SmallCheck and Lazy SmallCheck

automatic exhaustive testing for small values

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Motivation

Small Scope Hypothesis

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If a program fails to meet its specification in some cases,
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Contrapositive Corollary

If a program does not fail in any simple case,
it *hardly ever* fails in *any* case.

Success of QuickCheck

QuickCheck (Claessen & Hughes, ICFP'00):

- A combinator library for *random testing*.
- Exploits *type classes* to generate test values.
- Checks *universally quantified* properties.
- Reports *counter-example* if found, or N tests OK.
- Widely used; *often reported effective*.

Drawbacks of QuickCheck

Principally:

- *If failing cases are rare, none may be tested even though some of them are very simple.*

Also:

- Counter-examples are random *not minimal*.
- Some properties have *conditions hard to satisfy*.
- Writing good *custom generators can be tricky*.
- No assurance of *test-space coverage*.
- No support for *existential properties*.
- Counter-examples that are *functions are not displayed*.

Property-based Testing and QuickCheck

- `Arbitrary` types have random-value generators.
- `Testable` types represent properties.

  ```
  instance Testable Bool
  instance (Arbitrary a, Show a, Testable b)
    => Testable (a -> b)
  ```

- Any `Testable` property can be *tested automatically* for some pre-assigned number of random values using

  ```
  quickCheck :: Testable a => a -> IO ()
  ```

  a *class-polymorphic* test-driver.

Example

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- Specify an expected property:

  ```
  prop_isPrefix :: [Int] -> [Int] -> Bool
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  ```

- Test it automatically:

  ```
  > quickCheck prop_isPrefix
  OK, passed 100 tests.
  ```

  Or if `isPrefix` interprets arguments the other way round:

  ```
  Falsifiable, after 1 tests:
  [1]
  [2]
  ```

Arbitrary User-defined Types

```
data Prop = Var Name | Not Prop | Or Prop Prop
```

Defining a generator for such a recursive data type requires careful use of *controlling numeric parameters*.

```
instance Arbitrary Prop where
  arbitrary = sized arbProp
    where arbProp 0 = liftM Var arbitrary
          arbProp n = frequency
            [ (1,liftM Var arbitrary)
            , (2,liftM Not (arbProp (n-1)))
            , (4,liftM2 Or (arbProp (n `div` 2))
                           (arbProp (n `div` 2))) ]
```

Conditional Properties and Custom Generators

- QuickCheck defines an implication operator

  ```
  (==>) :: Testable a => Bool -> a -> Property
  ```

  where `Property` is a new `Testable` type.
- For example:

  ```
  type Set a = [a]
  insert :: Ord a => a -> Set a -> Set a

  prop_insertSet :: Char -> Set Char -> Property
  prop_insertSet c s =
    ordered s ==> ordered (insert c s)
  ```
- To avoid useless *unordered* lists, use a *custom generator*. But there are drawbacks: (1) defining it; (2) verifying it.

SmallCheck

Small Data Values

Algebraic data types

- Small bound on the *depth of constructor nesting*.
  Eg. `Or (Not (Var P)) (Var Q)` has depth 3.

Tuples

- Depth is the maximum component depth.

Numbers

- The depth of an *integer* `i` is its absolute value.
  (cf. $\texttt{Succ}^{\texttt{i}}$ `Zero`).
- The depth of a *floating point* number $\texttt{s} \times 2^{\texttt{e}}$ is the depth of the integer pair `(s,e)`. Eg. the floating point numbers of depth $<=2$ are `-4.0`, `-2.0`, `-1.0`, `-0.5`, `-0.25`, `0.0`, `0.25`, `0.5`, `1.0`, `2.0` and `4.0`.

Small Functions

Functions with data arguments

- Bound the depth of the body — treating case like a constructor with its alternatives as components.
  Eg. The `Bool->Bool` functions of depth 1 are:

```
\b -> case b of {True -> True  ; False -> True }
\b -> case b of {True -> True  ; False -> False}
\b -> case b of {True -> False ; False -> True }
\b -> case b of {True -> False ; False -> False}
```

Functions with functional arguments

- Defined generically — thank you Ralf!

Serial Types

- A series is a function from *depths* to *finite value-lists*.

```
type Series a = Int -> [a]
```

- A `Serial` type is one with a `series` method.

```
class Serial a where
  series :: Series a
```

- Sums and products are simply defined (no diagonalisation):

```
(\/) :: Series a -> Series a -> Series a
s1 \/ s2 = \d -> s1 d ++ s2 d

(><) :: Series a -> Series b -> Series (a, b)
s1 >< s2 = \d -> [(x,y) | x <- s1 d, y <- s2 d]
```

Defining Serial Instances

- Instances are predefined for `Prelude` types.
- Instances for new algebraic types follow a simple pattern. The `series` method uses generic `\/` and `cons<N>` combinators.

```
instance Serial Prop where
  series = cons1 Var \/ cons1 Not \/ cons2 Or
```

- The `coseries` method, generating functions, uses generic `alts<N>` combinators to generate case alternatives.
- The *Derive* tool *automates* instance definition — thank you Neil and Stefan!

Partial Extensions of Functional Values

- Are *all* binary operations on `Bool` associative?

```
prop_assoc op = \x y z ->
  (x `op` y) `op` z == x `op` (y `op` z)
  where typeInfo = op :: Bool -> Bool -> Bool
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- Testing finds and displays a failing case:

```
Main> smallCheckI prop_assoc
Depth 0:
  Failed test no. 22. Test values follow.
  {True->{True->True;False->True};
   False->{True->False;False->True}}
  False
  True
  False
```

Existential Properties

- Testing `exists f` succeeds if for *some small* argument `x` testing `f x` succeeds.

  ```
  exists :: (Show a, Serial a, Testable b) =>
    (a -> b) -> Property
  ```

Uniqueness

- Properties written using the translation
  $(\exists ! \mathtt{x}(\mathtt{P\ x})) \iff (\exists \mathtt{x}(\mathtt{P\ x})) \wedge (\forall \mathtt{y}(\mathtt{P\ y} \Rightarrow \mathtt{y} = \mathtt{x}))$
  are *awkward* to write & read, *inefficient* to test and *limited* to `Eq` types. A variant `exists1` requires a *unique witness*.

Depth

- A universal property may *pass shallow* tests but *fail deeper* ones. An existential property may *fail shallow* tests but *pass deeper* ones. The variant `existsDeeperBy dt` specifies in `dt::Int->Int` a *depth transformer*.

Example Revisited

- Consider the `isPrefix` specification:
  $\forall$xs$\forall$ys(isPrefix xs ys $\iff$ $\exists$xs'(xs++xs' $=$ ys))
- `prop_isPrefix` captures the $\Longleftarrow$ direction, but what about the $\Longrightarrow$ direction?

```
prop_isPrefixSound xs ys =
  isPrefix xs ys ==>
    exists $ \xs' -> xs++xs' == ys
```

- A QuickCheck user could write

```
prop_isPrefixSound' xs ys =
    isPrefix xs ys ==> xs ++ skolem xs ys == ys
  where skolem = drop . length
```

  but `skolem` has to be invented and defined — rarely so simple.

Dealing with Large Test Spaces

Depth-Adjustment and Filtering

- Generators of type `Int -> [t]` compose with *depth adjustment* functions of type `Int -> Int`, or with *filtering* functions of type `[t] -> [t]`. Eg:

```
instance Serial Prop where
  series = take 2 . cons1 Var
        \/          cons1 Not
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Bijective Representations

- Impose data invariants by using *testable bijections* from a shallower representation. Eg:

```
instance Serial OrdNats where
  series = map (OrdNats . scanl1 plus) . series
```

Lazy SmallCheck

Partial Values and Refinements

```
ordered [] = True
ordered [x] = True
ordered (x:y:zs) = x <= y && ordered (y:zs)
```

- If we evaluate `ordered 1:0:`$\bot$ it reduces to `False`. We conclude that `ordered 1:0:`*xs* is `False` for every *xs*.
- By applying a function to a *single* partially-defined input, we deduce its result over *many* fully-defined ones.
- If a property holds for some a partially-defined argument value then it holds for *all refinements* of it.
- Lazy SmallCheck uses this fact to *prune the test space* for *first-order, universal* properties.

Example Revisited

```
prop_insertSet c s =
  ordered s ==> ordered (insert c s)
```

- Testing with SmallCheck:

  ```
  Main> depthCheck 7 prop_insertSet
  Depth 7:
    Completed 109600 test(s) without failure.
    But 108576 did not meet ==> condition.
  ```

- Testing with Lazy SmallCheck

  ```
  Main> depthCheck 7 prop_insertSet
  OK, required 1716 tests at depth 7
  ```

Laziness is Delicate

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- Standard `&&` evaluates its left-hand argument first, and `allDiff` is less restrictive than `ordered`.

Parallel Conjunction

- The solution is *parallel refinement of conjuncts*.

```
isSet :: Ord a => Set a -> Property
isSet s = lift (ordered s) *&* lift (allDiff s)

prop_insertSet :: Char -> Set Char -> Property
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- Lists such as `1:0:`$\perp$ falsify `ordered` but not `allDiff`; lists such as `0:0:`$\perp$ falsify `allDiff` but not `ordered`.

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- Values of type `Cons a` describe how to construct and refine (partial) values of type `a`.

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data Cons a = Type :*: [[Term] -> a]
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- If a test evaluation reaches a `Hole`, a *position-carrying exception* is raised.
- By using a *universal* `Term` type, machinery such as refinement can be defined generically:

```
refine :: Term -> Pos -> [Term]
```

Comparative Evaluation

Red-black Trees (Okasaki)

```
data Colour = R | B
data Tree a = E | T Colour (Tree a) a (Tree a)
```

```
redBlack :: Ord a => Tree a -> Bool
redBlack t = ord t && black t && red t
```

With a *fault injected* into rebalancing, we test whether insertion preserves the `redBlack` data invariant:

```
prop_insertRB :: Int -> Tree Int -> Bool
prop_insertRB x t =
  redBlack t ==> redBlack (insert x t)
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QC *no counter-example* after 100,000 batches of 1000 tests.

SC *still testing* at depth 4 after 20 minutes.

LSC *level 4 counter-example* after a fraction of a second.

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  where ft = collate cs
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Condition can be falsified for *partially-defined* arguments.

SC Verifies to depth 5 in 8 sec; still testing depth 6 after 20 min.

LSC Verifies to depth 6 in 23 sec.

Mate Chess Solver

Conjecture: king and pawn alone cannot give checkmate

```
prop_checkmate b@(Board ws bs) =
  (  length ws == 2
  && Pawn `elem` map fst ws
  && validBoard b  ) ==> not (checkmate Black b)
```

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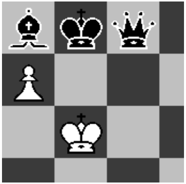

Conclusions and Future Work

Overall Conclusions

- SmallCheck, Lazy SmallCheck and QuickCheck are *complementary* approaches to property-based testing in Haskell.
- Each tool has strengths and weaknesses making it effective for *some kinds of properties* but ineffective for others.

To-do List Top Three

- Refine SmallCheck's treatment of functional values.
- Extend Lazy SmallCheck for higher-order and existential properties.
- Increase the *genericity of the property language* to enable free combinations of testing by different methods.

Availability

- SmallCheck and Lazy SmallCheck are freely available from `http://hackage.haskell.org/`.

Support Acknowledged

- Galois
- EPSRC