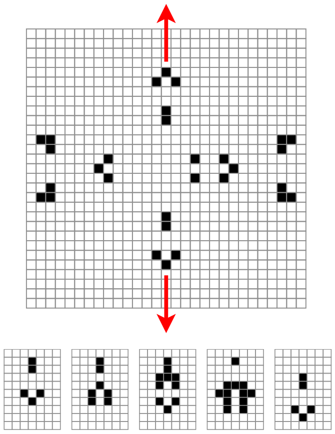

Figure 3: Basic glider-gun GGb shooting gliders Gb North and South, speed=c/2. *Inset*: the 4 phases of Gb South.

Searching for promising rules

The initially step was to search rule-space for emergent gliders and stable structure (eaters/reflectors) using the variability of input-entropy (Gomez-Soto and Wuensche, 2015; Wuensche, 2011, 1999) in figure 4. The search was restricted to isotropic rules only – equal outputs for any neighborhood rotation, reflection, or vertical flip – where rule-space is reduced to 2^{102}. The λ parameter (density of 1s) was also restricted to be similar to the game-of-Life. From a large rule sample, a shortlist of about 70 rules with both gliders and stable structures were identified in the ordered sector of figure 4. Five rules, with gliders travelling both orthogonally and diagonally, were selected for further study.

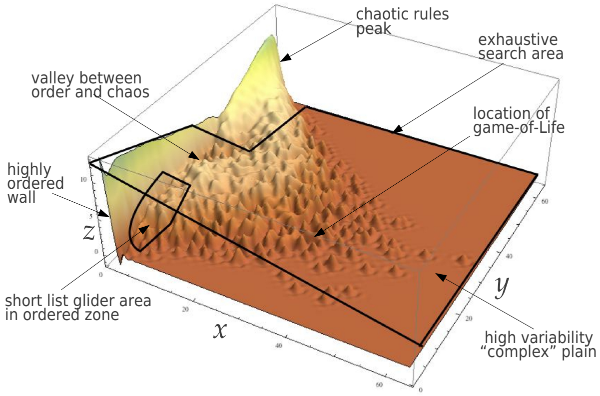

Figure 4: A scatter/frequency-plot (Gomez-Soto and Wuensche, 2015) of a sample of 93000+ rules. Min-max input-entropy variability x is plotted against mean entropy y, and (log) frequency (z) of rules on a 256x256 grid, which separates rule-space into fuzzy zones of chaos, order, and complexity. Promising rules where found in the ordered zone.

Constructing a reflecting/bouncing oscillator

The pivotal step in the project was to construct glider-guns based on simpler periodic oscillators. This involved building a periodic bouncing-colliding structure. From the short-list of five isotropic rules, we selected a rule, the X-rule precursor, with bouncing/reflecting behavior from spontaneously emergent objects: gliders Ga moving diagonally with speed=c/4, gliders Gc moving orthogonally with speed=c/2 (c is the speed of light), and three types of simple eaters/reflectors, (and rotations) 1 2 3. From Gc gliders and the eaters/reflectors we were able to construct the reflecting/bouncing oscillator in figure 5 where the distance between reflectors could be varied.

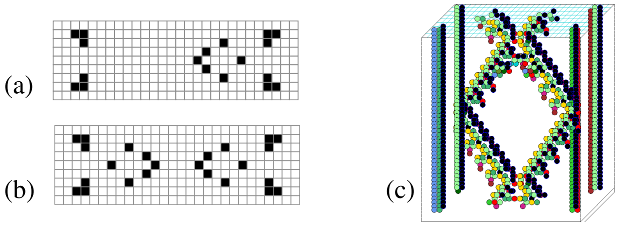

Figure 5: (a) A Gc gilder bouncing back and forth between two reflectors. (b) A reflecting/bouncing oscillator (RBO) – two Gc gilders reflecting and bouncing of each other, gap=20, period=30 – as yet no gliders are ejected. (c) a representation of RBO showing 2D time-steps.

Creating glider-guns – the X-rule

Our strategy for creating glider-guns was to mutate outputs in the X-rule precursor's rule-table with no restriction on preserving isotropy, but retaining the essence of reflecting/bouncing oscillators (RBOs) in figure 5, and crucially – ejecting gliders.

Using automatic methods for mutating and testing we obtained two different glider-guns (figures 2 and 3) in a rule later named the X-rule (figure 1). In the testing sequence other rules produced GGa (but not GGb) – we decided to focus on the X-rule because it supported two glider-guns, reasoning that two are better than one. The X-rule differs from its precursor by just 11 out of 512 neighborhood outputs – its dynamics gives the appearance of isotropy to a significant extent.

X-rule glider-guns have a special property in that the gap between reflectors can be enlarged from the minimum – 24 for GGa and 23 for GGb. Only increments of +4 are valid in each case to preserve the glider-gun, which increases the oscillation period and thus reduces the frequency of the resulting glider-stream.

Emergent structures in the X-rule universe

The X-rule conserves the two emergent glider types Ga and Gc , and the three eaters/reflectors 1 2 3 (and rotations) from its precursor. These emerge easily from a random initial seed because the eaters/reflectors and phase patterns of Ga are very simple, and Gc has a simple predecessor – the pattern Gc-p and its rotations. There are two more emergent gliders in the X-rule, Gb , an orthogonal glider moving only North and South with speed c/2 in 4 phases, and Gd , an orthogonal asymmetric glider moving only West and East with speed c/2 in 4 phases where the asymmetry alternates about a horizontal axis.

The outcome of glider-glider and glider-eater/reflector collisions is highly sensitive to collision phases, and the point and angle of impact. Gliders can self destruct, form stable structures, transform, combine, and bounce off at different angles. Eaters/reflectors can be destroyed or transformed. In order to create logical circuits a catalogue of the possible collisions and interactions is desirable, and a start was made in (Gomez-Soto and Wuensche, 2015).

Compound glider-guns

To date, a basic glider-gun for glider Gc has not been found, and the basic glider-guns are restricted to preferred directions because of non-isotropy. However, compound glider-guns for Gc and Ga allow any direction. These are constructed from two or more basic glider-guns and eaters/reflectors, positioned and synchronised precisely, making self-contained and sustainable multiple oscillating colliding compound structures. Figures 6 and 7 are snapshots of selected examples, shown on a 93×85 lattice. Gliders (and other mobile patterns) appear with green dynamic time-trails of 20 time-steps.

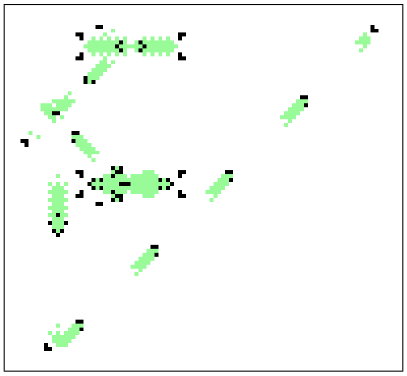

Figure 6: A compound glider-gun shooting Gc gliders towards the South, which bounce of a stable reflector to send Ga gliders NE.

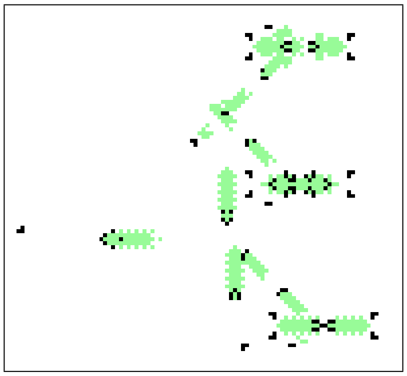

Figure 7: A compound glider-gun shooting Gc gliders towards the West made from a compound glider-gun shooting Gc gliders South, and a basic glider-gun below shooting Ga gliders NW.

Logical gates – logic universality

To demonstrate the X-rule's logic universality we followed the game-of-Life method using glider-guns as "pulse generators"(Berlekamp et al., 1982, Chapter 25), to construct logical gates NOT, AND, OR, and finally the functionally complete NAND gate – a combination of NAND gates can implement any logic circuit.

In a glider-stream, the presence of a glider represents 1, and a gap 0. When two suitably synchronised glider-streams intersect, gliders either collide and self-destruct leaving a gap, or a glider passes through a gap and survives. Logical gates are implemented by combining perfectly spaced and synchronised input streams with intersecting glider-streams generated by one or more glider-guns.

All the gates in various orientations have been demonstrated (Gomez-Soto and Wuensche, 2015). As an example we show a NAND gate with the output directed NW.

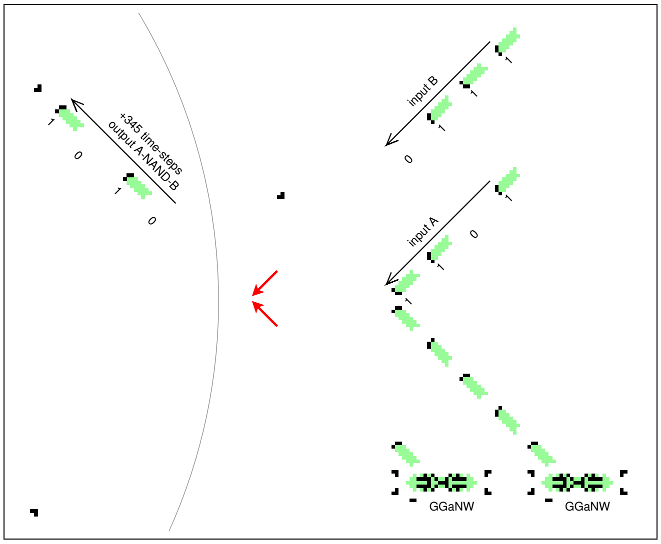

Figure 8: NAND gate: 1101 NAND 0111 giving the output 1010 heading NW. The first intersection is an AND gate, the second (at the red arrows) is a NOT gate.

Remarks

X-rule has underlying symmetry subject to marginal non-isotropy – the periodic oscillators based on reflecting/bouncing behaviour temporarily break symmetry to eject gliders. This is best perceived as dynamics, so the glider-guns, compound glider-guns, and logic-circuits will be shown in real time at ECAL15, using DDLab software.

Acknowledgements

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